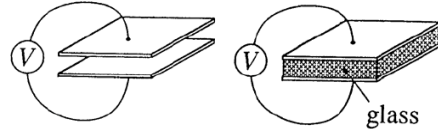


PRINT YOUR NAME \_\_\_\_\_

You have three hours to do this exam. You may use up to three pages of formulas.

**Section A (50 points): Conceptual Questions.** You will get 2 points for each correct answer and 3 additional points if you also give a correct brief explanation.

1. **T/F** T A parallel plate capacitor is attached to a battery that maintains a constant potential difference of  $V$  between the plates. If, while the battery is still connected, a glass slab is inserted so as to just fill the space between the capacitor plates, the stored energy will increase.



Stored energy is  $\frac{1}{2} CV^2$ . Here  $V$  is constant. When the glass is inserted,  $C$  increases by a factor  $K$ , so the stored energy increases by the same factor.

2. **T/F** T If you double a resistance  $R$  but keep the voltage  $V$  constant, the power dissipated in the resistor decreases by a factor of 2.

Power  $P = IV = V^2/R$ , so with  $V$  constant increasing  $R$  by a factor of 2 decreases the power by the same factor.

3. **T/F** T At \$0.15 per kWh, it costs \$33 to leave a 25 W porch light on for a year.

A 25 W light uses  $(0.025 \text{ kW})(24\text{h/day})(365.25 \text{ days}) = 220 \text{ kWh}$  per year, and at \$0.15 per kWh this costs  $\$0.15(220) = \$33$ . per year.

4. **T/F** T In the drawing at the right, a counterclockwise current is induced in the loop.



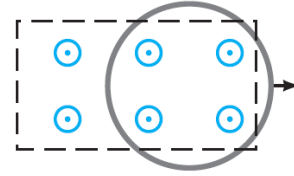
Increasing  $I$  in the wire causes an increasing magnetic field pointing into the paper in the loop. By Lenz's Law the induced current must indeed be counterclockwise to produce a magnetic field in the opposite direction.

5. **T/F** F If you double the current through a solenoid, you double the magnetic field inside it and you double the energy stored.

Magnetic field  $B \propto I$ , but the energy stored  $U_B \propto B^2$  so  $U_B \propto I^2$ . Thus if  $I$  doubles, then  $B$  doubles but  $U_B$  quadruples.

6. T/F T A proton and an electron are in an electric field created by charged plates. You release the proton from the positive side and the electron from the negative side. When the particles strike the opposite sides, they both have the same kinetic energy. The particles start with the same PE, so they acquire the same KE.

7. T/F F If a wire loop is being pulled through a magnetic field rising up out of the page that suddenly ends, a clockwise current is induced in the loop.



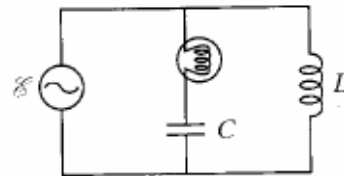
Lenz's Law says that the current induced in the loop must oppose the decrease in the "up" magnetic field. To produce such an "up" B, the current in the loop must be counterclockwise.

8. T/F T Two identical bar magnets are dropped at the same moment from a certain distance above the floor. There is nothing between magnet A and the floor, but magnet B falls through a copper loop. Magnet A hits the floor first.

When magnet B falls through the copper loop, eddy currents are induced that dissipate energy because of the resistance of the loop. Energy is conserved, and the energy dissipated must come from the kinetic energy of magnet B, which must slow down and thus hit the ground after magnet A. (An alternative to this energy analysis is the force analysis: the eddy currents produce a magnetic field opposing the magnetic field of magnet B. Like poles repel, and since  $F = ma$  this deceleration slows magnet B's fall.)

9. The light bulb has a resistance R, and the ac emf drives the circuit with a frequency  $\omega$ . The light bulb glows most brightly at

- very low frequencies.
- very high frequencies.
- the frequency  $\omega = (LC)^{-1/2}$ .

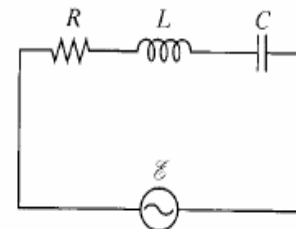


Select the correct answer b  
explain why

The reactance of the capacitor  $X_C = (\omega C)^{-1}$  is small at high frequencies  $\omega$ , while that of the inductor  $X_L = \omega L$  is high at high frequencies. Thus at high frequencies very little current flows through the inductor and most of the current flows through the lamp and the capacitor.

