Physics 224 - Spring 2010

Week 2
GENERAL RELATIVISTIC COSMOLOGY

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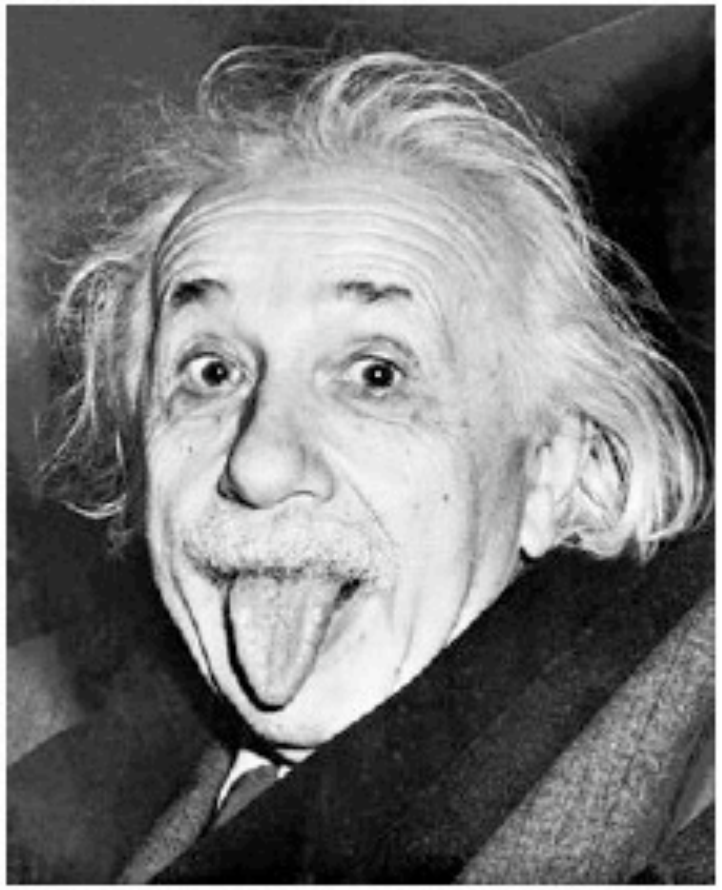
New observations hit the theory like an ice cold shower.
They show that cold dark matter has too little large scale power.
Says Peebles: "Cold dark matter? My feeblest innovation.
An overly aesthetic, theoretical abberation.
Our theories must have firmer empirical foundation.
Shed all this extra baggage, including the carry-ons.
Use particles we know, i.e., the baryons.
Others aren't convinced, and a few propose a mixture
of matter hot and cold, perhaps with strings or texture.
And nowadays some physicists are beginning to wonder
if it's time to resurrect Einstein's "greatest blunder."
Why seek exotic particles instead of just assume
that the dark matter's all around us -- it's what we call the vacuum?



Who's right? It's hard to know, 'til observation or experiment
gives overwhelming evidence that relieves our predicament.
The search is getting popular as many realize
that the detector of dark matter may well win the Nobel Prize.

So now you've heard my lecture, and it's time to end the session
with the standard closing line: Thank you, any questions?

SUMMARY

- We now know the cosmic recipe. Most of the universe is invisible stuff called "nonbaryonic dark matter" (25%) and "dark energy" (70%). Everything that we can see makes up only about 1/2% of the cosmic density, and invisible atoms about 4%. The earth and its inhabitants are made of the rarest stuff of all: heavy elements (0.01%).

- The ΛCDM Cold Dark Matter Double Dark theory based on this appears to be able to account for all the large scale features of the observable universe, including the details of the heat radiation of the Big Bang and the large scale distribution of galaxies.

- Constantly improving data are repeatedly testing this theory. The main ingredients have been checked several different ways. There exist no convincing disagreements, as far as I can see. Possible problems on subgalactic scales may be due to the poorly understood physics of gas, stars, and massive black holes.

- But we still don't know what the dark matter and dark energy are, nor really understand how galaxies form and evolve. There's lots more work for us to do!

Modern Cosmology

A series of major discoveries has laid a lasting foundation for cosmology. Einstein's general relativity (1916) provided the conceptual foundation for the modern picture. Then Hubble discovered that "spiral nebulae" are large galaxies like our own Milky Way (1925), and that distant galaxies are receding from the Milky Way with a speed proportional to their distance (1929), which means that we live in an expanding universe. The discovery of the cosmic background radiation (1965) showed that the universe began in a very dense, hot, and homogeneous state: the Big Bang. This was confirmed by the discovery that the cosmic background radiation has exactly the same spectrum as heat radiation (1989), and the measured abundances of the light elements agree with the predictions of Big Bang theory if the abundance of ordinary matter is about 4% of critical density. Most of the matter in the universe is invisible particles which move very sluggishly in the early universe ("Cold Dark Matter").

General Relativity

GRAVITY ACCORDING TO GENERAL RELATIVITY

PRINCIPLE OF EQUIVALENCE

CURVED SPACE TELLS MATTER HOW TO MOVE.

EINSTEIN FIELD EQUATIONS

MATTER TELLS SPACE HOW TO CURVE.

MASS + ENERGY DENSITY?

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

CURVATURE

COSMOLOGICAL CONSTANT

CURVED SPACE-TIME IS NOT JUST AN ARENA IN WHICH THINGS MOVE, IT IS DYNAMIC. CURVATURE CAN CAUSE HORIZONS, BEYOND WHICH INFORMATION CANNOT BE SENT.

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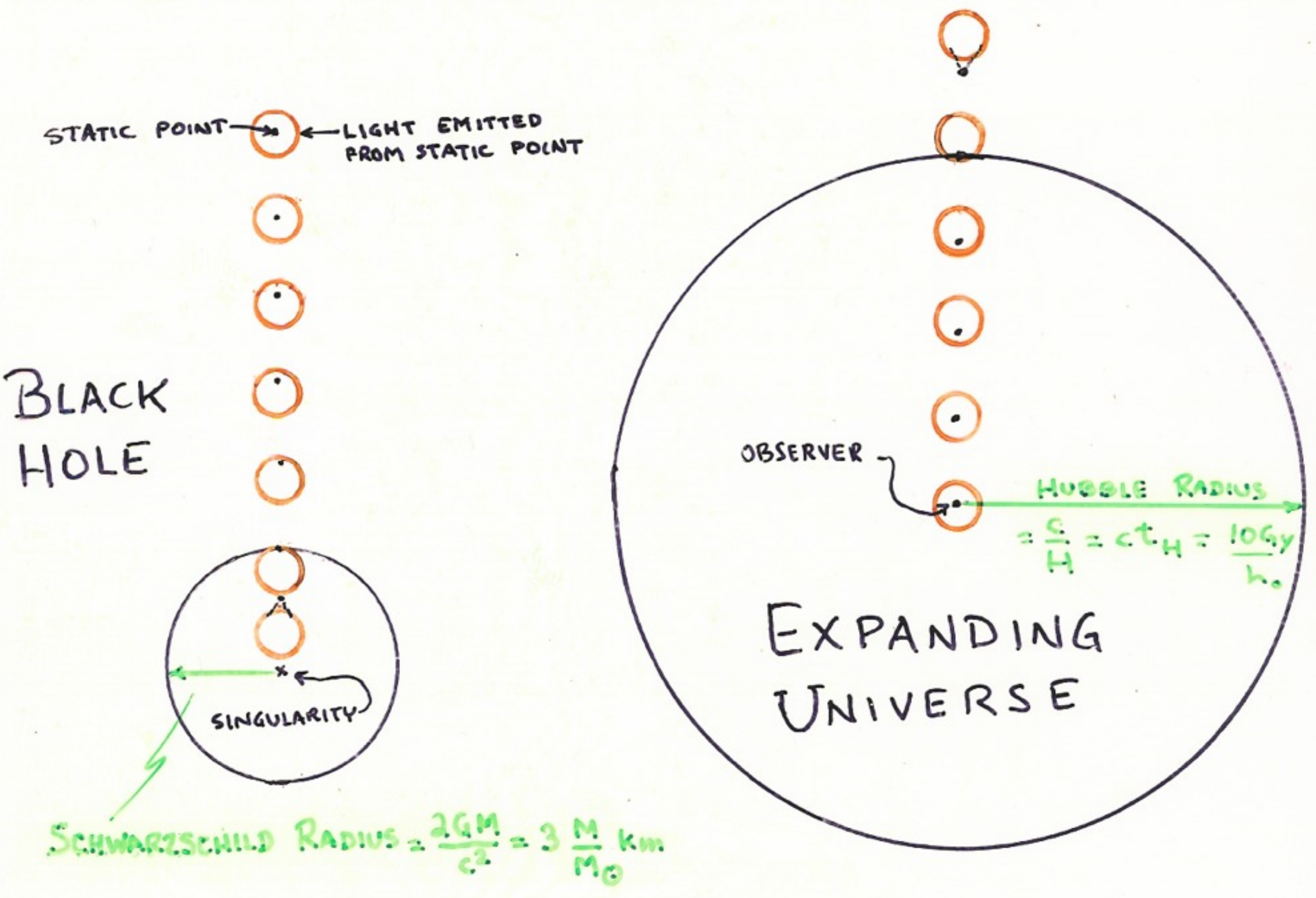

General Relativity

GR follows from the principle of equivalence and Einstein's equation $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi GT_{\mu\nu}$.* Einstein had intuited the local equivalence of gravity and acceleration in 1907 (Pais, p. 179), but it was not until November 1915 that he developed the final form of the GR equation.

It can be derived from the following assumptions (Weinberg, p. 153):
1. The l.h.s. $G_{\mu\nu}$ is a tensor
2. $G_{\mu\nu}$ consists only of terms linear in second derivatives or quadratic in first derivatives of the metric tensor $g_{\mu\nu}$ ($\Leftrightarrow$ $G_{\mu\nu}$ has dimension L^{-2})
3. Since $T_{\mu\nu}$ is symmetric in $\mu\nu$, so is $G_{\mu\nu}$
4. Since $T_{\mu\nu}$ is conserved (covariant derivative $T^{\mu}{}_{\nu;\mu}=0$) so also $G^{\mu}{}_{\nu;\mu}=0$
5. In the weak field limit where $g_{00} \approx -(1+2\phi)$, satisfying the Poisson equation $\nabla^2\phi=4\pi G\rho$ (i.e., $\nabla^2 g_{00} = -8\pi GT_{00}$), we must have $G_{00} = \nabla^2 g_{00}$

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*Note: we're here using the metric –1, 1,1,1 as in Dodelson, Weinberg.

Einstein's equation can also be derived from an action principle, varying the total action $I = I_M + I_G$, where I_M is the action of matter and I_G is that of gravity:

$$I_G = -\frac{1}{16\pi G}\int R(x)\sqrt{g(x)}\,d^4x$$

(see, e.g., Weinberg, p. 364). The curvature scalar $R \equiv R_{\mu\nu}\, g^{\mu\nu}$ is the obvious term to insert in I_G since a scalar connected with the metric is needed and it is the only one, unless higher powers R^2, R^3 or higher derivatives $\Box R$ are used, which will lead to higher-order or higher-derivative terms in the gravity equation.

Einstein realized in 1916 that the 5th postulate above isn't strictly necessary – merely that the equation reduce to the Newtonian Poisson equation within observational errors, which allows the inclusion of a small cosmological constant term. In the action derivation, such a term arises if we just add a constant to R.

General Relativity and Cosmology

GR: MATTER TELLS SPACE HOW TO CURVE

CURVED SPACE TELLS MATTER HOW TO MOVE

$$R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

Cosmological Principle: on large scales, space is uniform and isotropic. COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\,[dr^2\,(1 - kr^2)^{-1} + r^2\, d\Omega^2]$$

with curvature constant k = – 1, 0, or +1. Substituting this metric into the Einstein equation at left above, we get the Friedmann eq.

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Experimental and Historical Sciences

Physics, Chemistry — **Astronomy, Geology, Evolutionary Biology**

both make predictions about new knowledge, whether from experiments or from the past

Historical Explanation Is Always Inferential

Our age cannot look back to earlier things
Except where reasoning reveals their traces Lucretius

Patterns of Explanation Are the Same in the Historical Sciences as in the Experimental Sciences

Specific conditions + General laws ⇒ Particular event

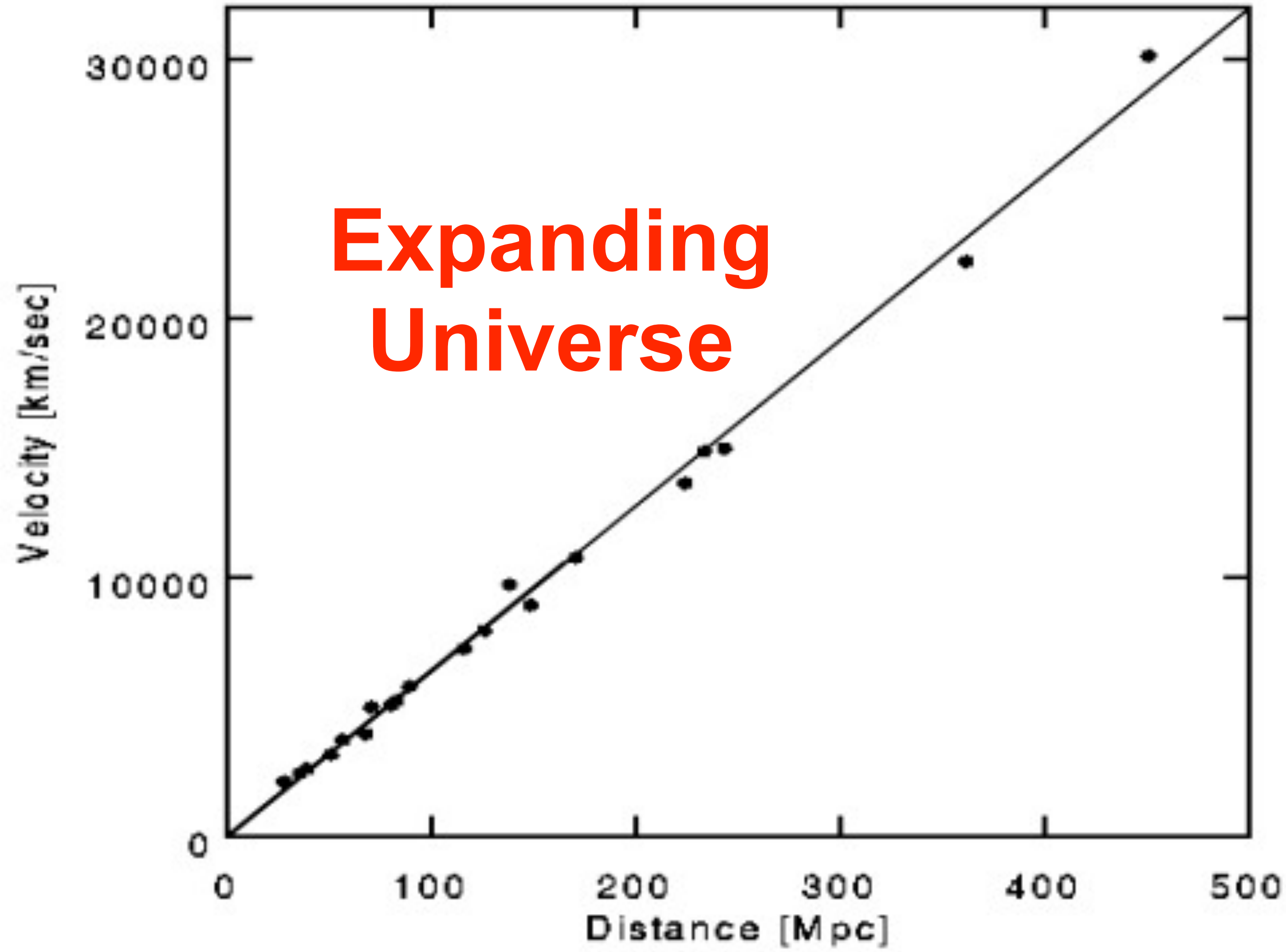
In history as anywhere else in empirical science, the explanation of a phenomenon consists in subsuming it under general empirical laws; and the criterion of its soundness is … exclusively whether it rests on empirically well confirmed assumptions concerning initial conditions and general laws.

C.G. Hempel, *Aspects of Scientific Explanation* (1965), p. 240.

