Physics 5C **Midterm 2** May 19, 2008

Solutions

1. True/False Questions (30 points).

a. F: Part of the energy is dissipated as heat in the resistor.

b. T: The force on a particle of charge q in electric and magnetic fields is given by the Lorentz law $\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. In order for the electron to travel in a straight line perpendicular to crossed electric and magnetic fields, $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$. This will also be true for a proton with the same velocity.

c. T: Hysteresis is the property of ferromagnets that they retain a residual magnetization in the direction of an imposed magnetic field when the imposed field is removed. Computer hard disks are coated with ferromagnetic material that remembers the orientation of the magnetic fields used to write binary data.

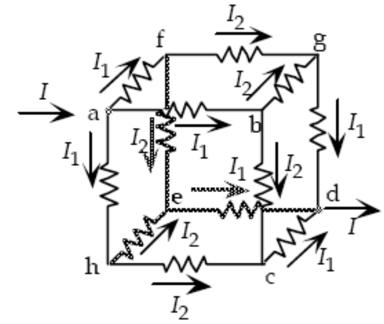
d. T: The magnetic field produced by the current I is up from the page at the position of the charge, so the force $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$ points toward the wire. (Parallel currents attract.)

e. F: The increasing current in the straight wire causes a magnetic field through the loop into the page. According to Lenz's Law, the current induced in the loop must oppose this magnetic field. The induced current is therefore counterclockwise according to the right hand rule.

f. T: The magnetic field in the solenoid is proportional to the current, so if the current doubles so does the magnetic field. The energy stored in a magnetic field **B** is proportional to B^2, so if the current doubles the energy increases by a factor of four.

Calculation problems (70 points).

2. If a voltage is imposed between points a = (0,1,1) and d = (1,0,0), the junctions at points (0,1,0), (0,0,1), and b = (1,1,1) are all at the same potential, so that the three resistors connecting point a to these junctions are effectively in parallel, with a net resistance = R/3. Also, the junctions at points (0,0,0), (1,1,0), and c = (1,0,1) are all at the same potential, so the six resistors connecting these junctions to the previous ones are all in parallel, with a net resistance = R/6. Finally the three resistors connecting (0,0,0), (1,1,0), and c = (1,0,1) to d = (1,0,0) are effectively in parallel, with a net resistance =R/3. These three effective resistances are in series, so the total effective resistance of the cube of resistors is R/3 + R/6 + R/3 = <u>5R/6</u>.



Alternative solution:

We use Kirchhoff' junction rule at points a and b and the loop rule around the loop abgda (through the power source) to write three equations for the three unknown currents. We solve these equations for the ratio of the emf to the current through the emf (I) to calculate the effective resistance.

$$I = 3I_1 \ [1] \ ; \ I_1 = 2I_2 \ [2] \qquad 0 = -2I_1R - I_2R + \mathcal{E} \ [3]$$

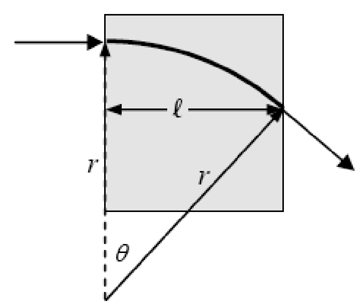
We insert Eq. [2] into Eq. [3] and solve for I_1. Inserting I_1 into Eq. [1] enables us to solve for the effective resistance:

$$0 = -2I_1R - \tfrac{1}{2}I_1R + \mathcal{E} \rightarrow I_1 = \frac{2\mathcal{E}}{5R} \ ; \ I = 3I_1 = \frac{6\mathcal{E}}{5R} \rightarrow R_{eq} = \frac{\mathcal{E}}{I} = \boxed{\tfrac{5}{6}R}$$

3. In this RC circuit, the charge Q on the capacitor satisfies IR + Q/C = $\mathcal{E}$, with current I = dQ/dt. The solution is Q = C $\mathcal{E}$ (1 – $e^{-t/\tau}$), where the time constant τ = RC. The maximum charge on the capacitor is Q_{max} = C $\mathcal{E}$. The energy stored in a capacitor is U = 1/2 Q^2/C, so we need to calculate at what time Q^2 = 0.75 $Q_{max}{}^2$, or Q/Q_{max} = 1 – $e^{-t/\tau}$ = √0.75 = 0.866, or $e^{-t/\tau}$ = 0.134, or (taking the natural log of both sides) t/τ = 2.01, or finally <u>t = 2.01 τ = 2.01 RC</u>. (The answer <u>t = 2.0 RC</u> is also acceptable.)

4. (a) The protons will follow a circular path as they move through the region of magnetic field, with a radius of curvature given by F = qvB = m v^2/r, so the radius of curvature is <u>r = mv/qB</u> .

(b) Protons moving faster will have a larger radius of curvature and go <u>above</u> the beam tube on the right. Protons moving slower will have a smaller radius of curvature and go <u>below</u> the beam tube.



(c) Since the exit velocity is perpendicular to the radius line from the center of curvature, the bending angle can be calculated:

$$\sin\theta = \frac{\ell}{r}$$

$$\theta = \sin^{-1}\frac{\ell}{r} = \sin^{-1}\frac{\ell qB}{mv} = \sin^{-1}\frac{\left(5.0\times10^{-2}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(0.38\,\text{T}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(0.85\times10^{7}\,\text{m/s}\right)} = \sin^{-1}0.214 = \boxed{12^\circ}$$

5. (a) The accelerating force on the bar is due to the magnetic force on the current. If the current is constant, the magnetic force will be constant, and so constant acceleration kinematics can be used:

$$v^2 = v_0^2 + 2a\Delta x \ \rightarrow \ a = \frac{v^2 - 0}{2\Delta x} = \frac{v^2}{2\Delta x}$$

$$F_{net} = ma = IdB \ \rightarrow \ I = \frac{ma}{dB} = \frac{m\left(\frac{v^2}{2\Delta x}\right)}{dB} = \frac{mv^2}{2\Delta x dB} = \frac{\left(1.5\times10^{-3}\,\text{kg}\right)\left(25\,\text{m/s}\right)^2}{2\left(1.0\,\text{m}\right)\left(0.24\,\text{m}\right)\left(1.8\,\text{T}\right)} = \boxed{1.1\,\text{A}}$$

(b) Using the right hand rule, for the force on the bar to be in the direction of the acceleration shown in the figure, the magnetic field must be <u>down</u>.

6.

If θ is the angle between the resistor wire and the moving bar, the area of the plane surface bounded by the closed circuit is

$$(\pi D^2)\frac{\theta}{2\pi} = \frac{1}{2}D^2\theta \ .$$

The magnetic flux through this surface is then $\Phi_B = \frac{1}{2}BD^2\theta$, and the magnitude of the induced EMF is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{1}{2}BD^2\frac{d\theta}{dt} = \frac{1}{2}BD^2\omega \ .$$

The induced current will be

$$I = \frac{|\mathcal{E}|}{R} = \frac{BD^2\omega}{2R} \ .$$

(By Lenz's law, this current will flow in a counterclockwise sense, so as to counter the changing flux due to the motion of the bar.)