

Astronomy 233 Winter 2009

Physical Cosmology

Weeks 7 & 8

*Cosmic Background, Phase
Transitions, Defects, & Inflation*

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CMB, Phase Transitions, Cosmic Defects, and Inflation

Weeks 7 and 8

Outline

Cosmic Microwave Background

WMAP 5-year Data and Papers Released

Grand Unification of Forces

Phase Transitions in the Early Universe

Topological Defects: Strings, Monopoles

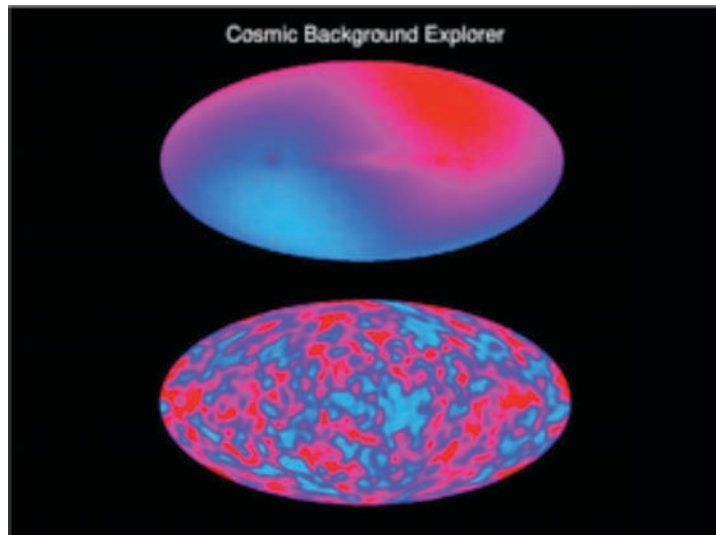
Cosmic Inflation

Cosmic Microwave Background

Early History

Although Penzias and Wilson discovered the CMB in 1965, Weinberg (p. 104) points out that Adams and McKellar had shown that the rotational spectra of cyanogen (CN) molecules observed in 1941 suggested that the background temperature is about 3K.

The COBE FIRAS measurements showed that the spectrum is that of thermal radiation with $T = 2.73\text{K}$.



The CMB dipole anisotropy was discovered by Paul Henry (1971) and Edward Conklin (1972), and confirmed by Conklin and Wilkinson (1977) and Smoot, Gorenstein, and Muller (1977) -- see <http://www.astro.ucla.edu/~wright/CMB-dipole-history.html>

The upper panel of the figure shows the CMB dipole anisotropy in the COBE data. It is usually subtracted when the temperature anisotropy map is displayed (lower panel).

CMB Temperature Anisotropy

Sachs & Wolfe (1967, ApJ, 147, 73) showed that on large angular scales the temperature anisotropy is $\Delta T/T = \phi/3c^2$. White & Hu give a pedagogical derivation in <http://background.uchicago.edu/~whu/Papers/sw.pdf>

PERTURBATIONS OF A COSMOLOGICAL MODEL AND ANGULAR VARIATIONS OF THE MICROWAVE BACKGROUND

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Received May 13, 1966

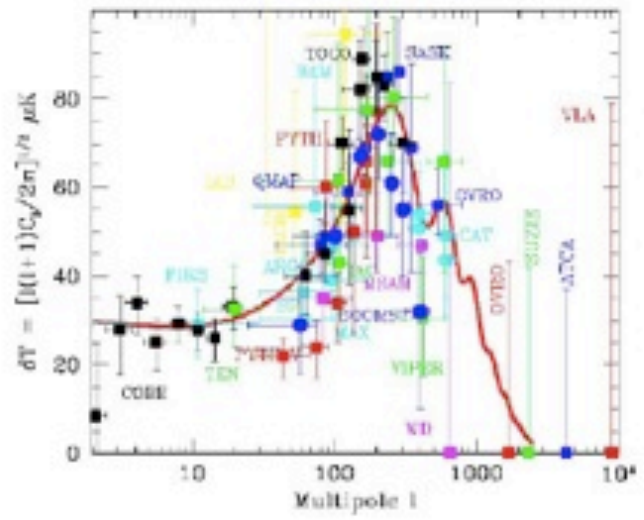
ABSTRACT

We consider general-relativistic, spatially homogeneous, and isotropic $k = 0$ cosmological models with either pressure zero or pressure one-third the energy density. The equations for general linearized perturbations away from these models are explicitly integrated to obtain density fluctuations, rotational perturbations, and gravitational waves. The equations for light rays in the perturbed models are integrated. The models are used to estimate the anisotropy of the microwave radiation, assuming this radiation is cosmological. It is estimated that density fluctuations now of order 10 per cent with characteristic lengths now of order 1000 Mpc would cause anisotropies of order 1 per cent in the observed microwave temperature due to the gravitational redshift and other general-relativistic effects. The $p = 0$ models are compared in detail with corresponding Newtonian models. The perturbed Newtonian models do not contain gravitational waves, but the density perturbations and rotational perturbations are surprisingly similar.

This was first convincingly seen by the COBE DMR experiment, reported by George Smoot on April 27, 1992. Their result $\Delta T/T = 10^{-5}$ had been predicted by the CDM model (Blumenthal, Faber, Primack, & Rees 1984). The search then began for smaller-angular-scale CMB anisotropies.



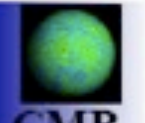
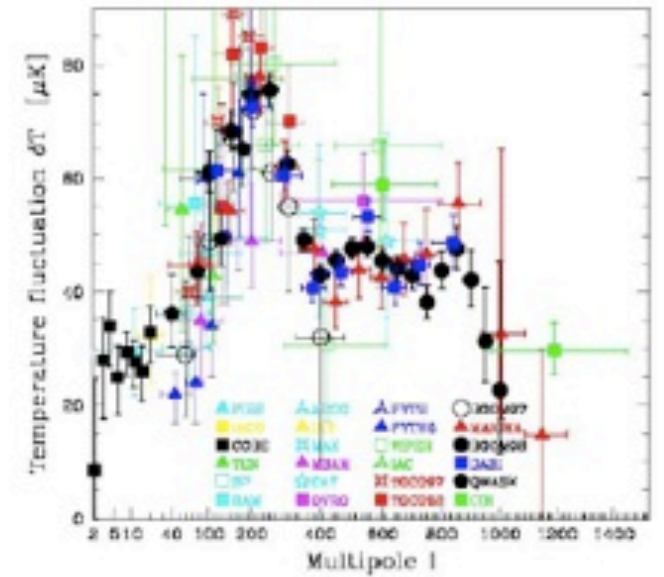
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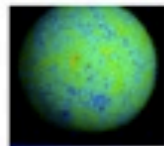
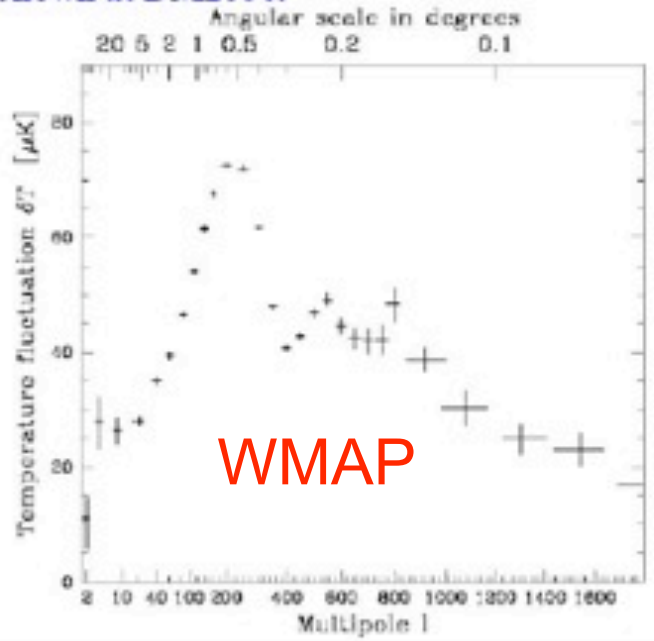
Max Tegmark



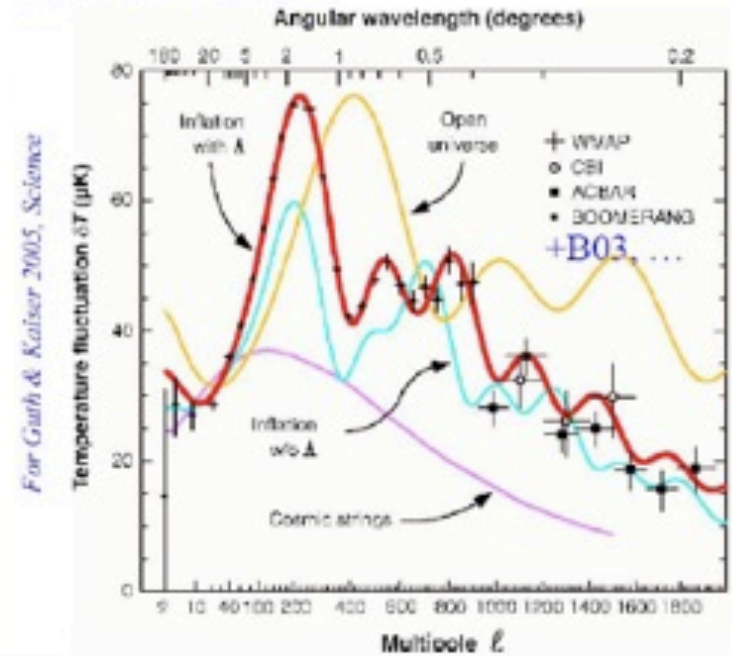
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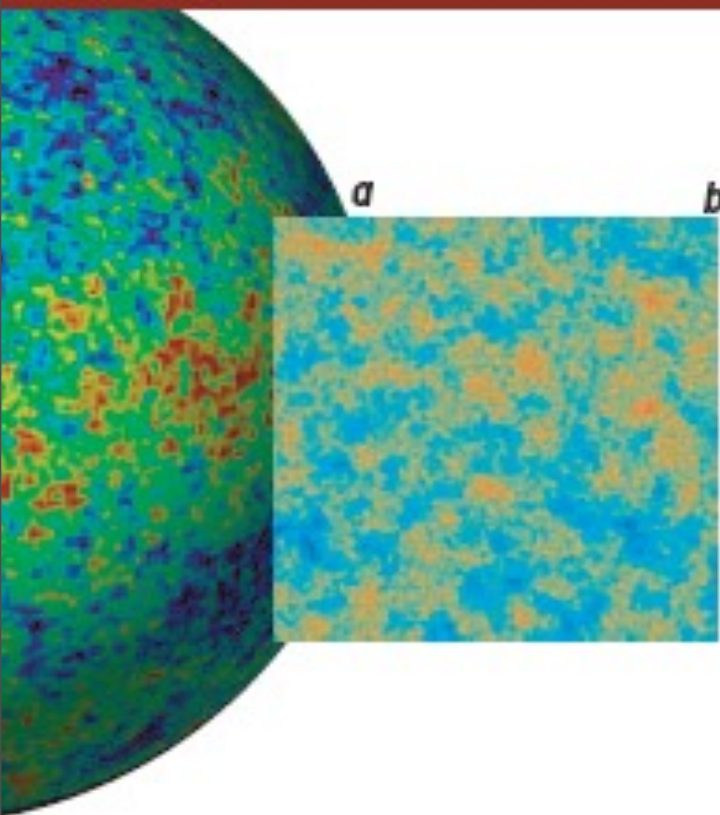
THE COSMIC SYMPHONY

By Wayne Hu and Martin White

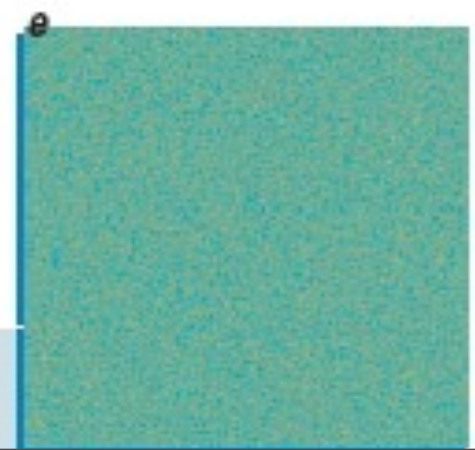
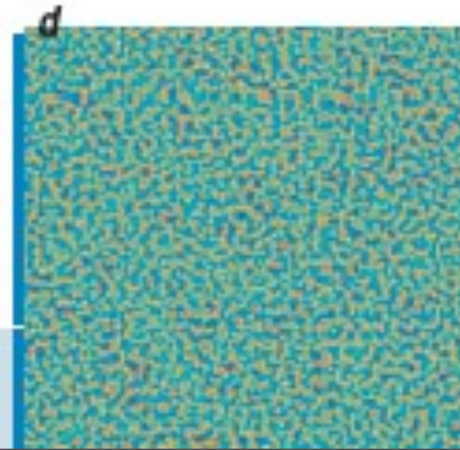
New observations of the cosmic microwave background radiation show that the early universe resounded with harmonious oscillations

Scientific American February 2004

THE POWER SPECTRUM



OBSERVATIONS OF THE CMB provide a map of temperature variations across the whole sky (*a*). When researchers analyze portions of that map (*b*), they use band filters to show how the temperature of the radiation varies at different scales. The variations are barely noticeable at large scales corresponding to regions that stretch about 30 degrees across the sky (*c*) and at small scales corresponding to regions about a tenth of a degree across (*e*). But the temperature differences are quite distinct for regions about one degree across (*d*). This first peak in the power spectrum (*graph at bottom*) reveals the compressions and rarefactions caused by the fundamental wave of the early universe; the subsequent peaks show the effects of the overtones.

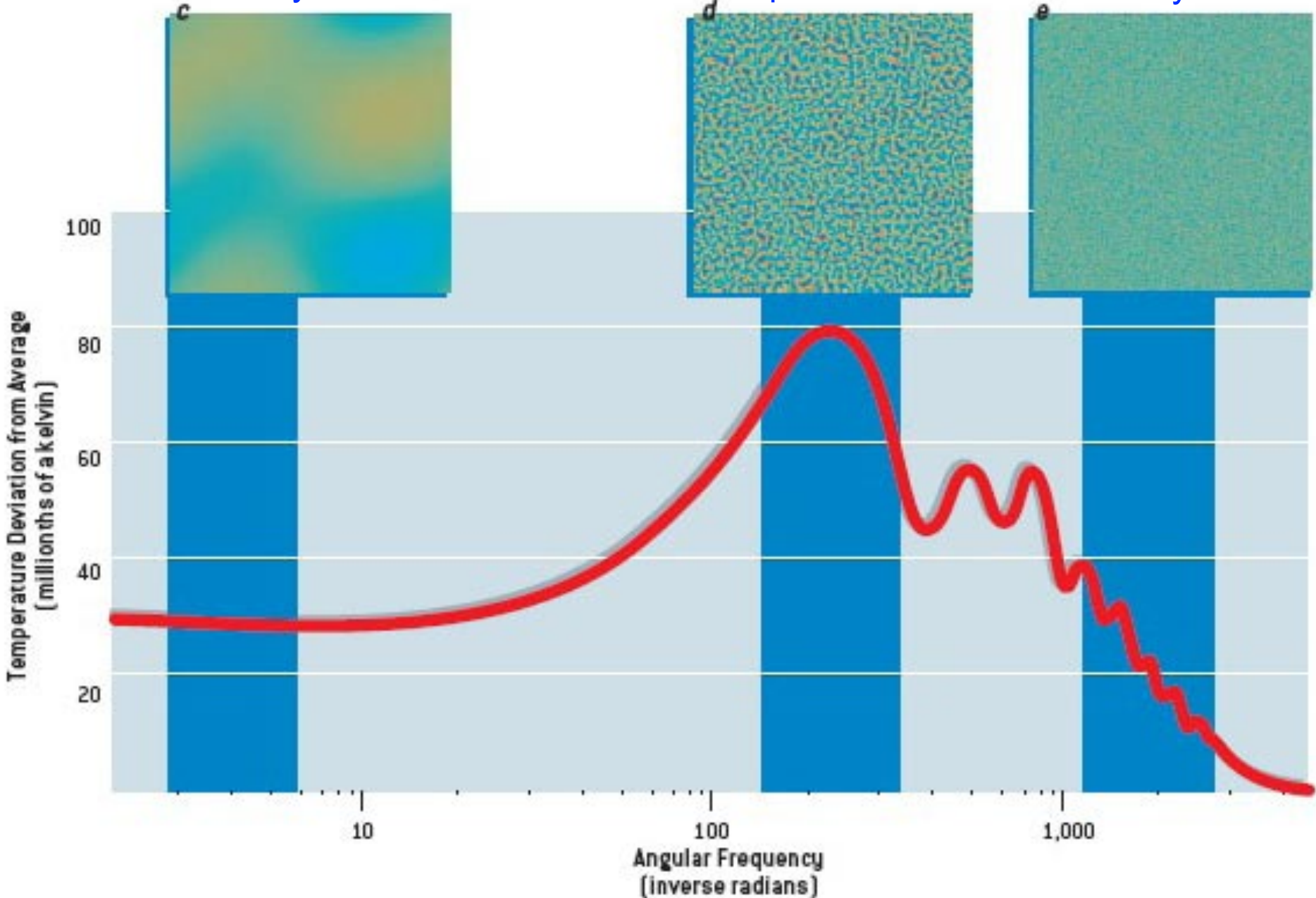


Angular Thermal Variations

30° barely visible

1° prominent

0.1° barely visible



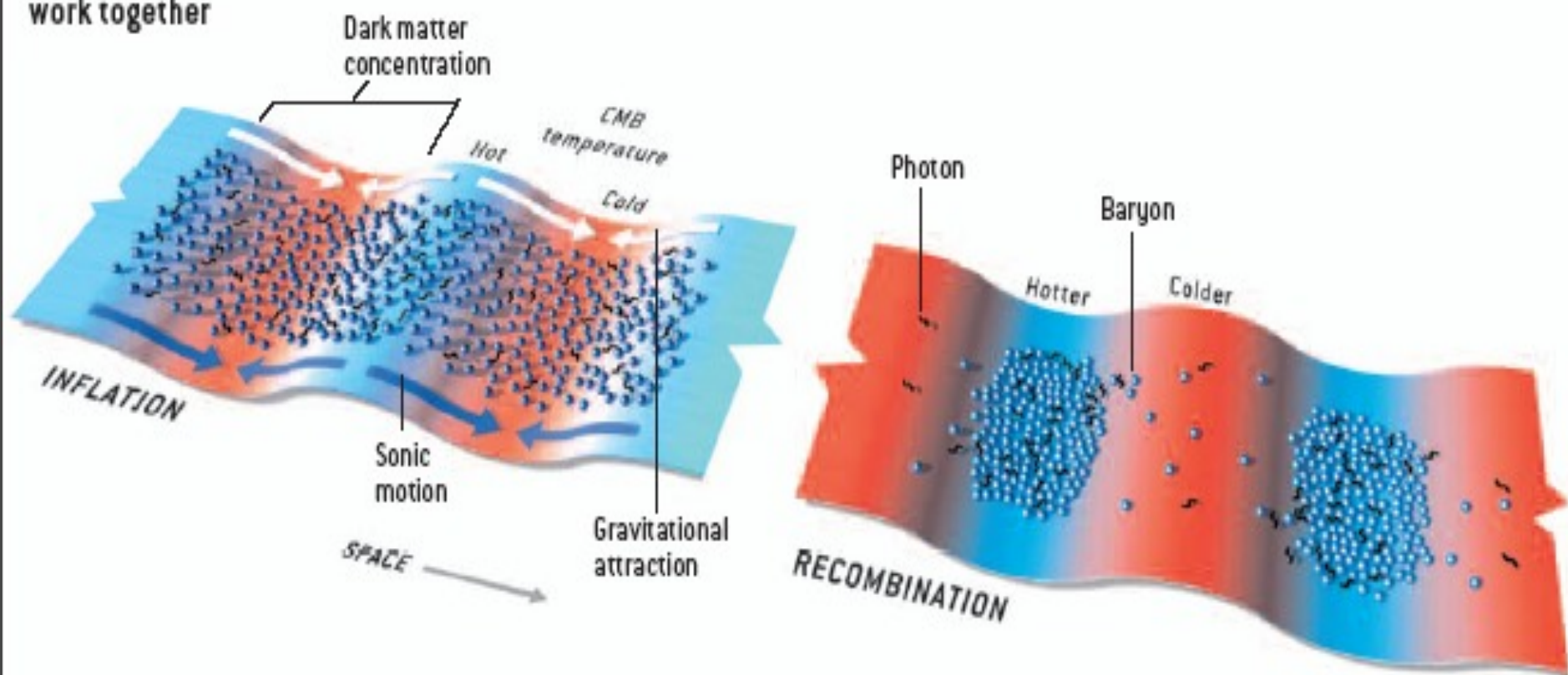
GRAVITATIONAL MODULATION

INFLUENCE OF DARK MATTER modulates the acoustic signals in the CMB. After inflation, denser regions of dark matter that have the same scale as the fundamental wave (represented as troughs in this potential-energy diagram) pull in baryons and photons by gravitational attraction. (The troughs are shown in

red because gravity also reduces the temperature of any escaping photons.) By the time of recombination, about 380,000 years later, gravity and sonic motion have worked together to raise the radiation temperature in the troughs (blue) and lower the temperature at the peaks (red).

FIRST PEAK

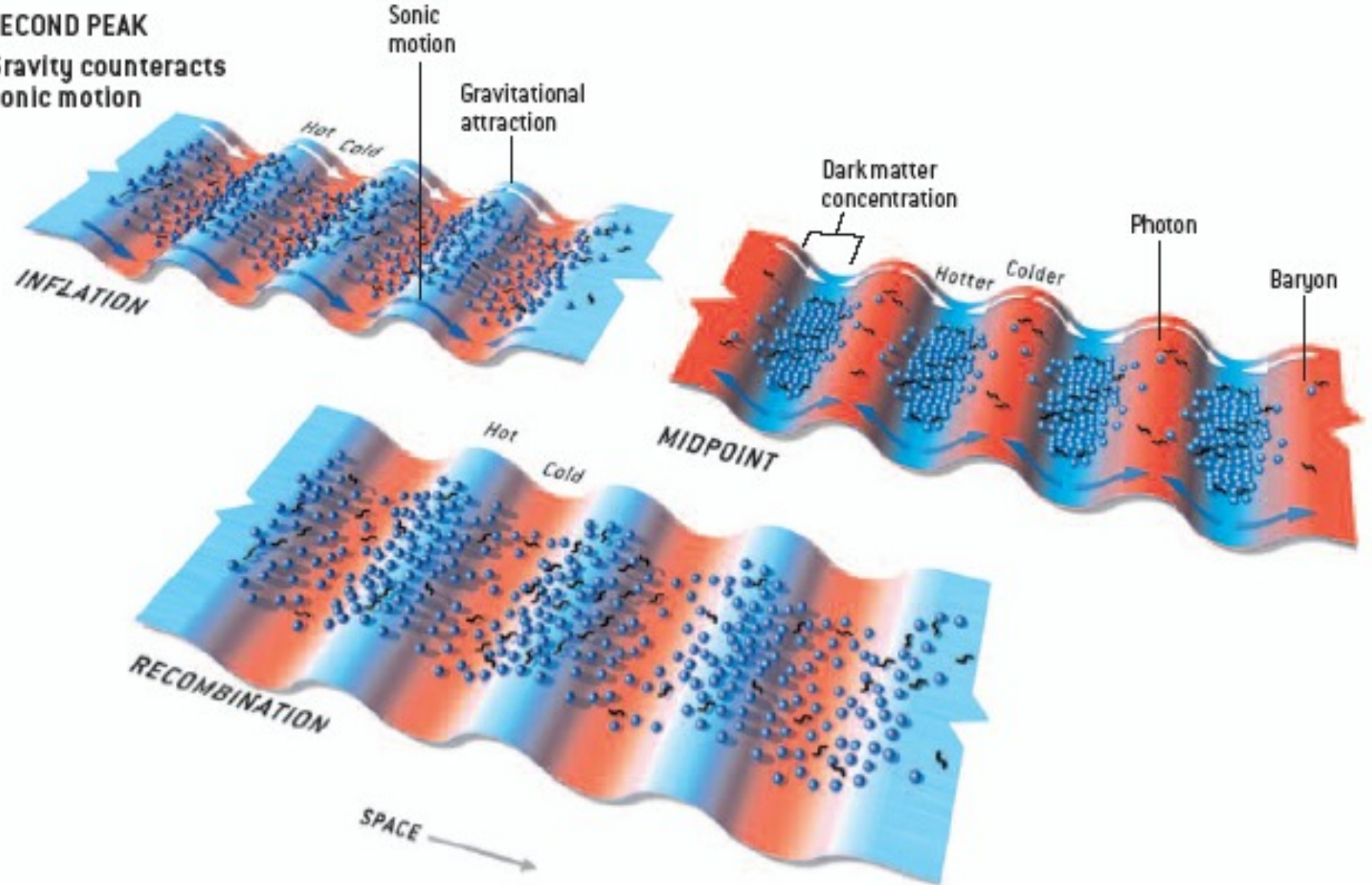
Gravity and sonic motion work together

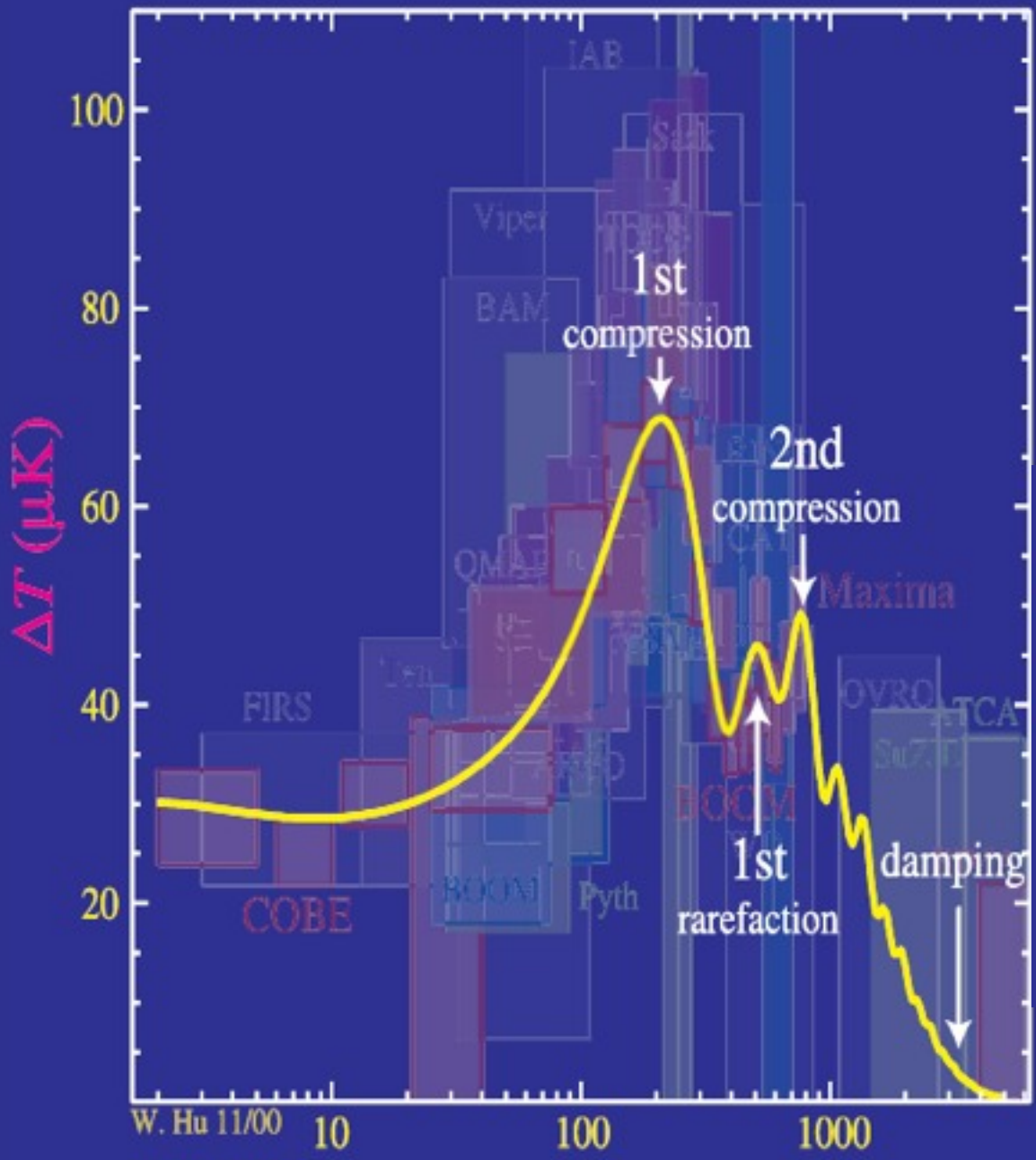


AT SMALLER SCALES, gravity and acoustic pressure sometimes end up at odds. Dark matter clumps corresponding to a second-peak wave maximize radiation temperature in the troughs long before recombination. After this midpoint, gas pressure pushes

baryons and photons out of the troughs (*blue arrows*) while gravity tries to pull them back in (*white arrows*). This tug-of-war decreases the temperature differences, which explains why the second peak in the power spectrum is lower than the first.