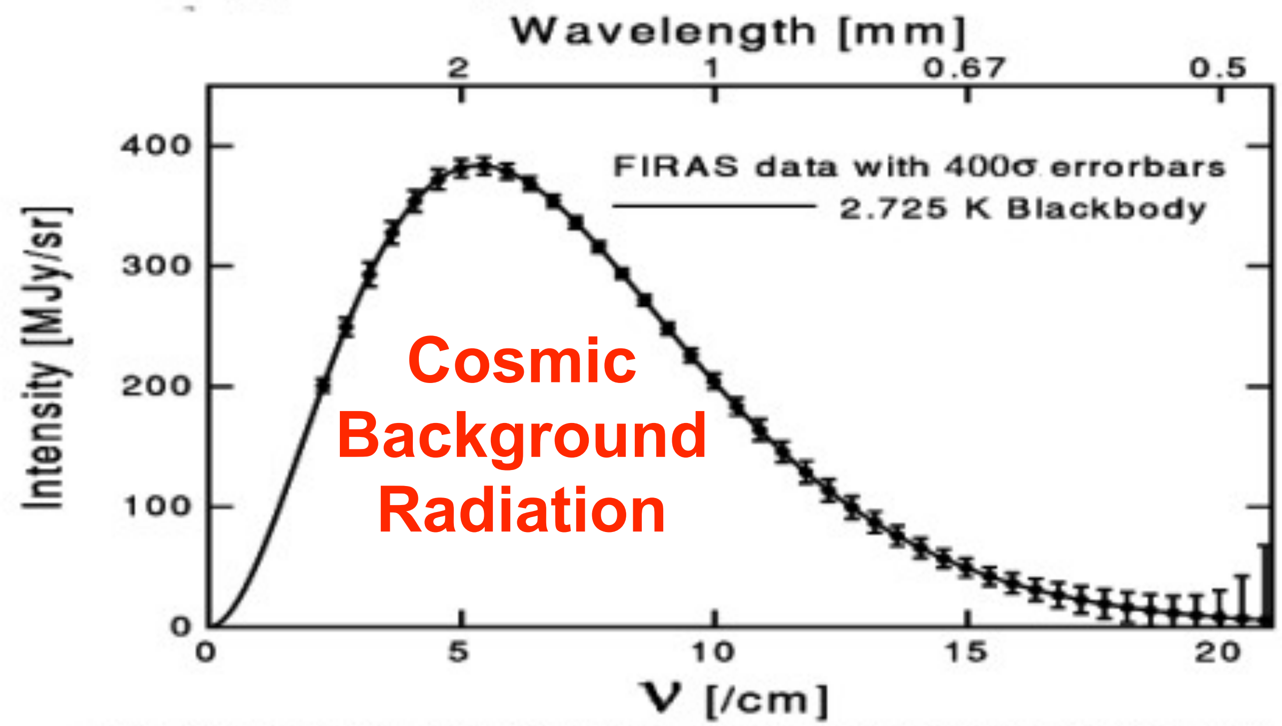
Successful Predictions of the Big Bang

First Prediction	First Confirmation
Expansion of the Universe	
Friedmann 1922, Lemaitre 1927 based on Einstein 1916	Hubble 1929
Cosmic Background Radiation	
Existence of CBR	
Gamow, Alpher, Hermann 1948	Penzias & Wilson 1965
CBR Thermal Spectrum	
Peebles 1966	COBE 1989
CBR Fluctuation Amplitude	
Cold Dark Matter theory 1984	COBE 1992
CBR Acoustic Peak	BOOMERANG 2000 MAXIMA 2000
Light Element Abundances	
Peebles 1966, Wagoner 1967	D/H Tytler et al.1997

Three Pillars of the Big Bang



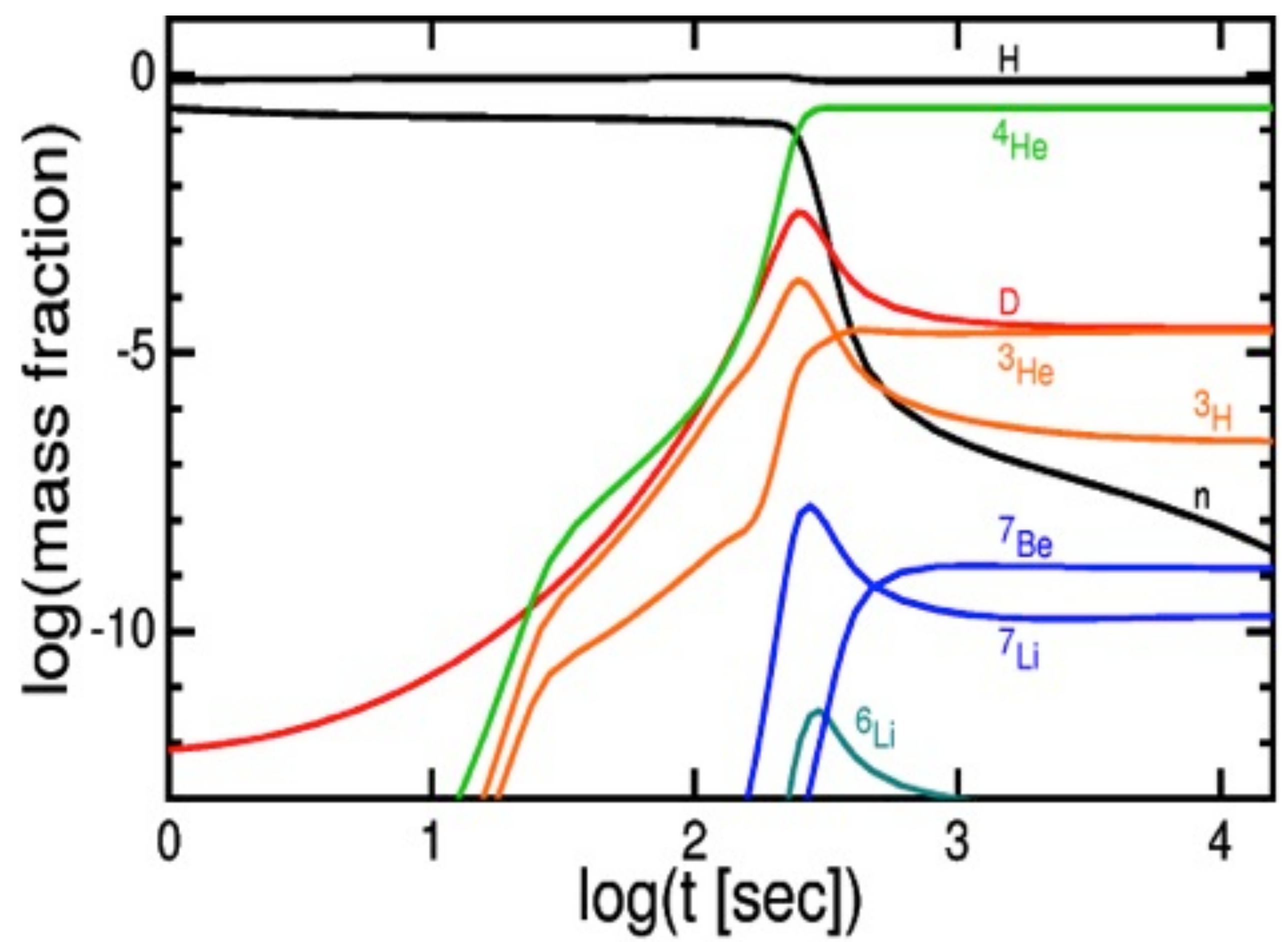
A modern illustration of Hubble's Law, displaying the increase of recession speed of galaxies growing in direct proportion to their distance.



The variation of the intensity of the microwave background radiation with its frequency, as observed by the COBE satellite from above the Earth's atmosphere. The observations (boxes) display a perfect fit with the (solid) curve expected from pure heat radiation with a temperature of 2.73°K.

Big Bang Nucleosynthesis

The detailed production of the lightest elements out of protons and neutrons during the first three minutes of the universe's history. The nuclear reactions occur rapidly when the temperature falls below a billion degrees Kelvin. Subsequently, the reactions are shut down, because of the rapidly falling temperature and density of matter in the expanding universe.



Caution: ^{7}Li may now be discordant

Mon. Not. R. astr. Soc. (1991) **251**, 128–136

Dynamical effects of the cosmological constant

Ofer Lahav,[1] Per B. Lilje,[2] Joel R. Primack[3] and Martin J. Rees[1]

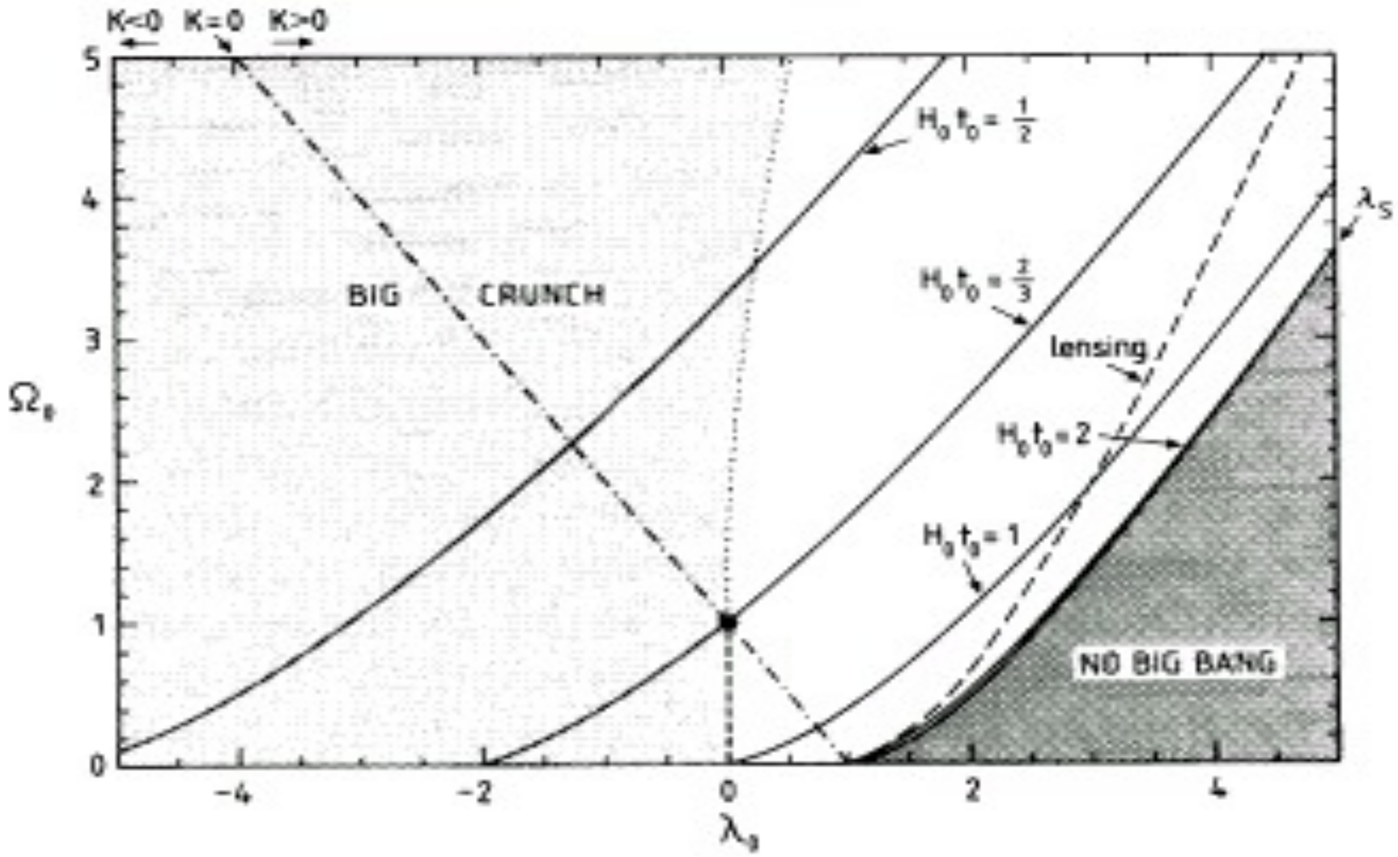
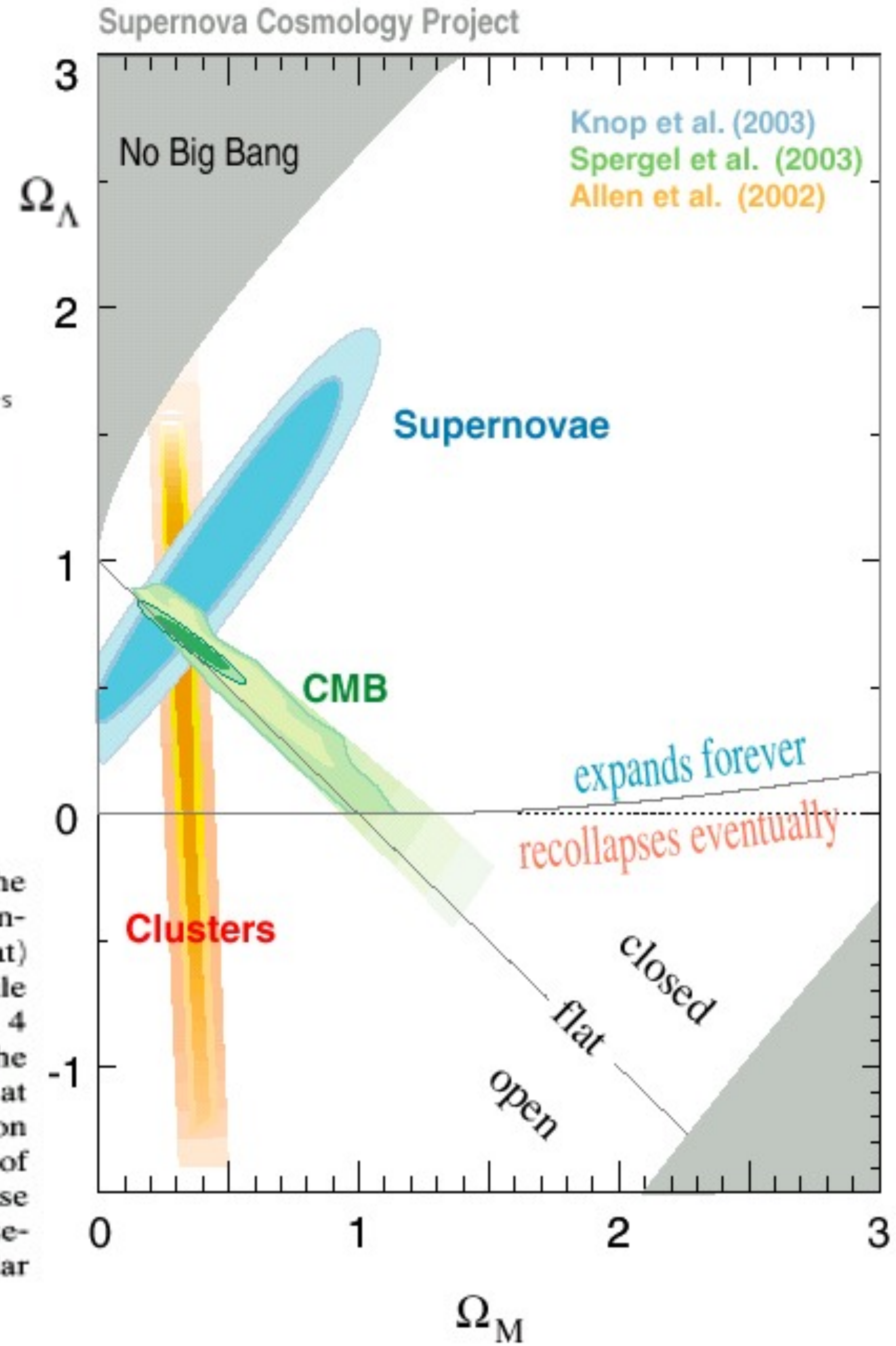


Figure 1. The phase-space of the density parameter Ω_0 and the cosmological constant $\lambda_0 = \Lambda/(3H_0^2)$ with various fundamental constraints. The dashed-dotted line indicates an inflationary (i.e. flat) universe. Note that some open models will have a Big Crunch, while some closed models will expand forever. The solid lines show 4 values for the age of the universe H_0t_0, and the dashed line is the constraint of Gott *et al.* (1989) from a normally lensed quasar at $z = 3.27$. The boundary (λ_s) of the shaded 'No Big Bang' region corresponds to a coasting phase in the past, while the boundary of the 'Big Crunch' (for $\Omega_0 > 1$) region corresponds to a coasting phase in the future. We see that the permitted range in the ($\lambda_0 - \Omega_0$) phase-space is fairly small, but allows values different from the popular point ($\Omega_0 = 1$, $\lambda_0 = 0$).



Friedmann-Robertson-Walker Framework (homogeneous, isotropic universe)

FRW E(00) $\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$ ⟵ Friedmann equation

FRW E(ii) $\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$

$$H_0 \equiv 100h\,\mathrm{km\,s^{-1}Mpc^{-2}} \equiv 70h_{70}\,\mathrm{km\,s^{-1}Mpc^{-2}}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_\odot \mathrm{Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi G p + \frac{2}{3}\Lambda$$

$$\text{Divide by } 2E(00) \Rightarrow \quad q_0 \equiv -\left(\frac{\ddot{a}}{a}\frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{-\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{da}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda)$ $\qquad H_0^{-1} = 9.78h^{-1}\mathrm{Gyr}$ $= 13.97\ h_{70}^{-1}$ Gyr

$f(1,0) = \frac{2}{3}$
$f(0,0) = 1$
$f(0,1) = \infty$
$f(0.3, 0.7) = 0.964$

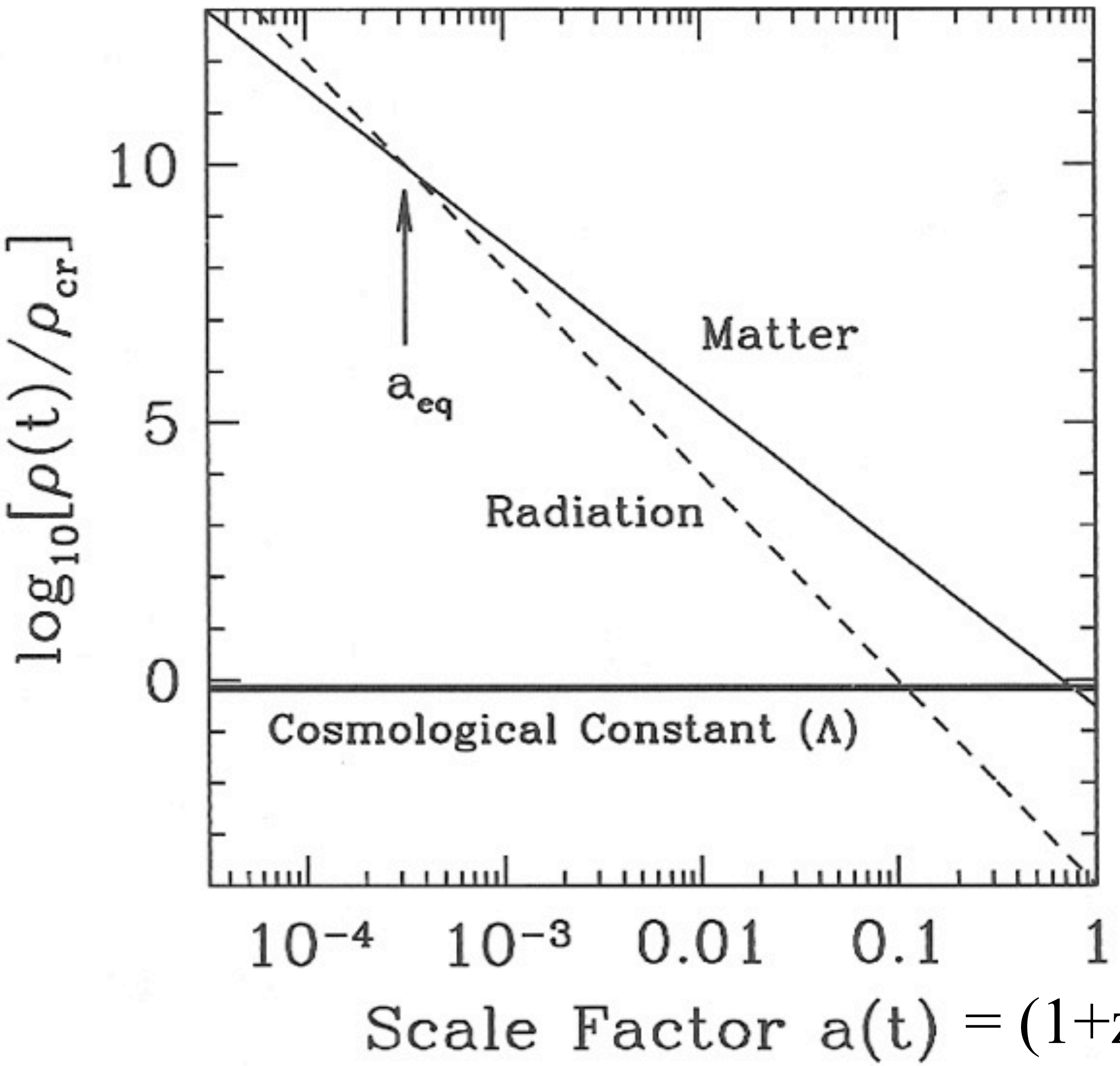
$$[E(00)a^3]' \text{ vs. } E(ii) \Rightarrow \quad \frac{\partial}{\partial a}(\rho a^3) = -3pa^2 \ (\text{“continuity”})$$

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

Matter: $p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0, t_0 eqs.)