10. For the RLC circuit shown, which of these statements is true:

- Energy oscillates between L and C.
- Energy lost in R is compensated by energy stored in C and L.
- The current through C is  $90^\circ$  out of phase with that through L.
- The voltage across C is  $180^\circ$  out of phase with that across L.
- All energy is dissipated in R.



List all correct answers a, d, e  
and explain briefly

a. Energy oscillates between magnetic energy in L and electric energy in C. d. Voltage leads current by  $90^\circ$  across L and lags by  $90^\circ$  across C. e. Only resistance dissipates.

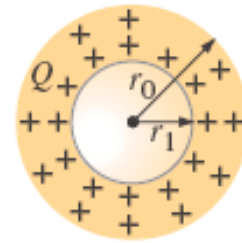
**Section B (125 points): Calculation problems.** Show all your work and make your method clear in order to get full credit. If you need additional paper print your name and the problem number on each sheet. Note:  $m_e = 9.11 \times 10^{-31}$  kg,  $e = 1.60 \times 10^{-19}$  C.

11. (15 points) Suppose that a charge  $-Q$  is located at  $x = -a$ , a charge  $+2Q$  is located at  $x = 0$ , and a charge  $-Q$  is located at  $x = +a$ .
- (a) Calculate the electric potential  $V(x)$  on the x-axis for  $x \gg a$ .
- (b) Calculate the electric field on the x-axis for  $x \gg a$  using Coulomb's law.
- (c) Calculate the electric field on the x-axis for  $x \gg a$  by differentiating the potential.

**Answer:**

- (a) Recall the binomial formula  $(1+\epsilon)^n = 1 + n\epsilon + \frac{n(n-1)}{2}\epsilon^2 + O(\epsilon^3)$ . Since  $x \gg a$ ,  $\epsilon = a/x \ll 1$  and it will suffice to work to order  $\epsilon^2$ .  $V(x) = kQ [-(a+x)^{-1} + 2/x - (a-x)^{-1}] = (kQ/x) [-(1+\epsilon)^{-1} + 2 - (1-\epsilon)^{-1}] = (kQ/x) [-(1-\epsilon+\epsilon^2) + 2 - (1+\epsilon+\epsilon^2)] = (kQ/x)(-2\epsilon^2) = -\frac{2kQa^2}{x^3}$ .
- (b)  $E_x(x) = kQ [-(a+x)^{-2} + 2/x^2 - (a-x)^{-2}] = (kQ/x^2) [-(1+\epsilon)^{-2} + 2 - (1-\epsilon)^{-2}] = (kQ/x^2) [-(1-2\epsilon+3\epsilon^2) + 2 - (1+2\epsilon+3\epsilon^2)] = (kQ/x^2)(-6\epsilon^2) = -\frac{6kQa^2}{x^4}$ . [Note: alternatively one could add dipoles to solve each part.]
- (c)  $E_x(x) = -dV/dx = -(-2kQ)(-3a^2/x^4) = -\frac{6kQa^2}{x^4}$ , in agreement with (b).

12. (15 points) Suppose that a nonconducting spherical shell of inner radius  $r_1$  and outer radius  $r_0$  has a charge  $Q$  uniformly distributed in the shell. Determine the electric field
- (a) for  $r < r_1$ , (b) for  $r_1 < r < r_0$ , and (c) for  $r_0 < r$ .



**Answer:**

Due to the spherical symmetry of the problem, Gauss's law using a spherical Gaussian surface yields the following.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2}$$

- (a) For the region  $0 < r < r_1$ , the enclosed charge is 0.

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \boxed{0}$$

- (b) For the region  $r_1 < r < r_0$ , the enclosed charge is the product of the volume charge density times

the volume of charged material enclosed. The charge density is given by  $\rho = \frac{Q}{\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_1^3}$

$$= \frac{3Q}{4\pi(r_0^3 - r_1^3)}$$

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{\rho V_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{\rho \left[ \frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3 \right]}{4\pi\epsilon_0 r^2} = \frac{\frac{3Q}{4\pi(r_0^3 - r_1^3)} \left[ \frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3 \right]}{4\pi\epsilon_0 r^2} = \boxed{\frac{Q}{4\pi\epsilon_0 r^2} \frac{(r^3 - r_1^3)}{(r_0^3 - r_1^3)}}$$

- (c) For the region  $r > r_0$ , the enclosed charge is the total charge,  $Q$ .