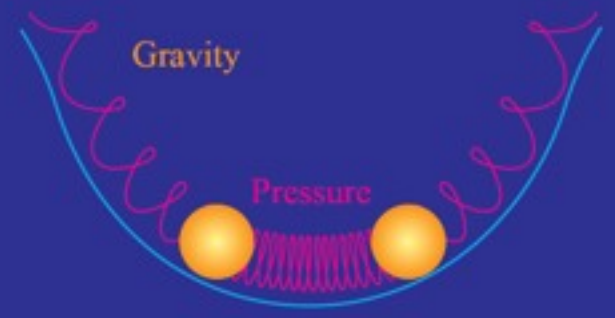
SECOND PEAK
Gravity counteracts
sonic motion





Gravitational Ringing

- Potential wells = inflationary seeds of structure
- Fluid falls into wells, pressure resists: acoustic oscillations



Extrema=Peaks

- First peak = mode that just compresses
- Second peak = mode that compresses then rarefies: twice the wavenumber
- Harmonic peaks: 1:2:3 in wavenumber



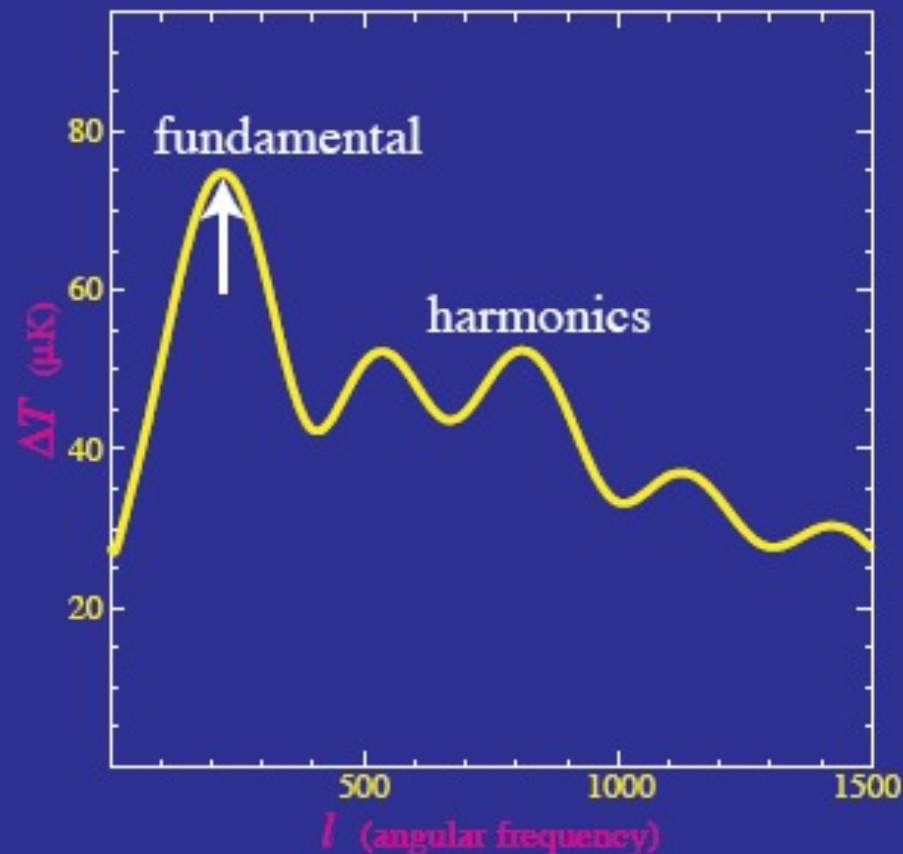
From Wayne Hu's Warner Prize

Lecture, AAS meeting Jan 2001

<http://background.uchicago.edu/~whu/Presentations/warnerprint.pdf>

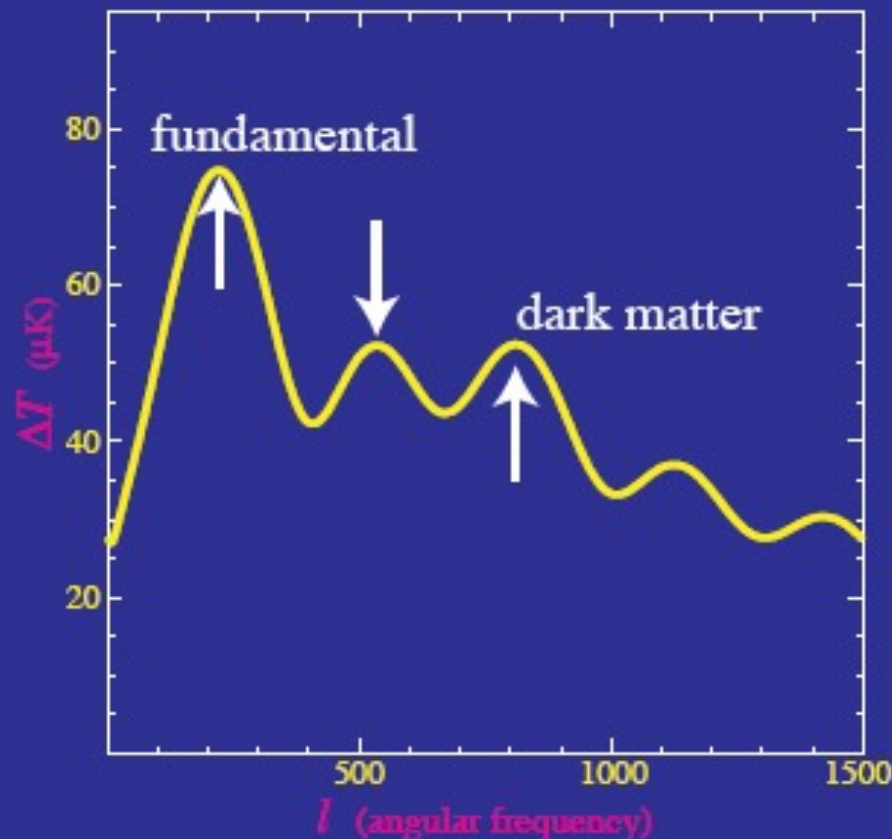
Sound Spectrum

- Spectrum of sound shows harmonics at integer ratios of the fundamental
- Other models that generate structure causally at intermediate times would not have these harmonics



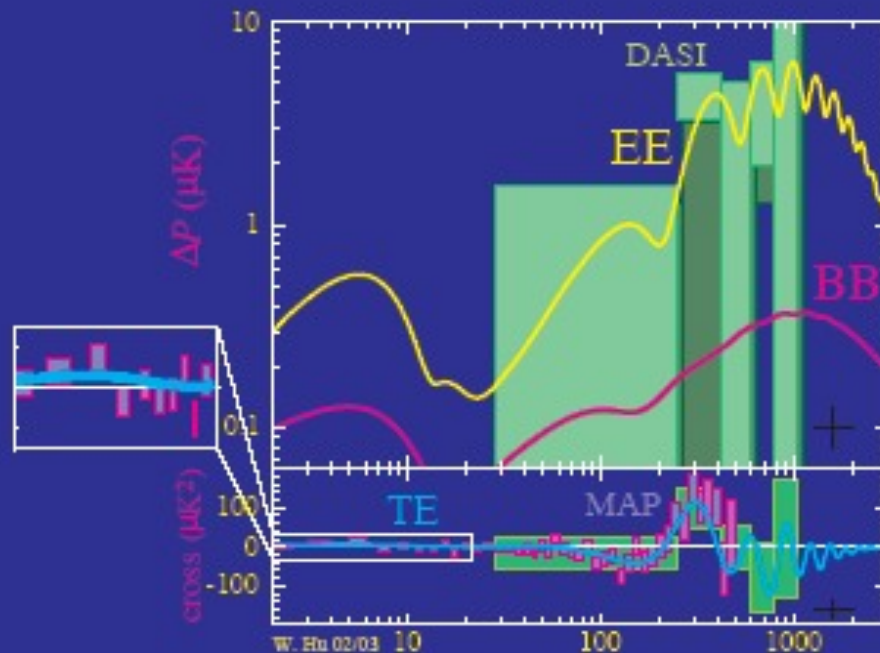
Dark Matter

- A third peak comparable to second peak indicates a dark matter density $\sim 5x$ that of ordinary matter
- Dark matter $\sim 25\%$ of the critical density



Predictive Power

- Model predicts the precise form of the damping of sound waves: observed ✓
- Model predicts that associated with the damping, the CMB becomes polarized: observed ✓
- Model predicts that temperature fluctuations correlated with local structure due to the dark energy: observed ✓



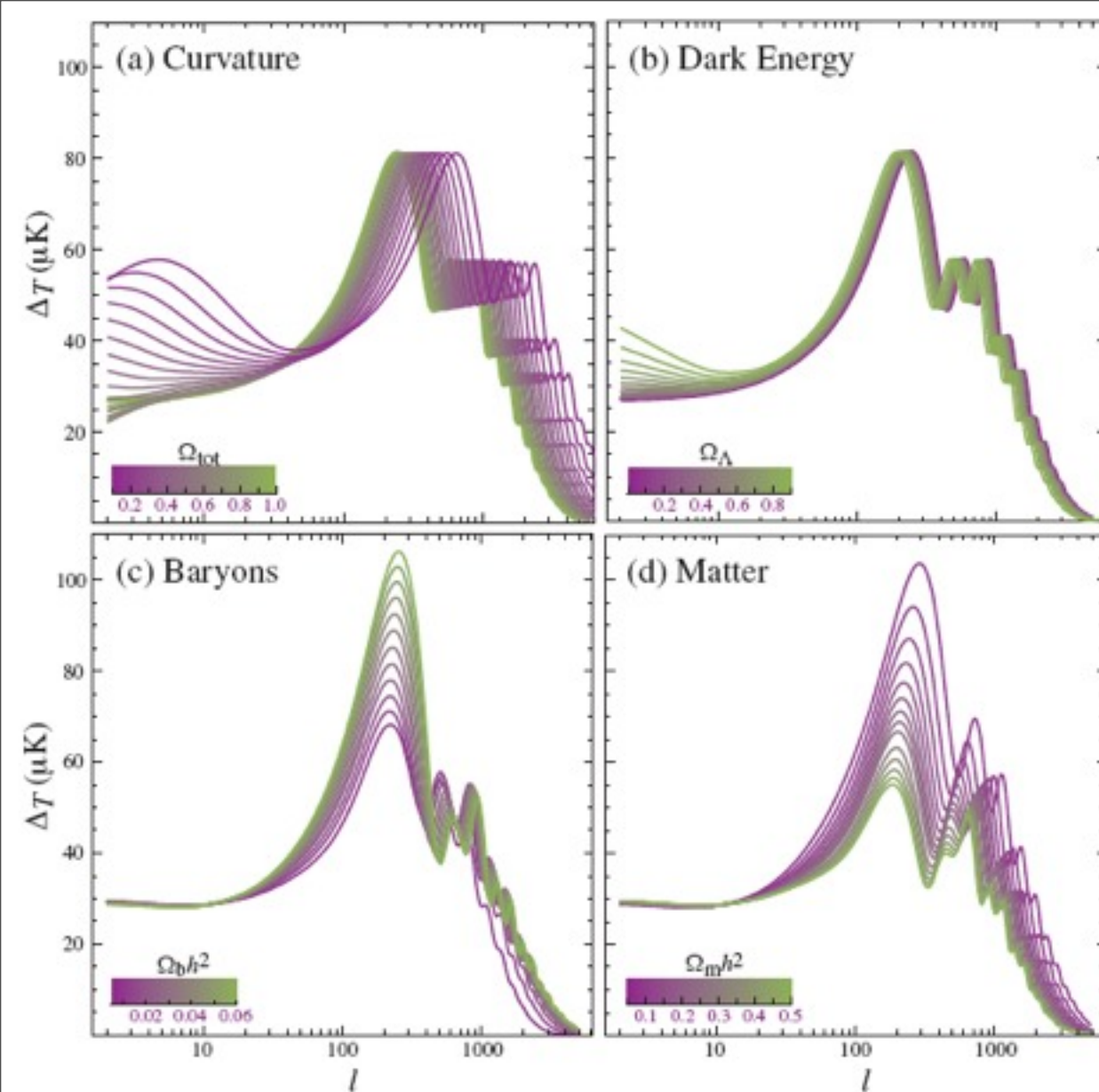


Plate 4: Sensitivity of the acoustic temperature spectrum to four fundamental cosmological parameters (a) the curvature as quantified by Ω_{tot} (b) the dark energy as quantified by the cosmological constant Ω_{Λ} ($w_{\Lambda} = -1$) (c) the physical baryon density $\Omega_b h^2$ (d) the physical matter density $\Omega_m h^2$, all varied around a fiducial model of $\Omega_{\text{tot}} = 1$, $\Omega_{\Lambda} = 0.65$, $\Omega_b h^2 = 0.02$, $\Omega_m h^2 = 0.147$, $n = 1$, $z_{\text{re}} = 0$, $E_i = 0$.

Annu. Rev. Astron. and
Astrophys. 2002

**Cosmic Microwave Background
Anisotropies** by **Wayne Hu** and
Scott Dodelson

For animation of the effects of changes in cosmological parameters on the CMB angular power spectrum and the matter power spectrum, plus links to many CMB websites, see Max Tegmark's and Wayne Hu's websites:

<http://space.mit.edu/home/tegmark/>

<http://background.uchicago.edu/~whu/physics/physics.html>

WMAP 5-year data and papers are at <http://lambda.gsfc.nasa.gov/>

G. Hinshaw et al. *ApJS*, 180, 225 (2009)

Five-Year Wilkinson Microwave Anisotropy Probe (WMAP¹) Observations: Data Processing, Sky Maps, & Basic Results

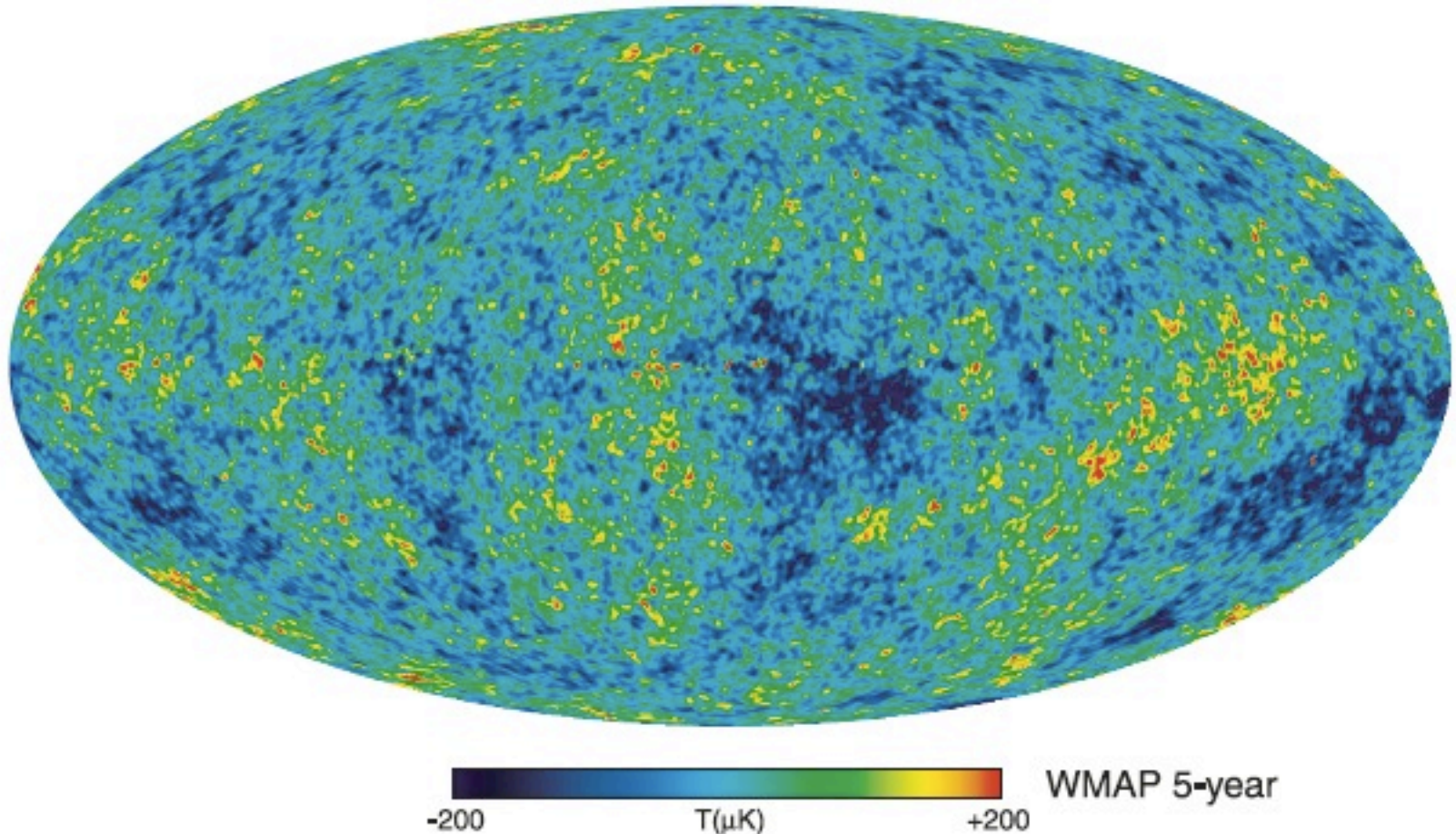


Fig. 12. The foreground-reduced Internal Linear Combination (ILC) map based on the 5 year WMAP data.

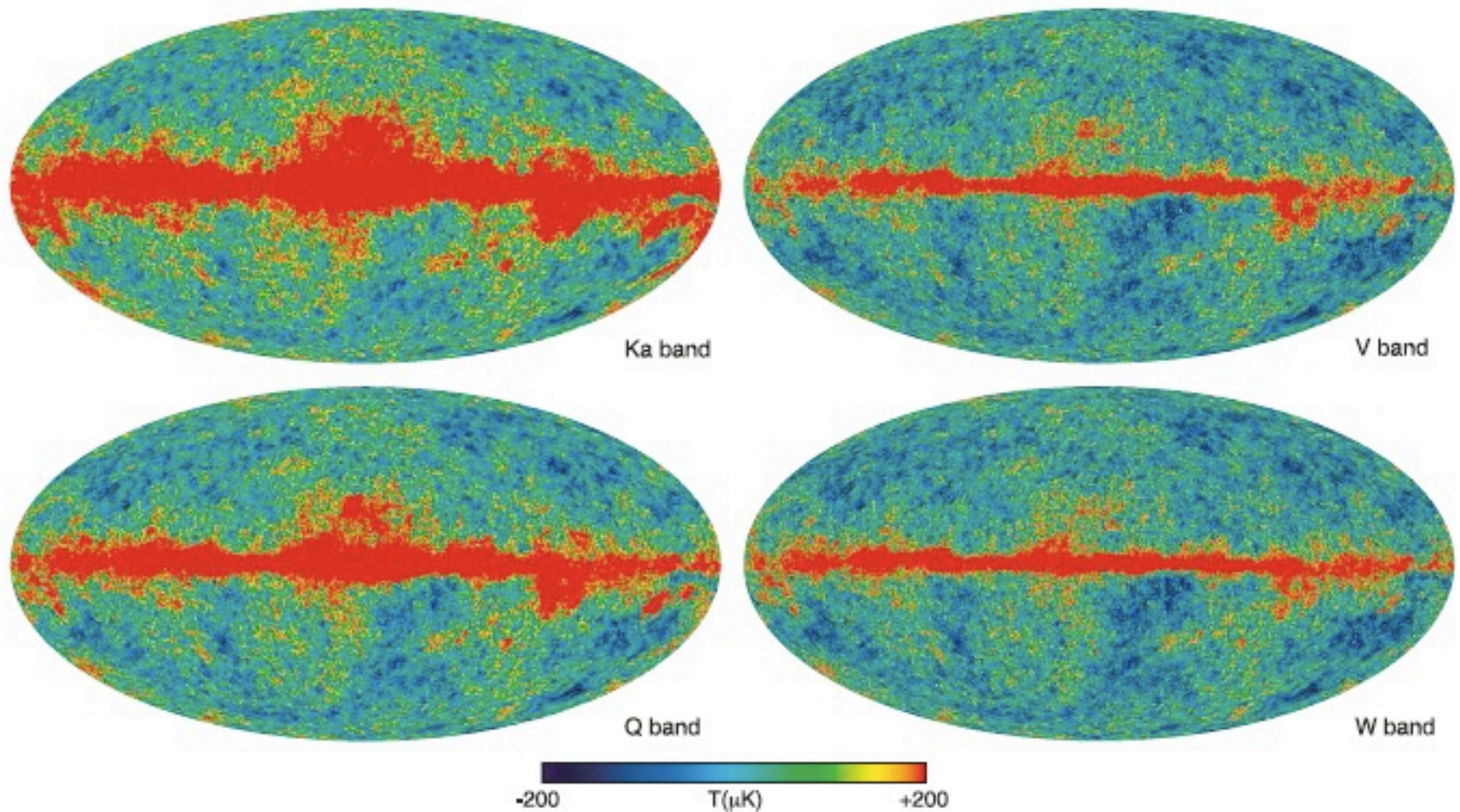


Fig. 1.— Five-year temperature sky maps in Galactic coordinates smoothed with a 0.2° Gaussian beam, shown in Mollweide projection. ~~top: K band (23 GHz)~~, middle-left: Ka band (33 GHz), bottom-left: Q band (41 GHz), middle-right: V band (61 GHz), bottom-right: W band (94 GHz). **Note: only V and W bands are used for cosmological analysis.**

