Name: \_\_\_\_\_

## **Final Instructions:**

- You have 3 hours for the exam.
- You are allowed to use both sides of one sheet of (standard letter size) paper that you have brought with you, for reference.
- No calculators. Where numerical answers are requested, approximate final answers are fine (unless the calculation is really trivial).
- You will receive 11 points for putting your name on the paper, accurately following directions, etc. Failure in any of these thing will lead to a partial or total loss of your 11 points.
- The exam is worth 200 points total. Partial credit will be given *if* your thought process is clear to the grader, and you are being graded not just on the final answer but also your thought process, so *show all work* as clearly as possible. (It might even help to throw a few words in to your solution here and there.) If you change your mind in the middle of a problem, please indicate the line of thought and answers that you wish us to consider.

Scores:	Multiple choice $(40)$	
	Free bonus $(11)$	
	Problem 1 (12)	
	Problem $2(16)$	
	Problem $3$ (16)	
	Problem 4 $(16)$	
	Problem 5 $(19)$	
	Problem 6 $(70)$	
	Total	
	Letter grade	

Section A (40 points): Multiple choice: choose the best answer. You will get 5 points for each correct answer.

- 1. For a gaussian surface through which the net flux is zero, the following four statements *could* be true. Which statement *must* be true?
  - (a) \_\_\_\_\_ No charges are inside the surface.
  - (b) \_\_\_\_\_ The net charge inside the surface is zero.
  - (c) \_\_\_\_\_ The electric field is the same everywhere on the surface.
  - (d) \_\_\_\_\_ No electric field lines leave the surface.

- 2. The ends of a resistanceless inductor are labeled A and B. The potential at A is higher than at B. Which of the following is consistent with this:
  - (a) \_\_\_\_\_ The current is constant and directed from A to B.
  - (b) \_\_\_\_\_ The current is constant and directed from B to A.
  - (c) \_\_\_\_\_ The current is directed from A to B and increasing.
  - (d) \_\_\_\_\_ The current is directed from A to B and decreasing.

- 3. A parallel plate capacitor with circular plates of radius  $r_0$  is being charged. The induced magnetic field between the plates is greatest at r =
  - (a)  $\_$  0 (the center)
  - (b) \_\_\_\_\_  $r_0/2$
  - (c) \_\_\_\_  $r_0$
  - (d) \_\_\_\_ more than  $r_0$

- 4. A particle of charge q is circulating in a uniform magnetic field of strength B. Which of the following will most increase the time it takes for the charge to make a full revolution?
  - (a) \_\_\_\_\_ Doubling the magnetic field.
  - (b) \_\_\_\_\_ Doubling the radius of revolution.
  - (c) \_\_\_\_\_ Doubling the speed of the particle.
  - (d) \_\_\_\_\_ Doubling the mass of the particle.

- 5. A hanging spring is attached to a powerful battery and a switch. When the switch is closed, a current suddenly flows through the spring. Does the spring
  - (a) <u>Compress.</u>
  - (b) \_\_\_\_ Expand.
  - (c) \_\_\_\_\_ Stay the same.

- 6. A long thin finite solenoid having n turns per unit length is tightly wrapped in N turns of insulated wire over a short portion of its length, with the resulting mutual inductance being M. The N turns of wire are removed and replaced by N turns of identical wire having twice the radius of the original wrapping. The mutual inductance of the new configuration compared to the original is:
  - (a) \_\_\_\_\_ the same
  - (b) \_\_\_\_\_ greater due to the greater area of the secondary coil
  - (c) \_\_\_\_\_ twice the original
  - (d) \_\_\_\_\_ less than the original although possibly only slightly