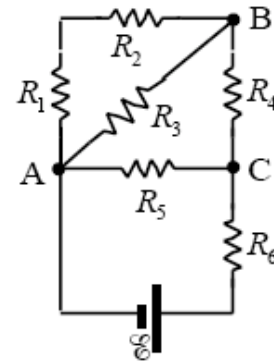
$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

13. (15 points) The voltage in a power outlet is 110V rms at 60Hz in the US and Canada.
- What is the maximum voltage that comes out of the wall socket at any instant of time?
  - What is the resistance of a 4-slice, 1600W toaster?
  - What is the capacitance that (with no resistor present) will allow the same amplitude current to flow when plugged into the wall socket?

**Answer:**

- $V_{\text{rms}} = V_0/\sqrt{2}$ , where  $V_0$  is the amplitude and also the maximum, so  $V_0 = \underline{156 \text{ V}}$ .
- Average power  $= \langle P \rangle = V_{\text{rms}}^2/R$ , so  $R = \underline{7.6 \Omega}$ .
- $I_0 = V_0/Z$ , where the impedance  $Z = [R^2 + (\omega L - 1/\omega C)^2]^{1/2}$ . Thus if  $L = 0$ , a capacitance of  $C = 1/(\omega R) = 1/(2\pi f R)$  is equivalent to a resistance of  $R$ ; in this case  $C = \underline{3.5 \times 10^{-4} \text{ F}}$ .

14. (10 points) In the figure at the right assume  $R = 1.2 \text{ k}\Omega$ .  
What is the equivalent resistance of the circuit connected to the battery?



**Answer:**

We reduce the circuit to a single loop by combining series and parallel combinations. We label a combined resistance with the subscripts of the resistors used in the combination.

$R_1$  and  $R_2$  are in series.

$$R_{12} = R_1 + R_2 = R + R = 2R$$

$R_{12}$  and  $R_3$  are in parallel.

$$R_{123} = \left( \frac{1}{R_{12}} + \frac{1}{R_3} \right)^{-1} = \left( \frac{1}{2R} + \frac{1}{R} \right)^{-1} = \frac{2}{3}R$$

$R_{123}$  and  $R_4$  are in series.

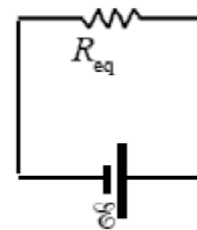
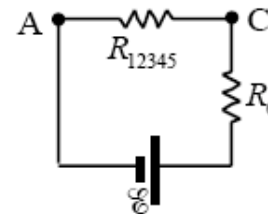
$$R_{1234} = R_{123} + R_4 = \frac{2}{3}R + R = \frac{5}{3}R$$

$R_{1234}$  and  $R_5$  are in parallel.

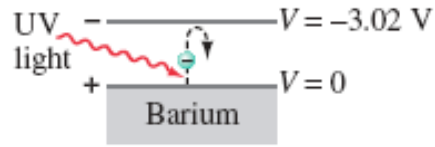
$$R_{12345} = \left( \frac{1}{R_{1234}} + \frac{1}{R_5} \right)^{-1} = \left( \frac{1}{\frac{5}{3}R} + \frac{1}{R} \right)^{-1} = \frac{5}{8}R$$

$R_{12345}$  and  $R_6$  are in series, producing the equivalent resistance.

$$R_{\text{eq}} = \frac{5}{8}R + R = \boxed{\frac{13}{8}R}$$



15. (10 points) In a photocell, ultraviolet (UV) light provides enough energy to some electrons in barium metal to eject them from the surface at high speed. To measure the maximum energy of the electrons, another plate above the barium surface is kept at a negative enough potential that the emitted electrons are slowed down and stopped, and return to the barium surface. If the plate voltage is  $-3.02\text{ V}$  (compared to the barium) when the fastest electrons are stopped, what was the maximum speed of these electrons when they were emitted?