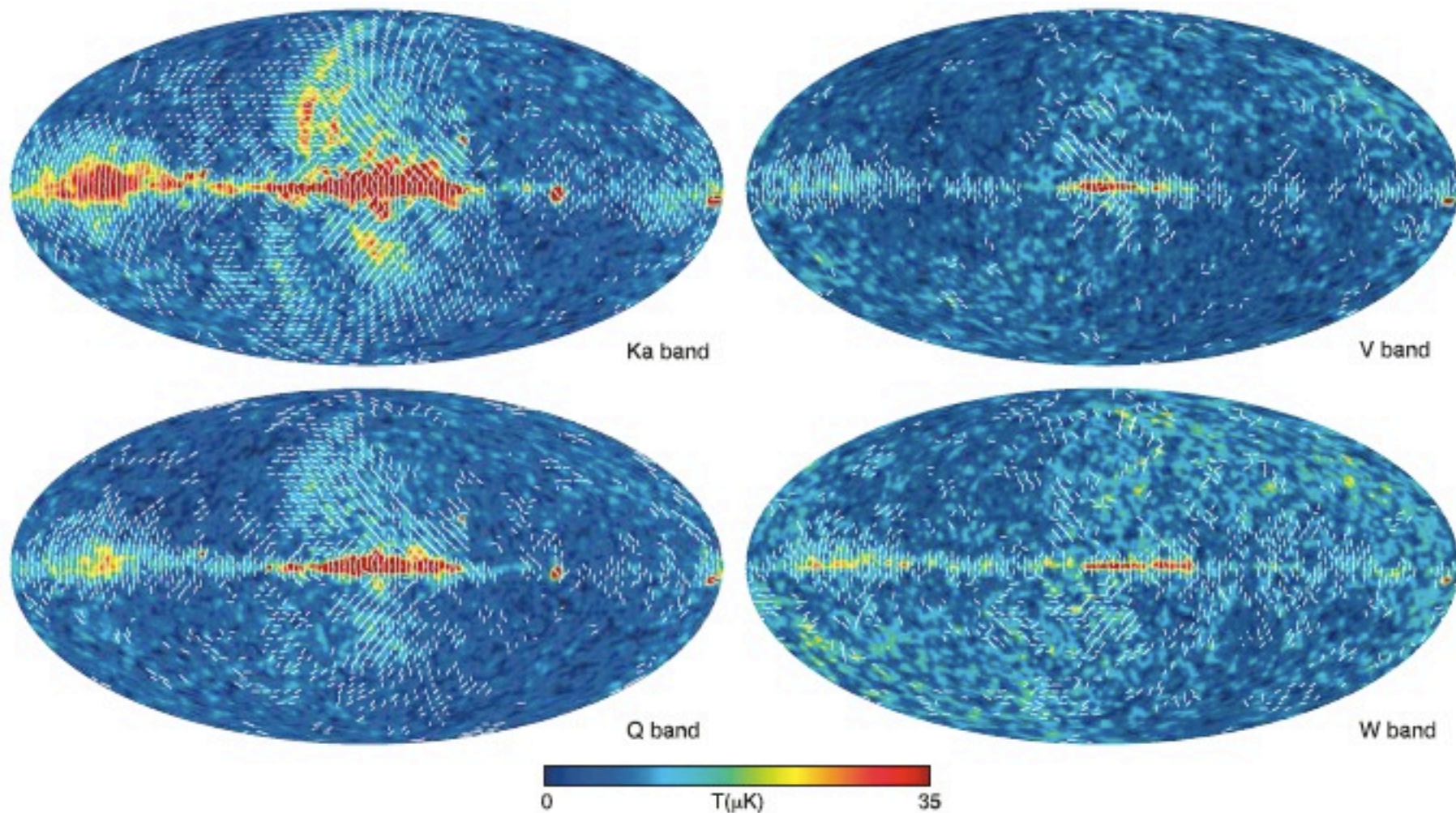
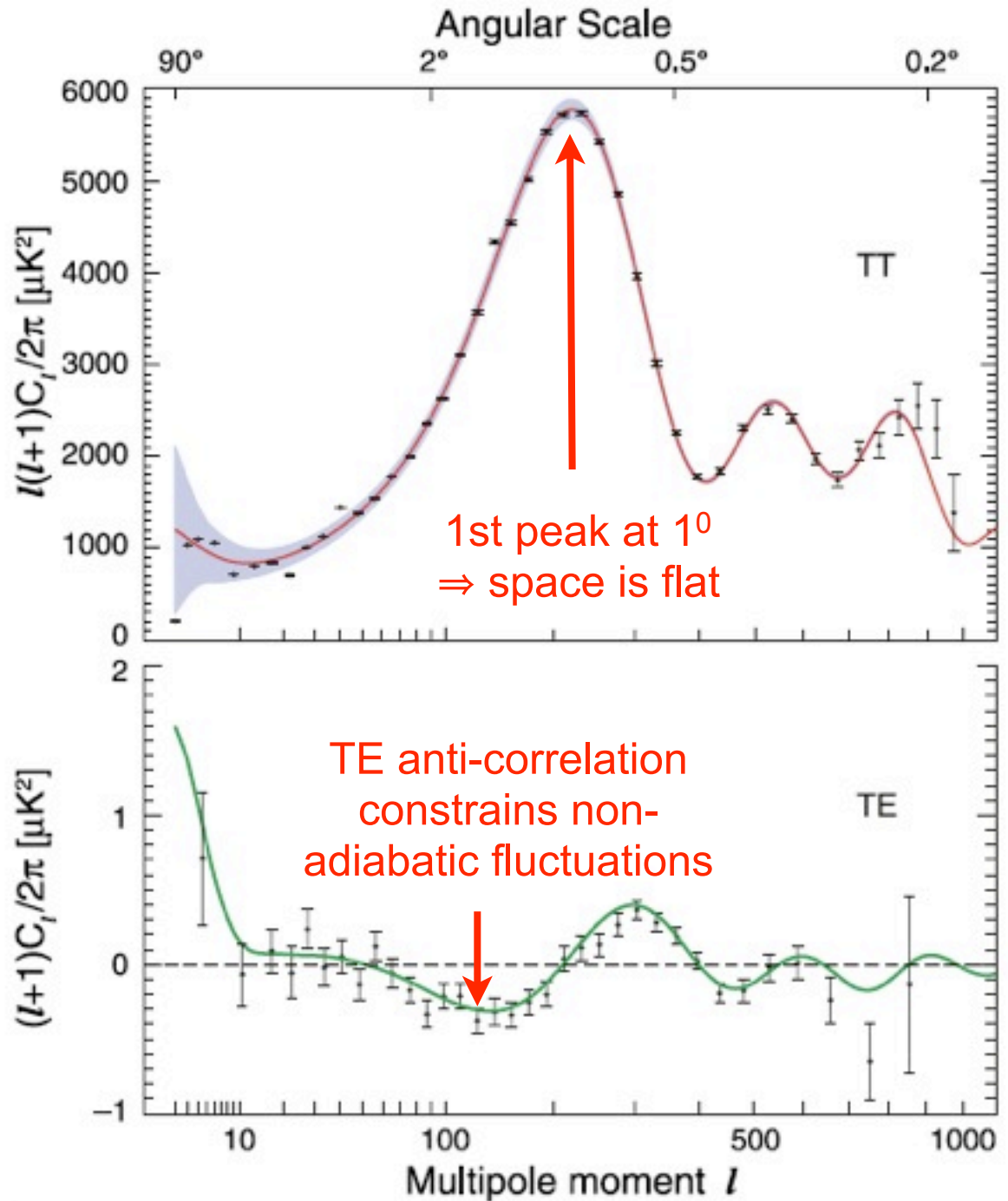


Fig. 4.— Five-year polarization sky maps in Galactic coordinates smoothed to an effective Gaussian beam of 2.0° , shown in Mollweide projection. The color scale indicates polarized intensity, $P = \sqrt{Q^2 + U^2}$, and the line segments indicate polarization direction in pixels whose signal-to-noise exceeds 1. ~~top: K band~~ (23 GHz), *middle-left*: Ka band (33 GHz), *bottom-left*: Q band (41 GHz), *middle-right*: V band (61 GHz), *bottom-right*: W band (94 GHz). G. Hinshaw et al. *ApJS*, 180, 225 (2009)

Fig. 13. The temperature (TT) and temperature-polarization correlation (TE) power spectra based on the 5 year WMAP data. The addition of 2 years of data provide more sensitive measurements of the third peak in TT and the high- l TE spectrum, especially the second trough.



The cosmological implications of the 5 year *WMAP* data are discussed in detail in Dunkley et al. (2008) and Komatsu et al. (2008). The now-standard cosmological model: a flat universe dominated by vacuum energy and dark matter, seeded by nearly scale-invariant, adiabatic, Gaussian random-phase fluctuations, continues to fit the 5 year data. *WMAP* has now determined the key parameters of this model to high precision; a summary of the 5 year parameter results is given in Table 6. The most notable improvements are the measurements of the dark matter density, $\Omega_c h^2$, and the amplitude of matter fluctuations today, σ_8 . The former is determined with 6% uncertainty using *WMAP* data only (Dunkley et al. 2008), and with 3% uncertainty when *WMAP* data is combined with BAO and SNe constraints (Komatsu et al. 2008). The latter is measured to 5% with *WMAP* data, and to 3% when combined with other data. The redshift of reionization is $z_{\text{reion}} = 11.0 \pm 1.4$, if the universe were reionized instantaneously. The 2σ lower limit is $z_{\text{reion}} > 8.2$, and instantaneous reionization at $z_{\text{reion}} = 6$ is rejected at 3.5σ . The *WMAP* data continues to favor models with a tilted primordial spectrum, $n_s = 0.963^{+0.014}_{-0.015}$. Dunkley et al. (2008) discuss how the Λ CDM model continues to fit a host of other astronomical data as well.

Moving beyond the standard Λ CDM model, when *WMAP* data is combined with BAO and SNe observations (Komatsu et al. 2008), we find no evidence for running in the spectral index of scalar fluctuations, $dn_s/d\ln k = -0.032^{+0.021}_{-0.020}$ (68% CL). The new limit on the tensor-to-scalar ratio is $r < 0.20$ (95% CL), and we obtain tight, simultaneous limits on the (constant) dark energy equation of state and the spatial curvature of the universe: $-0.11 < 1 + w < 0.14$ (95% CL) and $-0.0175 < \Omega_k < 0.0085$ (95% CL). The angular power spectrum now exhibits the signature of the cosmic neutrino background: the number of relativistic degrees of freedom, expressed in units of the effective number of neutrino species, is found to be $N_{\text{eff}} = 4.4 \pm 1.5$ (68% CL), consistent with the standard value of 3.04. Models with $N_{\text{eff}} = 0$ are disfavored at $>99.5\%$ confidence. A summary of the key cosmological parameter values is given in Table 6, where we provide estimates using *WMAP* data alone and *WMAP* data combined with BAO and SNe observations. A complete tabulation of all parameter values for each model and dataset combination we studied is available on LAMBDA.

Description	Symbol	WMAP-only	WMAP+BAO+SN
Parameters for Standard Λ CDM Model ^a			
Age of universe	t_0	13.69 ± 0.13 Gyr	13.73 ± 0.12 Gyr
Hubble constant	H_0	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.1 ± 1.3 km/s/Mpc
Baryon density	Ω_b	0.0441 ± 0.0030	0.0462 ± 0.0015
Physical baryon density	$\Omega_b h^2$	0.02273 ± 0.00062	0.02265 ± 0.00059
Dark matter density	Ω_c	0.214 ± 0.027	0.233 ± 0.013
Physical dark matter density	$\Omega_c h^2$	0.1099 ± 0.0062	0.1143 ± 0.0034
Dark energy density	Ω_Λ	0.742 ± 0.030	0.721 ± 0.015
Curvature fluctuation amplitude, $k_0 = 0.002 \text{ Mpc}^{-1}$ ^b	$\Delta_{\mathcal{R}}^2$	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.796 ± 0.036	0.817 ± 0.026
$l(l+1)C_{220}^{TT}/2\pi$	C_{220}	$5756 \pm 42 \mu\text{K}^2$	$5748 \pm 41 \mu\text{K}^2$
Scalar spectral index	n_s	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
Redshift of matter-radiation equality	z_{eq}	3176^{+151}_{-150}	3280^{+88}_{-89}
Angular diameter distance to matter-radiation eq. ^c	$d_A(z_{\text{eq}})$	14279^{+186}_{-189} Mpc	14172^{+141}_{-139} Mpc
Redshift of decoupling	z_*	1090.51 ± 0.95	$1091.00^{+0.72}_{-0.73}$
Age at decoupling	t_*	380081^{+5843}_{-5841} yr	375938^{+3148}_{-3115} yr
Angular diameter distance to decoupling ^{c,d}	$d_A(z_*)$	14115^{+188}_{-191} Mpc	14006^{+142}_{-141} Mpc
Sound horizon at decoupling ^d	$r_s(z_*)$	146.8 ± 1.8 Mpc	145.6 ± 1.2 Mpc
Acoustic scale at decoupling ^d	$l_A(z_*)$	$302.08^{+0.83}_{-0.84}$	$302.11^{+0.84}_{-0.82}$
Reionization optical depth	τ	0.087 ± 0.017	0.084 ± 0.016
Redshift of reionization	z_{reion}	11.0 ± 1.4	10.8 ± 1.4
Age at reionization	t_{reion}	427^{+88}_{-65} Myr	432^{+90}_{-67} Myr

Parameters for Extended Models ^e

Total density ^f	Ω_{tot}	$1.099^{+0.100}_{-0.085}$	1.0052 ± 0.0064
Equation of state ^g	w	$-1.06^{+0.41}_{-0.42}$	$-0.972^{+0.061}_{-0.060}$
Tensor to scalar ratio, $k_0 = 0.002 \text{ Mpc}^{-1}$ ^{b,h}	r	< 0.43 (95% CL)	< 0.20 (95% CL)
Running of spectral index, $k_0 = 0.002 \text{ Mpc}^{-1}$ ^{b,i}	$dn_s/d \ln k$	-0.037 ± 0.028	$-0.032^{+0.021}_{-0.020}$
Neutrino density ^j	$\Omega_\nu h^2$	< 0.014 (95% CL)	< 0.0065 (95% CL)
Neutrino mass ^j	$\sum m_\nu$	$< 1.3 \text{ eV}$ (95% CL)	$< 0.61 \text{ eV}$ (95% CL)
Number of light neutrino families ^k	N_{eff}	> 2.3 (95% CL)	4.4 ± 1.5

^aThe parameters reported in the first section assume the 6 parameter Λ CDM model, first using WMAP data only (Dunkley et al. 2008), then using WMAP+BAO+SN data (Komatsu et al. 2008).

^b $k = 0.002 \text{ Mpc}^{-1} \longleftrightarrow l_{\text{eff}} \approx 30$.

^cComoving angular diameter distance.

^d $l_A(z_*) \equiv \pi d_A(z_*) r_s(z_*)^{-1}$.

^eThe parameters reported in the second section place limits on deviations from the Λ CDM model, first using WMAP data only (Dunkley et al. 2008), then using WMAP+BAO+SN data (Komatsu et al. 2008). A complete listing of all parameter values and uncertainties for each of the extended models studied is available on LAMBDA.

^fAllows non-zero curvature, $\Omega_k \neq 0$.

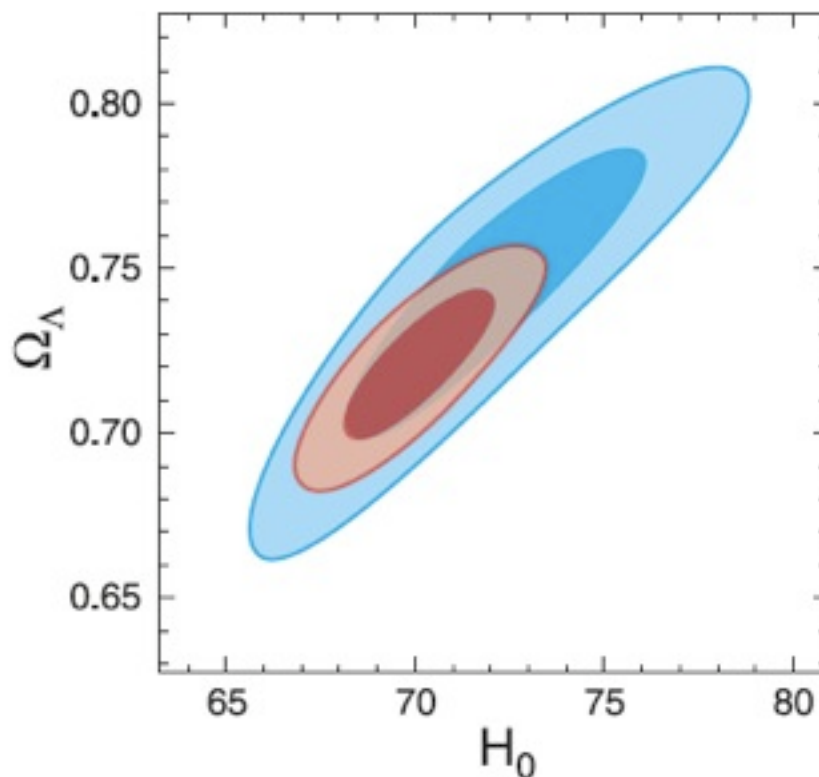
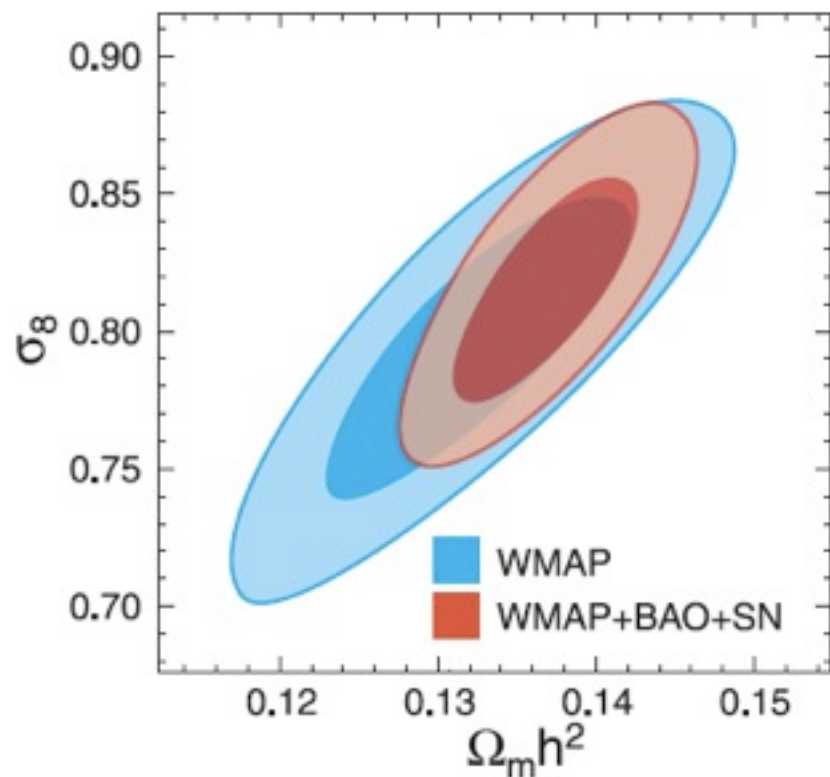
^gAllows $w \neq -1$, but assumes w is constant.

^hAllows tensor modes but no running in scalar spectral index.

ⁱAllows running in scalar spectral index but no tensor modes.

^jAllows a massive neutrino component, $\Omega_\nu \neq 0$.

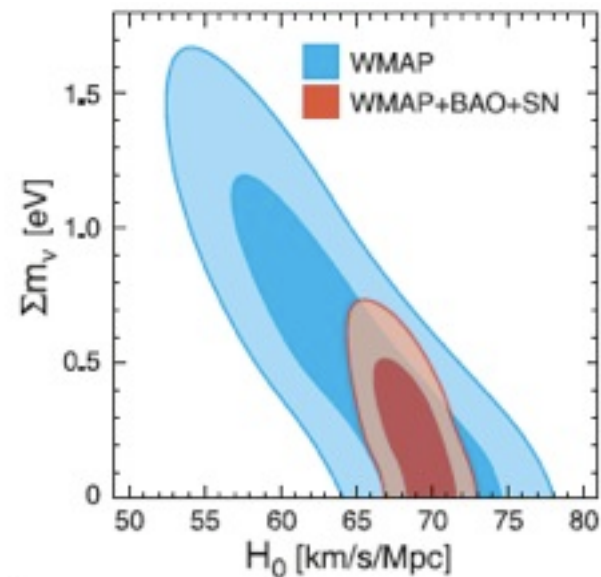
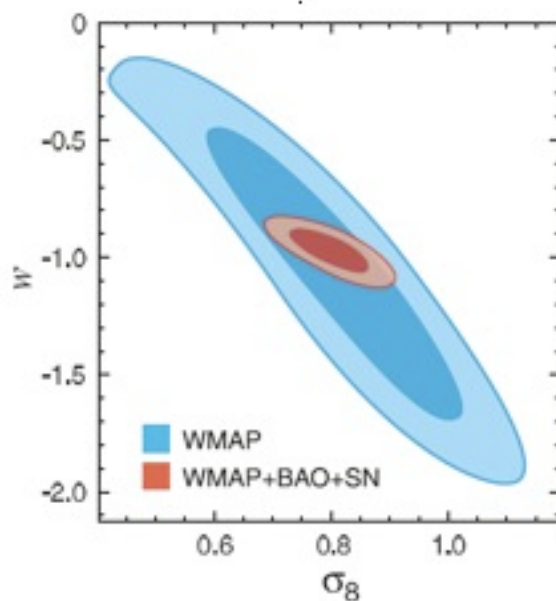
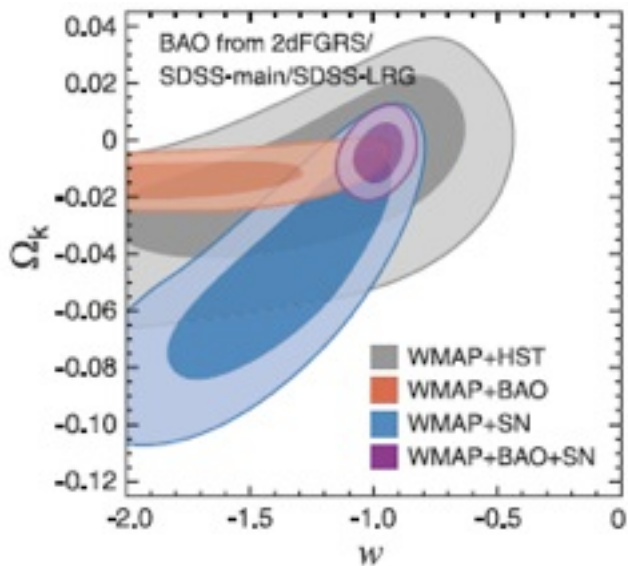
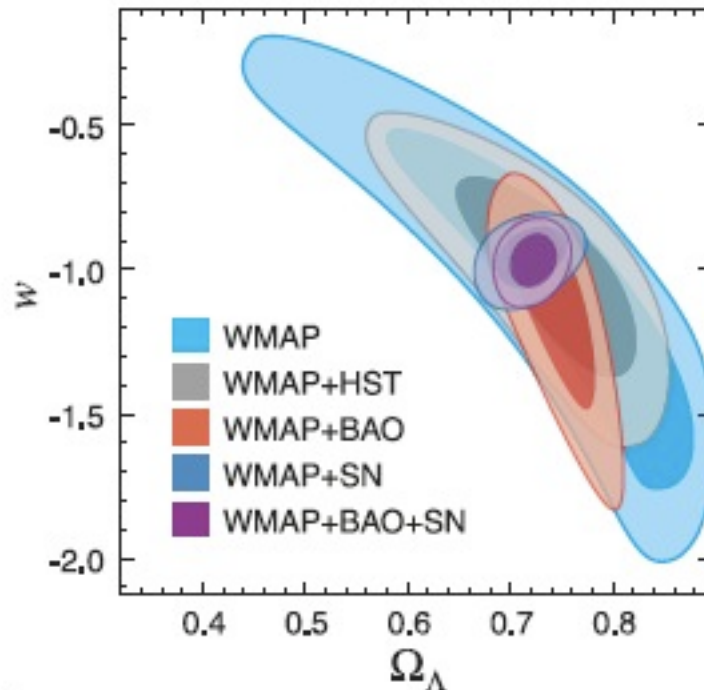
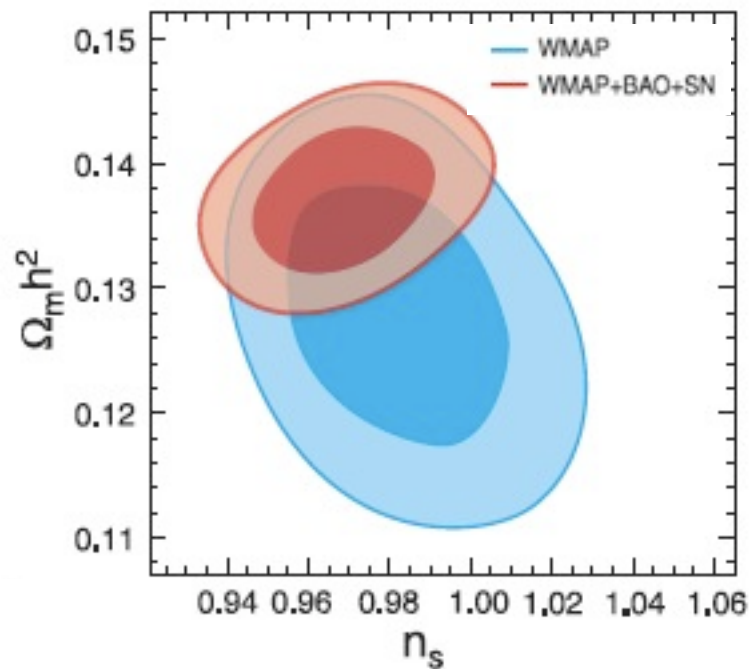
^kAllows N_{eff} number of relativistic species. The last column adds the HST prior to the other data sets.



7. CONCLUSION

With 5 years of integration, the *WMAP* temperature and polarization data have improved significantly. An improved determination of the third acoustic peak has enabled us to reduce the uncertainty in the amplitude of matter fluctuation, parametrized by σ_8 , by a factor of 1.4 from the *WMAP* 3-year result. The E-mode po-

larization is now detected at 5 standard deviations (c.f., 3.0 standard deviations for the 3-year data; Page et al. 2007), which rules out an instantaneous reionization at $z_{\text{reion}} = 6$ at the 3.5σ level. Overall, the *WMAP* 5-year data continue to support the simplest, 6-parameter Λ CDM model (Dunkley et al. 2008).



J. Dunkley, et.al. Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Likelihoods and Parameters from WMAP Data

Final paragraph of Conclusions:

Considering a range of extended models, we continue to find that **the standard Λ CDM model is consistently preferred by the data.** The improved measurement of the third peak now requires the existence of light relativistic species, assumed to be neutrinos, at high confidence. The standard scenario has three neutrino species, but the three-year WMAP data could not rule out models with none. **The CDM model also continues to succeed in fitting a substantial array of other observations.** Certain tensions between other observations and those of WMAP, such as the amplitude of matter fluctuations measured by weak lensing surveys and using the Ly- α forest, and the primordial lithium abundance, have either been resolved with improved understanding of systematics, or show promise of being explained by recent observations. With further WMAP observations we will better probe both the universe at a range of epochs, measuring fluctuation characteristics to probe the initial inflationary process, or other non-inflationary scenario, improving measurements of the composition of the universe at the recombination era, and characterizing the reionization process in the universe.