Radiation: $p = \frac{\rho}{3},\ k = 0 \Rightarrow \rho \propto a^{-4}$

Evolution of Densities of Radiation, Matter, & Λ



$= (1+z)^{-1}$ z = redshift

Since matter or dark energy has dominated the cosmic density since t_{eq} = 47,000 yr (z_{eq} = 3500), we can neglect it in calculating the age of the universe t_0.

Figure 1.3. Energy density vs scale factor for different constituents of a flat universe. Shown are nonrelativistic matter, radiation, and a cosmological constant. All are in units of the critical density today. Even though matter and cosmological constant dominate today, at early times, the radiation density was largest. The epoch at which matter and radiation are equal is a_{eq}.

Dodelson, Chapter 1

LCDM Benchmark Cosmological Model: Ingredients & Epochs

	List of Ingredients
photons:	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos:	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation:	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic matter:	$\Omega_{\text{bary},0} = 0.04$
nonbaryonic dark matter:	$\Omega_{\text{dm},0} = 0.26$
total matter:	$\Omega_{m,0} = 0.30$
cosmological constant:	$\Omega_{\Lambda,0} \approx 0.70$

Important Epochs

radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^{4}$ yr
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.5$ Gyr

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Benchmark Model: Scale Factor vs. Time

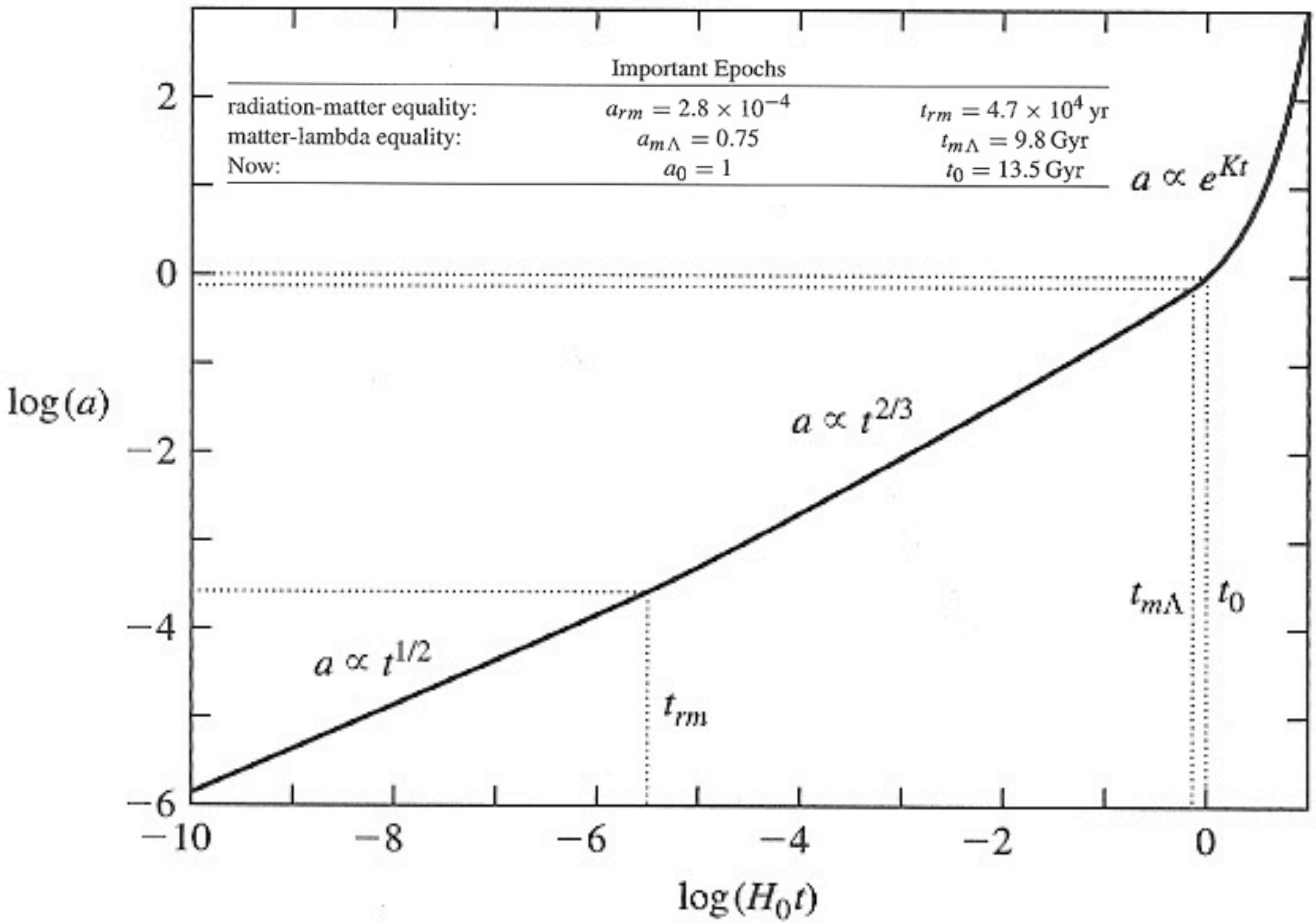
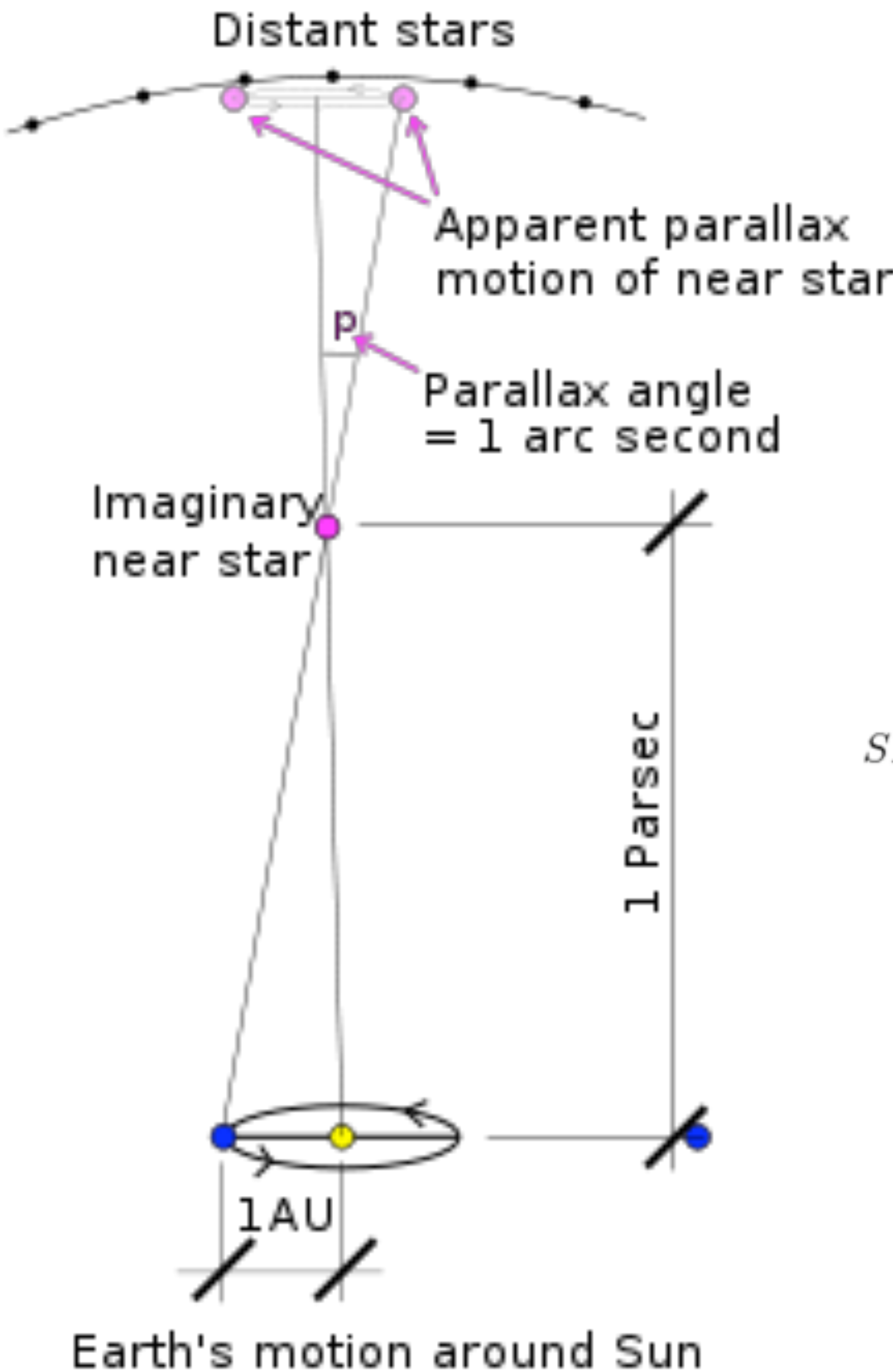
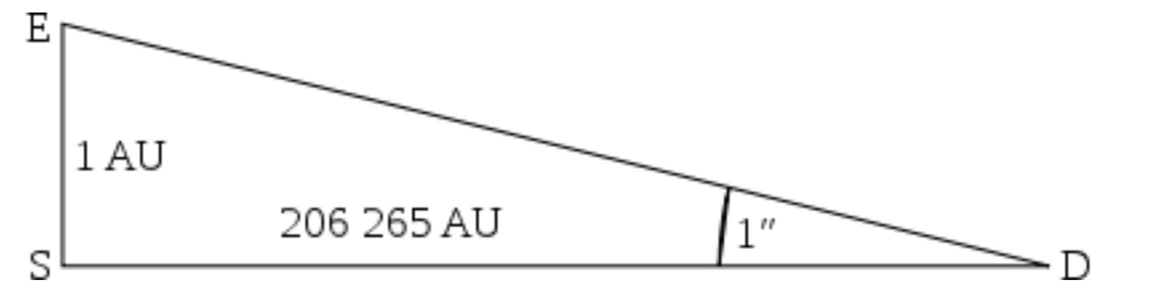


FIGURE 6.5 The scale factor a as a function of time t (measured in units of the Hubble time), computed for the Benchmark Model. The dotted lines indicate the time of radiation-matter equality, $a_{rm} = 2.8 \times 10^{-4}$, the time of matter-lambda equality, $a_{m\Lambda} = 0.75$, and the present moment, $a_0 = 1$.

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)





In the diagram above *(not to scale)*, **S** represents the Sun and **E** the Earth at one point in its orbit. Thus the distance **ES** is one astronomical unit (AU). The angle **SDE** is one arcsecond (1/3600 of a degree) so by definition **D** is a point in space at a distance of one parsec from the Sun. By trigonometry, the distance **SD** is

$$SD = \frac{\mathrm{ES}}{\tan 1''} \approx \frac{\mathrm{ES}}{1''} = \frac{360 \times 60 \times 60}{2\pi}\,\mathrm{AU} = \frac{648{,}000}{\pi}\,\mathrm{AU}$$

One AU = 149,597,870,691 m, so

1 parsec ≈ $3.085\,678 \times 10^{16}$ meters
≈ 3.261 564 light-years.

Measuring Distances in the Universe

Primary Distance Indicators

Trigonometric parallax

α Centauri 1.35 pc - first measured by Thomas Henderson 1832

61 Cygni 3.48 pc - by Friedrich Wilhelm Bessel in 1838

Only a few stars to < 30 pc, until the Hipparcos satellite 1997 measured distances of 118,000 stars to about 100 pc, about 20,000 stars to <10%.

Proper motions

Moving cluster method

Mainly for the Hyades, at about 100 pc. Now supplanted by Hipparcos.

Distance to Cepheid ζ Geminorum = 336 ± 44 pc

Using Doppler to measure change of diameter, and interferometry to measure change of angular diameter.

Similar methods for Type II SN, for stars in orbit about the Sagittarius A* SMBH (gives distance 8.0 ± 0.4 kpc to Galactic Center), for radio maser in NGC 4258 (7.2 ± 0.5 Mpc), etc.

Apparent Luminosity of various types of stars

$L = 10^{-2M/5}\, 3.02\times10^{35}$ erg sec^{-1} where M_{vis} = + 4.82 for the sun

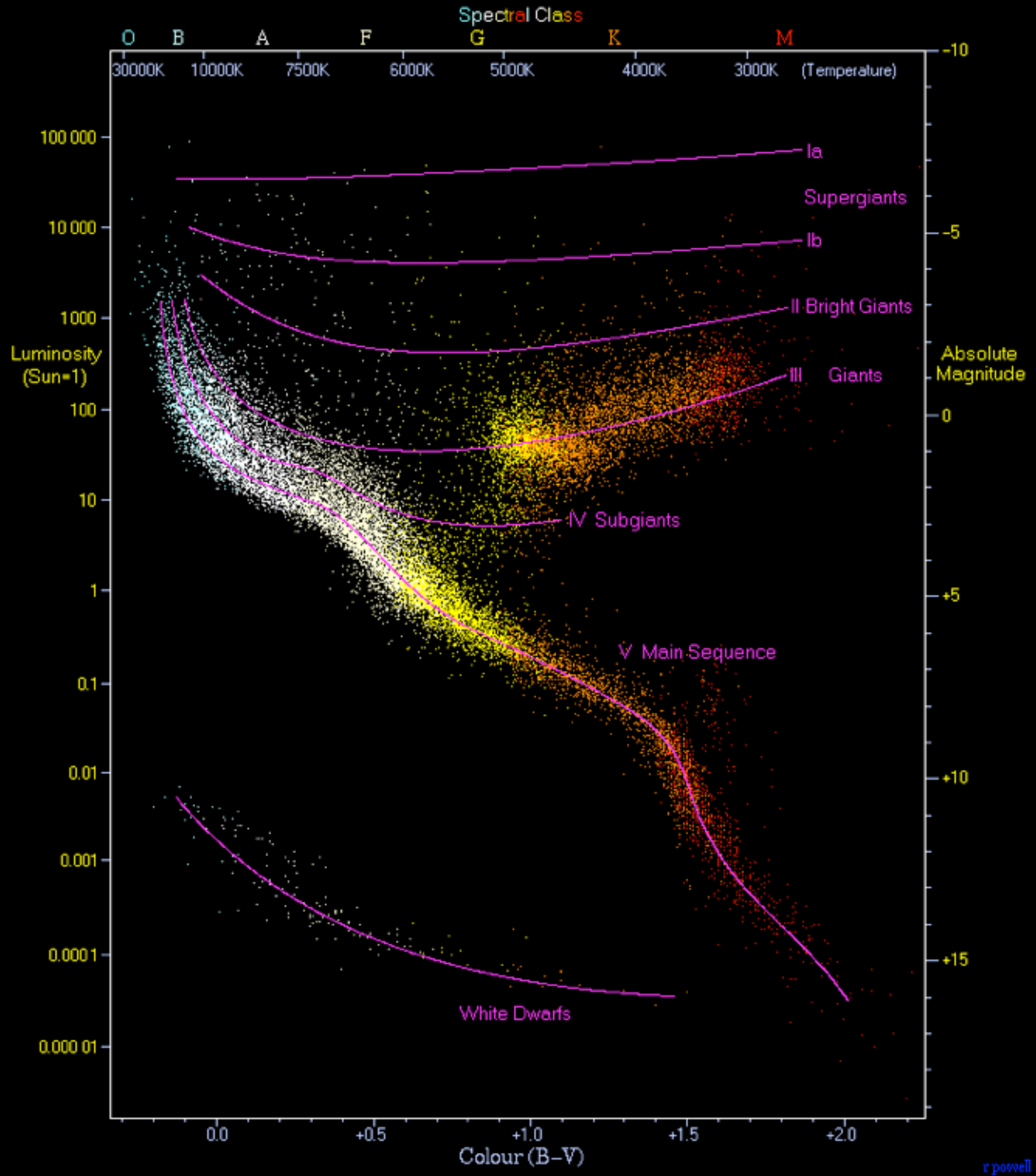
Apparent luminosity $\ell = L\,(4\pi d^2)^{-1}$ for nearby objects,
related to apparent magnitude m by $\ell = 10^{-2m/5}\,(2.52\times10^{-5}$ erg cm^{-2} s$^{-1})$