- 7. Consider the bent wire shown in Fig. 2 (see a few pages below), with current I flowing in the direction shown. Three circular loops are shown drawn to scale; loops (i) and (ii) enclose the top and bottom wires, respectively, and loop (iii) encloses both. Consider  $B_{\text{max}}$ , the maximum *B*-field strength around a given loop, and also the quantity  $I_{\text{enc}}$ , as it appears in Ampere's law. then:
  - (a)  $\underline{B}_{\text{max}}$  is greater for loops (i) and (ii) than for loop (iii), but  $I_{\text{enc}}$  is largest in loop (iii).
  - (b)  $B_{\text{max}}$  is smaller for loops (i) and (ii) than for loop (iii), and  $I_{\text{enc}}$  is largest in loop (iii).
  - (c)  $B_{\text{max}}$  is smaller for loops (i) and (ii) than for loop (iii), but  $I_{\text{enc}}$  is smallest in loop (iii).
  - (d)  $B_{\text{max}}$  is greater for loops (i) and (ii) than for loop (iii), and  $I_{\text{enc}}$  is largest in loop (iii).

8. Some stuff happens, involving capacitors, currents, resistors and inductors of values C, I, R, and L, respectively. A quantity results, which is:

$$X = 4\pi \frac{Q^2}{(\epsilon_0 \mu_0)^{1/2} I^2} \frac{R}{L^{3/2} C}.$$

This quantity is:

- (a) \_\_\_\_\_ A current
- (b) \_\_\_\_\_ A velocity
- (c) \_\_\_\_\_ A force
- (d) \_\_\_\_\_ A capacitance
- (e) \_\_\_\_\_ A power

## Section B: Problems

- 1. (12 points) On the island of Pala, AC power is delivered at 23 Hz and 42 Volts rms. You have brought a hotpot that draws 600 W when plugged in at home (with 60 Hz, 110 V rms power).
  - (a) If you consider your hotpot to be a pure resistance R, how much power is delivered to your it in Pala?
  - (b) Suppose the same hotpot also had a capacitor of capacitance C in it, connected in series to the resistor. What would be the current amplitude  $I_0$  and power burned in Pala in this case? (Don't bother numerically evaluating just write the symbolic expression.)

2. (16 points) Figure 1 shows a circuit with EMF  $\mathcal{E} = 7V$ . If  $R = 1 \Omega$ , which is the current I' in the indicated wire segment?



FIG. 1: For problem 2

- 3. (16 points) Consider the 'hairpin' of wire shown in Fig. 2, with current I = 1 A flowing in the direction shown, and the radius R = 0.4 m. Note that the wires continue indefinitely to the right.
  - (a) What is the field amplitude and direction at point *a*, very far away from the turn in the hairpin?
  - (b) What is the field amplitude and direction at point *b* due to the two straight wire segments only?
  - (c) Would the field direction due to just one of the two wire segments alone be in the same direction as the combined field of both? Why or why not?
  - (d) What is the total field amplitude and direction at point b?



FIG. 2: For multiple-choice #7 and problem 3.

(more space for prob. 3)

- 4. (16 points) A solid nonconducting sphere carries a uniform charge per unit volume  $\rho$ . Let  $\vec{r}$  be the vector from the sphere's center to a general point P within the sphere.
  - (a) Show that the electric field at P is given by  $\vec{E} = \rho \vec{r}/3\epsilon_0$ .
  - (b) A spherical cavity is created in the sphere; the vector  $\vec{a}$  points from the sphere's center to the cavity's center. Find the electric field at all points within the cavity; your result should depend *only* on  $\vec{a}$ ,  $\rho$ , and  $\epsilon_0$ . (Hint: use superposition, and keep careful track of you vectors.)

- 5. (19 points) Two small metal spheres, of charge  $\pm Q$ , are floating in space. They have a potential difference  $\Delta V$  and a separation d.
  - (a) Using our usual formulas for electric potential, what is  $\Delta V$ ?
  - (b) What is the capacitance of the configuration?
  - (c) How much energy is stored in this capacitor?
  - (d) What is the potential energy of the configuration, considering the two spheres to be point charges?
  - (e) The last two quantities both represent amounts of energy stored by the system. Do they match? If not, discuss why not.

(more space for prob. 5)

6. (70 points, 8 parts) Somewhere in the south Pacific ocean, in a bunker under a Hatch, there is an enormous magnet (hereafter "The Device") of unknown nature. It is possible that an Event involving this electromagnet led to the crash of Oceanic Flight 815 from Sydney to Los Angeles. You are to investigate this.