Outline

Cosmic Microwave Background

WMAP 5-year Data and Papers Released

Grand Unification of Forces

Phase Transitions in the Early Universe

Topological Defects: Strings, Monopoles*

Cosmic Inflation

*Note: I edited much of the material in the following Topological Defects slides from the website http://www.damtp.cam.ac.uk/user/gr/public/cs_top.html

Grand Unification

The basic premise of grand unification is that the known symmetries of the elementary particles result from a larger (and so far unknown) symmetry group G . Whenever a phase transition occurs, part of this symmetry is lost, so the symmetry group changes. This can be represented mathematically as

$$G \rightarrow H \rightarrow \dots \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1).$$

Here, each arrow represents a symmetry breaking phase transition where matter changes form and the groups - G , H , $SU(3)$, etc. - represent the different types of matter, specifically the symmetries that the matter exhibits and they are associated with the different fundamental forces of nature.

The liquid phase of water is rotationally symmetric, that is, it looks the same around each point regardless of the direction in which we look. We could represent this large three-dimensional symmetry by the group G (actually $SO(3)$). The solid form of frozen water, however, is not uniform in all directions; the ice crystal has preferential lattice directions along which the water molecules align. The group describing these different discrete directions H , say, will be smaller than G . Through the process of freezing, therefore, the original symmetry G is broken down to H .

Grand Unified Theory

GUT refers to a theory in physics that unifies the strong interaction and electroweak interaction. Several such theories have been proposed, but none is currently universally accepted. The (future) theory that will also include gravity is termed theory of everything. Some common GUT models are:

- Georgi-Glashow (1974) model -- $SU(5)$
- $SO(10)$
- Flipped $SU(5)$ -- $SU(5) \times U(1)$
- Pati-Salam model -- $SU(4) \times SU(2) \times SU(2)$
- E_6

GUT models generically predict the existence of topological defects such as monopoles, cosmic strings, domain walls, and others. None have been observed and their absence is known as the monopole problem in cosmology.

There is still no hard evidence nature is described by a GUT theory. In fact, since the Higgs particle hasn't been discovered yet, it's not even certain if the Standard Model is fully accurate.

Topological Defects

These arise when some n-component scalar field $\phi_i(\mathbf{x}) = 0$ because of topological trapping that occurs as a result of a phase transition in the early universe (as I will explain shortly).

If the ϕ field is complex then $n=2$, and $\phi_i(\mathbf{x}) = 0$ occurs along a linear locus of points, a **string**, in three dimensional space. This corresponds to a 2-dimensional world-sheet in the 3+1 dimensions of spacetime.

If the ϕ field has three components, then $\phi_i(\mathbf{x}) = 0$ occurs at a point in three dimensional space, a **monopole**. This corresponds to a 1-dimensional world-line in the 3+1 dimensions of spacetime.

If the ϕ field has four components, then $\phi_i(\mathbf{x}) = 0$ occurs at a point in space-time, an **instanton**. A related concept is **texture**.

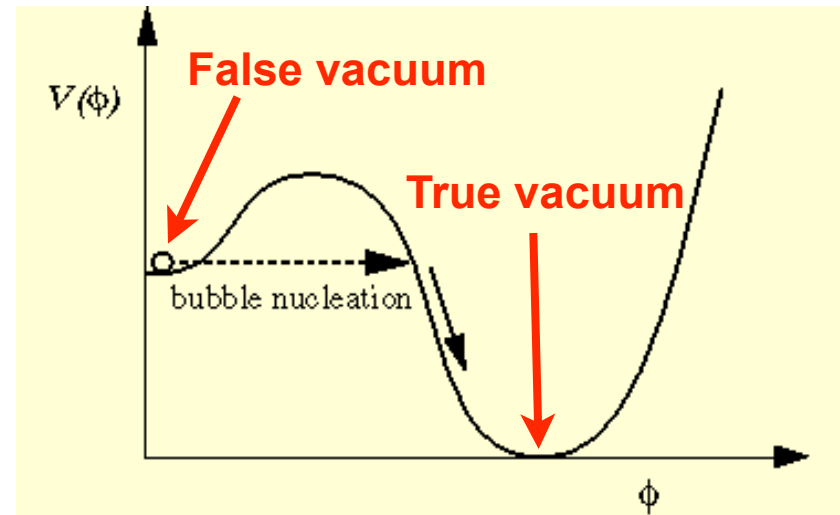
Phase transitions

The cosmological significance of symmetry breaking is due to the fact that symmetries are restored at high temperature (just as it is for liquid water when ice melts). For extremely high temperatures in the early universe, we will even achieve a grand unified state G. Viewed from the moment of creation forward, the universe will pass through a succession of phase transitions at which the strong nuclear force will become differentiated and then the weak nuclear force and electromagnetism.

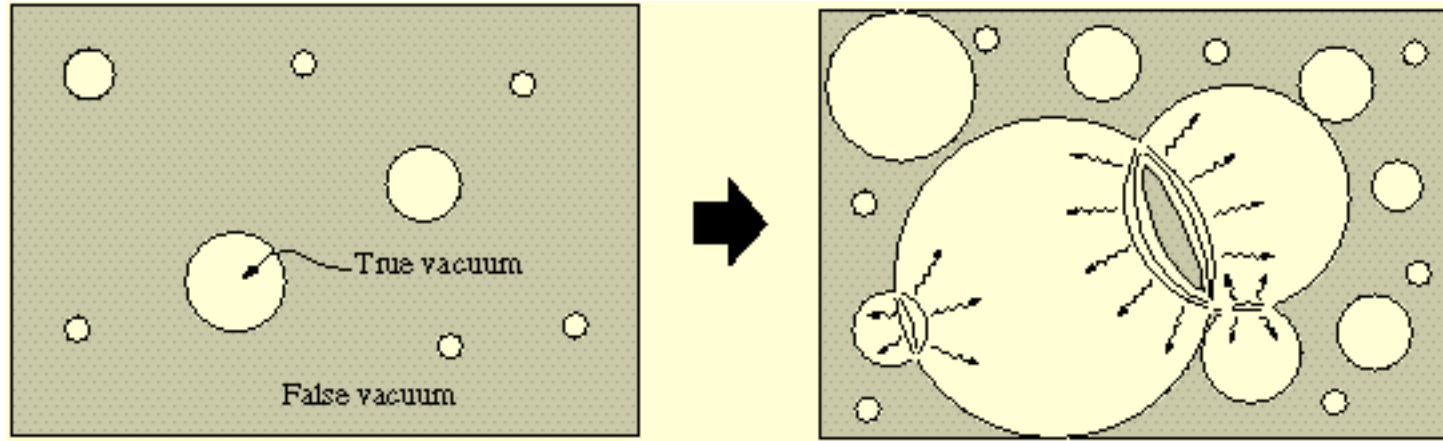
Phase transitions can have a wide variety of important implications including the formation of topological defects -cosmic strings, domain walls, monopoles and textures, or it may even trigger a period of exponential expansion (inflation).

Phase transitions can be either dramatic - first order - or smooth - second order.

During a first-order phase transition, the matter fields get trapped in a 'false vacuum' state from which they can only escape by nucleating bubbles of the new phase, that is, the 'true vacuum' state.



First-order phase transitions (illustrated below) occur through the formation of bubbles of the new phase in the middle of the old phase; these bubbles then expand and collide until the old phase disappears completely and the phase transition is complete.



First-order phase transitions proceed by bubble nucleation. A bubble of the new phase (the true vacuum) forms and then expands until the old phase (the false vacuum) disappears. A useful analogue is boiling water in which bubbles of steam form and expand as they rise to the surface.

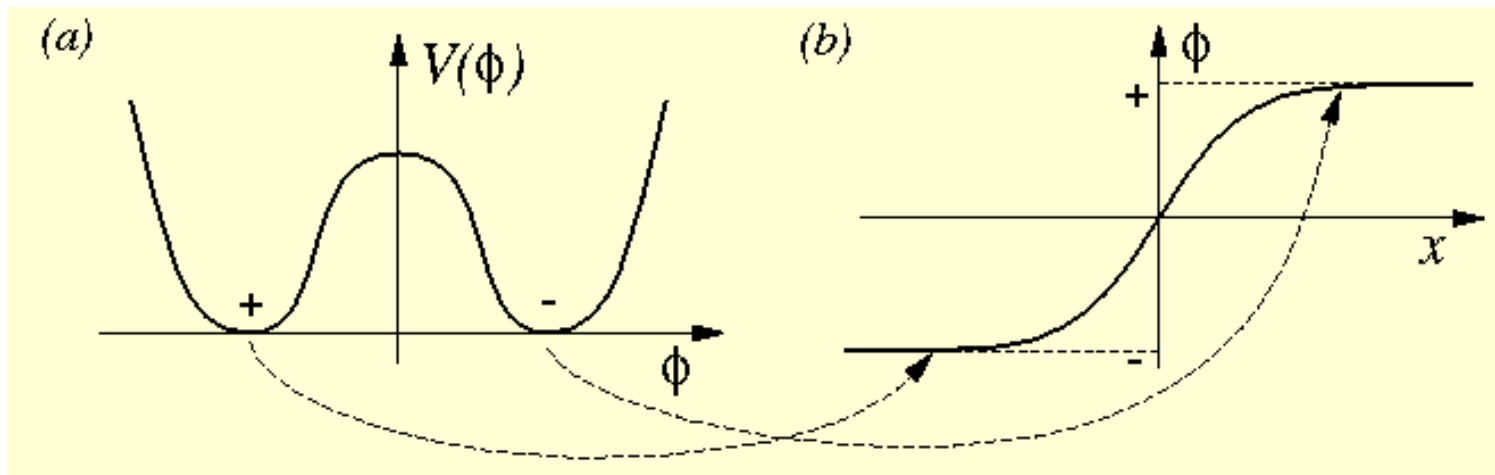
Second-order phase transitions, on the other hand, proceed smoothly. The old phase transforms itself into the new phase in a continuous manner. There is energy (specific heat of vaporization, for example) associated with a first order phase transition.

Either type of phase transition can produce stable configurations called “topological defects.”

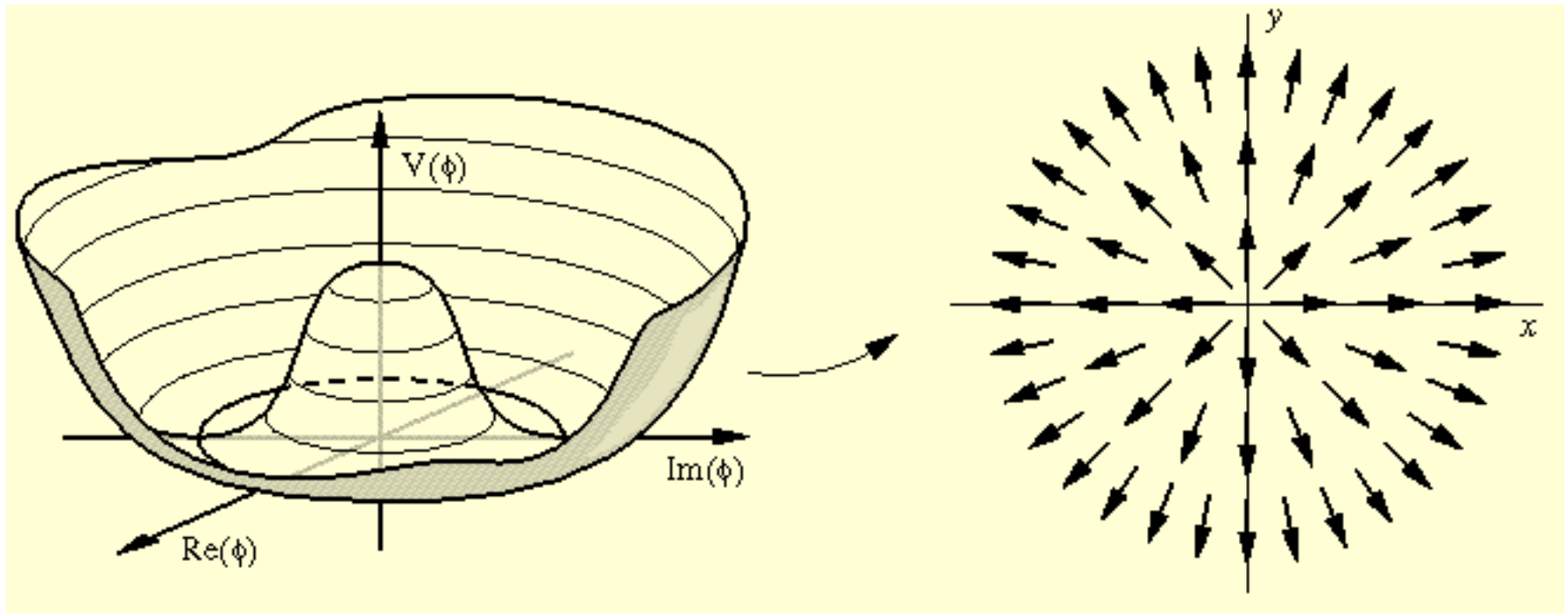
Cosmic Strings & Other Topological Defects

Topological defects are stable configurations that are in the original, symmetric or old phase, but nevertheless for topological reasons they persist after a phase transition to the asymmetric or new phase is completed - because to unwind them would require a great deal of energy. There are a number of possible types of defects, such as domain walls, cosmic strings, monopoles, and textures. The type of defect is determined by the symmetry properties of the matter and the nature of the phase transition.

Domain walls: These are two-dimensional objects that form when a discrete symmetry is broken at a phase transition. A network of domain walls effectively partitions the universe into various 'cells'. Domain walls have some rather peculiar properties. For example, the gravitational field of a domain wall is repulsive rather than attractive.

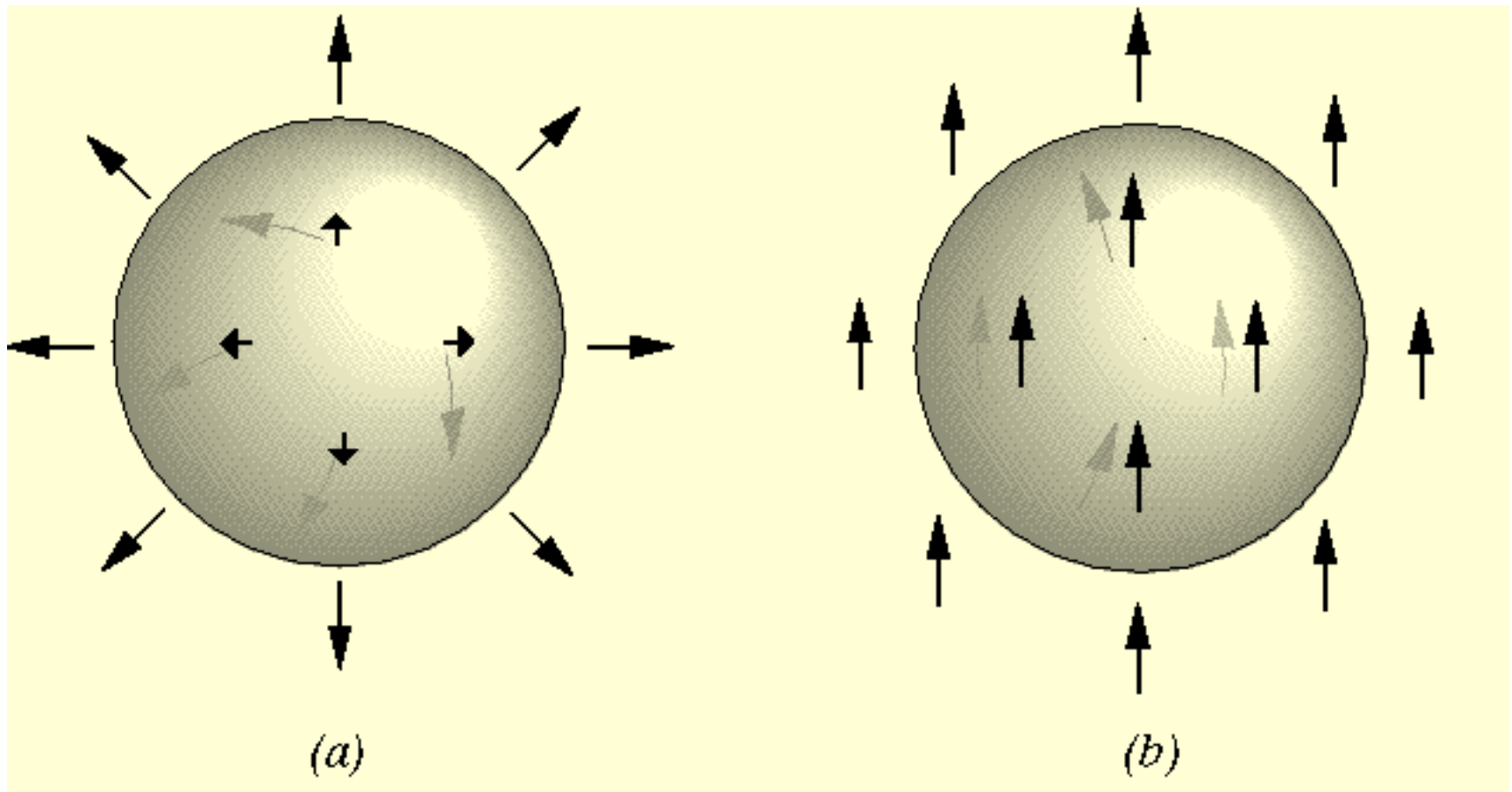


Cosmic strings: These are one-dimensional (that is, line-like) objects which form when an axial or cylindrical symmetry is broken. Strings can be associated with grand unified particle physics models, or they can form at the electroweak scale. They are very thin and may stretch across the visible universe. A typical GUT string has a thickness that is less than a trillion times smaller than the radius of a hydrogen atom, but a 10 km length of one such string would weigh as much as the earth itself!

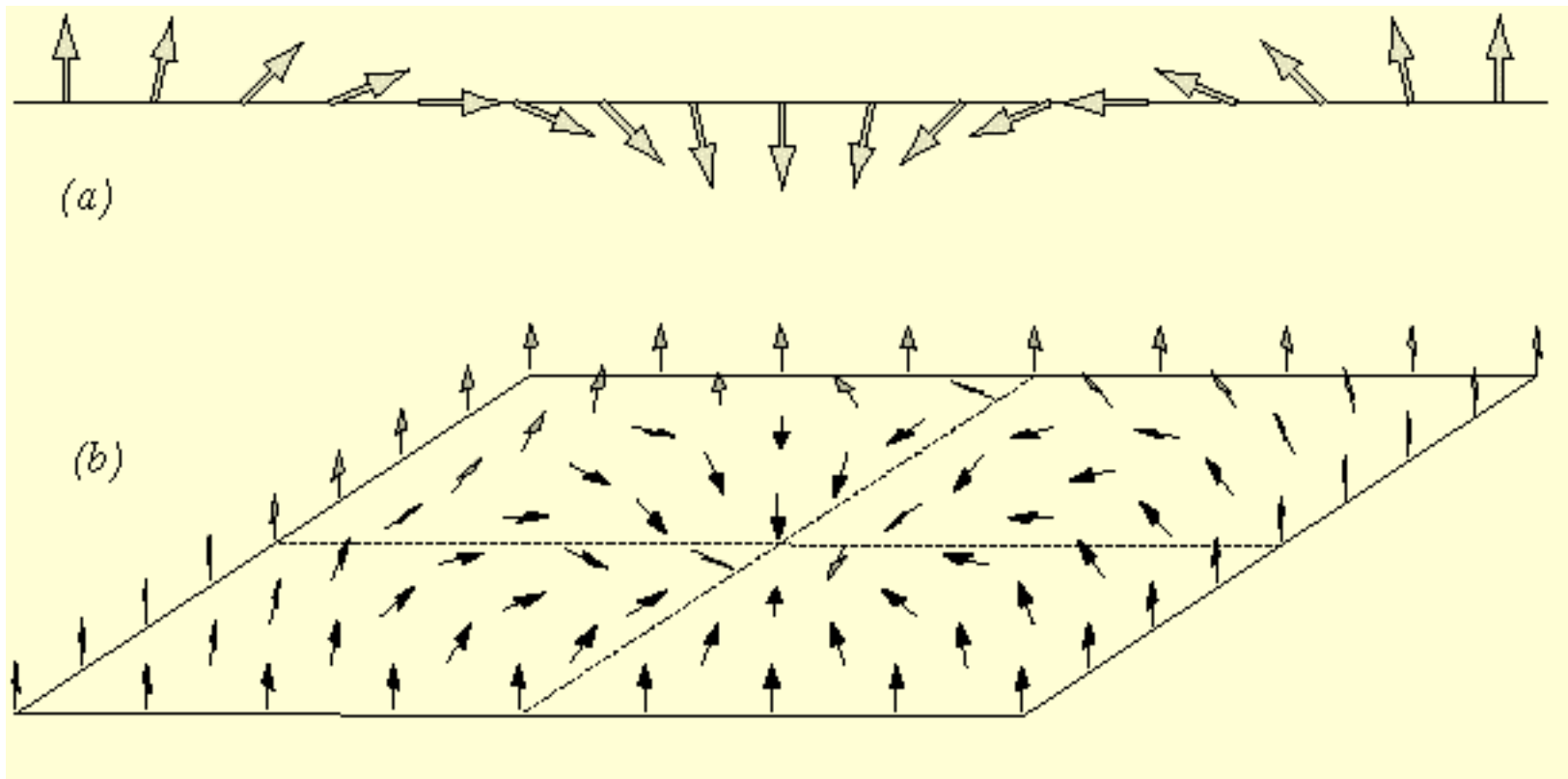


Cosmic strings are associated with models in which the set of minima are not simply-connected, that is, the vacuum manifold has 'holes' in it. The minimum energy states on the left form a circle and the string corresponds to a non-trivial winding around this.

Monopoles: These are zero-dimensional (point-like) objects which form when a spherical symmetry is broken. Monopoles are predicted to be supermassive and carry magnetic charge. The existence of monopoles is an inevitable prediction of grand unified theories (GUTs - more on this shortly); why the universe isn't filled with them is one of the puzzles of the standard cosmology.



Textures: These form when larger, more complicated symmetry groups are completely broken. Textures are delocalized topological defects which are unstable to collapse. A speculation that the largest “cold spot” in the WMAP CMB data was caused by cosmic textures was published by Cruz et al. (2007, Science 318, 1612).



Examples of delocalized texture configurations in one and two dimensions.

CMB Lensing and the WMAP Cold Spot

Sudeep Das^{1,*} and David N. Spergel^{1,2,3,†}

Cosmologists have suggested a number of intriguing hypotheses for the origin of the “WMAP cold spot”, the coldest extended region seen in the CMB sky, including a very large void and a collapsing texture. Either hypothesis predicts a distinctive CMB lensing signal. We show that the upcoming generation of high resolution CMB experiments such as ACT and SPT should be able to detect the signatures of either textures or large voids. If either signal is detected, it would have profound implications for cosmology.

Some theorists have speculated that the Cold Spot is a secondary effect, generated at some intermediate distance between us and the last scattering surface. One such model proposes that the Cold Spot may have been caused by the Rees-Sciama effect due to an underdense void of comoving radius $\sim 200h^{-1}\text{Mpc}$ and fractional density contrast $\delta \sim -0.3$ at redshift of $z < 1$ [8, 9]. Interestingly, [10] reported a detection of an underdense region with similar characteristics in the distribution of extragalactic radio sources in the NRAO VLA Sky Survey in the direction of the Cold Spot, a claim which has recently been challenged [11]. An alternative view [12] proposes that the spot was caused by the interaction of the CMB photons with a cosmic texture, a type of topological defect that can give rise to hot and cold spots in the CMB [13]. Bayesian analysis by [14] claims that the texture hypothesis seems to be favored over the void explanation, mainly because such large voids as required by the latter is highly unlikely to form in a CDM structure formation scenario.

[8] K. T. Inoue and J. Silk, ApJ 648, 23 (2006), arXiv:astro-ph/0602478.

[9] K. T. Inoue and J. Silk, ApJ 664, 650 (2007), arXiv:astro-ph/0612347.

[10] L. Rudnick, S. Brown, and L. R. Williams, ApJ 671, 40 (2007).

[11] K. M. Smith and D. Huterer, ArXiv:0805.2751.

[12] M. Cruz, N. Turok, P. Vielva, E. Martínez-González, and M. Hobson, Science 318, 1612 (2007).

[13] N. Turok and D. Spergel, Physical Review Letters 64, 2736 (1990).

[14] M. Cruz, E. Martínez-González, P. Vielva, J. M. Diego, M. Hobson, and N. Turok, ArXiv:0804.2904.

A Cosmic Microwave Background Feature Consistent with a Cosmic Texture

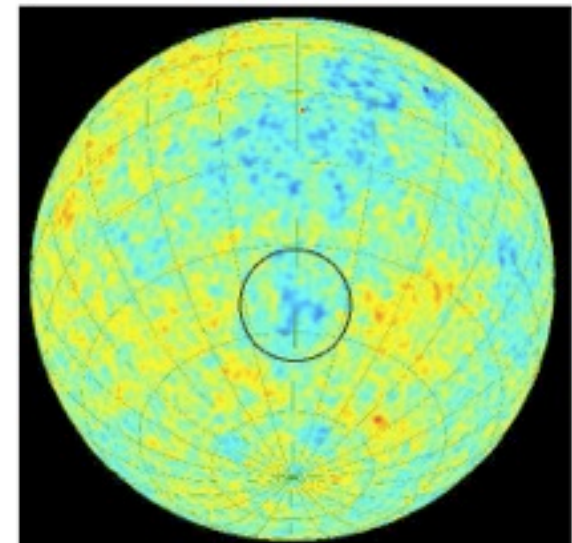
M. Cruz,^{1,2*} N. Turok,³ P. Vielva,¹ E. Martínez-González,¹ M. Hobson⁴ SCIENCE VOL 318 7 DECEMBER 2007

The Cosmic Microwave Background provides our most ancient image of the universe and our best tool for studying its early evolution. Theories of high-energy physics predict the formation of various types of topological defects in the very early universe, including cosmic texture, which would generate hot and cold spots in the Cosmic Microwave Background. We show through a Bayesian statistical analysis that the most prominent 5°-radius cold spot observed in all-sky images, which is otherwise hard to explain, is compatible with having been caused by a texture. From this model, we constrain the fundamental symmetry-breaking energy scale to be $\phi_0 \approx 8.7 \times 10^{15}$ gigaelectron volts. If confirmed, this detection of a cosmic defect will probe physics at energies exceeding any conceivable terrestrial experiment.