Distance modulus m - M related to distance by $d = 10^{1 + (m - M)/5}$ pc

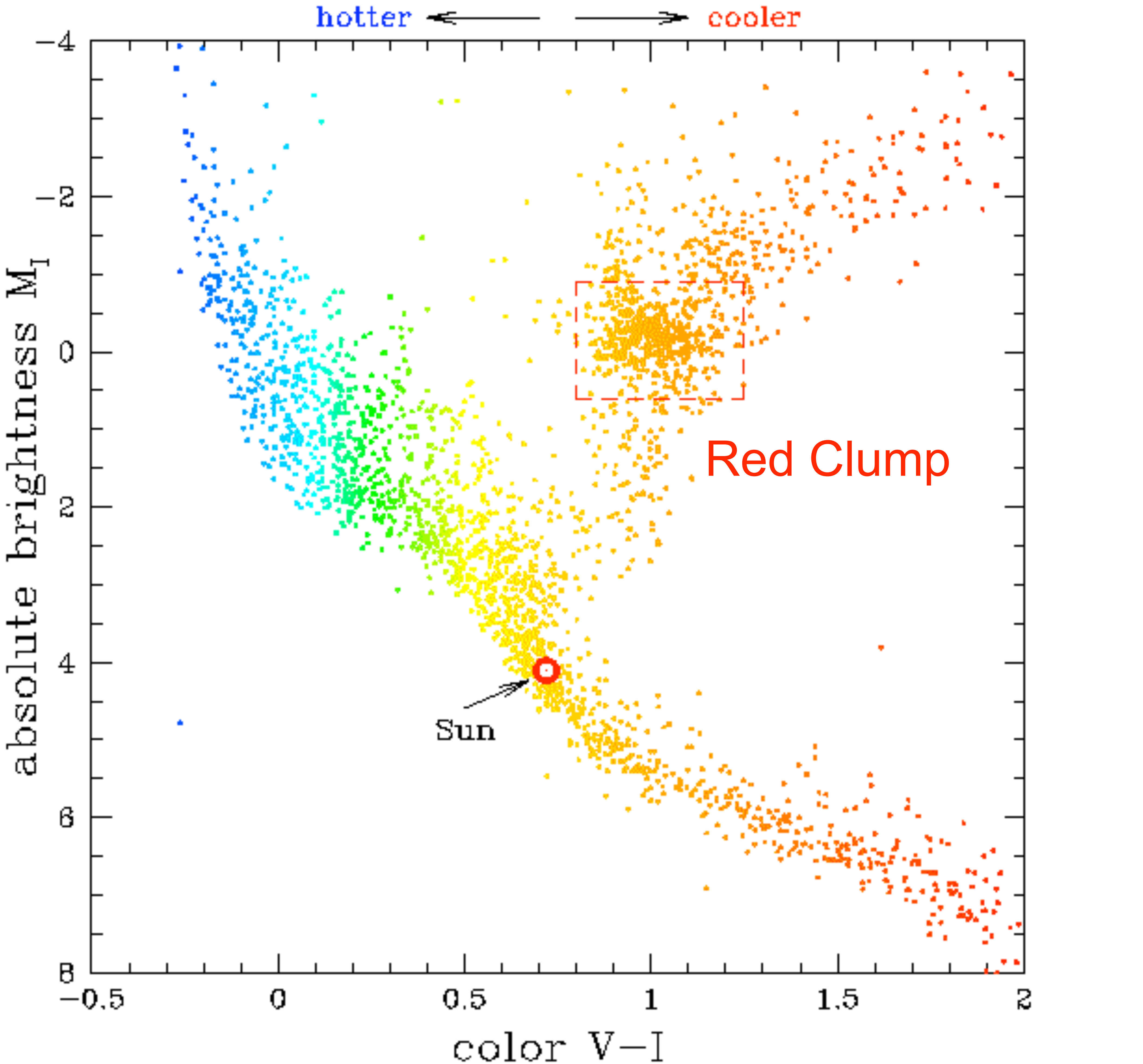
Main sequence stars were calibrated by Hipparchos distances and the Hubble Space Telescope Fine Guidance Sensor
Red clump (He burning) **stars**.
RR Lyrae Stars - variables with periods 0.2 - 0.8 days
Eclipsing binaries - v from Doppler, ellipticity from v(t), radius of primary from duration of eclipse, T from spectrum, gives $L = \sigma T^4 \pi R^2$
Cepheid variables - bright variable stars with periods 2 - 45 days

Henrietta Swan Leavitt in 1912 discovered the Cepheid period-luminosity relation in the SMC, now derived mainly from the LMC. This was the basis for Hubble's 1923 finding that M31 is far outside the Milky Way. Best value today for the LMC distance modulus m - M = 18.50 (see Weinberg, *Cosmology*, p. 25), or d_{LMC} = 50.1 kpc.

Hertzsprung-Russell Diagram



Hertzsprung-Russell Diagram



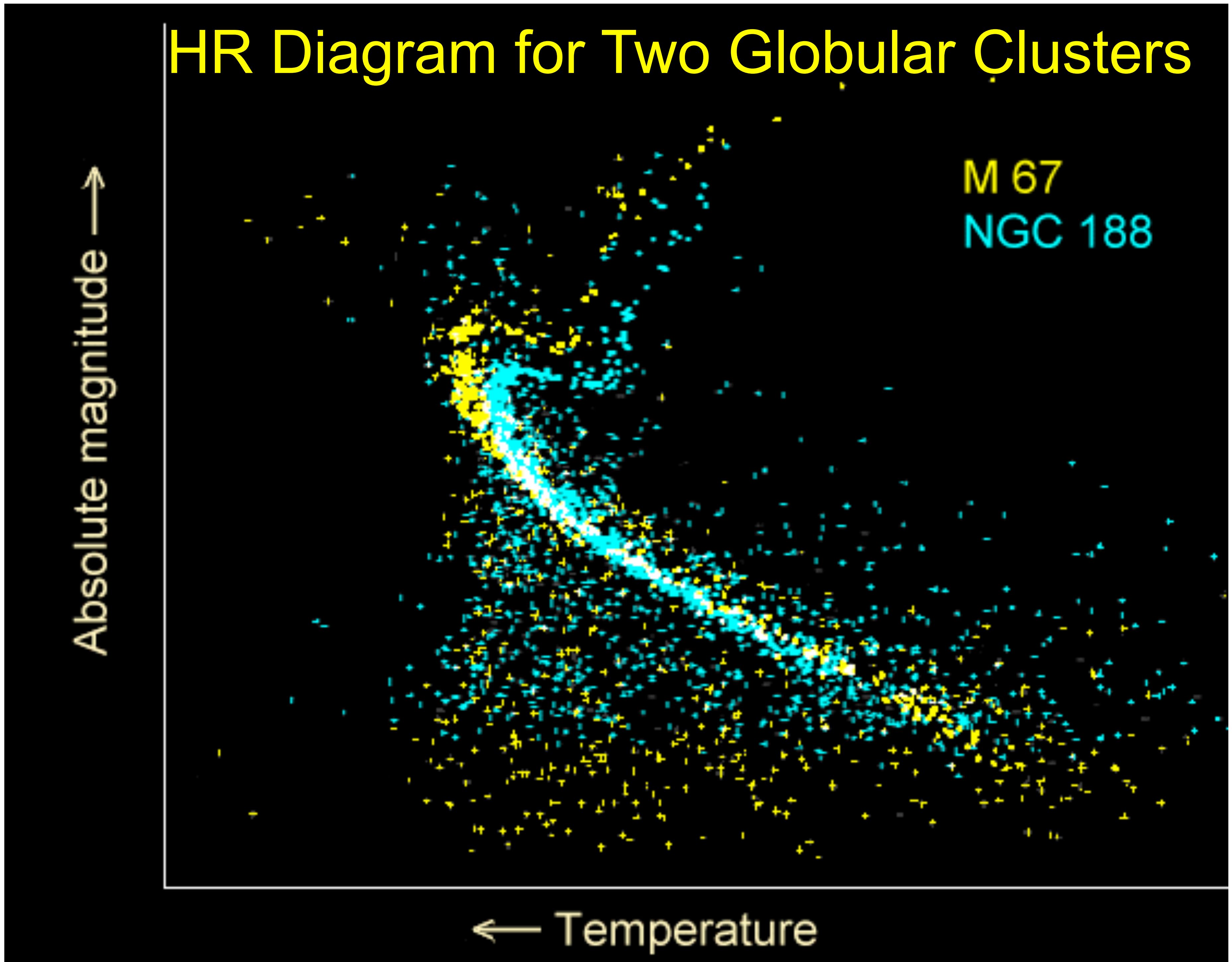
Secondary Distance Indicators

- Tully-Fisher relation
- Faber-Jackson relation
- Fundamental plane
- Type Ia supernovae
- Surface brightness fluctuations

The Age of the Universe

In the mid-1990s there was a crisis in cosmology, because the age of the old Globular Cluster stars in the Milky Way, then estimated to be 16±3 Gyr, was higher than the expansion age of the universe, which for a critical density ($\Omega_m = 1$) universe is 9±2 Gyr (with the Hubble parameter h=0.72±0.07).

But when the data from the Hipparcos astrometric satellite became available in 1997, it showed that the distance to the Globular Clusters had been underestimated, which implied that their ages are 12±3 Gyr.



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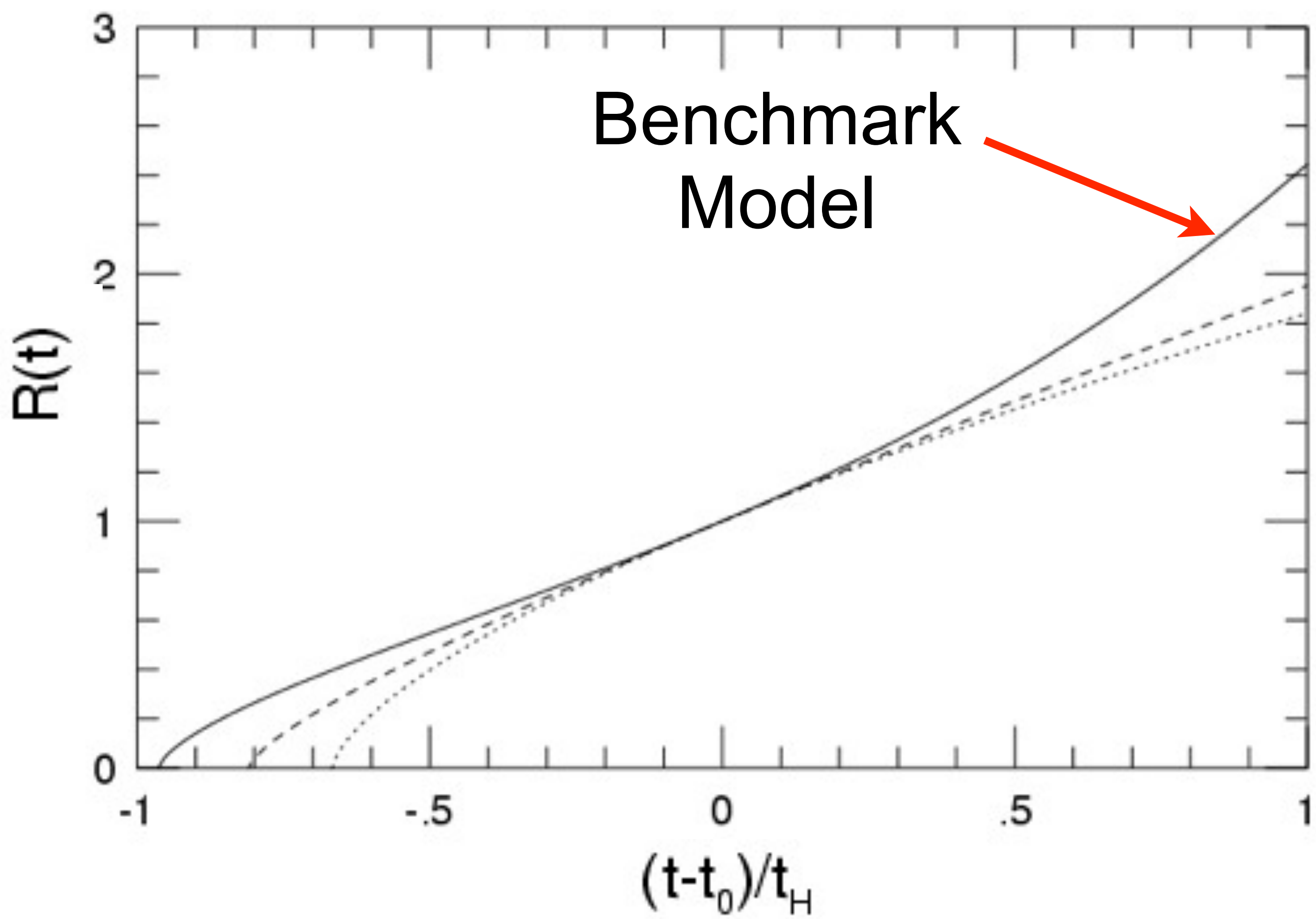
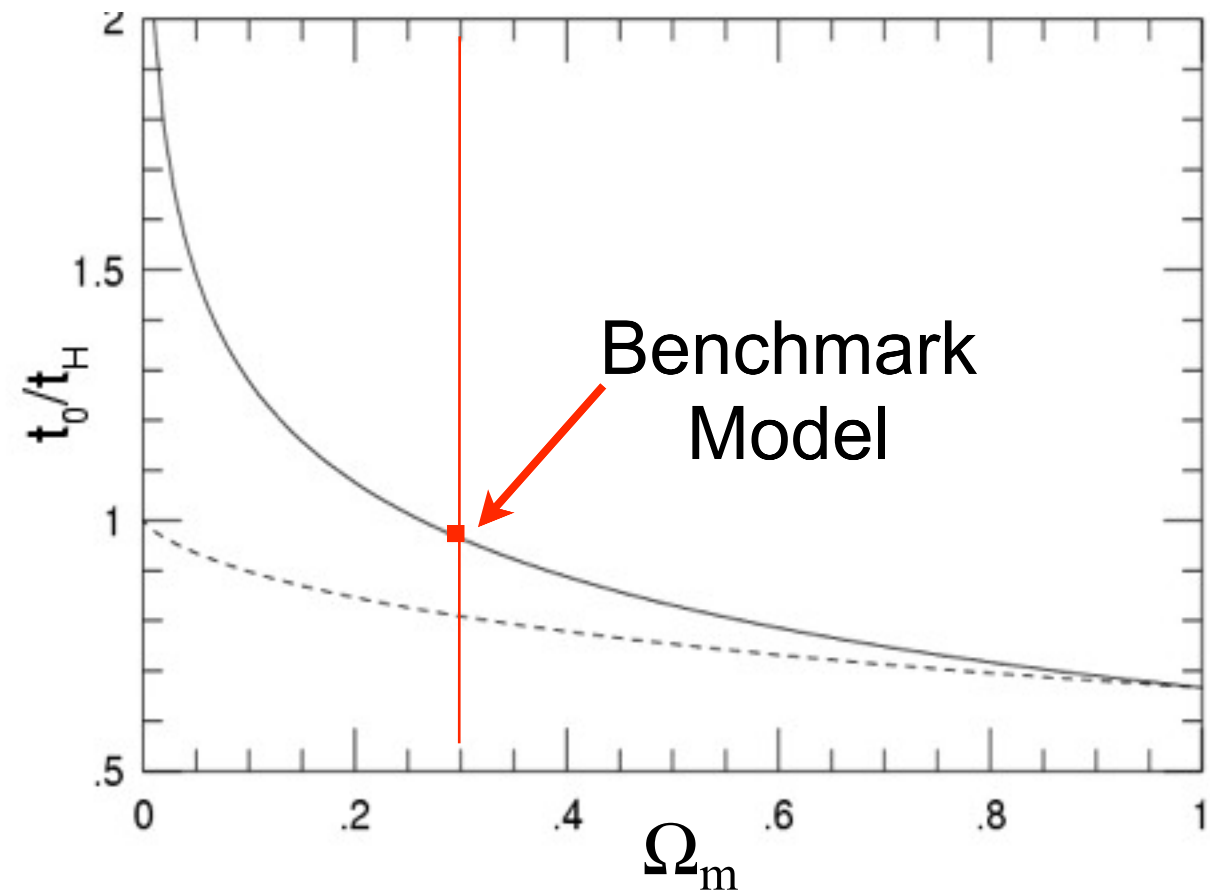
Several lines of evidence now show that the universe does not have $\Omega_m = 1$ but rather $\Omega_{tot} = \Omega_m + \Omega_\Lambda = 1.0$ with $\Omega_m \approx 0.3$, which gives an expansion age of about 14 Gyr.

Moreover, a new type of age measurement based on radioactive decay of Thorium-232 (half-life 14.1 Gyr) measured in a number of stars gives a completely independent age of 14±3 Gyr. A similar measurement, based on the first detection in a star of Uranium-238 (half-life 4.47 Gyr), gives 12.5±3 Gyr.

All the recent measurements of the age of the universe are thus in excellent agreement. It is reassuring that three completely different clocks – stellar evolution, expansion of the universe, and radioactive decay – agree so well.