**Answer:**

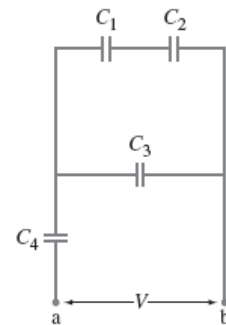
The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$\text{KE}_{\text{initial}} = \text{PE}_{\text{final}} \rightarrow \frac{1}{2}mv^2 = qV \rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(-1.60 \times 10^{-19} \text{ C})(-3.02 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = \boxed{1.03 \times 10^6 \text{ m/s}}$$

16. (15 points) In the figure at the right, suppose that all the capacitances are equal to  $C$ .

- (a) Determine the equivalent capacitance between points a and b.  
 (b) Determine the potential difference across each capacitor and the charge on each in terms of  $V$ .



**Answer:**

- (a) From the diagram, we see that  $C_1$  and  $C_2$  are in series. That combination is in parallel with  $C_3$ , and then that combination is in series with  $C_4$ . Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \rightarrow C_{12} = \frac{1}{2}C ; C_{123} = C_{12} + C_3 = \frac{1}{2}C + C = \frac{3}{2}C ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \rightarrow C_{1234} = \boxed{\frac{3}{5}C}$$

- (b) The charge on the equivalent capacitor  $C_{1234}$  is given by  $Q_{1234} = C_{1234}V = \frac{3}{5}CV$ . This is the charge on both of the series components of  $C_{1234}$ .

$$Q_{123} = \frac{3}{5}CV = C_{123}V_{123} = \frac{3}{2}CV_{123} \rightarrow V_{123} = \frac{2}{5}V$$

$$Q_4 = \frac{3}{5}CV = C_4V_4 \rightarrow V_4 = \frac{3}{5}V$$

The voltage across the equivalent capacitor  $C_{123}$  is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of  $C_{123}$  is the same as the total charge on the two components,  $\frac{3}{5}CV$ .

$$V_{123} = \frac{2}{5}V = V_{12} ; Q_{12} = C_{12}V_{12} = \left(\frac{1}{2}C\right)\left(\frac{2}{5}V\right) = \frac{1}{5}CV$$

$$V_{123} = \frac{2}{5}V = V_3 ; Q_3 = C_3V_3 = C\left(\frac{2}{5}V\right) = \frac{2}{5}CV$$

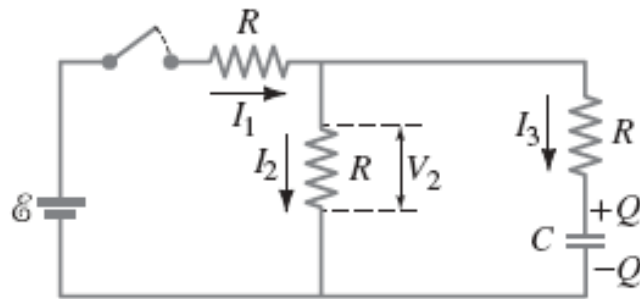
Finally, the charge on the equivalent capacitor  $C_{12}$  is the charge on both of the series components of  $C_{12}$ .

$$Q_{12} = \frac{1}{5}CV = Q_1 = C_1V_1 \rightarrow V_1 = \frac{1}{5}V ; Q_{12} = \frac{1}{5}CV = Q_2 = C_2V_2 \rightarrow V_2 = \frac{1}{5}V$$

Here are all the results, gathered together.

|  |
|--|
| $Q_1 = Q_2 = \frac{1}{5}CV ; Q_3 = \frac{2}{5}CV ; Q_4 = \frac{3}{5}CV$ $V_1 = V_2 = \frac{1}{5}V ; V_3 = \frac{2}{5}V ; V_4 = \frac{3}{5}V$ |
|--|

17. (15 points) Consider the circuit shown at the right, where all resistors have the same resistance  $R$ . At  $t = 0$  with the capacitor  $C$  uncharged, the switch is closed.



(a) At  $t = 0$  the three currents can be determined by analyzing a simpler, but equivalent, circuit.

Identify this simpler circuit and use it to find the values of  $I_1$ ,  $I_2$ , and  $I_3$  at  $t = 0$ .

(b) At  $t = \infty$ , the currents can be determined by analyzing another equivalent circuit.