The Axis of Evil revisited

Kate Land, Joao Magueijo, 2007 MNRAS, 378, 153

Abstract: In light of the three-year data release from WMAP we re-examine the evidence for the "Axis of Evil" (AOE) [anomalous alignment of CMB multipoles in the direction $l \approx -100$, $b = 60$]. We discover that previous statistics are not robust with respect to the data-sets available and different treatments of the galactic plane. We identify the cause of the instability and implement an alternative "model selection" approach. A comparison to Gaussian isotropic simulations find the features significant at the 94-98% level, depending on the particular AOE model. The Bayesian evidence finds lower significance, ranging from "substantial" to no evidence for the most general AOE model.



The zone of the CS has been placed at the center of the black circle.

The CMB cold spot: texture, cluster or void?

M. Cruz,^{1*} E. Martínez-González,¹ P. Vielva,¹ J. M. Diego,¹ M. Hobson,² N. Turok³

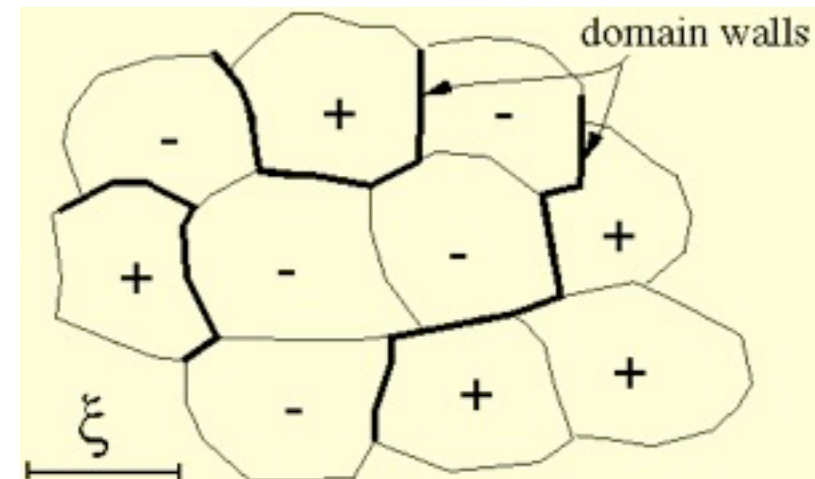
The non-Gaussian cold spot found in the WMAP data has created controversy about its origin. Here we calculate the Bayesian posterior probability ratios for three different models that could explain the cold spot. A recent work claimed that the Spot could be caused by a cosmic texture, while other papers suggest that it could be due to the gravitational effect produced by an anomalously large void. Also the Sunyaev-Zeldovich effect caused by a cluster is taken into account as a possible origin. We perform a template fitting on a 20° radius patch centered at Galactic coordinates ($b = -57^\circ, l = 209^\circ$) and calculate the posterior probability ratios for the void and Sunyaev-Zeldovich models, comparing the results to those obtained with texture. Taking realistic priors for the parameters, the texture interpretation is favored, while the void and Sunyaev-Zeldovich hypotheses are discarded. The temperature decrement produced by voids or clusters is negligible considering realistic values for the parameters.

Why do cosmic topological defects form?

If cosmic strings or other topological defects *can* form at a cosmological phase transition then they *will* form. This as first pointed out by Kibble and, in a cosmological context, the defect formation process is known as the **Kibble mechanism**.

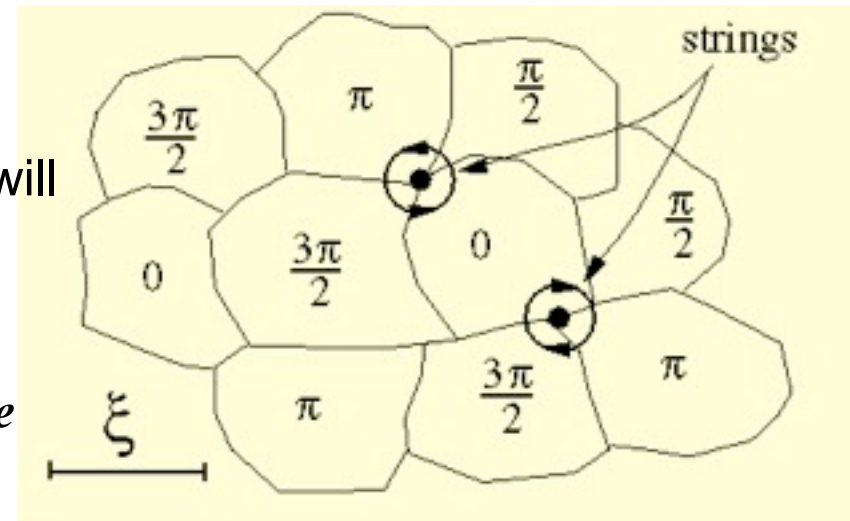
The simple fact is that causal effects in the early universe can only propagate (as at any time) as the speed of light c . This means that at a time t , regions of the universe separated by more than a distance $d=ct$ can know nothing about each other. In a symmetry breaking phase transition, different regions of the universe will choose to fall into different minima in the set of possible states (this set is known to mathematicians as the vacuum manifold). Topological defects are precisely the “boundaries” between these regions with different choices of minima, and their formation is therefore an inevitable consequence of the fact that different regions cannot agree on their choices.

For example, in a theory with two minima, plus $+$ and minus $-$, then neighboring regions separated by more than ct will tend to fall randomly into the different states (as shown below). Interpolating between these different minima will be a **domain wall**.



Cosmic strings will arise in slightly more complicated theories in which the minimum energy states possess 'holes'. The strings will simply correspond to non-trivial 'windings' around these holes (as illustrated at right).

The Kibble mechanism for the formation of cosmic strings.



Topological defects can provide a unique link to the physics of the very early universe. Furthermore, they can crucially affect the evolution of the universe, so their study is an unavoidable part of any serious attempt to understand the early universe. The **cosmological consequences** vary with the type of defect considered. Domain walls and monopoles are cosmologically catastrophic. Any cosmological model in which they form will evolve in a way that contradicts the basic observational facts that we know about the universe. Such models must therefore be ruled out! Cosmic inflation was invented to solve this problem.

Cosmic strings and textures are (possibly) much more benign. Among other things, they were until recently thought to be a possible source of the fluctuations that led to the formation of the large-scale structures we observe today, as well as the anisotropies in the Cosmic Microwave Background. However, the CMB anisotropies have turned out not to agree with the predictions of this theory.

Cosmic String Dynamics and Evolution

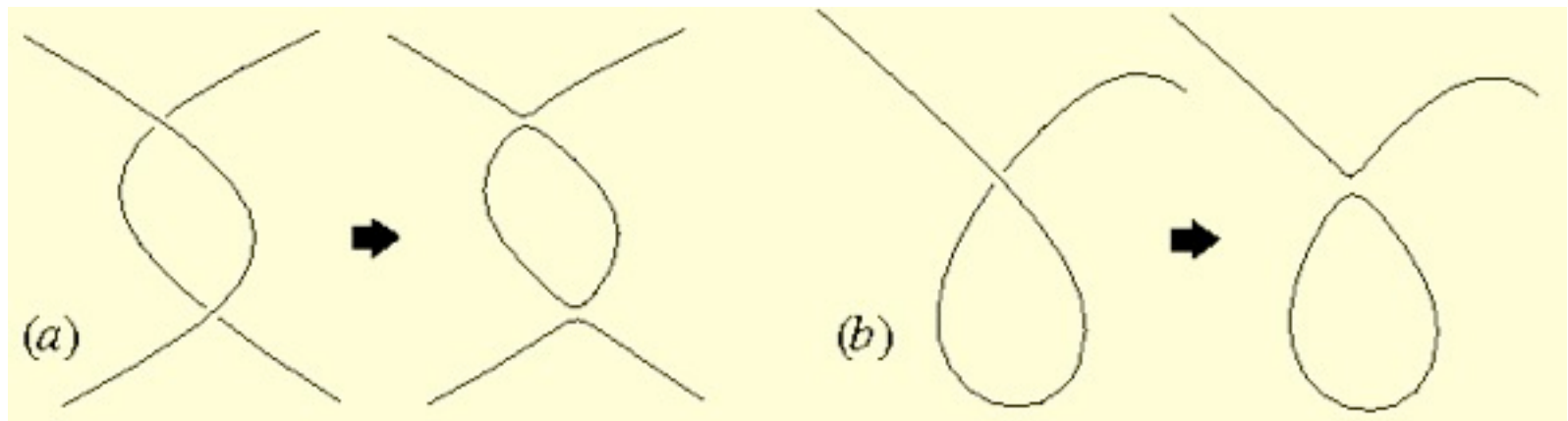
The evolution of cosmic string network is the relatively complicated result of only three rather simple and fundamental processes: cosmological expansion, intercommuting & loop production, and radiation.

Cosmological expansion

The overall expansion of the universe will 'stretch' the strings, just like any other object that is not gravitationally bound. You can easily understand this through the well-known analogy of the expanding balloon. If you draw a line on the surface of the balloon and then blow it up, you will see that the length of your 'string' will grow at the same rate as the radius of the balloon.

Intercommuting & loop production

Whenever two long strings cross each other, they exchange ends, or 'intercommute' (case (a) in the figure below). In particular, a long string can intercommute with itself, in which case a loop will be produced (this is case (b) below).



Radiation from strings

Both long cosmic strings and small loops will emit radiation. In most cosmological scenarios this will be **gravitational radiation**, but electromagnetic radiation or axions can also be emitted in some cases (for some specific phase transitions).

The effect of radiation is much more dramatic for loops, since they lose all their energy this way, and eventually disappear. Here you can see what happens in the case of two interlocked loops.

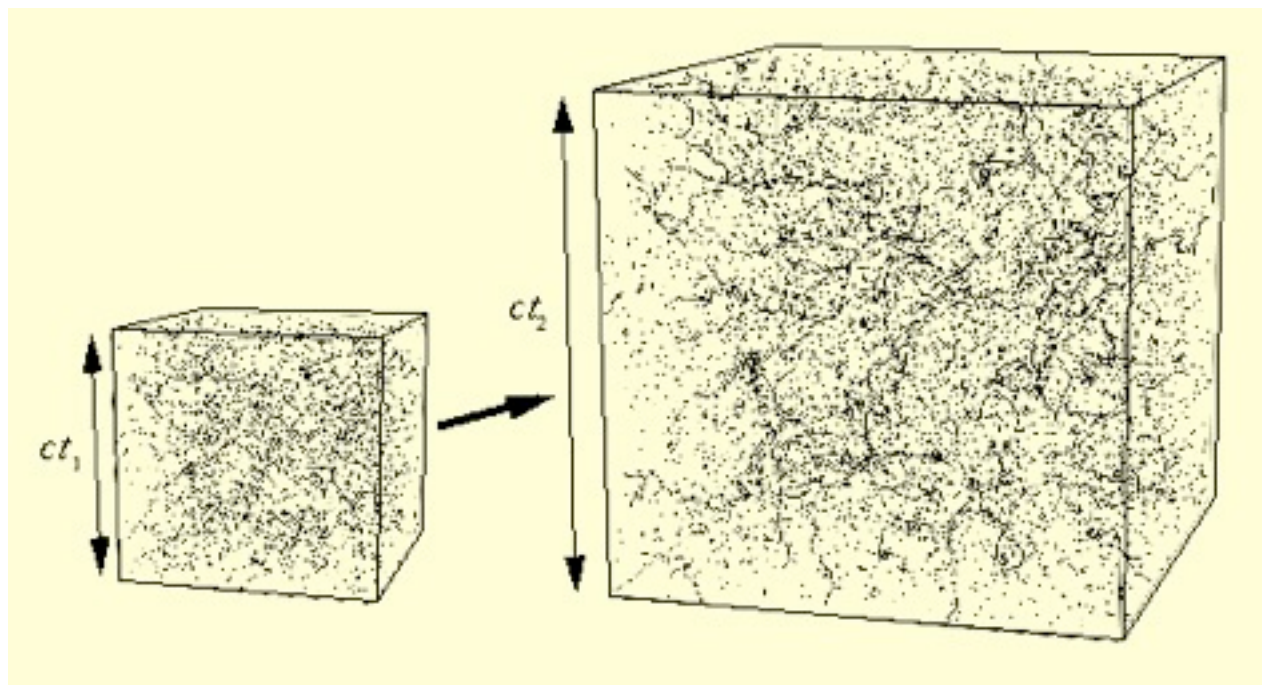
This configuration is unlikely to happen in a cosmological setting, but it is nevertheless quite enlightening. Notice the succession of complicated dynamic processes before the loop finally disappears!

After formation, an initially high density string network begins to chop itself up by producing small loops. These loops oscillate rapidly (relativistically) and decay away into gravitational waves.

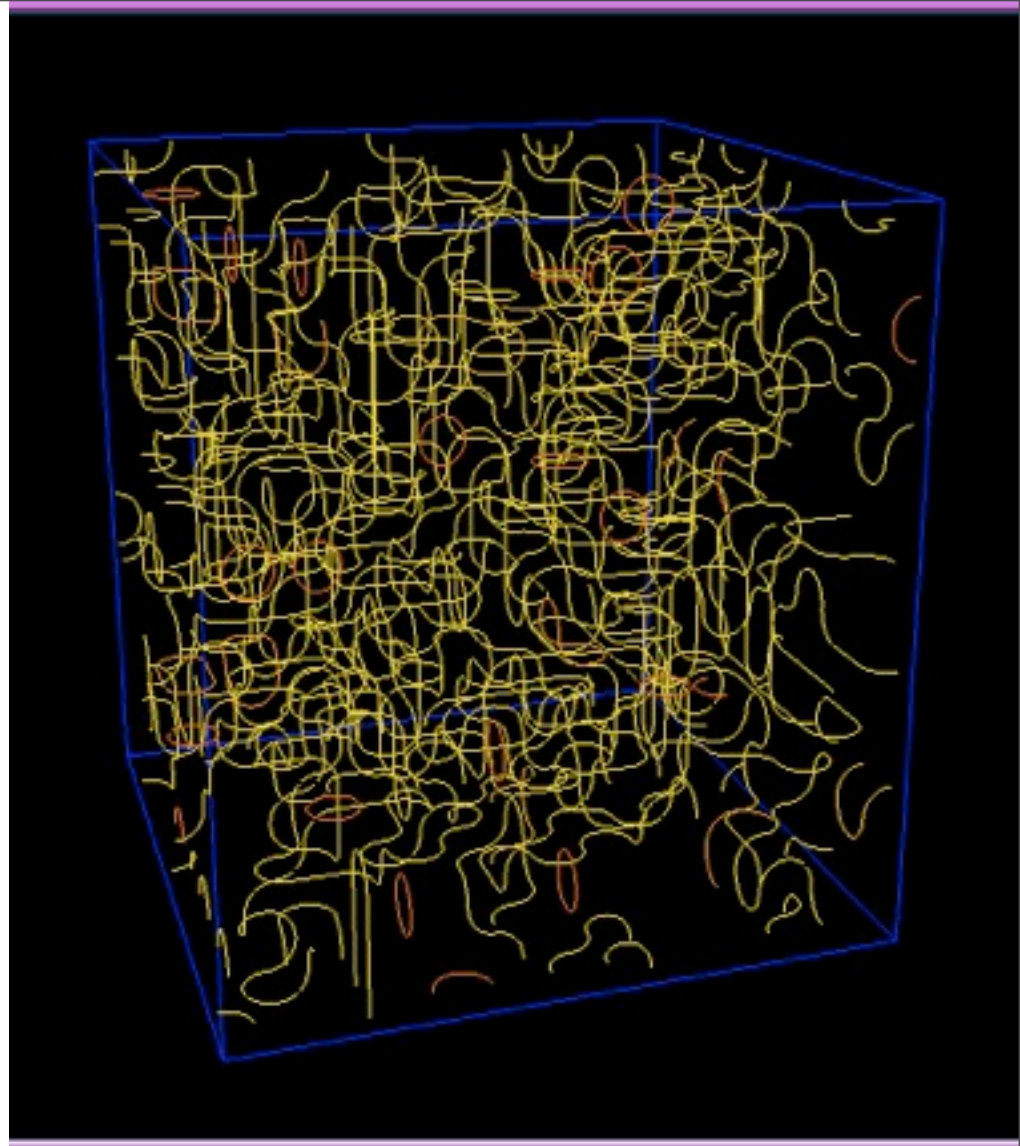


The net result is that the strings become more and more dilute with time as the universe expands. From an enormous density at formation, mathematical modelling suggests that today there would only be about 10 long strings stretching across the observed universe, together with about a thousand small loops!

In fact the network dynamics is such that the string density will eventually stabilize at an exactly constant level relative to the rest of the radiation and matter energy density in the universe. Thus the string evolution is described as '**scaling**' or scale-invariant, that is, the properties of the network look the same at any particular time t if they are scaled (or multiplied) by the change in the time. This is schematically represented below:

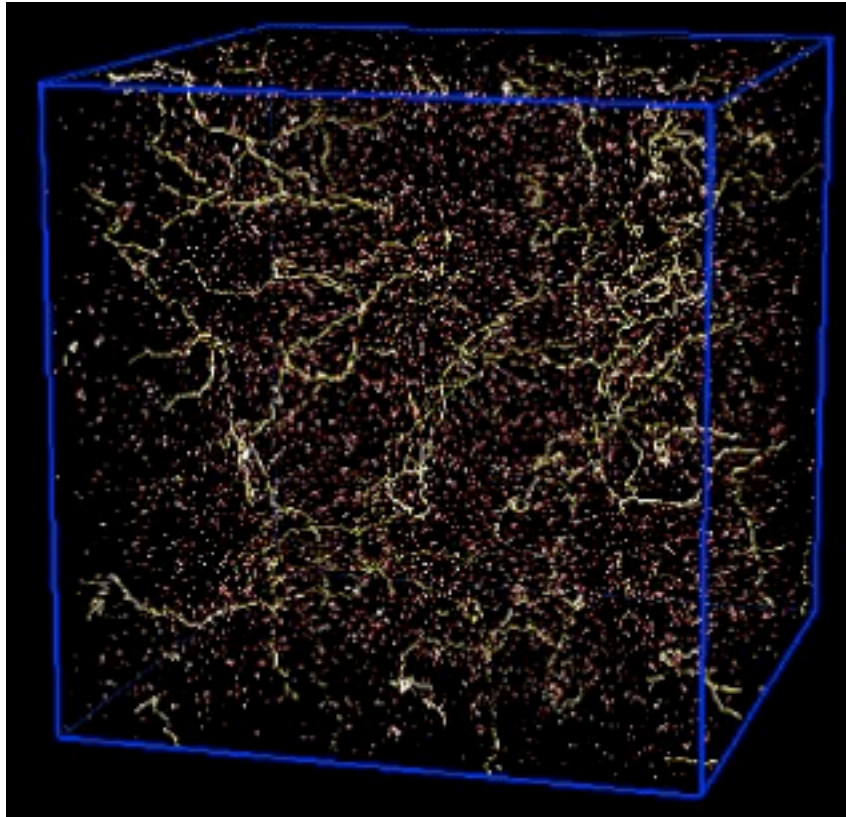


Because strings are extremely complex non-linear objects, the only rigorous way to study their evolution and cosmological consequences is to simulate in on the computer. One of the aims of performing numerical simulations of the evolution of cosmic string networks is to subsequently use the resulting information as an input to build (relatively) simpler semianalytic models that reproduce (in an averaged sense) the crucial properties of these objects. One starts by generating an initial “box of stings” containing a configuration of strings such as one would expect to find after a phase transition in the early universe. Then one evolves this initial box, by using the laws of motion of the strings.

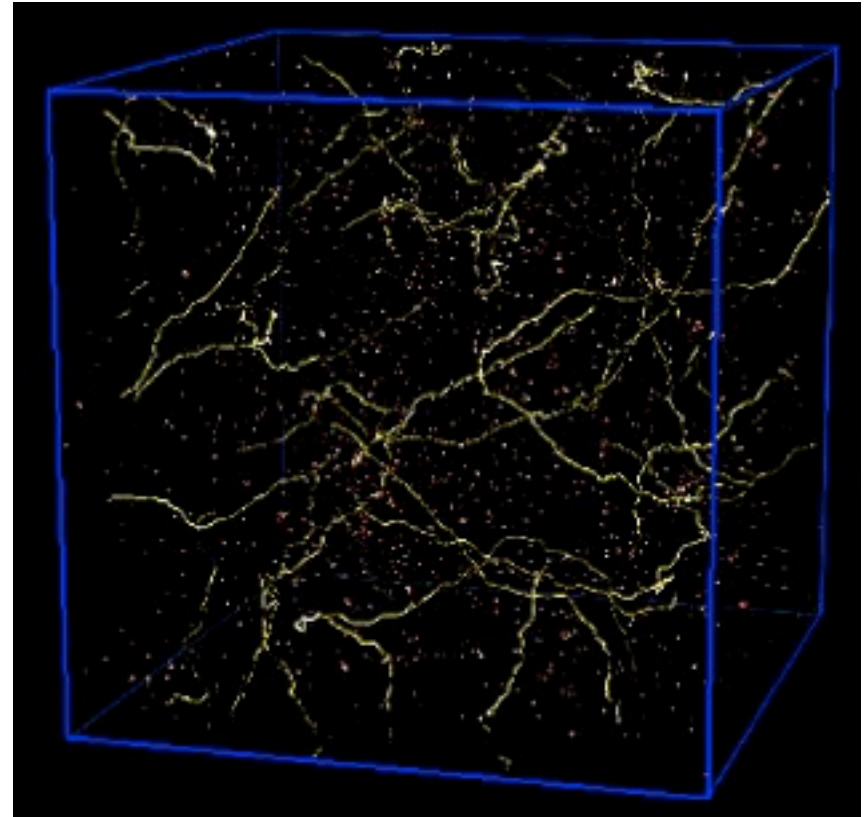


In this and all other pictures and movies below long strings are shown in yellow, while small loops have a color code going from yellow to red according to their size (red loops being the smallest).

Why do the two boxes below look different? Because the rate at which the universe is expanding is different.