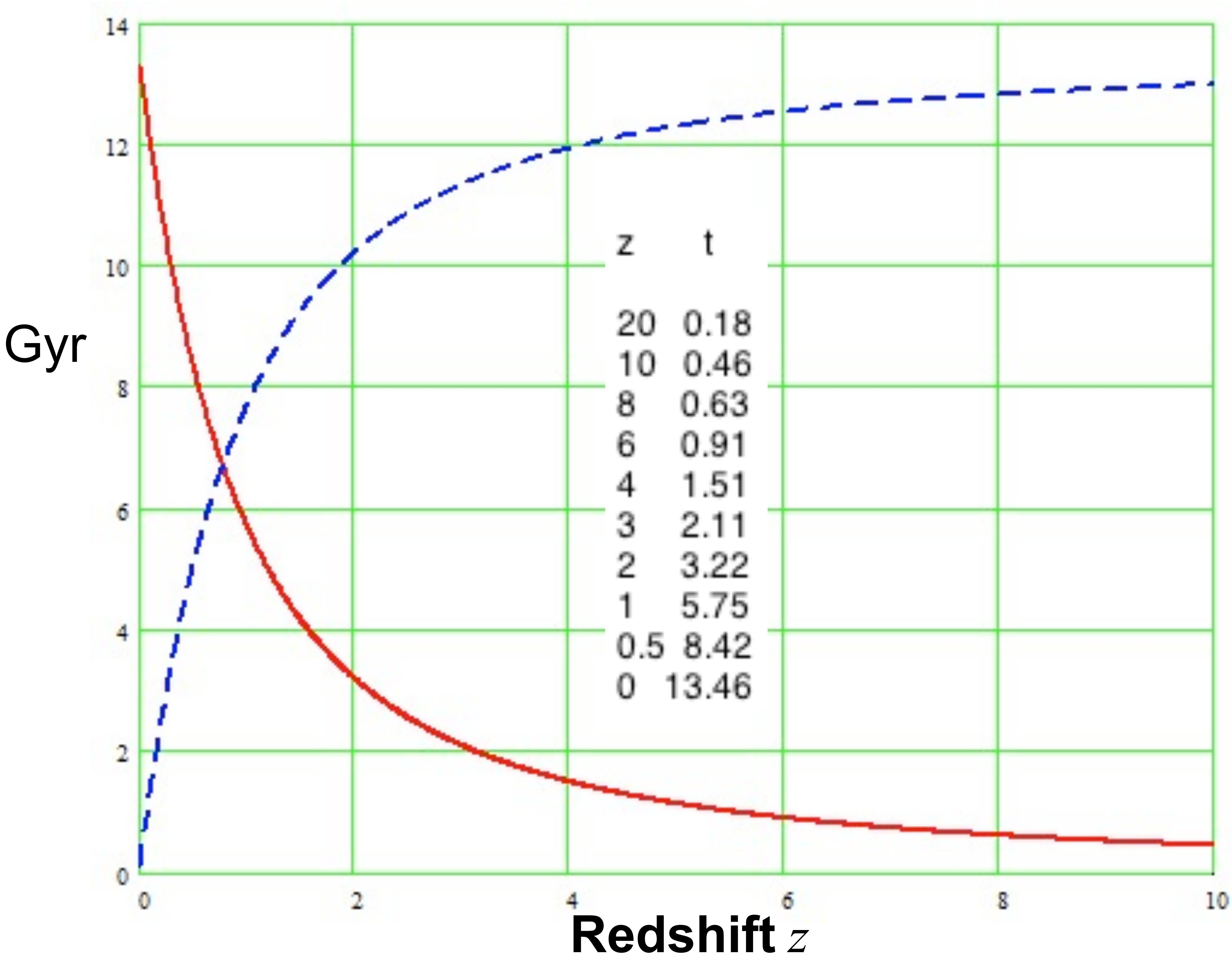
Age of the Universe t_0 in FRW Cosmologies





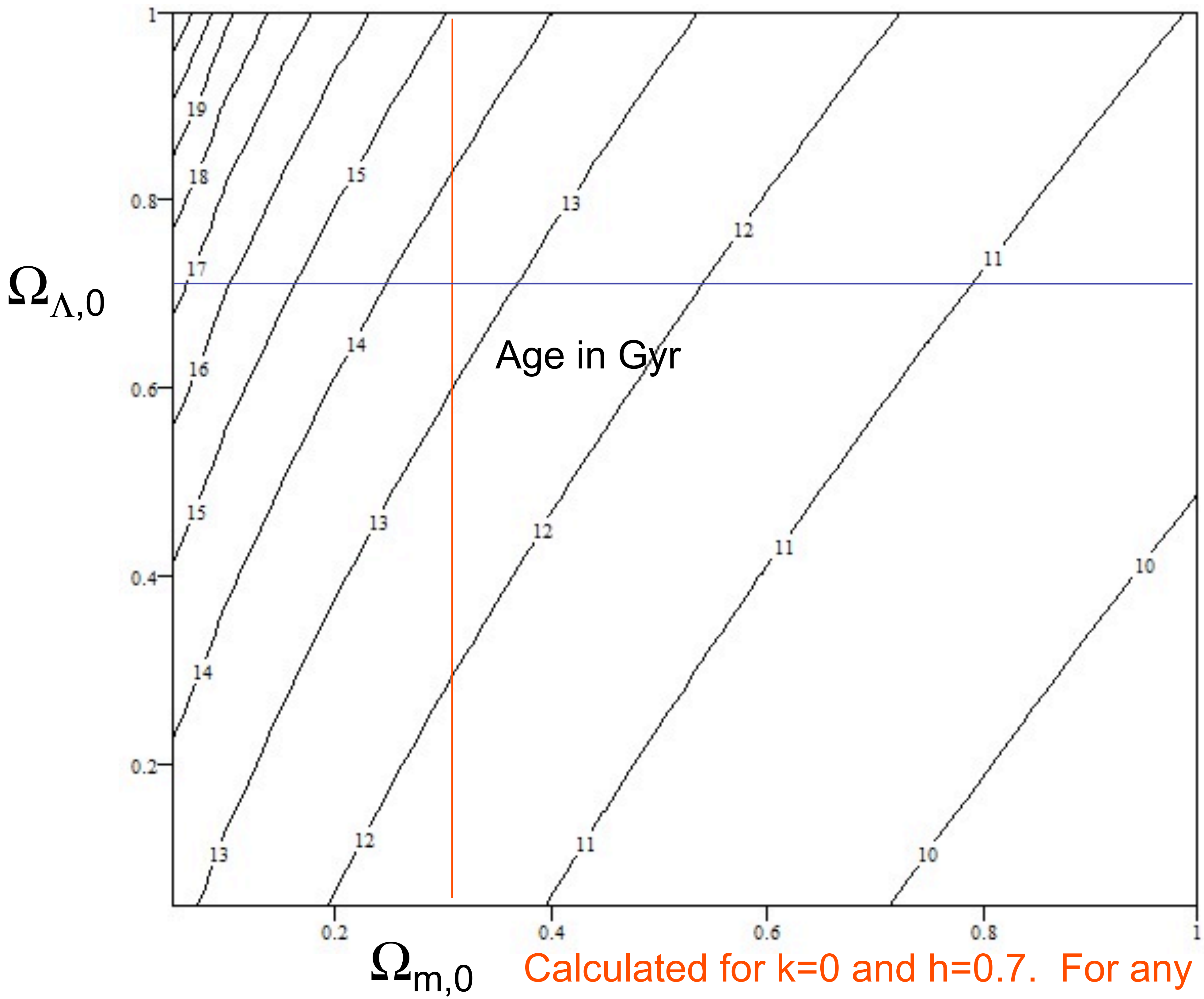
(a) Evolution of the scale factor $a(t)$ plotted vs. the time after the present $(t - t_0)$ in units of Hubble time $t_H \equiv H_0^{-1} = 9.78h^{-1}$ Gyr for three different cosmologies: Einstein-de Sitter ($\Omega_0 = 1, \Omega_\Lambda = 0$ dotted curve), negative curvature ($\Omega_0 = 0.3, \Omega_\Lambda = 0$: dashed curve), and low-Ω_0 flat ($\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$: solid curve). (b) Age of the universe today t_0 in units of Hubble time t_H as a function of Ω_0 for $\Lambda = 0$ (dashed curve) and flat $\Omega_0 + \Omega_\Lambda = 1$ (solid curve) cosmologies.

Age of the Universe and Lookback Time



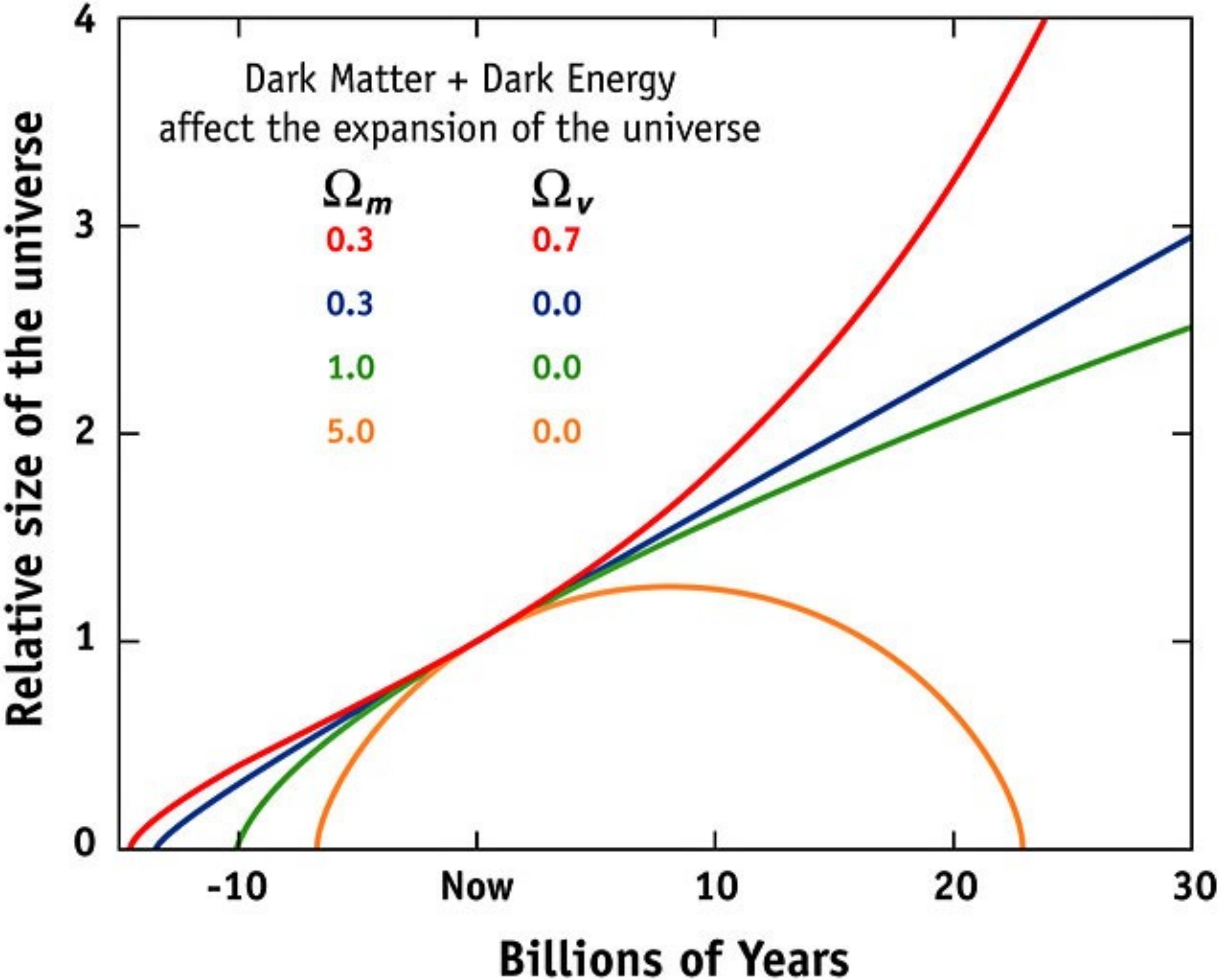
These are for the Benchmark Model $\Omega_{m,0}$=0.3, $\Omega_{\Lambda,0}$=0.7, h=0.7.

Age t_0 of the Double Dark Universe



Calculated for k=0 and h=0.7. For any other value of the Hubble parameter, multiply the age by (h/0.7).

History of Cosmic Expansion for General Ω_M & Ω_Λ



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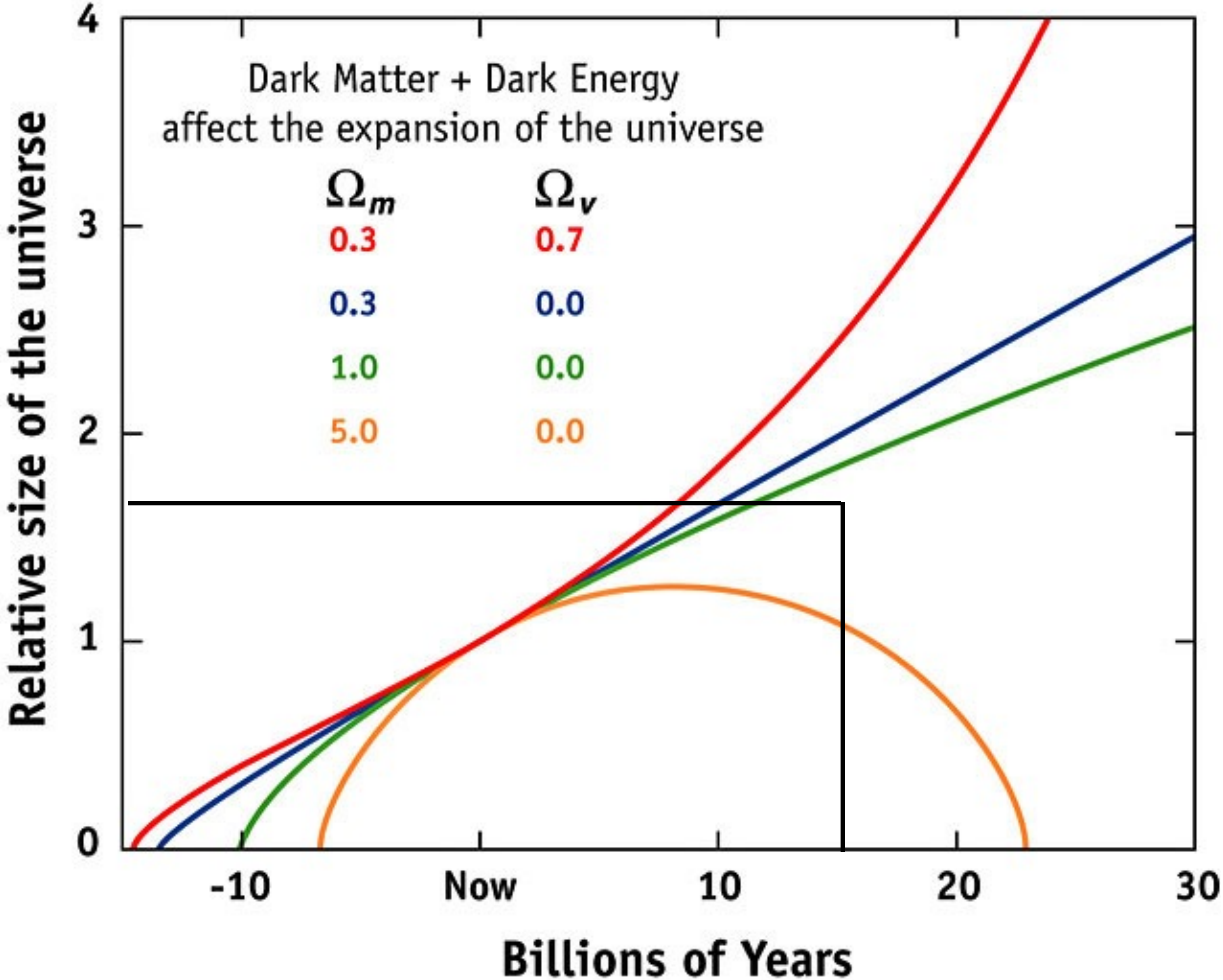
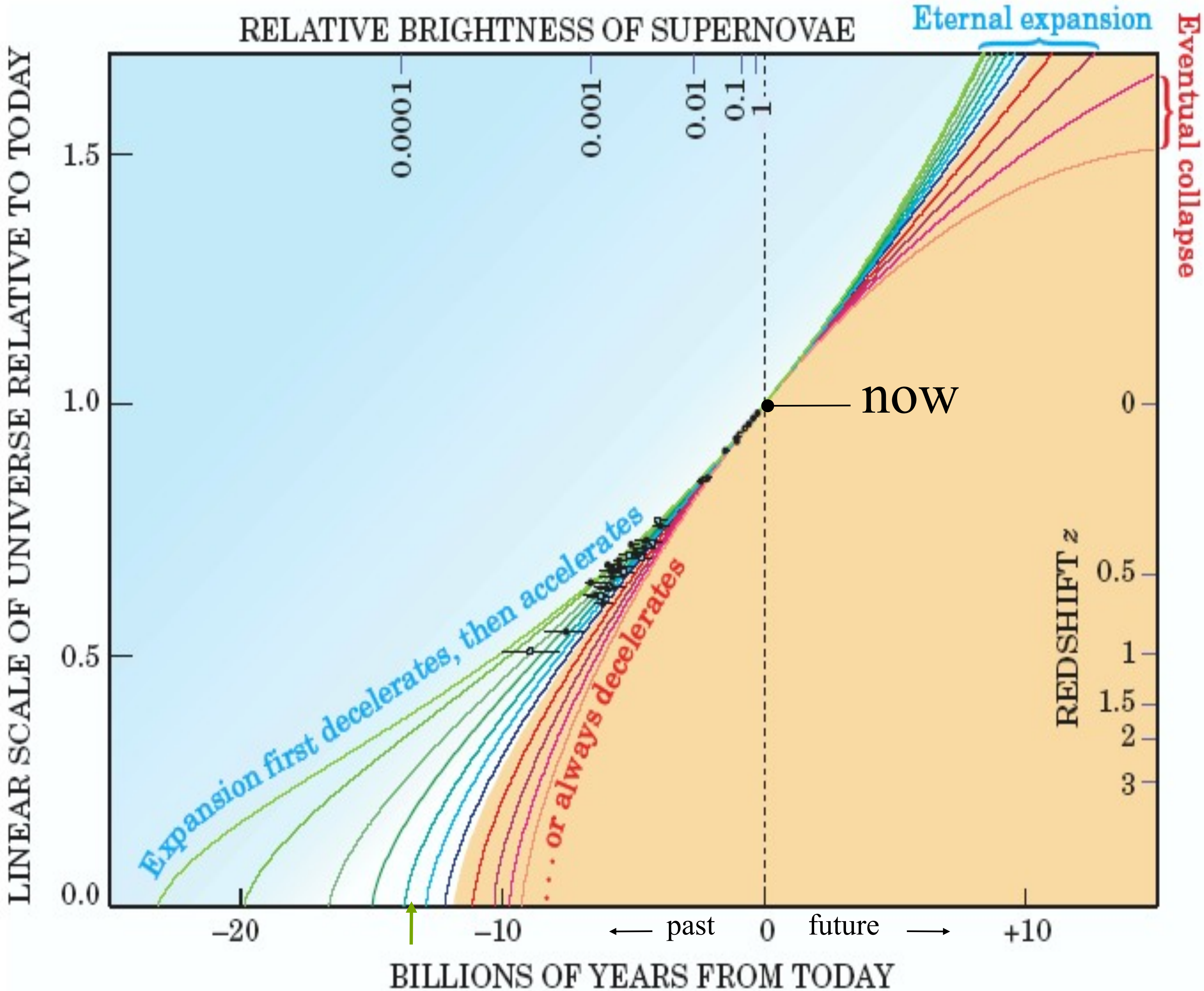