(Note: you can answer later parts in terms of the variables given in earlier parts, so don't stop if you get stuck.)

(Note: while finding the numerical answers may be fun, remember that you will get almost all of the credit for the correct symbolic expressions. If you do evaluate, don't get hung up on precise numbers, since the spirit of this problem is *estimating* things.)

(Note: if you are not a *Lost* fan you can do just as well on this, so don't worry.)

(a) Let's model the Device as a coil of many current loops each of radius R = 10 m. As you recall from class, the magnetic field strength a distance x from the center of one such loop, in a direction perpendicular to the plane of the loop, is:

$$B = \mu_0 I R^2 / 2(R^2 + x^2)^{3/2}.$$

Since the plane was at an altitude  $h \simeq 10,000 \, m \gg R$ , write down this field strength for  $x \gg R$ , if there are N turns of wire around the loop.

- (b) One theory is that the plane was simply pulled down out of the sky by the magnet. I won't make you show it (you could), but this turns our to require a field of  $B_{\text{yank}} \sim 1 T$  in the vicinity of the plane. Another possibility is that the field somehow disrupted the electronics in the airplane. Suppose, for example, that a some key components of the airplane's circuitry are connected by a piece of wire of length L = 10 m that runs perpendicular to the plane's flight direction, and which normally has a potential difference across it of V = 5 V. If the airplane is flying at v = 200 m/s, what magnitude  $B_{\text{disrupt}}$  of a magnetic field (assume it is uniform and pointed up and thus perpendicular to the wire) would be required to induce a similar EMF across the wire? Is  $B_{\text{disrupt}}$  or  $B_{\text{yank}}$  larger?
- (c) We might also theorize that a magnetic field significantly larger than that of the Earth simply threw the plane's navigational system into total confusion. Let's call this field  $B_{\text{confuse}} \sim 5 \times B_{\text{earth}} \sim 2.5 \times 10^{-4} T$ . This is certainly much smaller than  $B_{\text{yank}}$ . Let  $B_{\text{crash}}$  be the smaller of  $B_{\text{disrupt}}$  and  $B_{\text{confuse}}$ . Let's think about what could cause such a *B*-field. If N = 10000, what current is required in the loop to produce this field? (Express in terms of  $B_{\text{crash}}$ , then numerically).

(d) We'd like to know the inductance of the Device, but we don't know the inductance of a current loop. For a decent estimate, assume the loop is instead a solenoid of radius r = 10 m and length r, and compute its inductance. Then, assume for the moment that the Device is made up of steel wire of resistivity  $\rho = 10^{-7} \Omega m$  and cross-sectional area  $A = 10^{-4} m^2$ ; what would its resistance R be? Does this make sense when combined with the current calculated above? Perhaps the Device is superconducting...

- (e) Consider, then, the Device to be an EMF E attached in series to an almost perfectly superconducting loop that has resistance  $R = 10^{-5} \Omega$  and inductance L. Suppose the EMF is connected at time t = 0 (as a result, say, of someone failing to enter certain numbers into a computer), and that the current necessary to supply  $B_{\text{crash}}$  to the airplane is reached 30 seconds later. What is the amplitude of the EMF? (You may want to use the approximation  $\exp x \simeq 1 + x$  for small x.)
- (f) In a second, later Event, things go on a bit longer, leading to a catastrophic explosive 'discharge' initiated by a failsafe system. Among other things, this discharge released a lot of energy. Using the same assumptions as in the last question, calculate the total energy stored in the magnetic field at time t = 200 s.

- (g) Suppose the discharge lasts 10 s, during which the energy  $U_B$  you just calculated is released at a uniform rate into a spherically outgoing electromagnetic wave. 20 km away, at a small suburban town oddly situated on this mysterious island, what is the intensity (in W/m<sup>2</sup>) of radiation during the discharge?
- (h) So far away, we can model the wave as a plane wave. What is the maximum amplitude  $E_0$  of the electric field in this wave? Does it seem possible that this can create enough voltage (say 50 V) across say, a 1 cm circuit element to "fry" it in this electromagnetic pulse?

Extra paper