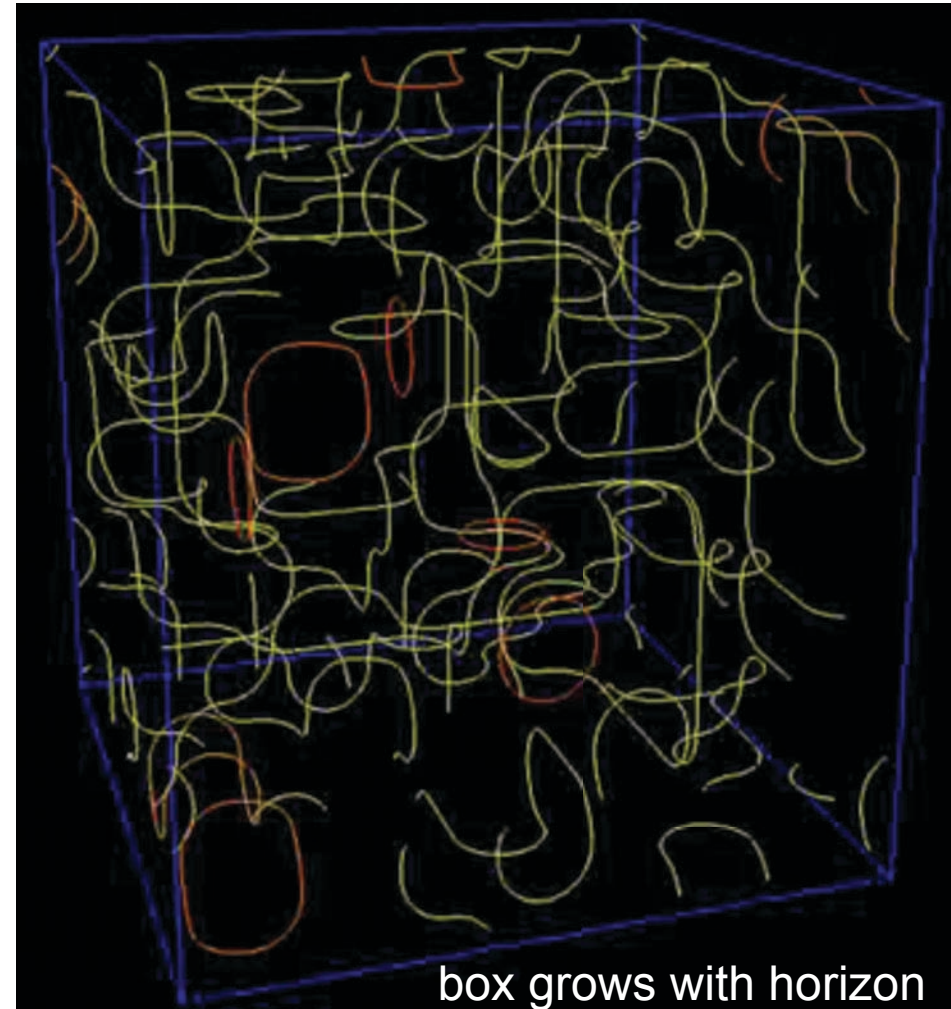
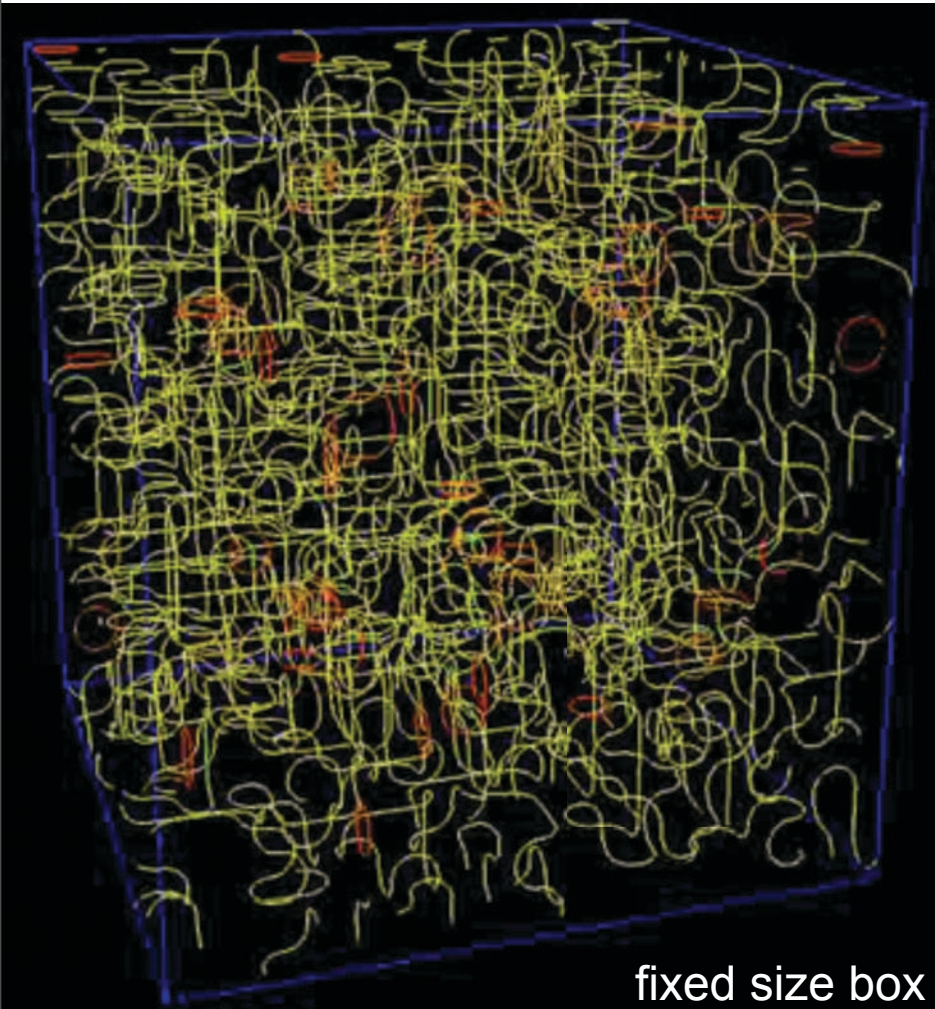


Snapshot of a string network in the **radiation era**. Note the high density of small loops and the 'wiggleness' of the long strings in the network. The box size is about $2c t$. (B. Allen & E. P. Shellard)



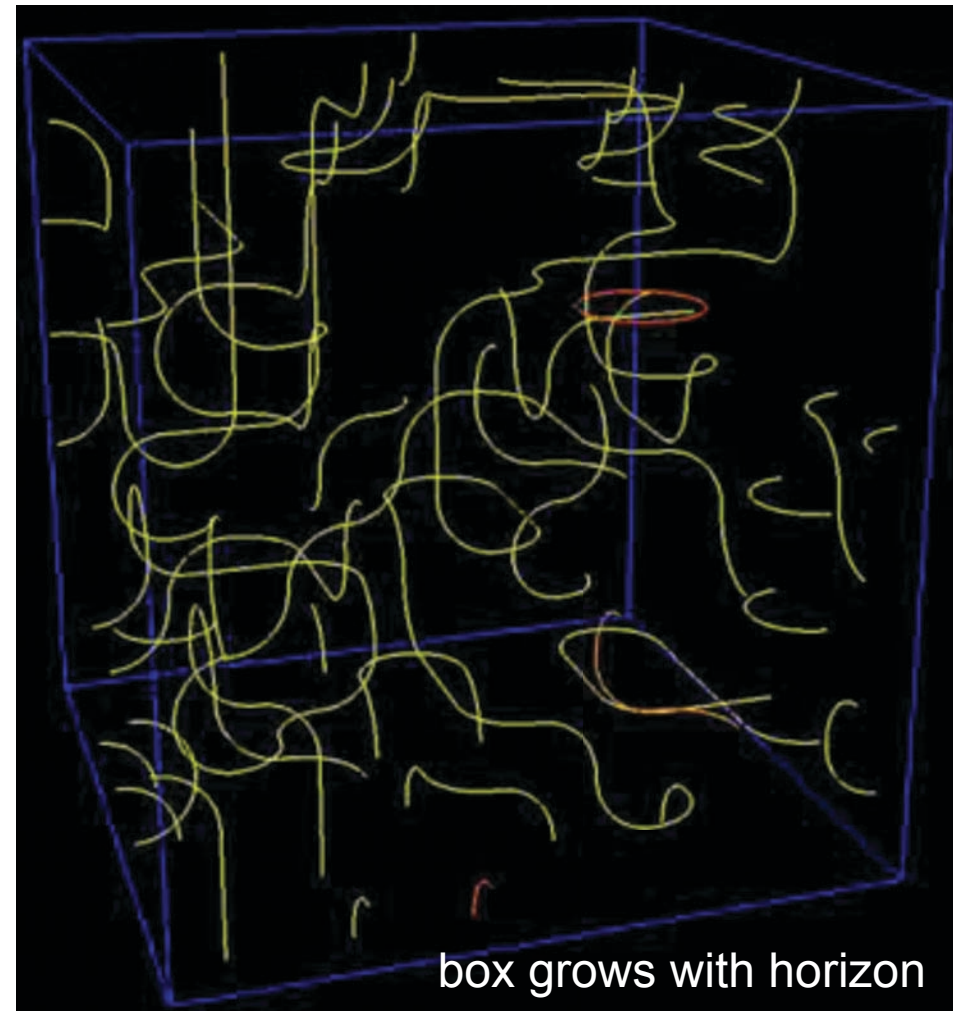
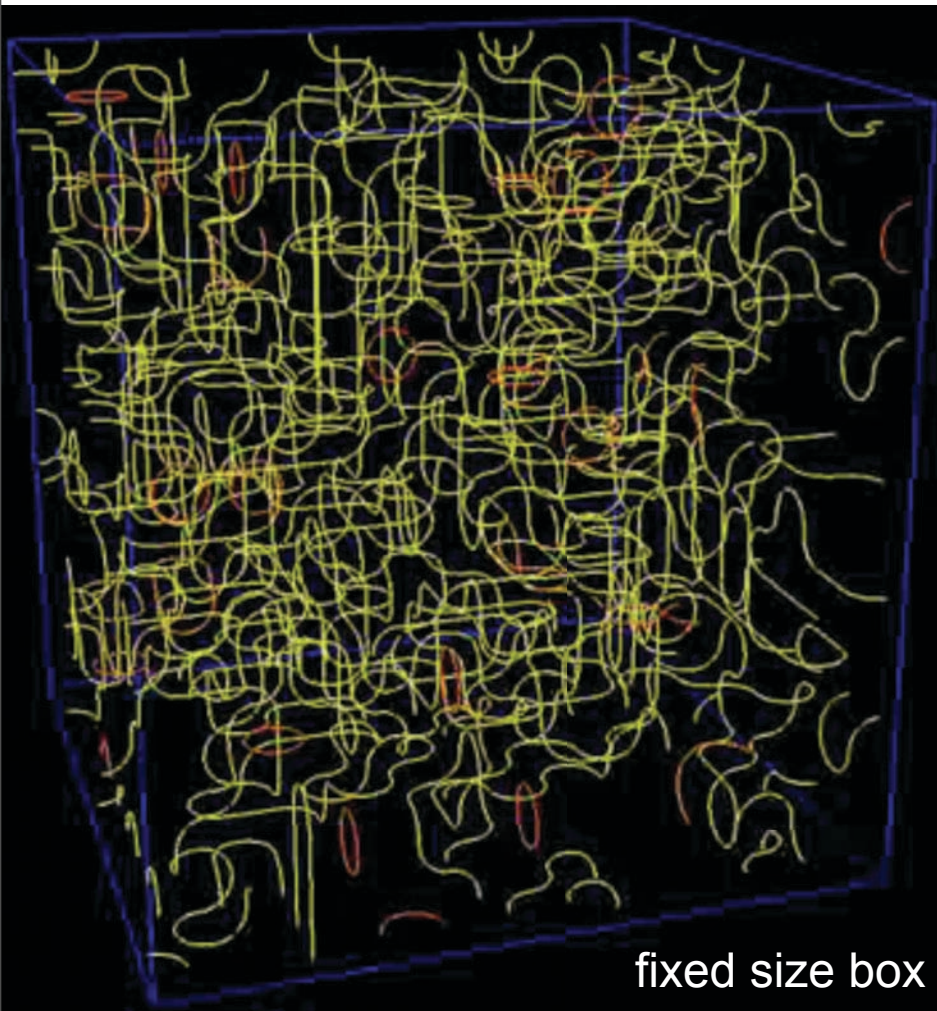
Snapshot of a string network in the **matter era**. Compare with the radiation case at left. Notice the lower density of both long strings and loops, as well as the lower 'wiggleness' of the former. The box size is again about $2c t$.

Two movies of the evolution of a cosmic string network in the **radiation era**. In the movie on the left the box has a fixed size (so you will see fewer and fewer strings as it evolves), while in the one on the right it grows as the comoving horizon. (C. Martins & E. P. Shellard)



Notice that the number of long strings in the box that grows with the horizon remains roughly constant, in agreement with the scaling hypothesis. This is because the additional length in strings is quickly converted into small loops.

Two movies of the evolution of a cosmic string network in the **matter era**. In the movie on the left the box has a fixed size (so you will see fewer and fewer strings as it evolves), while in the one on the right it grows as the comoving horizon. (C. Martins & E. P. Shellard)



Notice that the number of long strings in the box that grows with the horizon again remains roughly constant, in agreement with the scaling hypothesis. This is because the additional length in strings is quickly converted into

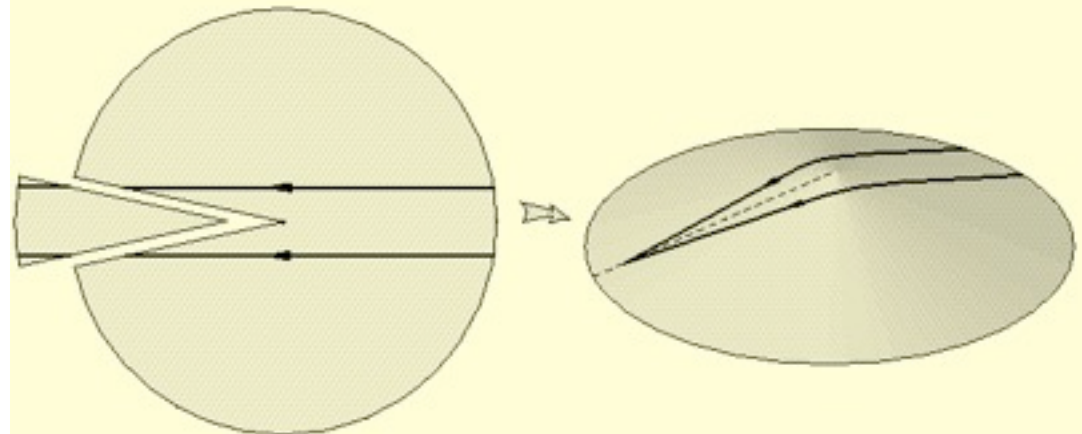
When strings evolve, scaling from smaller scales to larger ones, they create perturbations in the matter energy density of the universe. Because of their tension, cosmic strings pull straight as they come inside the horizon. Although there is no gravitational force from a static string, such moving cosmic strings produce wakes toward which matter falls, thus serving as seeds for structure formation. For a static string along the z axis of mass μ per unit length, the energy momentum tensor is

$$T^{\mu\nu} = \mu \text{diag}(1, 0, 0, -1)\delta(x)\delta(y)$$

and the metric is

$$ds^2 = dt^2 - dz^2 - dR^2 - (1 - 4G\mu)^2 R^2 d\varphi^2$$

$G\mu \approx (M_{\text{GUT}}/M_{\text{Pl}})^2 \approx 10^{-6}$ is just the magnitude needed for GUT string structure formation. There is an **angular defect** of $8\pi G\mu = 5.18''$ ($10^6 G\mu$). This implies that the geodesic path of light is curved towards a string when light passes by it. Two copies of a galaxy near a cosmic string will appear to observers on the other side of the string.



Cosmic Strings Summary

Cosmic strings arise in spontaneously broken (SB) gauge theories

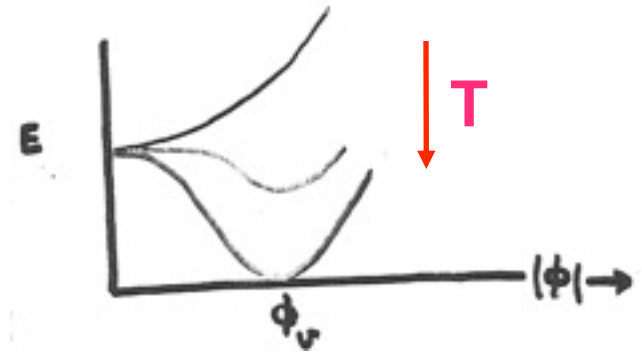
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |\partial_\mu \phi|^2 - \lambda (|\phi|^2 - \phi_v^2)^2$$

as a consequence of causality in the expanding universe.

As the temperature T falls, a complex scalar field ϕ gets a nonzero expectation value

$$\phi(x) = \phi_v e^{i\theta(x)}$$

The phase θ will inevitably be different in regions separated by distances greater than the horizon size when the SB phase transition occurred. If θ runs over $0 \rightarrow 2\pi$ as x goes around a loop in space, the loop encloses a string.



By 2000, it was clear that **cosmic defects are not the main source of the CMB anisotropies.**

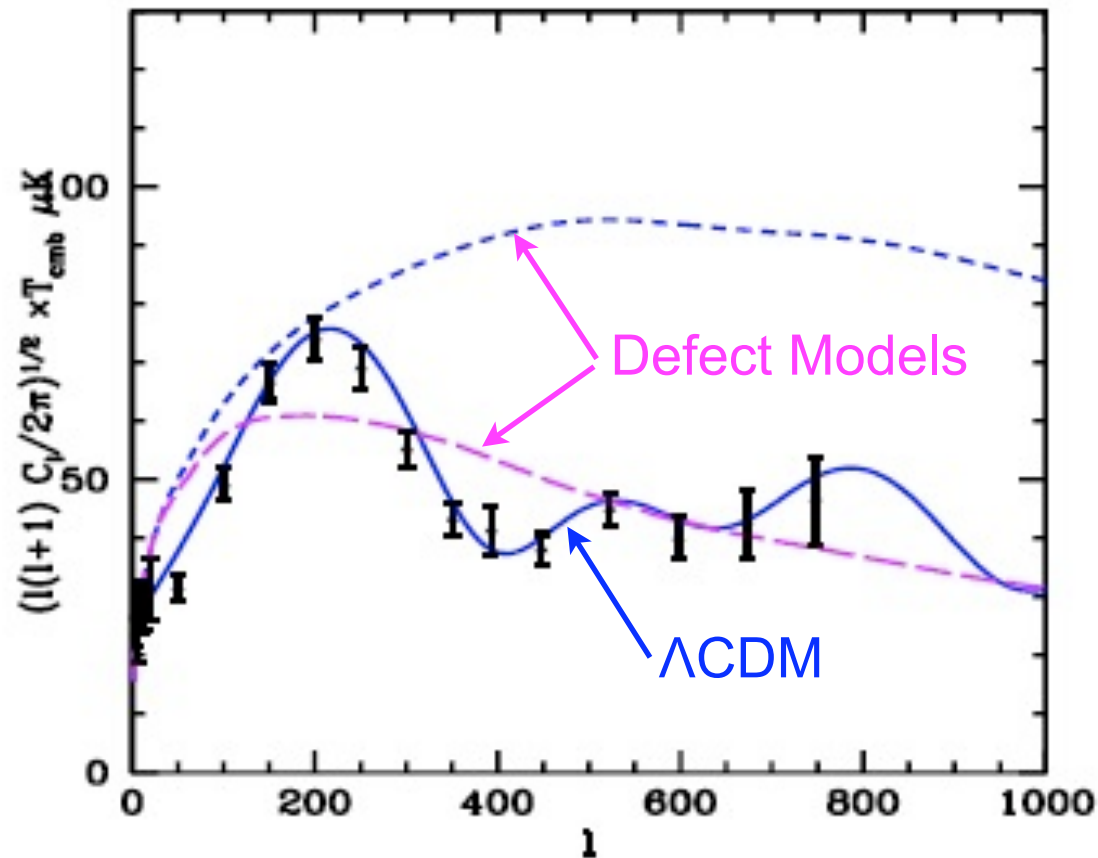


Figure 3: Current data (as compiled by Knox[22]) with two defect models (dashed) and an inflation-based model (solid). The upper defect model has a standard ionization history and the lower model has an ionization history specifically designed to produce a sharper, shifted peak.

Andreas Albrecht, Defect models of cosmic structure in light of the new CMB data, XXXVth Rencontres de Moriond "Energy Densities in the Universe" (2000).

Fitting CMB data with cosmic strings and inflation, by Neil Bevis, Mark Hindmarsh, Martin Kunz, and Jon Urrestilla 2008 PRL100.021301

The inflationary paradigm is successful in providing a match to measurements of the cosmic microwave background (CMB) radiation, and it appears that **any successful theory of high energy physics must be able to incorporate inflation**. While ad hoc single-field inflation can provide a match to the data, **more theoretically motivated models commonly predict the existence of cosmic strings**. These strings are prevalent in supersymmetric inflation models and occur frequently in grand-unified theories (GUTs). String theory can also yield strings of cosmic extent. Hence the observational consequences of cosmic strings are important, including their sourcing of additional anisotropies in the CMB radiation. In this letter we present a multi-parameter fit to CMB data for models incorporating cosmic strings. It is the first such analysis to use simulations of a fully dynamical network of local cosmic strings, and the first to incorporate their microphysics with a field theory. It yields conclusions which differ in significant detail from previous analyses based upon simplified models: **we find that the CMB data moderately favor a 10% contribution from strings to the temperature power spectrum** measured at multipole $\ell = 10$ with a corresponding spectral index of primordial scalar perturbations $n_s \simeq 1$. There are also important implications for models of inflation with blue power spectra ($n_s > 1$). These are disfavored by CMB data under the concordance model (power-law Λ CDM which gives $n_s = 0.951 \pm 0.015 \text{ } ^{-0.019}$) and previous work seemed to show that this remains largely the case even if cosmic strings are allowed. However with our more complete CMB calculations, we find that the CMB puts no pressure on such models if they produce cosmic strings. Our conclusions are slightly modified when additional non-CMB data are included, with the preference for strings then reduced.

GUT Monopoles

A simple SO(3) GUT illustrates how nonsingular monopoles arise. The Lagrangian is

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{8}\lambda(\Phi^a\Phi^a - \sigma^2)^2, \\ F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - e\epsilon_{abc}A_\mu^b A_\nu^c, \\ D_\mu\Phi^a &= \partial_\mu\Phi^a - e\epsilon_{abc}A_\mu^b\Phi^c.\end{aligned}$$

The masses of the resulting charged vector and Higgs bosons after spontaneous symmetry breaking are

$$\begin{aligned}M_V^2 &= e^2\sigma^2, \\ M_S^2 &= \lambda\sigma^2.\end{aligned}$$

If the Higgs field Φ^a happens to rotate about a sphere in SO(3) space as one moves around a sphere about any particular point in \mathbf{x} -space, then it must vanish at the particular point. Remarkably, if we identify the massless vector field as the photon, this configuration corresponds to a nonsingular magnetic monopole, as was independently discovered by 'tHooft and Polyakov. The monopole has magnetic charge twice the minimum Dirac value, $g = 2\pi/e = (4\pi/e^2)(e/2) \approx 67.5 e$.

The singular magnetic field is cut off at scale σ , and as a result the GUT monopole has mass $M_{\text{monopole}} \approx M_V/\alpha \approx M_{\text{GUT}}/\alpha \approx 10^{18} \text{ GeV}$.

The first accurate calculation of the mass of the 't Hooft - Polyakov non-singular monopole was Bais & Primack (Phys. Rev. D13:819,1976).

GUT Monopole Problem

The Kibble mechanism produces \sim one GUT monopole per horizon volume when the GUT phase transition occurs. These GUT monopoles have a number density over entropy

$$n_M/s \sim 10^2 (T_{\text{GUT}}/M_{\text{Pl}})^3 \sim 10^{-13}$$

(compared to $n_B/s \sim 10^{-9}$ for baryons) Their annihilation is inefficient since they are so massive, and as a result they are about as abundant as gold atoms but 10^{16} times more massive, so they “overclose” the universe. This catastrophe must be avoided! **This was Alan Guth's initial motivation for inventing cosmic inflation.**

Motivations for Inflation

	PROBLEM SOLVED
Horizon	Homogeneity, Isotropy, Uniform T
Flatness/Age	Expansion and gravity balance
“Dragons”	Monopoles, domain walls, ... banished
Structure	Small fluctuations to evolve into galaxies, clusters, voids

Cosmological constant $\Lambda > 0 \Rightarrow$ space repels space, so the more space the more repulsion, \Rightarrow de Sitter exponential expansion $a \propto e^{\sqrt{\Lambda}t}$.

Inflation is exponentially accelerating expansion caused by effective cosmological constant (“false vacuum” energy) associated with hypothetical scalar field (“inflaton”).

	FORCES OF NATURE	Spin
Known	Gravity	2
	Strong, weak, and electromagnetic	1
Goal of LHC	Mass (Higgs Boson)	0
Early universe	Inflation (Inflaton)	0

Inflation lasting only $\sim 10^{-32}$ s suffices to solve all the problems listed above. Universe must then convert to ordinary expansion through conversion of false to true vacuum (“re-”heating).

Inflation Basics

The basic idea of inflation is that before the universe entered the present adiabatically expanding Friedmann era, it underwent a period of de Sitter exponential expansion of the scale factor, termed *inflation* (Guth 1981). Actually, inflation is never precisely de Sitter, and any superluminal (faster-than-light) expansion is now called inflation. Inflation was originally invented to solve the problem of too many GUT monopoles, which, as mentioned in the previous section, would otherwise be disastrous for cosmology.

The de Sitter cosmology corresponds to the solution of Friedmann's equation in an empty universe (i.e., with $\rho = 0$) with vanishing curvature ($k = 0$) and positive cosmological constant ($\Lambda > 0$). The solution is $a = a_0 e^{Ht}$, with constant Hubble parameter $H = (\Lambda/3)^{1/2}$. There are analogous solutions for $k = +1$ and $k = -1$ with $a \propto \cosh Ht$ and $a \propto \sinh Ht$ respectively. The scale factor expands exponentially because the positive cosmological constant corresponds effectively to a negative pressure. de Sitter space is discussed in textbooks on general relativity (for example, Rindler 1977, Hawking & Ellis 1973) mainly for its geometrical interest. Until cosmological inflation was considered, the chief significance of the de Sitter solution in cosmology was that it is a limit to which all indefinitely expanding models with $\Lambda > 0$ must tend, since as $a \rightarrow \infty$, the cosmological constant term ultimately dominates the right hand side of the Friedmann equation.

As Guth (1981) emphasized, the de Sitter solution might also have been important in the very early universe because the vacuum energy that plays such an important role in spontaneously broken gauge theories also acts as an effective cosmological constant. A period of de Sitter inflation preceding ordinary radiation-dominated Friedmann expansion could explain several features of the observed universe that otherwise appear to require very special initial conditions: the horizon, flatness/age, monopole, and structure formation problems. (See Table 1.6.)

Let us illustrate how inflation can help with the horizon problem. At recombination ($p^+ + e^- \rightarrow H$), which occurs at $a/a_0 \approx 10^{-3}$, the mass encompassed by the horizon was $M_H \approx 10^{18} M_\odot$, compared to $M_{H,0} \approx 10^{22} M_\odot$ today. Equivalently, the angular size today of the causally connected regions at recombination is only $\Delta\theta \sim 3^\circ$. Yet the fluctuation in temperature of the cosmic background radiation from different regions is very small: $\Delta T/T \sim 10^{-5}$. How could regions far out of causal contact have come to temperatures that are so precisely equal? This is the “horizon problem”. With inflation, it is no problem because the entire observable universe initially lay inside a single causally connected region that subsequently inflated to a gigantic scale. Similarly, inflation exponentially dilutes any preceding density of monopoles or other unwanted relics (a modern version of the “dragons” that decorated the unexplored borders of old maps).