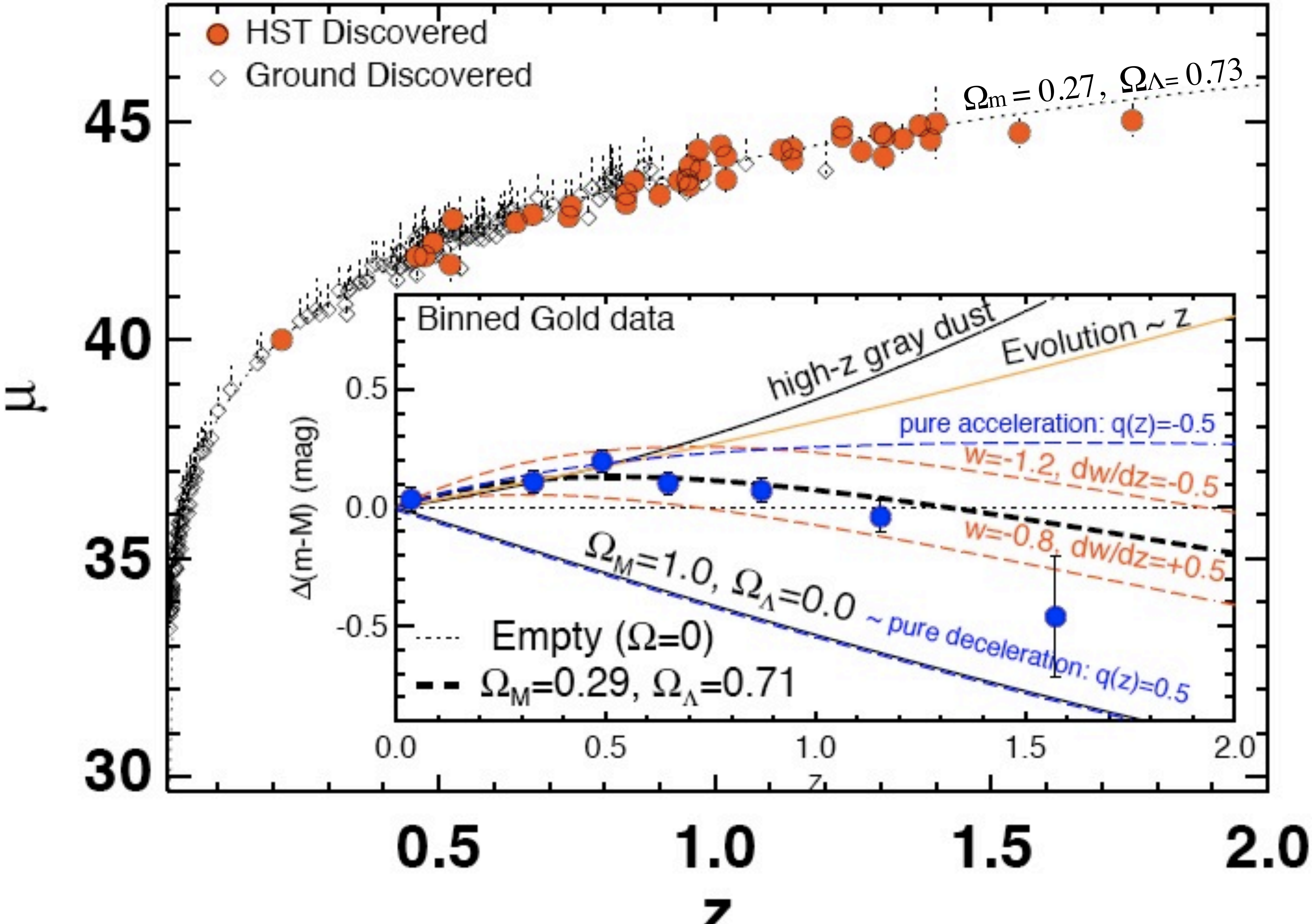
History of Cosmic Expansion for $\Omega_\Lambda = 1 - \Omega_M$

With $\Omega_\Lambda = 0$ the age of the decelerating universe would be only 9 Gyr, but $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ gives an age of 14Gyr, consistent with stellar and radioactive decay ages

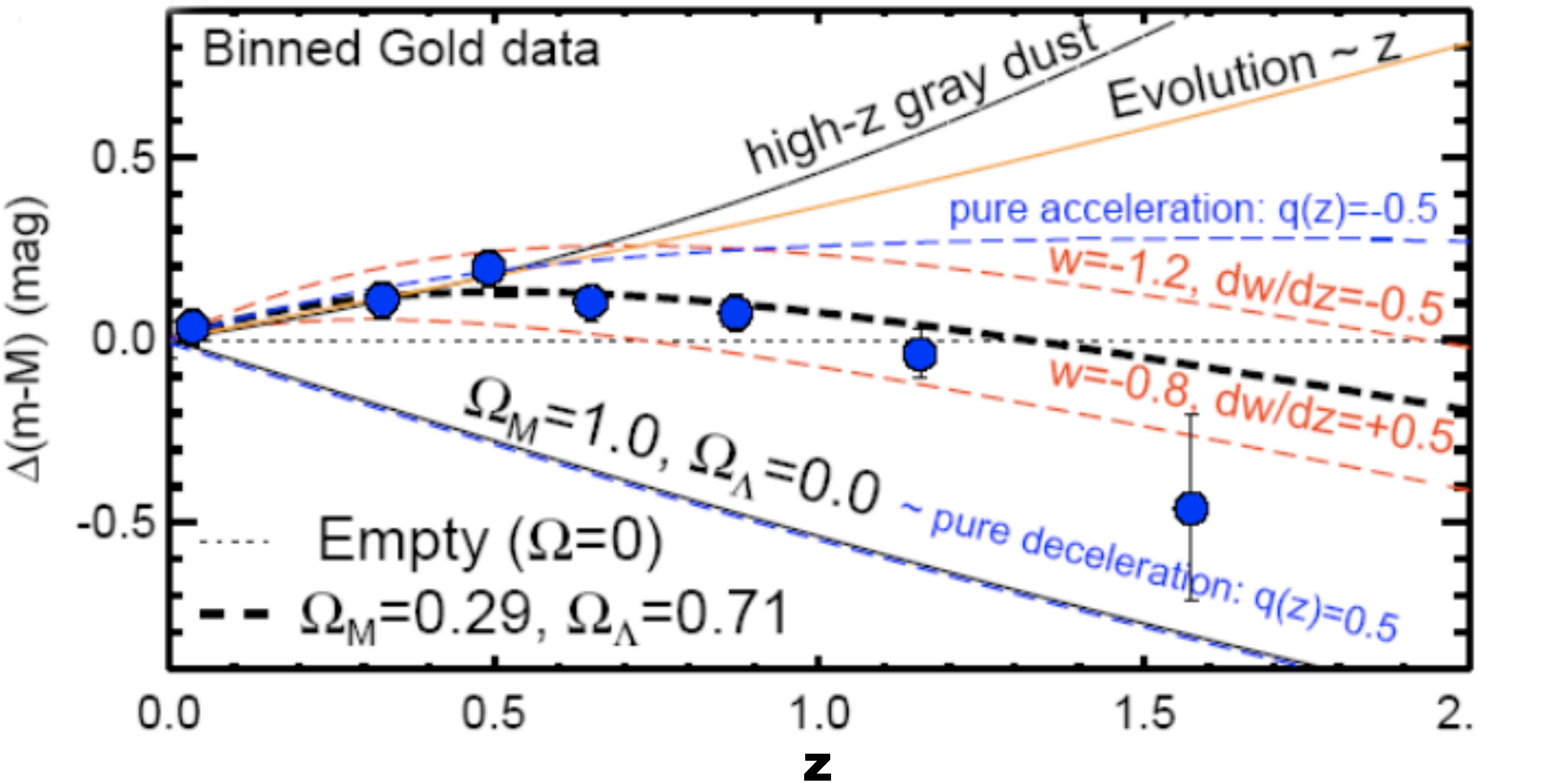
Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor R of the universe is taken to be 1 at present, so it equals $1/(1 + z)$. The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from 0.95 ρ_c (top curve) down to 0.4 ρ_c. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from 0.8 ρ_c up to 1.4 ρ_c. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.



Saul Perlmutter, ***Physics Today***, Apr 2003



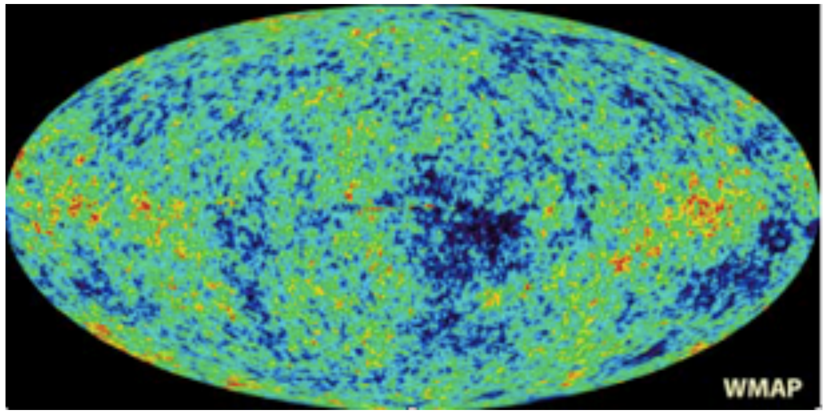
SNe Ia from ground-based discoveries in the Gold sample are shown as diamonds, HST-discovered SNe Ia are shown as filled symbols. Overplotted is the best fit for a flat cosmology: $\Omega_m = 0.27,\ \Omega_\Lambda = 0.73$.
Inset: Residual Hubble diagram and models after subtracting empty Universe model. The Gold sample is binned in equal spans of $n\Delta z = 6$ where n is the number of SNe in a bin and z is the redshift range of the bin. Fig. 6 of A. Riess et al. 2007, ApJ, 659, 98.



From the A. Riess et al. 2007, ApJ, 659, 98 Abstract: **The unique leverage of the HST high-redshift SNe Ia provides the first meaningful constraint on the dark energy equation-of-state parameter at z ≥ 1. The result remains consistent with a cosmological constant (w(z) = −1), and rules out rapidly evolving dark energy (dw/dz >> 1). The defining property of dark energy, its negative pressure, appears to be present at z > 1, in the epoch preceding acceleration, with ⌋ 98% confidence in our primary fit. Moreover, the z > 1 sample-averaged spectral energy distribution is consistent with that of the typical SN Ia over the last 10 Gyr, indicating that any spectral evolution of the properties of SNe Ia with redshift is still below our detection threshold.**

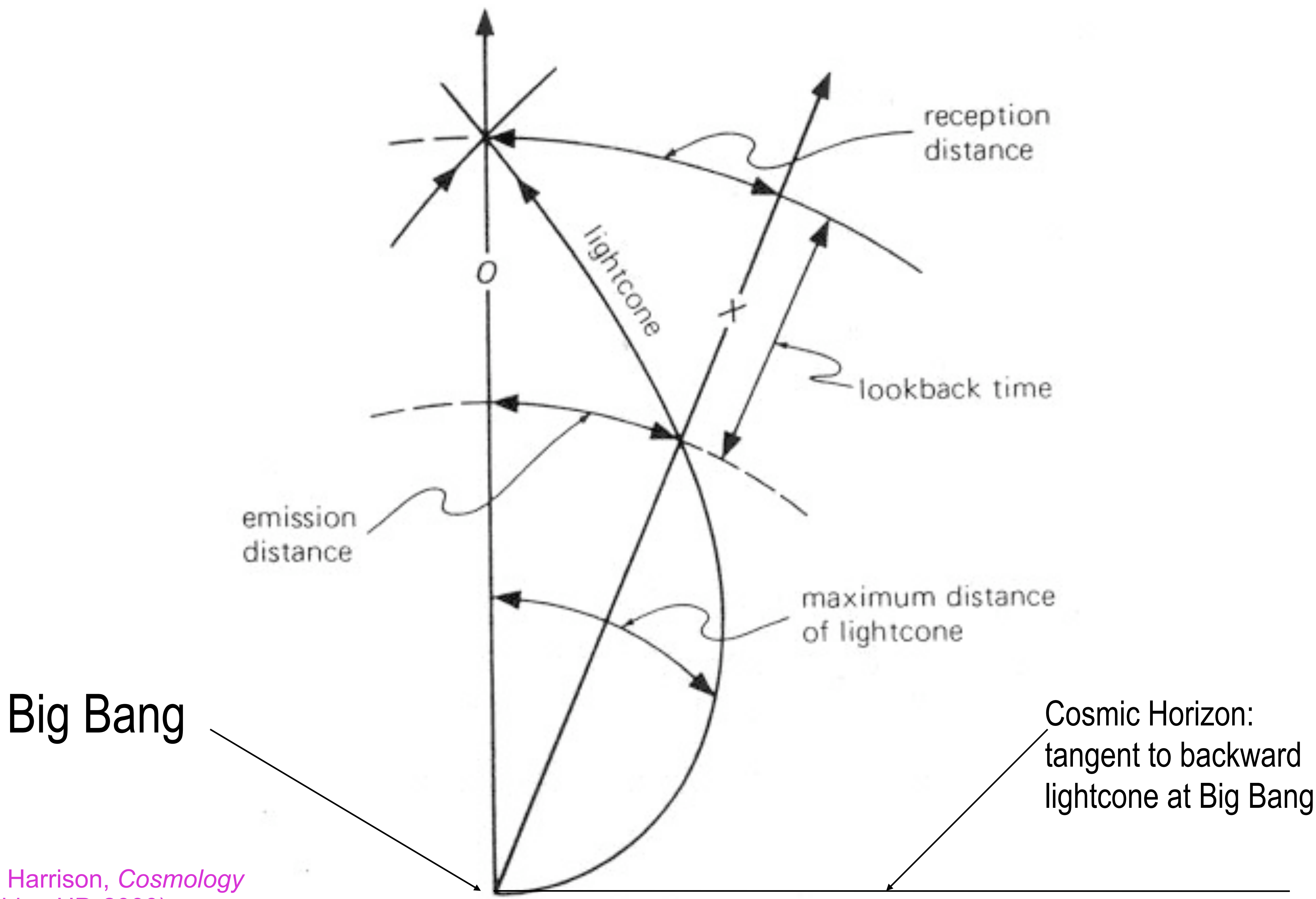
Brief History of the Universe

- Cosmic Inflation generates density fluctuations
- Symmetry breaking: more matter than antimatter
- All antimatter annihilates with almost all the matter (by ~1s)
- Big Bang Nucleosynthesis makes light nuclei (~3 min)
- Electrons and light nuclei combine to form atoms, and the cosmic background radiation fills the newly transparent universe (400,000 yr)



- Galaxies and larger structures form (~1 Gyr)
- Carbon, oxygen, iron, ... are made in stars
- Earth-like planets form around metal-rich stars
- Life somehow starts (~3.7 Gyr ago) and evolves on earth

Picturing the History of the Universe: The Backward Lightcone



From E. Harrison, *Cosmology* (Cambridge UP, 2000).

Picturing the History of the Universe: The Backward Lightcone

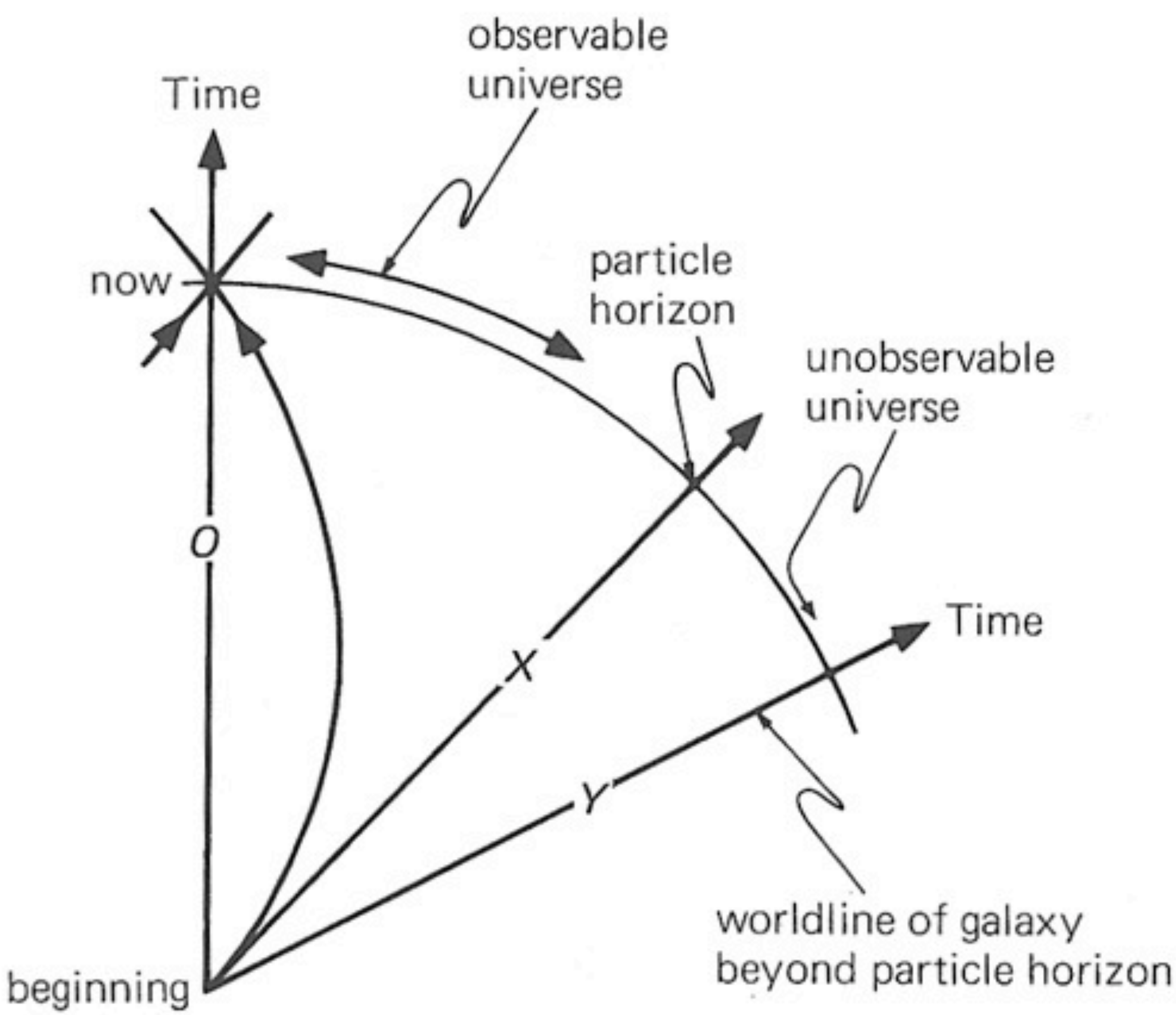


Figure 21.11. At the instant labeled "now" the particle horizon is at worldline X. In a big bang universe, all galaxies at the particle horizon have infinite redshift.