Identify this simpler circuit and use it in finding the values of  $I_1$ ,  $I_2$ , and  $I_3$  at  $t = \infty$ .

(c) At  $t = \infty$ , what is the potential difference across the capacitor?

**Answer:**

(a) At  $t = 0$ , the capacitor is uncharged and so there is no voltage difference across it. The capacitor is a “short”, and so a simpler circuit can be drawn just by eliminating the capacitor. In this simpler circuit, the two resistors on the right are in parallel with each other, and then in series with the resistor by the switch. The current through the resistor by the switch splits equally when it reaches the junction of the parallel resistors.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{3}{2}R \rightarrow I_1 = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{\frac{3}{2}R} = \boxed{\frac{2\mathcal{E}}{3R}} ; I_2 = I_3 = \frac{1}{2}I_1 = \boxed{\frac{\mathcal{E}}{3R}}$$

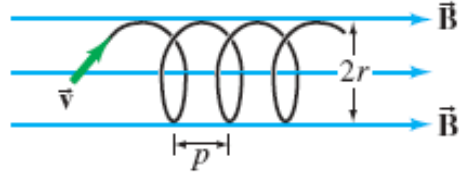
(b) At  $t = \infty$ , the capacitor will be fully charged and there will be no current in the branch containing the capacitor, and so a simpler circuit can be drawn by eliminating that branch. In this simpler circuit, the two resistors are in series, and they both have the same current.

$$R_{\text{eq}} = R + R = 2R \rightarrow I_1 = I_2 = \frac{\mathcal{E}}{R_{\text{eq}}} = \boxed{\frac{\mathcal{E}}{2R}} ; I_3 = \boxed{0}$$

(c) At  $t = \infty$ , since there is no current through the branch containing the capacitor, there is no potential drop across that resistor. Therefore the voltage difference across the capacitor equals the voltage difference across the resistor through which  $I_2$  flows.

$$V_C = V_{R_2} = I_2R = \left(\frac{\mathcal{E}}{2R}\right)R = \boxed{\frac{1}{2}\mathcal{E}}$$

18. (10 points) An electron enters a uniform magnetic field at a  $45^\circ$  angle to the magnetic field  $\mathbf{B}$ . Determine the radius  $r$  and pitch  $p$  (distance between loops) of the electron's helical path assuming its speed is  $3.0 \times 10^6$  m/s.



**Answer:**

The centripetal force is caused by the magnetic field, and is given by Eq. 27-5b.

$$F = qvB \sin \theta = qv_{\perp}B = m \frac{v_{\perp}^2}{r} \rightarrow$$

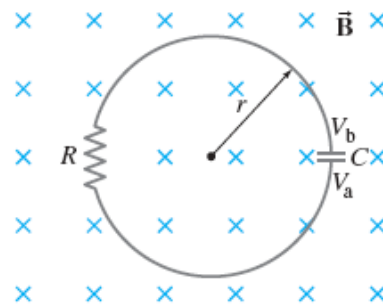
$$r = \frac{mv_{\perp}}{qB} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.0 \times 10^6 \text{ m/s}) \sin 45^\circ}{(1.60 \times 10^{-19} \text{ C})(0.28 \text{ T})} = 4.314 \times 10^{-5} \text{ m} \approx \boxed{4.3 \times 10^{-5} \text{ m}}$$

The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2\pi \frac{mv_{\perp}}{qB}}{v_{\perp}} = \frac{2\pi m}{qB}$$

$$p = v_{\parallel}T = v \cos 45^\circ \left( \frac{2\pi m}{qB} \right) = (3.0 \times 10^6 \text{ m/s}) \cos 45^\circ \frac{2\pi (9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.28 \text{ T})} = \boxed{2.7 \times 10^{-4} \text{ m}}$$

19. (10 points) A circular-shaped circuit of radius  $r$ , containing a resistance  $R$  and capacitance  $C$ , is situated with its plane perpendicular to the spatially uniform magnetic field within which it is immersed. The magnetic field is directed into the page as shown. Starting at time  $t = 0$ , the voltage difference  $V_{ba} = V_b - V_a$  across the capacitor plates is observed to increase with time according to  $V_{ba} = V_0(1 - e^{-t/\tau})$  where  $V_0$  and  $\tau$  are positive constants. Determine  $dB/dt$ , the rate at which the magnetic field magnitude changes with time. Is  $B$  becoming larger or smaller as time increases?