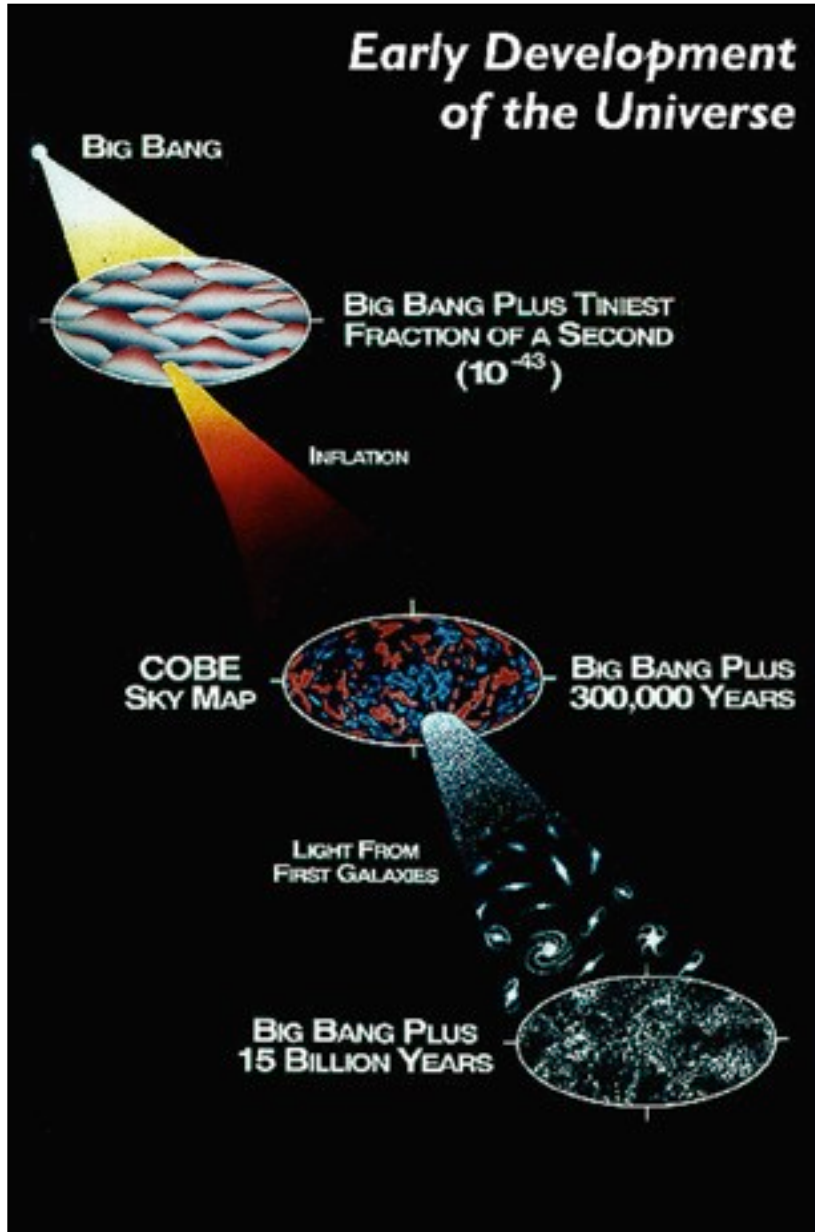
In the first inflationary models, the dynamics of the very early universe was typically controlled by the self-energy of the Higgs field associated with the breaking of a Grand Unified Theory (GUT) into the standard 3-2-1 model: $GUT \rightarrow SU(3)_{color} \otimes [SU(2) \otimes U(1)]_{electroweak}$. This occurs when the cosmological temperature drops to the unification scale $T_{GUT} \sim 10^{14}$ GeV at about 10^{-35} s after the Big Bang. Guth (1981) initially considered a scheme in which inflation occurs while the universe is trapped in an unstable state (with the GUT unbroken) on the wrong side of a maximum in the Higgs potential. This turns out not to work: the transition from a de Sitter to a Friedmann universe never finishes (Guth & Weinberg 1981). The solution in the “new inflation” scheme (Linde 1982; Albrecht and Steinhardt 1982) is for inflation to occur *after* barrier penetration (if any). It is necessary that the potential of the scalar field controlling inflation (“*inflaton*”) be nearly flat (i.e., decrease very slowly with increasing inflaton field) for the inflationary period to last long enough. This nearly flat part of the potential must then be followed by a very steep minimum, in order that the energy contained in the Higgs potential be rapidly shared with the other degrees of freedom (“reheating”). A more general approach, “chaotic” inflation, has been worked out by Linde (1983, 1990) and others; this works for a wide range of inflationary potentials, including simple power laws such as $\lambda\phi^4$. However, for the amplitude of the fluctuations to be small enough for consistency with observations, it is necessary that the inflaton self-coupling be very small, for example $\lambda \sim 10^{-14}$ for the ϕ^4 model. This requirement prevents a Higgs field from being the inflaton, since Higgs fields by definition have gauge couplings to the gauge field (which are expected to be of order unity), and these would generate self-couplings of similar magnitude even if none were present.

It turns out to be necessary to inflate by a factor $\gtrsim e^{66}$ in order to solve the flatness problem, i.e., that $\Omega_0 \sim 1$. (With $H^{-1} \sim 10^{-34}$ s during the de Sitter phase, this implies that the inflationary period needs to last for only a relatively small time $\tau \gtrsim 10^{-32}$ s.) The “flatness problem” is essentially the question why the universe did not become curvature dominated long ago. Neglecting the cosmological constant on the assumption that it is unimportant after the inflationary epoch, the Friedmann equation can be written

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \frac{\pi^2}{30} g(T) T^4 - \frac{kT^2}{(aT)^2}$$

where the first term on the right hand side is the contribution of the energy density in relativistic particles and $g(T)$ is the effective number of degrees of freedom. The second term on the right hand side is the curvature term. Since $aT \approx \text{constant}$ for adiabatic expansion, it is clear that as the temperature T drops, the curvature term becomes increasingly important. The quantity $K \equiv k/(aT)^2$ is a dimensionless measure of the curvature. Today, $|K| = |\Omega - 1| H_o^2 / T_o^2 \leq 2 \times 10^{-58}$. Unless the curvature exactly vanishes, the most “natural” value for K is perhaps $K \sim 1$. Since inflation increases a by a tremendous factor $e^{H\tau}$ at essentially constant T (after reheating), it increases aT by the same tremendous factor and thereby decreases the curvature by that factor squared. Setting $e^{-2H\tau} \lesssim 2 \times 10^{-58}$ gives the needed amount of inflation: $H\tau \gtrsim 66$. This much inflation turns out to be enough to take care of the other cosmological problems mentioned above as well.

Generating the Primordial Density Fluctuations



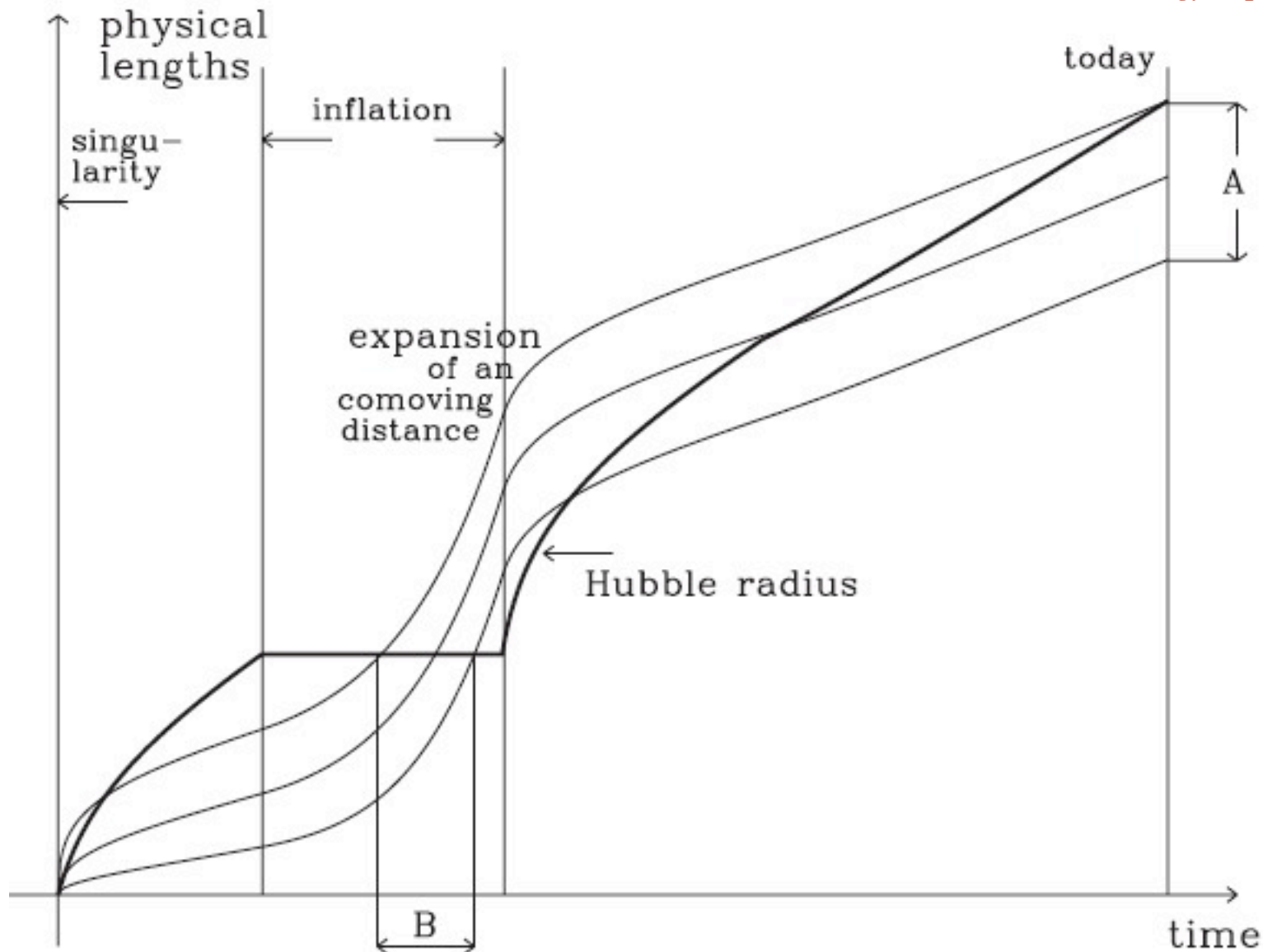
Early phase of exponential expansion
(Inflationary epoch)



Zero-point fluctuations of quantum
fields are stretched and frozen

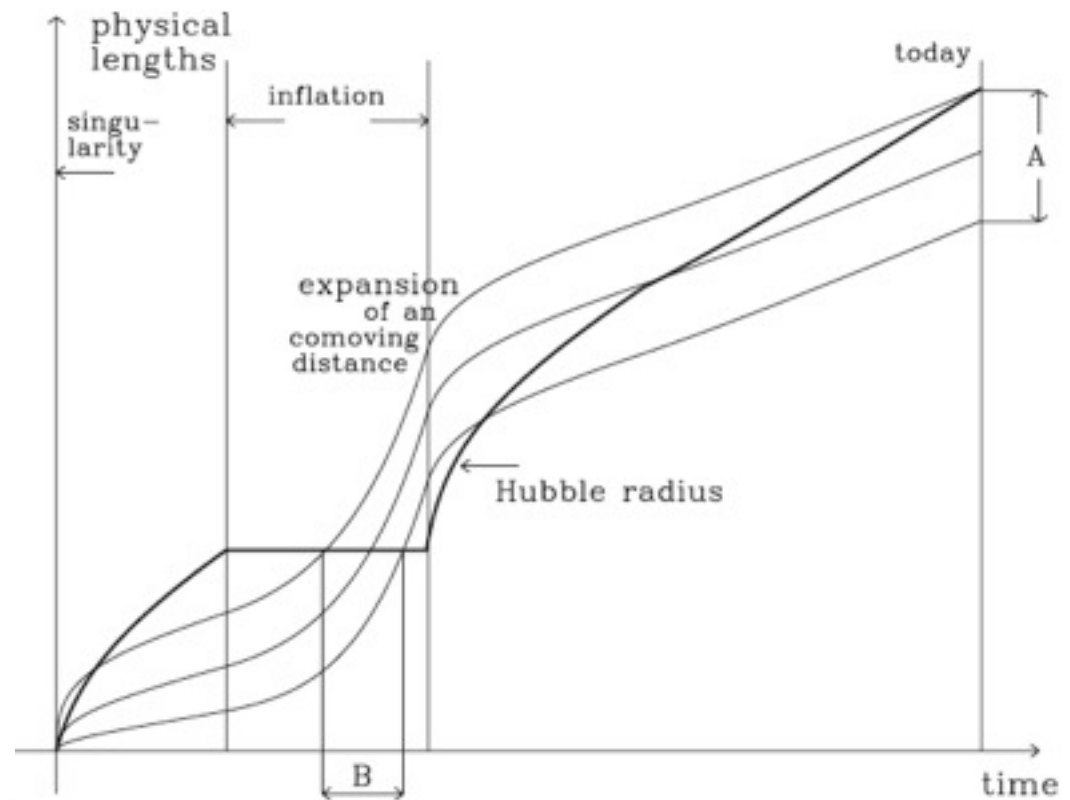


Cosmic density fluctuations are
frozen quantum fluctuations



The evolution of the scales of perturbations. The larger scales overtake the Hubble radius at an early time and fall below it again later. They measure the inflation at an earlier time than do the smaller scales, which overtake the Hubble radius during inflation later and fall below it again earlier. The region A of scales that are accessible to evaluation today corresponds to a time span B of the inflation and related values of the inflaton field; for this time span, we can tell something – at least in principle – about the potential of the inflaton.

Inflationary Fluctuations



Thus far, it has been sketched how inflation stretches, flattens, and smooths out the universe, thus greatly increasing the domain of initial conditions that could correspond to the universe that we observe today. But inflation also can explain the origin of the fluctuations necessary in the gravitational instability picture of galaxy and cluster formation. Recall that the very existence of these fluctuations is a problem in the standard Big Bang picture, since these fluctuations are much larger than the horizon at early times. How could they have arisen?

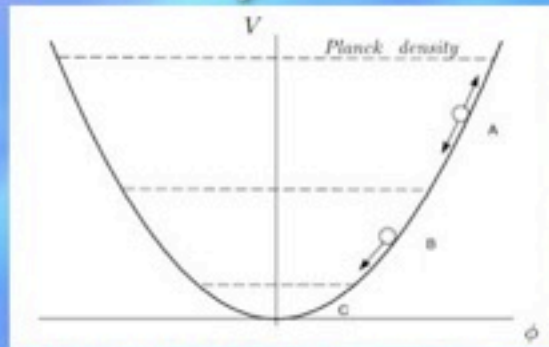
The answer in the inflationary universe scenario is that they arise from quantum fluctuations in the inflaton field ϕ whose vacuum energy drives inflation. The scalar fluctuations $\delta\phi$ during the de Sitter phase are of the order of the Hawking temperature $H/2\pi$. Because of these fluctuations, there is a time spread $\Delta t \approx \delta\phi/\dot{\phi}$ during which different regions of the same size complete the transition to the Friedmann phase. The result is that the density fluctuations when a region of a particular size re-enters the horizon are equal to (Guth & Pi 1982; see Linde 1990 for alternative approaches) $\delta_H \equiv (\delta\rho/\rho)_H \sim \Delta t/t_H = H\Delta t$. The time spread Δt can be estimated from the equation of motion of ϕ (the free Klein-Gordon equation in an expanding universe): $\ddot{\phi} + 3H\dot{\phi} = -(\partial V/\partial\phi)$. Neglecting the $\ddot{\phi}$ term, since the scalar potential V must be very flat in order for enough inflation to occur (this is called the “slow roll” approximation), $\dot{\phi} \approx -V'/(3H)$, so $\delta_H \sim H^3/V' \sim V^{3/2}/V'$. Unless there is a special feature in the potential $V(\phi)$ as ϕ rolls through the scales of importance in cosmology (producing such “designer inflation” features generally requires fine tuning — see e.g. Hodges et al. 1990), V and V' will hardly vary there and hence δ_H will be essentially constant. These are fluctuations of all the contents of the universe, so they are adiabatic fluctuations.

Thus *inflationary models typically predict a nearly constant curvature spectrum* $\delta_H = \text{constant}$ *of adiabatic fluctuations*. Some time ago Harrison (1970), Zel'dovich (1972), and others had emphasized that this is the only scale-invariant (i.e., power-law) fluctuation spectrum that avoids trouble at both large and small scales. If $\delta_H \propto M_H^{-\alpha}$, where M_H is the mass inside the horizon, then if $-\alpha$ is too large the universe will be less homogeneous on large than small scales, contrary to observation; and if α is too large, fluctuations on sufficiently small scales will enter the horizon with $\delta_H \gg 1$ and collapse to black holes (see e.g. Carr, Gilbert, & Lidsey 1995, Bullock & Primack 1996); thus $\alpha \approx 0$. The $\alpha = 0$ case has come to be known as the Zel'dovich spectrum.

Inflation predicts more: it allows the calculation of the value of the constant δ_H in terms of the properties of the scalar potential $V(\phi)$. Indeed, this proved to be embarrassing, at least initially, since the Coleman-Weinberg potential, the first potential studied in the context of the new inflation scenario, results in $\delta_H \sim 10^2$ (Guth & Pi 1982) some six orders of magnitude too large. But this does not seem to be an insurmountable difficulty; as was mentioned above, chaotic inflation works, with a sufficiently small self-coupling. Thus inflation at present appears to be a plausible solution to the problem of providing reasonable cosmological initial conditions (although it sheds no light at all on the fundamental question why the cosmological constant is so small now). Many variations of the basic idea of inflation have been worked out.

Comparing different inflationary models:

- Chaotic inflation** can start in the smallest domain of size 10^{-33} cm with total mass $\sim M_p$ (less than a milligram) and entropy $O(1)$



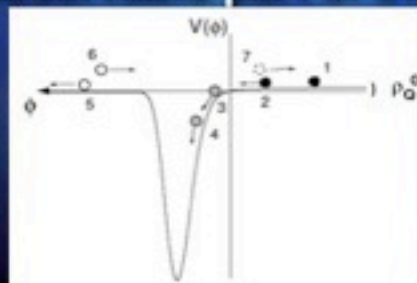
Solves flatness, mass and entropy problem

- New inflation** can start only in a domain with mass 6 orders of magnitude greater than M_p and entropy greater than 10^7



Not very good with solving flatness, mass and entropy problem

- Cyclic inflation** can occur only in the domain of size greater than the size of the observable part of the universe, with mass $> 10^{55}$ g and entropy $> 10^{87}$



Does not solve flatness, mass and entropy problem

Is the simplest chaotic inflation natural?

Andrei Linde

- Often repeated (but incorrect) argument:

$$V = V_0 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum C_n \frac{\phi^n}{M_p^n}$$

Thus one could expect that the theory is ill-defined at $\phi > M_p$

However, quantum corrections are in fact proportional to

$$\left(\frac{V}{M_p^4}\right)^n$$

and to

$$\left(\frac{m^2(\phi)}{M_p^2}\right)^n$$

These terms are harmless for sub-Planckian masses and densities, even if the scalar field itself is very large.

Many Inflation Models

following
Andrei Linde's
classification

HOW INFLATION BEGINS

Old Inflation T_{initial} high, $\phi_{\text{in}} \approx 0$ is false vacuum until phase transition
Ends by bubble creation; Reheat by bubble collisions

New Inflation Slow roll down $V(\phi)$, no phase transition

Chaotic Inflation Similar to New Inflation, but ϕ_{in} essentially arbitrary:
any region with $\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i\phi)^2 \lesssim V(\phi)$ inflates

Extended Inflation Like Old Inflation, but slower (e.g., power $a \propto t^p$),
so phase transition can finish

POTENTIAL $V(\phi)$ DURING INFLATION

Chaotic typically $V(\phi) = \Lambda\phi^n$, can also use $V = V_0 e^{\alpha\phi}$, etc.
 $\Rightarrow a \propto t^p$, $p = 16\pi/\alpha^2 \gg 1$

HOW INFLATION ENDS

First-order phase transition — e.g., Old or Extended inflation

Faster rolling \rightarrow oscillation — e.g., Chaotic $V(\phi)^2 \Lambda\phi^n$

“Waterfall” — rapid roll of σ triggered by slow roll of ϕ

(RE)HEATING

Decay of inflatons

“Preheating” by parametric resonance, then decay

BEFORE INFLATION?

Eternal Inflation? Can be caused by

- Quantum $\delta\phi \sim H/2\pi >$ rolling $\Delta\phi = \phi\Delta t = \phi H^{-1} \approx V'/V$
- Monopoles or other topological defects

Eternal Inflation

Vilenkin (1983) and Linde (1986, 1990) pointed out that if one extrapolates inflation backward to try to imagine what might have preceded it, in many versions of inflation the answer is “eternal inflation”: in most of the volume of the universe inflation is still happening, and our part of the expanding universe (a region encompassing far more than our entire cosmic horizon) arose from a tiny part of such a region. To see how eternal inflation works, consider the simple chaotic model with $V(\phi) = (m^2/2)\phi^2$. During the de Sitter Hubble time H^{-1} , where as usual $H^2 = (8\pi G/3)V$, the slow rolling of ϕ down the potential will reduce it by

$$\Delta\phi = \dot{\phi}\Delta t = -\frac{V'}{3H}\Delta t = \frac{m_{pl}^2}{4\pi\phi}. \quad (1.7)$$

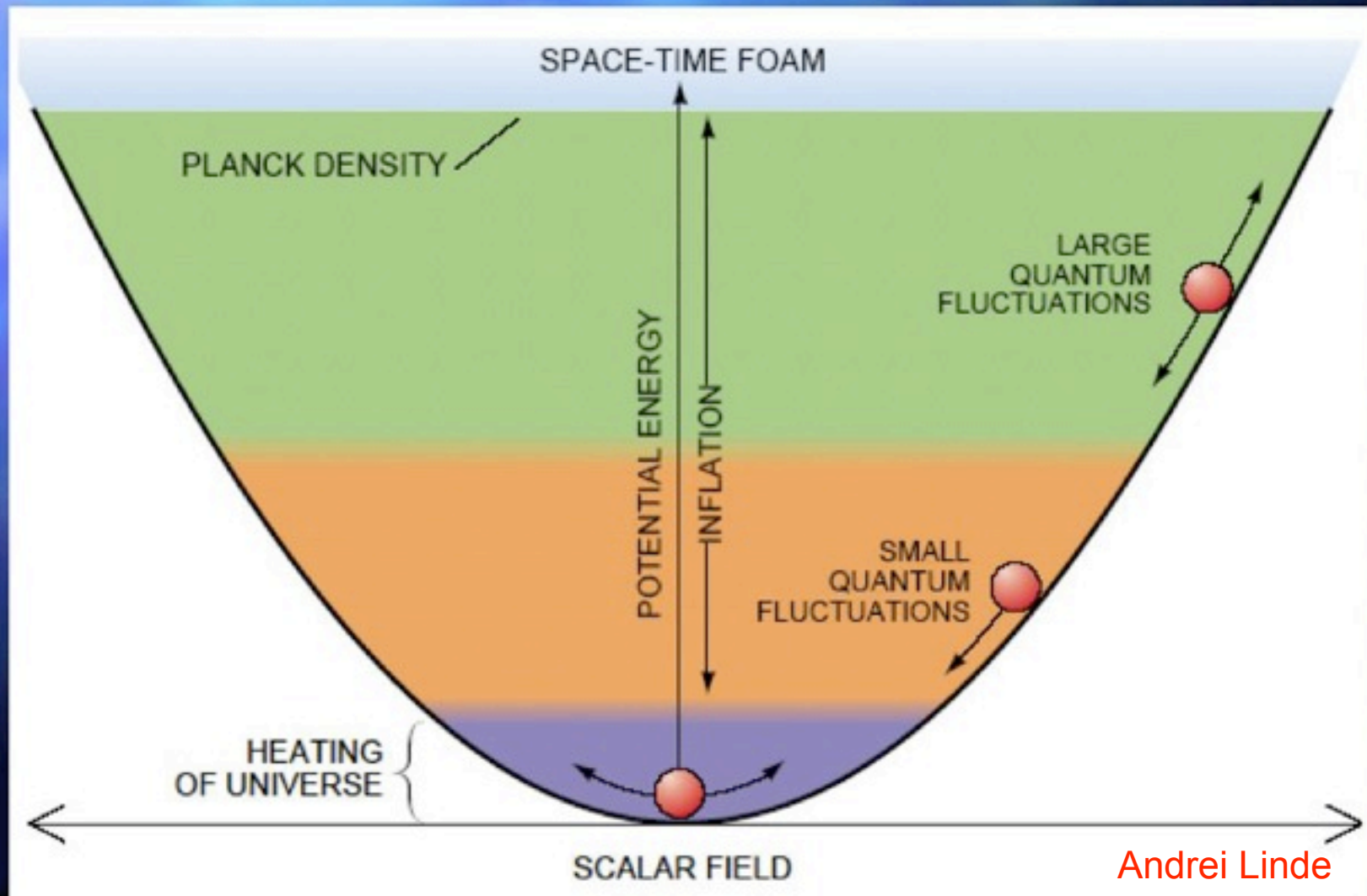
Here m_{pl} is the Planck mass ($m_{\text{Planck}} = 1/G^{1/2}$). But there will also be quantum fluctuations that will change ϕ up or down by

$$\delta\phi = \frac{H}{2\pi} = \frac{m\phi}{\sqrt{3\pi}m_{pl}} \quad (1.8)$$

These will be equal for $\phi_* = m_{pl}^{3/2}/2m^{1/2}$, $V(\phi_*) = (m/8m_{pl})m_{pl}^4$. If $\phi \gtrsim \phi_*$, *positive quantum fluctuations dominate* the evolution: after $\Delta t \sim H^{-1}$, an initial region becomes $\sim e^3$ regions of size $\sim H^{-1}$, in half of which ϕ increases to $\phi + \delta\phi$. Since $H \propto \phi$, this drives inflation faster in these regions.

Inflation as a theory of a harmonic oscillator

$$V(\phi) = \frac{m^2}{2}\phi^2$$



Supersymmetric Inflation

When Pagels and I (1982) first suggested that the lightest supersymmetric partner particle (LSP), stable because of R-parity, might be the dark matter particle, that particle was the gravitino in the early version of supersymmetry then in fashion. Weinberg (1982) immediately pointed out that if the gravitino were not the LSP, it could be a source of real trouble because of its long lifetime $\sim M_{\text{Pl}}^2/m_{3/2}^3 \sim (m_{3/2}/\text{TeV})^{-3}10^3$ s, a consequence of its gravitational-strength coupling to other fields. Subsequently, it was realized that supersymmetric theories can naturally solve the gauge hierarchy problem, explaining why the electroweak scale $M_{\text{EW}} \sim 10^2$ GeV is so much smaller than the GUT or Planck scales. In this version of supersymmetry, which has now become the standard one, the gravitino mass will typically be $m_{3/2} \sim \text{TeV}$; and the late decay of even a relatively small number of such massive particles can wreck BBN and/or the thermal spectrum of the CBR. The only way to prevent this is to make sure that the reheating temperature after inflation is sufficiently low: $T_{\text{RH}} \lesssim 2 \times 10^9$ GeV (for $m_{3/2} = \text{TeV}$) (Ellis, Kim, & Nanopoulos 1984, Ellis et al. 1992).