From E. Harrison, *Cosmology* (Cambridge UP, 2000).

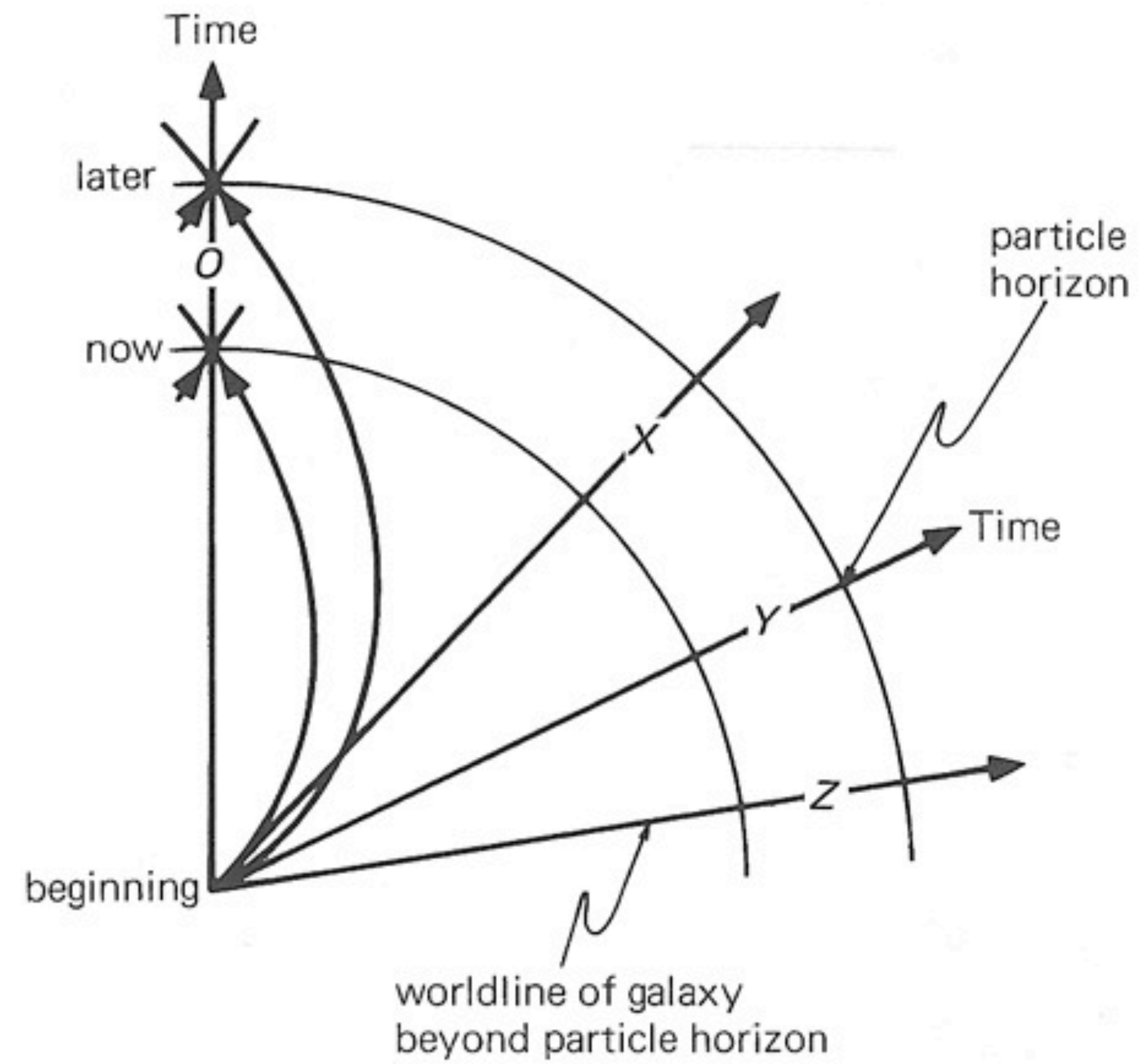


Figure 21.12. At the instant labeled "later" the particle horizon has receded to world line Y. Notice the distance of the particle horizon is always a reception distance, and the particle horizon always overtakes the galaxies and always the fraction of the universe observed increases.

Distances in an Expanding Universe

Proper distance = physical distance = d_p

$d_p(t_0)$ = (physical distance at t_0) = $a(t_0)\, r_e = r_e$

$\chi(t_e)$ = (comoving distance of galaxy emitting at time t_e)

$$\chi(t_e) = \int_0^{r_e} dr = r_e = c\int_{t_e}^{t_0} dt/a = c\int_{a_e}^{1} da/(a^2H)$$

because

$$dt = (dt/da)\, da = (a\, dt/da)\, da/a = da/(aH)$$

$d_p(t_e)$ = (physical distance at t_e) = $a(t_e)\, r_e = a_e\, r_e$

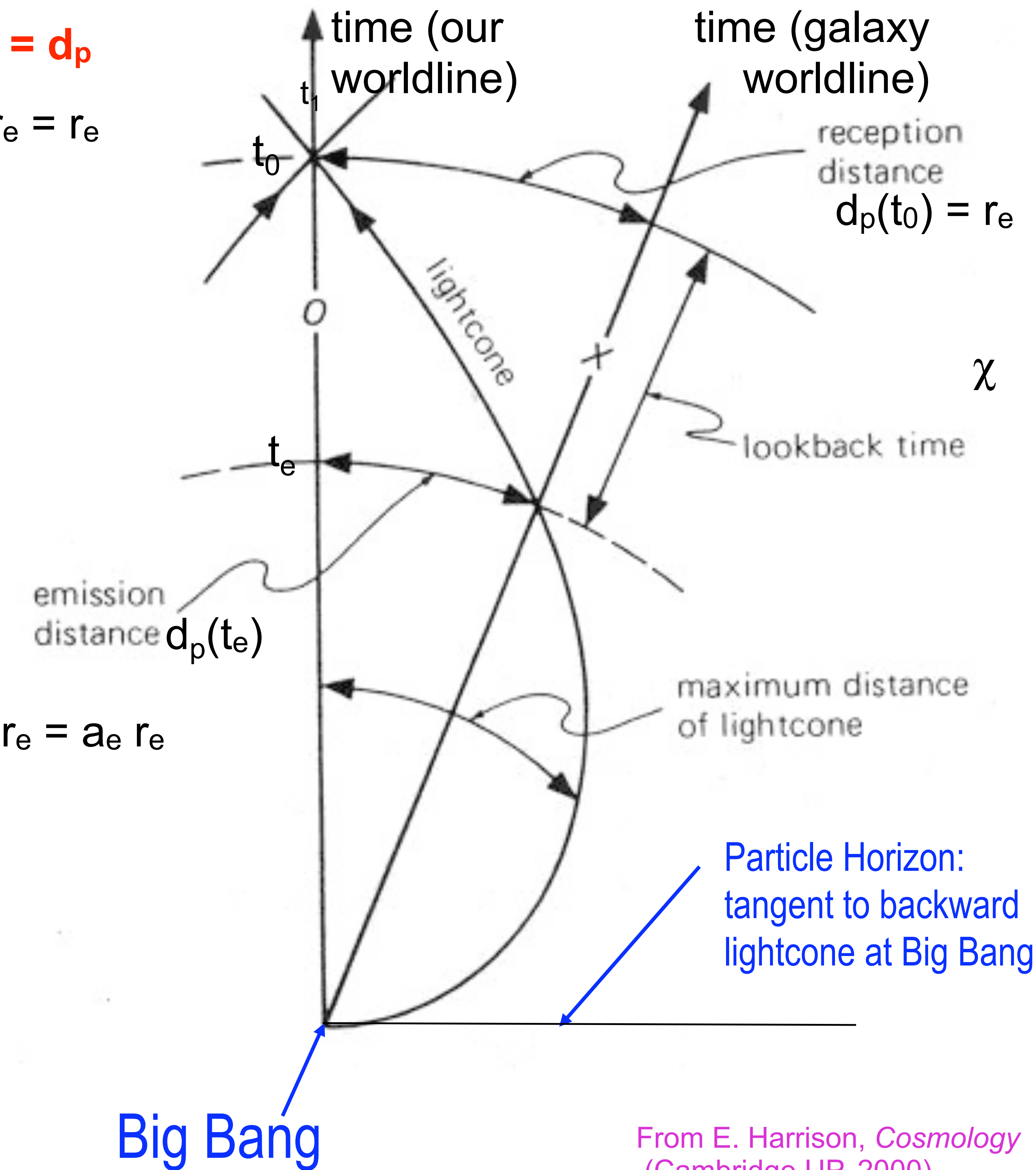
The Hubble radius $d_H = c\, H_0^{-1}$ =

= 4.29 h_{70}^{-1} Gpc = 13.97 h_{70}^{-1} Glyr

For E-dS, where $H = H_0\, a^{-3/2}$,

$$\chi(t_e) = r_e = d_p(t_0) = 2d_H\,(1 - a_e^{1/2})$$

$$d_p(t_e) = 2d_H\, a_e\,(1 - a_e^{1/2})$$



From E. Harrison, *Cosmology* (Cambridge UP, 2000).

Horizons

$$ds^2 = dt^2 - dX^2 = dt^2 - R^2 dx^2 = R^2(d\eta^2 - dx^2)$$

conformal time $d\eta = dt/R$ comoving coord. $dx = dX/R$

PARTICLE HORIZON

Spherical surface that at time t separates ***worldlines*** into observed vs. unobserved

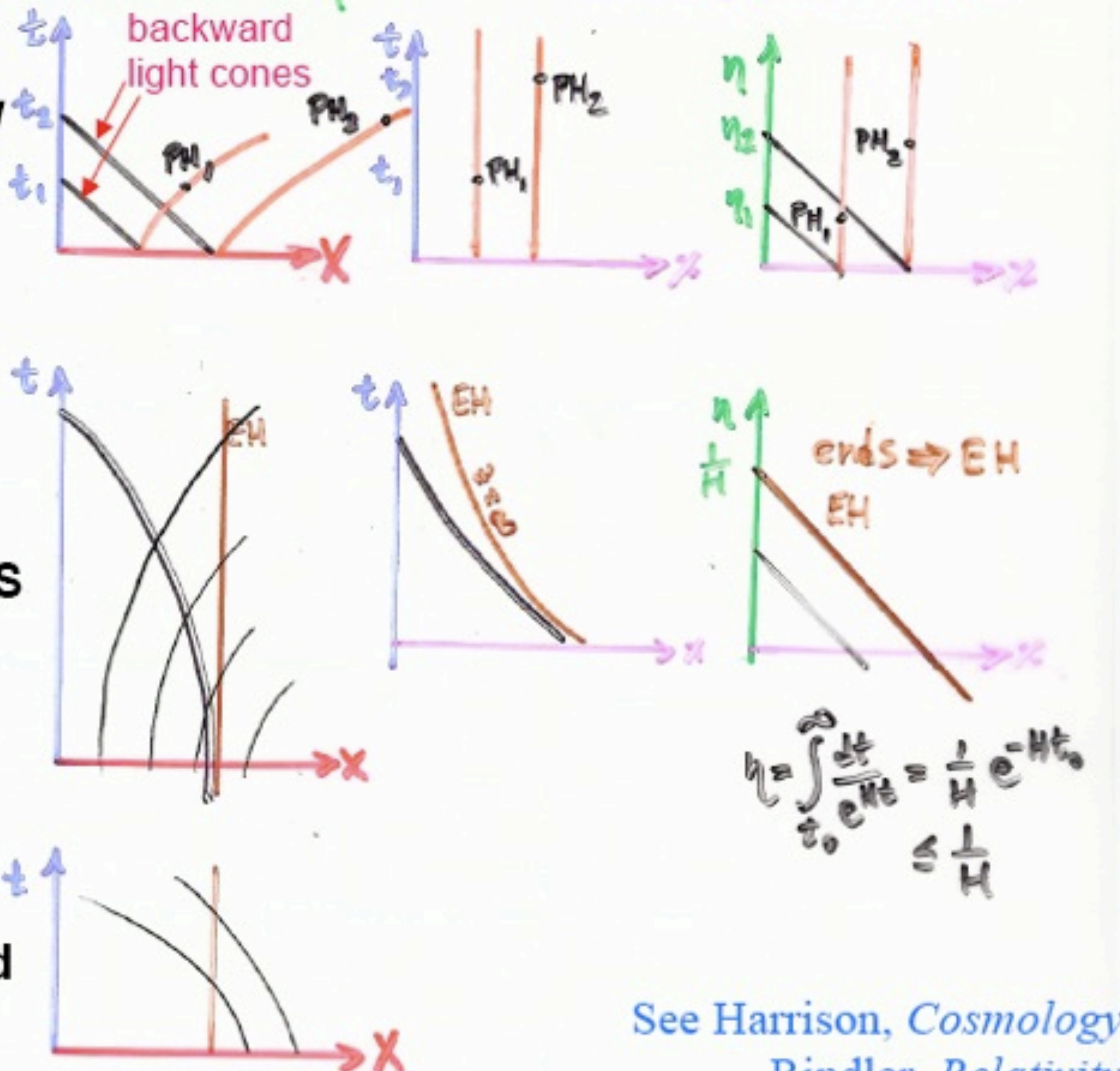
EVENT HORIZON

Backward lightcone that separates *events* that will someday be observed from those never observed

FRW

deS

Schwarzschild



$$\eta = \int_{t_0}^{\infty} \frac{dt}{e^{Ht}} = \frac{1}{H} e^{-Ht_0} \leq \frac{1}{H}$$

See Harrison, *Cosmology*
Rindler, *Relativity*

Our Particle Horizon

FRW: $ds^2 = -c^2\, dt^2 + a(t)^2\, [dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\ \ d\phi^2]$ for curvature K=0 so $\sqrt{g_{rr}} = a(t)$

Particle Horizon

$$\mathbf{d_p}(\text{horizon}) = (\text{physical distance at time } t_0) = a(t_0)\, r_p = r_p$$

$$\mathbf{d_p}(\text{horizon}) = \int_0^{r_{horizon}} dr = r_{horizon} = c\int_0^{t_0} dt/a = c\int_0^{1} da/(a^2 H)$$

For E-dS, where $H = H_0\, a^{-3/2}$,

$$r_{horizon} = \lim_{a_e \to 0} 2d_H\, (1 - a_e^{1/2}) = 2d_H =$$

$$= 8.58\ h_{70}^{-1}\ \text{Gpc} = 27.94\ h_{70}^{-1}\ \text{Glyr}$$

For the Benchmark Model with h=0.70,

$r_{horizon}$ = 13.9 Gpc = 45.2 Glyr.