**Answer:**

The emf around the loop is equal to the time derivative of the flux, as in Eq. 29-2a. Since the area of the coil is constant, the time derivative of the flux is equal to the derivative of the magnetic field multiplied by the area of the loop. To calculate the emf in the loop we add the voltage drop across the capacitor to the voltage drop across the resistor. The current in the loop is the derivative of the charge on the capacitor (Eq. 24-1).

$$I = \frac{dQ}{dt} = \frac{dCV}{dt} = \frac{d}{dt} [CV_0(1 - e^{-t/\tau})] = \frac{CV_0}{\tau} e^{-t/\tau} = \frac{V_0}{R} e^{-t/\tau}$$

$$\mathcal{E} = IR + V_C = \left( \frac{V_0}{R} e^{-t/\tau} \right) R + V_0(1 - e^{-t/\tau}) = V_0 = \frac{d\Phi_B}{dt} = A \frac{dB}{dt} = \pi r^2 \frac{dB}{dt} \rightarrow \boxed{\frac{dB}{dt} = \frac{V_0}{\pi r^2}}$$

Since the charge is building up on the top plate of the capacitor, the induced current is flowing clockwise. By Lenz's law this produces a downward flux, so the external downward magnetic field must be decreasing.

20. (10 points) Proposals have been made to use light pressure from the Sun to propel spacecraft around the solar system. Assume that solar radiation at the distance of the earth has 1 kW of power per square meter facing the sun.

(a) How much force would light pressure exert on a 1 km square highly reflective sail facing the sun that starts near the earth? (For this and the next part, ignore gravity.)

(b) If this solar sail is part of a 5000 kg spacecraft, how much would a force of this magnitude increase the speed of the spacecraft away from the sun in one year? How far would the spacecraft have gone at the end of the year (assuming the light pressure is constant)?

**Answer:**

Pressure  $P$  is force per unit area, and the light pressure  $P = 2S/c$  where  $S$  is the Poynting vector and  $c$  is the speed of light; the factor of 2 is because the light is reflected. Thus the force  $F = 2SA/c$ , where  $A$  is the sail area  $A = 10^6 \text{ m}^2$ . We find the acceleration  $a$  from Newton's 2<sup>nd</sup> Law  $F = ma$ , and assuming that the force is constant we find the spacecraft speed after time  $t$  from  $v = at$ ; then the distance covered is  $d = 1/2 vt = 1/2 at^2$ .

(a) The pressure  $P = 2S/c = 2(1000 \text{ W/m}^2)(3 \times 10^8 \text{ m/s})^{-1} = 6 \times 10^{-6} \text{ N}$ , so the force is  $F = PA = (6 \times 10^{-6} \text{ N})(10^6 \text{ m}^2) = \underline{6 \text{ N}}$ .

(b) The acceleration  $a = F/m = 6\text{N}/(5000 \text{ kg}) = 1.2 \times 10^{-3} \text{ m/s}^2$ , so after  $t = 1 \text{ year} = (3600\text{s/hr})(24\text{hr/day})(365\text{d}) = 3 \times 10^7 \text{ s}$ . Thus if the spacecraft starts from rest near the earth, after a year its speed away from the sun would be  $v = at = (1.2 \times 10^{-3} \text{ m/s}^2)(3 \times 10^7 \text{ s}) = 4 \times 10^4 \text{ m/s} = \underline{40 \text{ km/s}}$ .

The distance covered would be  $d = 1/2 at^2 = 0.5 (1.2 \times 10^{-3} \text{ m/s}^2) (3 \times 10^7 \text{ s})^2 = \underline{5 \times 10^{11} \text{ m}} = \underline{5 \times 10^8 \text{ km}} = \underline{500 \text{ million kilometers}}$ , more than three times the earth-sun distance of 150 million kilometers. Thus it is not realistic to assume that the solar light pressure would be constant, since the light intensity falls off as  $1/r^2$  where  $r$  is the distance from the sun. If the spacecraft were more than 3 times farther from the sun than the earth, the light intensity would be less than  $1/4^2 = 1/16$  what it is near the earth, so the pressure would be less than 1/16 what we calculated above.