Basic Predictions of Inflation

1. **Flat universe.** This is perhaps the most fundamental prediction of inflation. Through the Friedmann equation it implies that the total energy density is always equal to the critical energy density; it does not however predict the form (or forms) that the critical density takes on today or at any earlier or later epoch.
2. **Nearly scale-invariant spectrum of Gaussian density perturbations.** These density perturbations (scalar metric perturbations) arise from quantum-mechanical fluctuations in the field that drives inflation; they begin on very tiny scales (of the order of 10^{-23} cm, and are stretched to astrophysical size by the tremendous growth of the scale factor during inflation (factor of e^{60} or greater). Scale invariant refers to the fact that the fluctuations in the gravitational potential are independent of length scale; or equivalently that the horizon-crossing amplitudes of the density perturbations are independent of length scale. While the shape of the spectrum of density perturbations is common to all models, the overall amplitude is model dependent. Achieving density perturbations that are consistent with the observed anisotropy of the CBR and large enough to produce the structure seen in the Universe today requires a horizon crossing amplitude of around 2×10^{-5} .
3. **Nearly scale-invariant spectrum of gravitational waves**, from quantum-mechanical fluctuations in the metric itself. These can be detected as CMB “B-mode” polarization, or using special gravity wave detectors such as LIGO and LISA.

Inflationary Predictions:

Andrei Linde

COBE-normalized spectrum of perturbations of metric

Flat spectrum of metric perturbations with $n_s = 0.95$

No tensor perturbations

Inflation in this scenario is eternal

(topological inflation)

Parameters require fine-tuning with accuracy $O(0.1\%)$, which may not be a problem if one takes into account string theory landscape

Density Fluctuations from Inflation

The relationship between the inflationary potential and the power spectrum of density perturbations today ($P(k) \equiv \langle |\delta_k|^2 \rangle$) is given by

$$P(k) = \frac{1024\pi^3}{75} \frac{k}{H_0^4} \frac{V_*^3}{m_{\text{Pl}}^6 V_*'^2} \left(\frac{k}{k_*}\right)^{n-1} T^2(k)$$
$$n - 1 = -\frac{1}{8\pi} \left(\frac{m_{\text{Pl}} V_*'}{V_*}\right)^2 + \frac{m_{\text{Pl}}}{4\pi} \left(\frac{m_{\text{Pl}} V_*'}{V_*}\right)' \quad \text{generally nonzero}$$
$$\frac{dn}{d \ln k} = -\frac{1}{32\pi^2} \left(\frac{m_{\text{Pl}}^3 V_*'''}{V_*}\right) \left(\frac{m_{\text{Pl}} V_*'}{V_*}\right) + \frac{1}{8\pi^2} \left(\frac{m_{\text{Pl}}^2 V_*''}{V_*}\right) \left(\frac{m_{\text{Pl}} V_*'}{V_*}\right)^2 - \frac{3}{32\pi^2} \left(m_{\text{Pl}} \frac{V_*'}{V_*}\right)^4$$
$$T(q) = \frac{\ln(1 + 2.34q) / 2.34q}{[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{1/4}},$$

where $V(\phi)$ is the inflationary potential, prime denotes $d/d\phi$, V_* is the value of the scalar potential when the scale k_* crossed outside the horizon during inflation, $T(k)$ is the transfer function which accounts for the evolution of the mode k from horizon crossing until the present, $q = k/h\Gamma$, and $\Gamma \simeq \Omega_M h$ is the “shape” parameter

Gravity Waves from Inflation

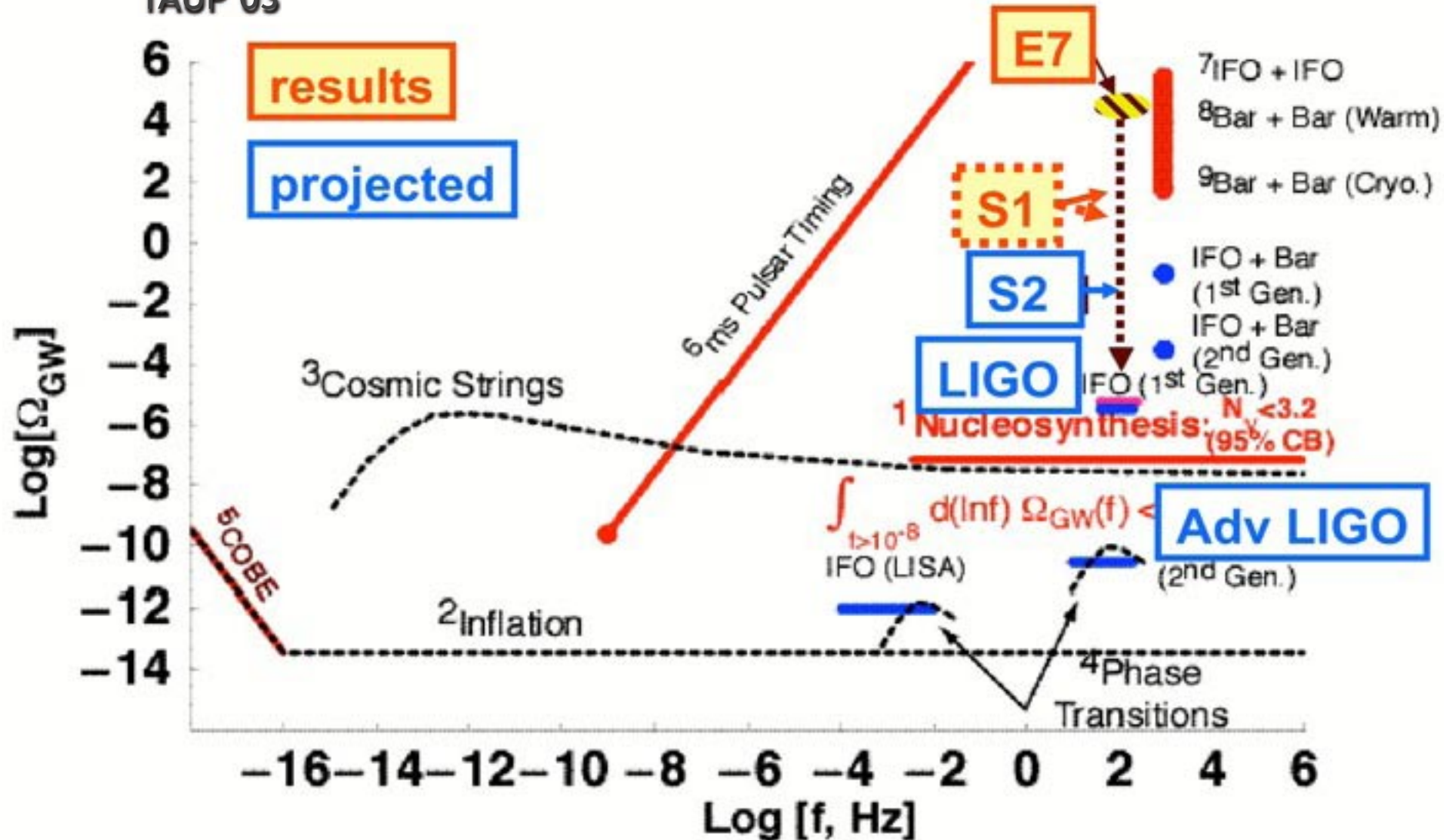
Unlike the scalar perturbations, which must have an amplitude of around 10^{-5} to seed structure formation, there is no astrophysical clue as to the amplitude of the tensor perturbations. They can be characterized by their power spectrum today

$$\begin{aligned}P_T(k) &\equiv \langle |h_k|^2 \rangle = \frac{8}{3\pi} \frac{V_*}{m_{\text{Pl}}^4} \left(\frac{k}{k_*} \right)^{n_T-3} T_T^2(k) \\n_T &= -\frac{1}{8\pi} \left(\frac{m_{\text{Pl}} V_*'}{V_*} \right)^2 \\ \frac{dn_T}{d \ln k} &= \frac{1}{32\pi^2} \left(\frac{m_{\text{Pl}}^2 V''}{V} \right) \left(\frac{m_{\text{Pl}} V_*'}{V} \right)^2 - \frac{1}{32\pi^2} \left(\frac{m_{\text{Pl}} V_*'}{V} \right)^4 = -n_T[(n-1) - n_T] \\ T_T(k) &\simeq \left[1 + \frac{4}{3} \frac{k}{k_{\text{EQ}}} + \frac{5}{2} \left(\frac{k}{k_{\text{EQ}}} \right)^2 \right]^{1/2},\end{aligned}\tag{11}$$

where $T_T(k)$ is the transfer function for gravity waves and describes the evolution of mode k from horizon crossing until the present, $k_{\text{EQ}} = 6.22 \times 10^{-2} \text{ Mpc}^{-1} (\Omega_M h^2 / \sqrt{g_* / 3.36})$ is the scale that crossed the horizon at matter-radiation equality, Ω_M is the fraction of critical density in matter, and g_* counts the effective number of relativistic degrees of freedom (3.36 for photons and three light neutrino species). The quantity $k^{3/2} |h_k| / \sqrt{2\pi^2}$ corresponds to the dimensionless strain (metric perturbation) on length scale $\lambda = 2\pi/k$.

Stochastic Background

B. Barish
TAUP 03



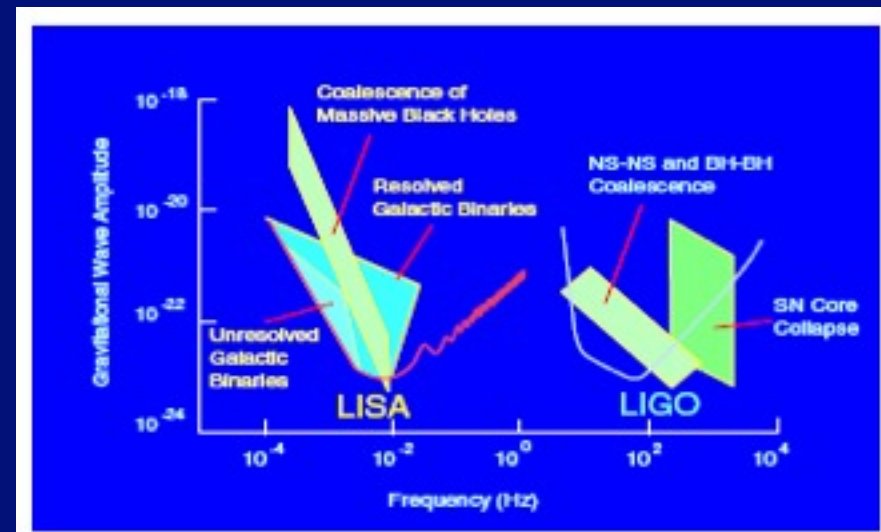
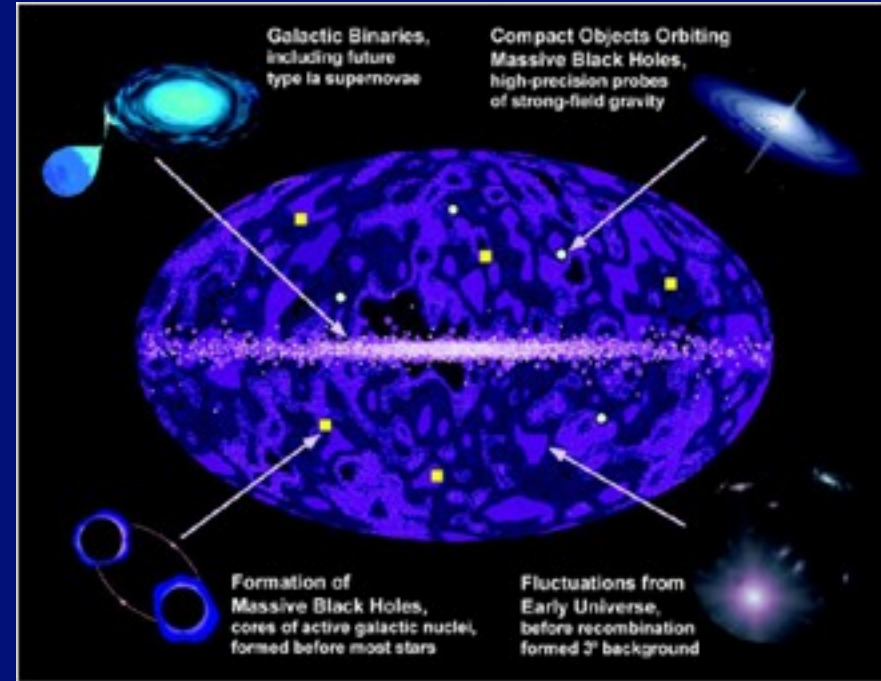
LISA: Science Goals

- Beyond Einstein science

- determine how and when massive black holes form
- investigate whether general relativity correctly describes gravity under extreme conditions
- determine how black hole growth is related to galaxy evolution
- determine if black holes are correctly described by general relativity
- investigate whether there are gravitational waves from the early universe
- determine the distance scale of the universe

- Broader science

- determine the distribution of binary systems of white dwarfs and neutron stars in our Galaxy



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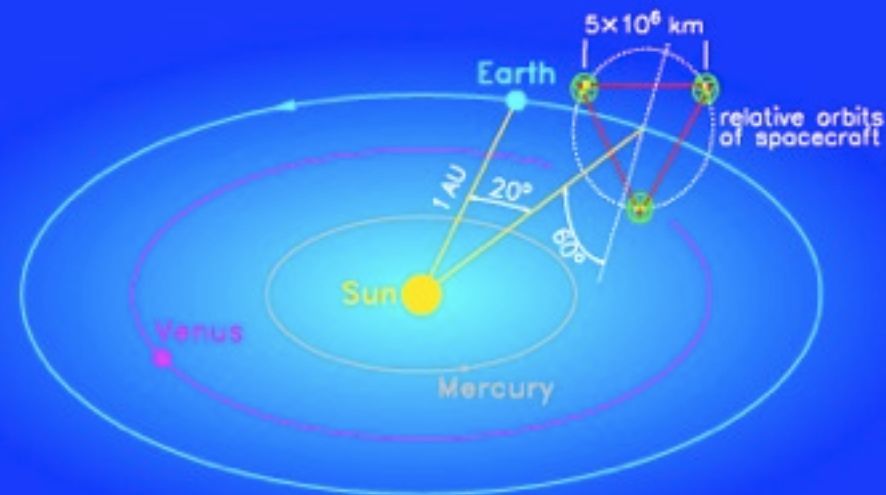
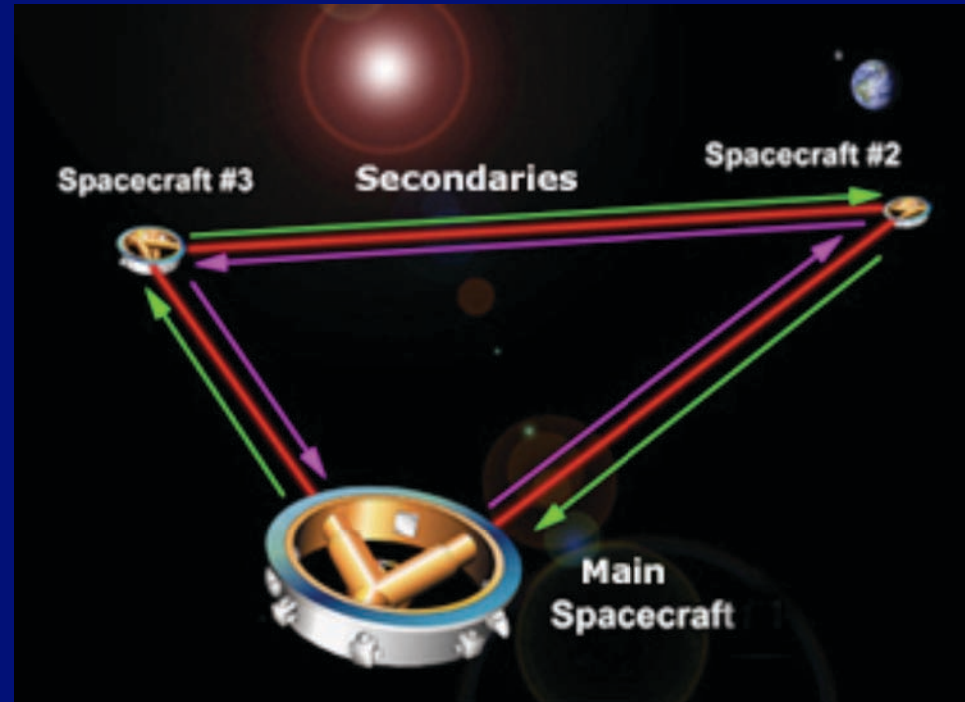
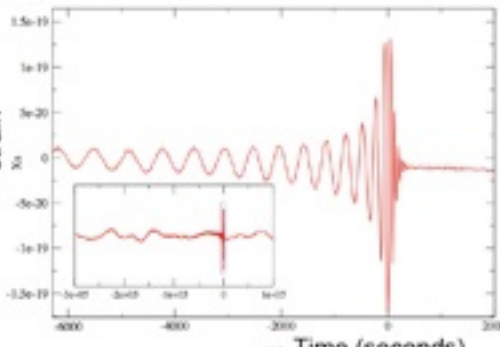
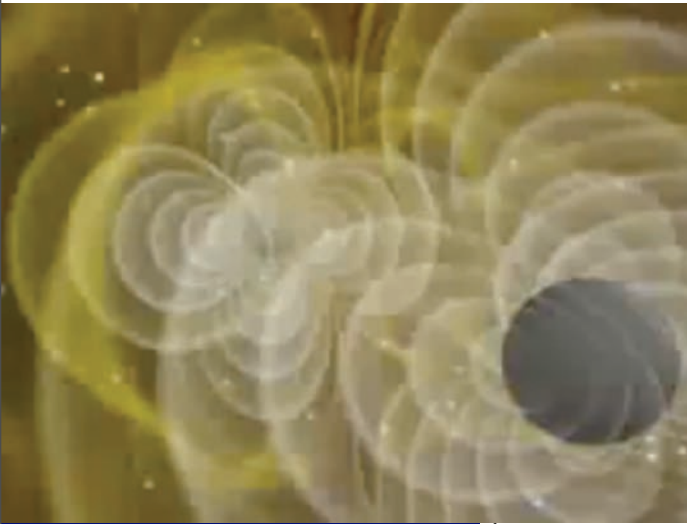
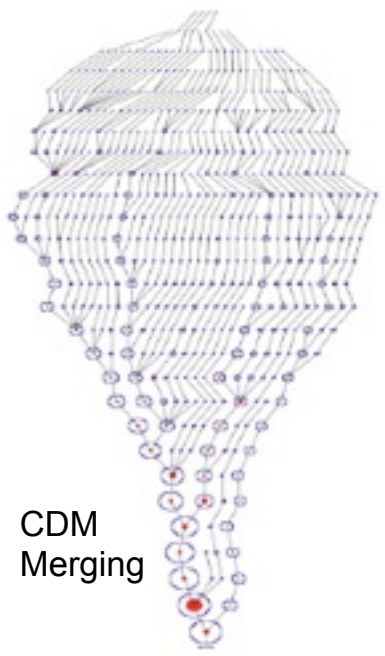


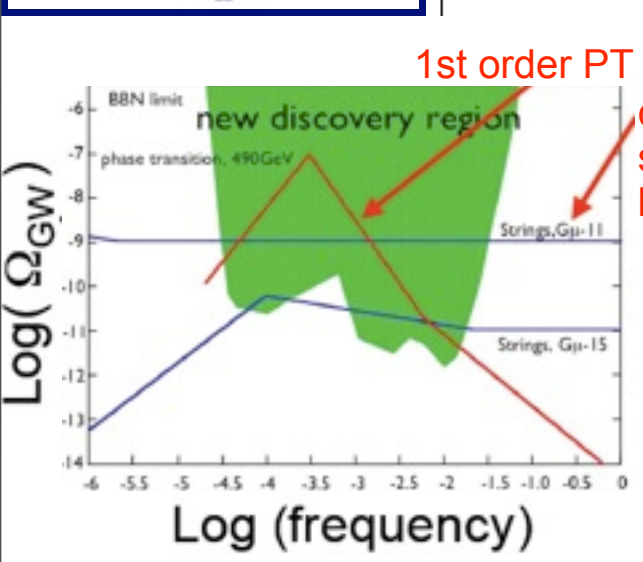
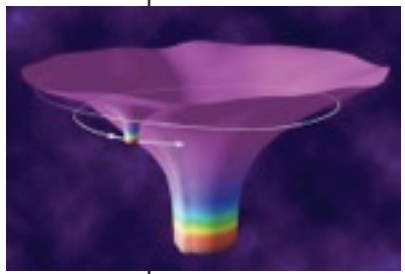
TABLE 2.F.3 LISA: Beyond Einstein Science Programs

Science	Program	Program Characteristics		Program Significance
<p>Science Definition Programs</p> 	<p>Formation of Massive Black Holes</p>	<p>Science Question</p>	<p>How and when do massive black holes form?</p>	<p>Observations will detect massive black hole binary mergers to $z=15$ and shed light on when massive black holes formed</p>
		<p>Measurements</p>	<p>Gravitational waveform shape as a function of time from massive black-hole binary inspiral and merger</p>	
		<p>Quantities Determined</p>	<p>Mass and spin of black holes as a function of distance</p>	
	<p>Test General Relativity in the Strong-Field Regime</p>	<p>Science Question</p>	<p>Does general relativity correctly describe gravity under extreme conditions?</p>	<p>Measurement of the detailed gravitational waveform will test whether general relativity accurately describes gravity under the most extreme conditions</p>
		<p>Measurements</p>	<p>Gravitational waveform shape as a function of time from massive black-hole binary inspiral and merger</p>	
		<p>Quantities Determined</p>	<p>Evolution of dynamical spacetime geometry, mass and spin of initial and final holes</p>	
<p>History of galaxy and black hole co-evolution</p>	<p>History of galaxy and black hole co-evolution</p>	<p>Science Question</p>	<p>How is black hole growth related to galaxy evolution?</p>	<p>Observations will trace the evolution of massive black hole masses as a function of distance or time, and will shed light on how black hole growth and galactic evolution may be linked</p>
		<p>Measurements</p>	<p>Gravitational waveform shape as a function of time from massive black-hole binary inspiral and merger</p>	

Joan Centrella et al.
 NASA/GSFC
 Visualization: Chris Henze, NASA/Ames



Science	Program	Program Characteristics		Program Significance
		Quantities Determined	Mass as a function of distance	
Additional Beyond Einstein Science	Map black-hole spacetimes	Science Question	Are black holes correctly described by general relativity?	Observations will yield maps of the spacetime geometry surrounding massive black holes, and will test whether they are described by the Kerr geometry predicted by general relativity. They will also measure the parameters (mass, spin, shape) of the holes, and test whether they obey the no-hair theorems of GR
		Measurements	Gravitational waveform shape from small bodies spiraling into massive black holes (EMRI)	
		Quantities Determined	Mass, spin, multipole moments, spacetime geometry close to hole	



Cosmological backgrounds	Science Question	Are there gravitational waves from the early universe?	First-order phase transitions or cosmic strings in the early universe could leave a background of detectable waves
	Measurements	Stochastic background of gravitational waves	
	Quantities Determined	Effective energy density of waves vs. frequency	

Cosmography, Dark energy	Science Question	What is the distance scale of the universe?	If redshift of source or host galaxy can be determined, then precise, calibration-free measurements of the Hubble parameter and other cosmological parameters could be done, significantly constraining dark energy
	Measurements	Gravitational waveform shape and amplitude measurements yield luminosity distance of sources directly	
	Quantities Determined	Luminosity distance	

$$\text{Distance} \cong c \frac{1}{\text{frequency}^2 \times t_{\text{chirp}} \times \text{amplitude}}$$

Inflation Summary

The key features of all inflation scenarios are a period of superluminal expansion, followed by (“re-”)heating which converts the energy stored in the inflaton field (for example) into the thermal energy of the hot big bang.

Inflation is *generic*: it fits into many versions of particle physics, and it can even be made rather natural in modern supersymmetric theories as we have seen. The simplest models have inflated away all relics of any pre-inflationary era and result in a flat universe after inflation, i.e., $\Omega = 1$ (or more generally $\Omega_0 + \Omega_\Lambda = 1$). Inflation also produces scalar (density) fluctuations that have a primordial spectrum

$$\left(\frac{\delta\rho}{\rho}\right)^2 \sim \left(\frac{V^{3/2}}{m_{Pl}^3 V'}\right)^2 \propto k^{n_p}, \quad (1.12)$$

where V is the inflaton potential and n_p is the primordial spectral index, which is expected to be near unity (near-Zel’dovich spectrum). Inflation also produces tensor (gravity wave) fluctuations, with spectrum

$$P_t(k) \sim \left(\frac{V}{m_{Pl}}\right)^2 \propto k^{n_t}, \quad (1.13)$$

where the tensor spectral index $n_t \approx (1 - n_p)$ in many models.

The quantity $(1 - n_p)$ is often called the “tilt” of the spectrum; the larger the tilt, the more fluctuations on small spatial scales (corresponding to large k) are suppressed compared to those on larger scales. The scalar and tensor waves are generated by independent quantum fluctuations during inflation, and so their contributions to the CMB temperature fluctuations add in quadrature. The ratio of these contributions to the quadrupole anisotropy amplitude Q is often called $T/S \equiv Q_t^2/Q_s^2$; thus the primordial scalar fluctuation power is decreased by the ratio $1/(1+T/S)$ for the same COBE normalization, compared to the situation with no gravity waves ($T = 0$). In power-law inflation, $T/S = 7(1 - n_p)$. This is an approximate equality in other popular inflation models such as chaotic inflation with $V(\phi) = m^2\phi^2$ or $\lambda\phi^4$. But note that the tensor wave amplitude is just the inflaton potential during inflation divided by the Planck mass, so the gravity wave contribution is negligible in theories like the supersymmetric model discussed above in which inflation occurs at an energy scale far below m_{Pl} . Because gravity waves just redshift after they come inside the horizon, the tensor contributions to CMB anisotropies corresponding to angular wavenumbers $\ell \gg 20$, which came inside the horizon long ago, are strongly suppressed compared to those of scalar fluctuations.