Homework Set 3

DUE: Wednesday March 11

1. (a) Find the deviation from flatness, $1 - \Omega(z)$, at $z_{dec} = 1100$ and at $z = 10^9$, assuming that $\Omega_{m,0} = 0.1$ and $\Omega_{rad,0} = 4 \times 10^{-5}$.

(b) Find the deviation from flatness if $\Omega_{tot} = \Omega_\Lambda = 0.1$, and discuss the difference from the previous case.

2. (a) For the Einstein-de Sitter cosmology ($\Omega_m = 1$, $\Omega_\Lambda = 0$), calculate the physical radius $r_h(z)$ of the particle horizon at z_{eq} and z_{dec}.

(b) For the Einstein-de Sitter cosmology, calculate the comoving radii $\chi(z)$ and corresponding masses $M(z)$ enclosed by those horizons.

(c) Calculate the angle those horizons subtend today, $\theta_h = r_h(z)/r_a(z)$, using the angular distance d_A for an Einstein-de Sitter cosmology, $d_A(z) = (2c/H_0)a(1 - a^{1/2})$, where $a = 1/(1 + z)$ as usual.

3. Suppose that monopoles formed at the GUT epoch when $T = T_{GUT} \sim 10^{16}$ GeV, with one monopole of mass $m_M = T_{GUT}$ per horizon volume. How many e-foldings of inflation would be required to drive the current density of monopoles below the bound $\Omega_{M,0} < 10^{-6}$, where $\Omega_{M,0}$ is the density of monopoles today in units of critical density. Assume that inflation occurred immediately after the formation of monopoles.

4. Explain why gravitational lensing of a galaxy by a cosmic string would produce two identical undistorted images of the galaxy, and work out the separation of the images for plausible values of the parameters. Discuss methods to determine whether an observed pair of images is caused by a cosmic string.

5. Use the slow-roll approximation to calculate the inflaton field $\phi(t)$ as a function of time for a potential $V(\phi) = g\phi^n$, where g and n are positive constants.