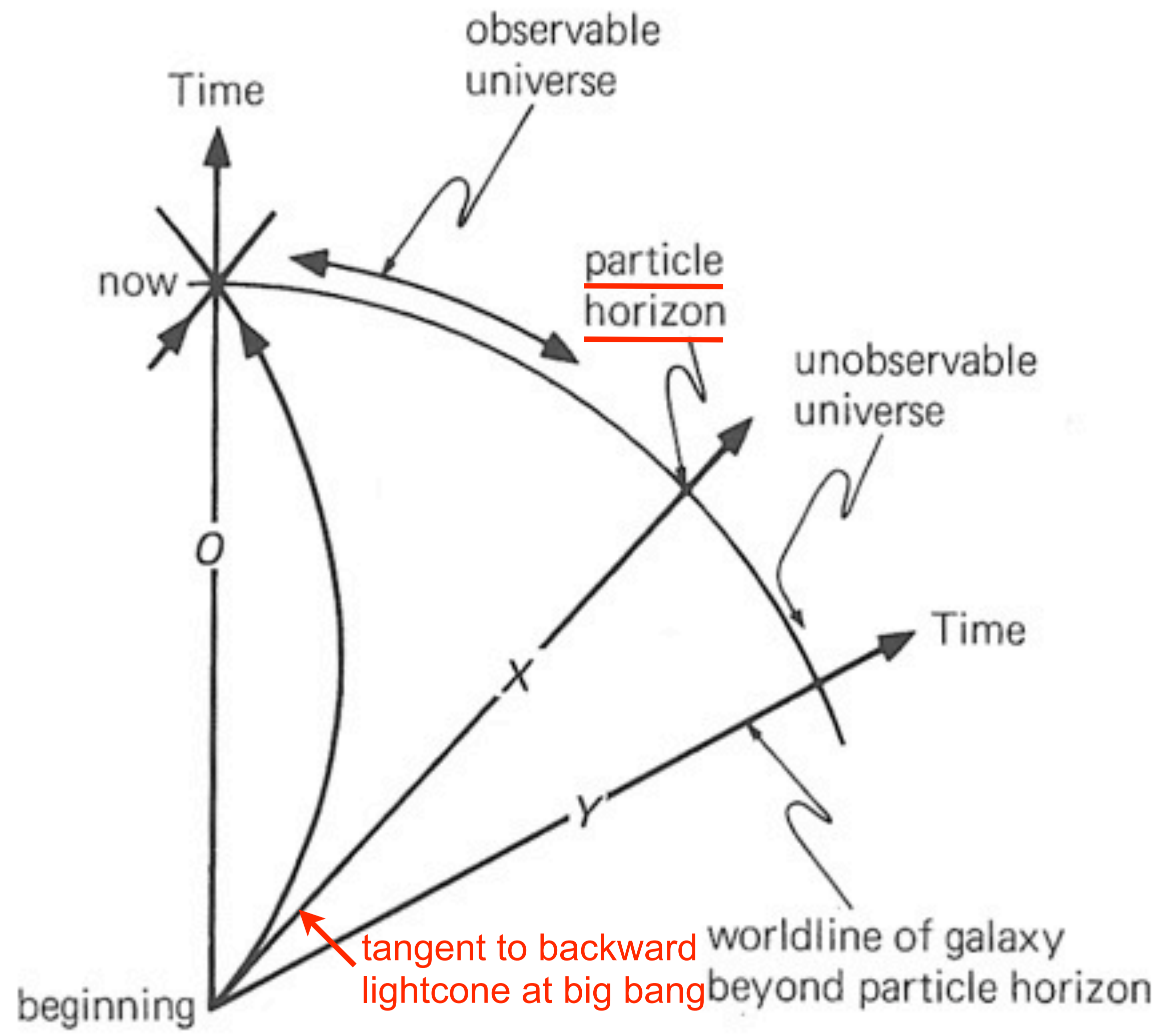
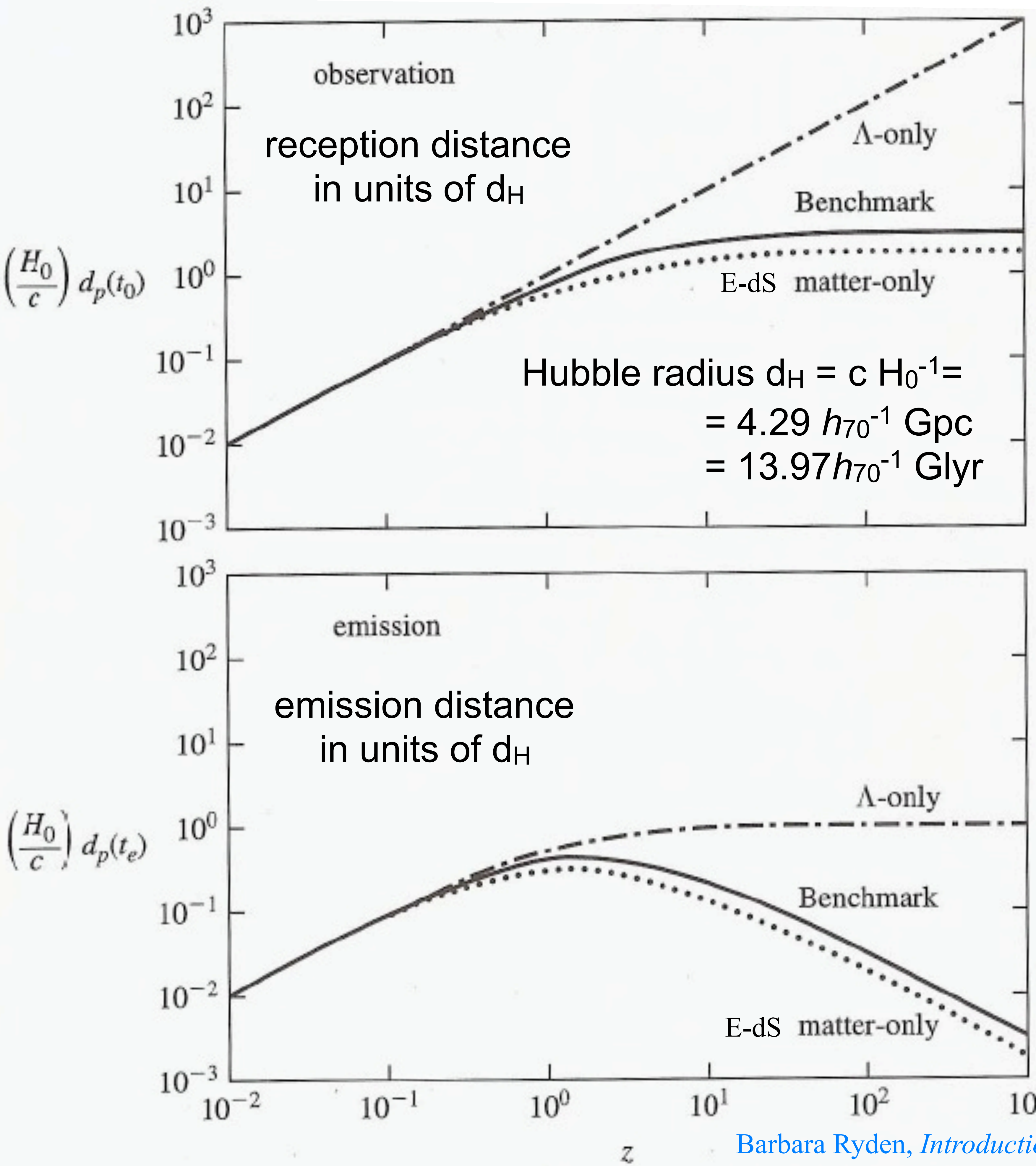
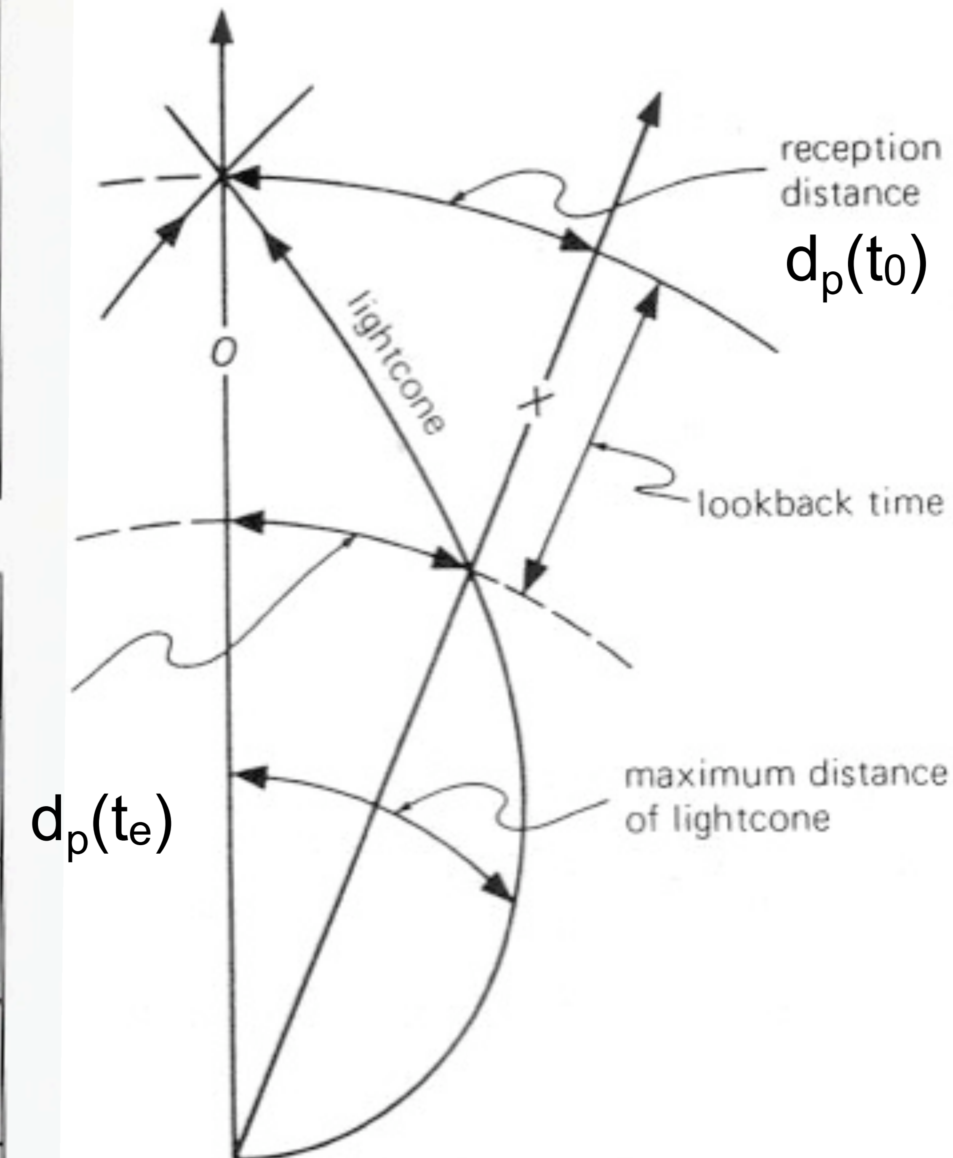


Figure 21.11. At the instant labeled "now" the particle horizon is at worldline X. In a big bang universe, all galaxies at the particle horizon have infinite redshift.

For the WMAP5 parameters h = 0.70, M = 0.28, K = 0, $r_{horizon}$ = 14.3 Gpc = 46.5 Glyr.

Distances in an Expanding Universe





For E-dS, where $H = H_0\, a^{-3/2}$,

$\chi(t_e) = r_e = d_p(t_0) = 2d_H\, (1-a_e^{1/2})$

$d_p(t_e) = 2d_H\, a_e\, (1-a_e^{1/2})$

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Distances in an Expanding Universe

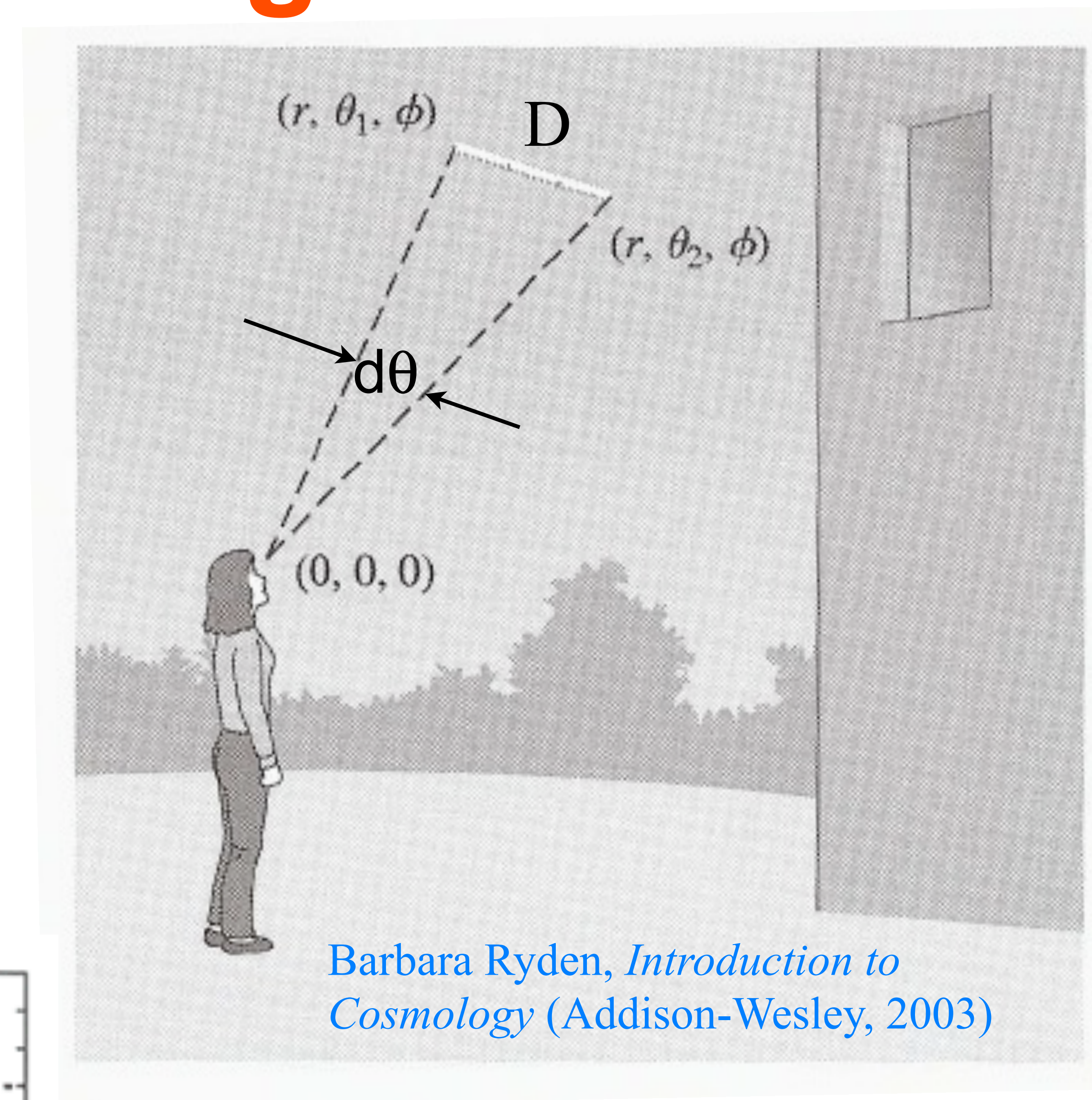
Angular Diameter Distance

From the FRW metric, the distance D across a source at comoving distance $r = r_e$ which subtends an angle $d\theta = \theta_1 - \theta_2$ is
$D = a(t)\, r\, d\theta$, or $d\theta = D/[a(t)\, r]$.

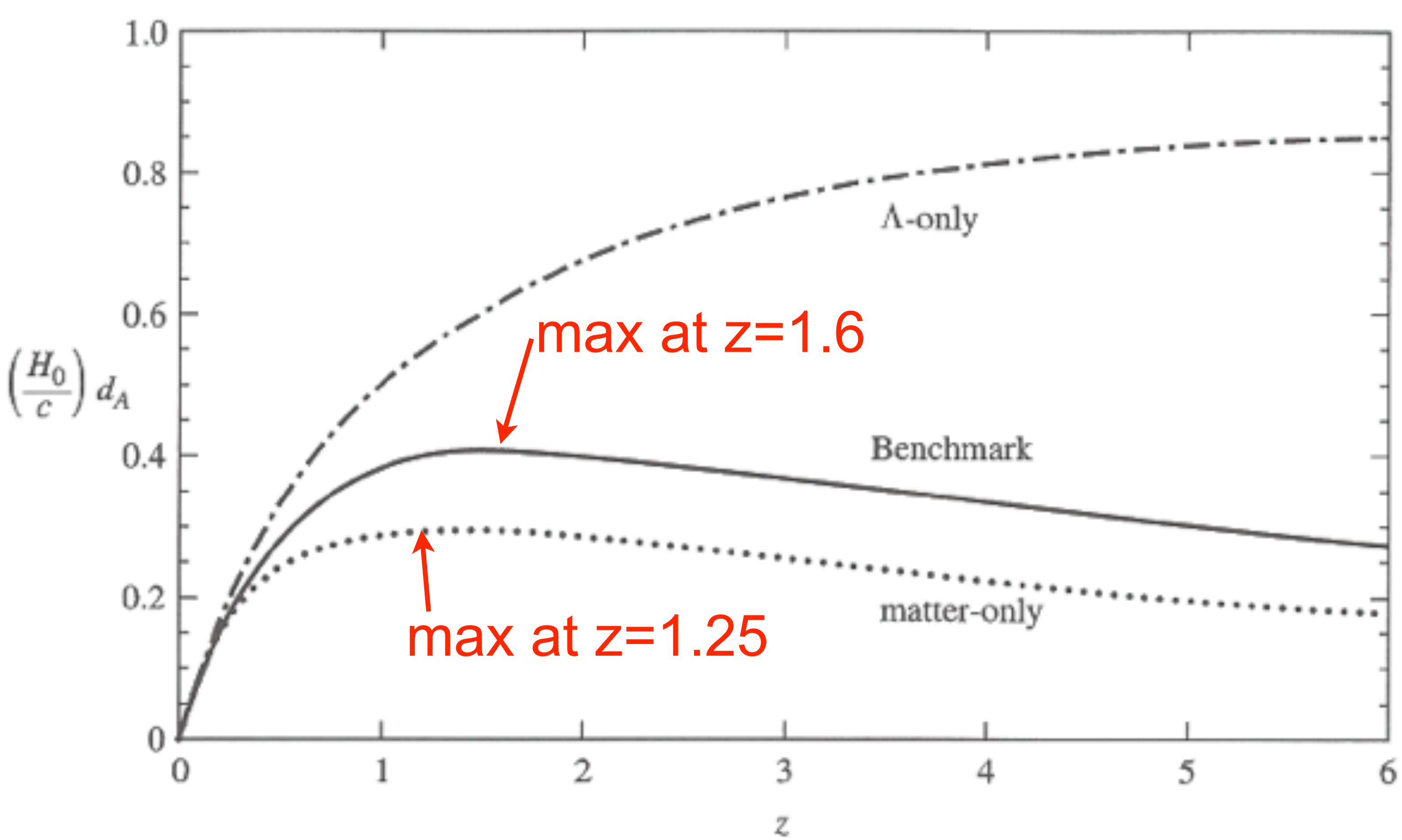
The **angular diameter distance d_A** is defined by $d_A = D/d\theta$, so

$$\mathbf{d_A = a(t_e)\, r_e = r_e/(1+z_e) = d_p(t_e)}\ .$$

This has a maximum, and $d\theta$ a minimum.



Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)



For the Benchmark Model

redshift z	D↔1 arcsec
0.1	1.8 kpc
0.2	3.3
0.5	6.1
1	8.0
2	8.4
3	7.7
4	7.0
6	5.7

Distances in an Expanding Universe

In Euclidean space, the **luminosity** L of a source at distance d is related to the **apparent luminosity** ℓ by ℓ = Power / Area = $L / 4\pi d^2$

The luminosity distance $\mathbf{d_L}$ is defined by

$$d_L = (L / 4\pi\ell)^{1/2} .$$

Weinberg, *Cosmology*, pp. 31-32, shows that in FRW

$$\ell = \text{Power/Area} = L / 4\pi d_L^2$$

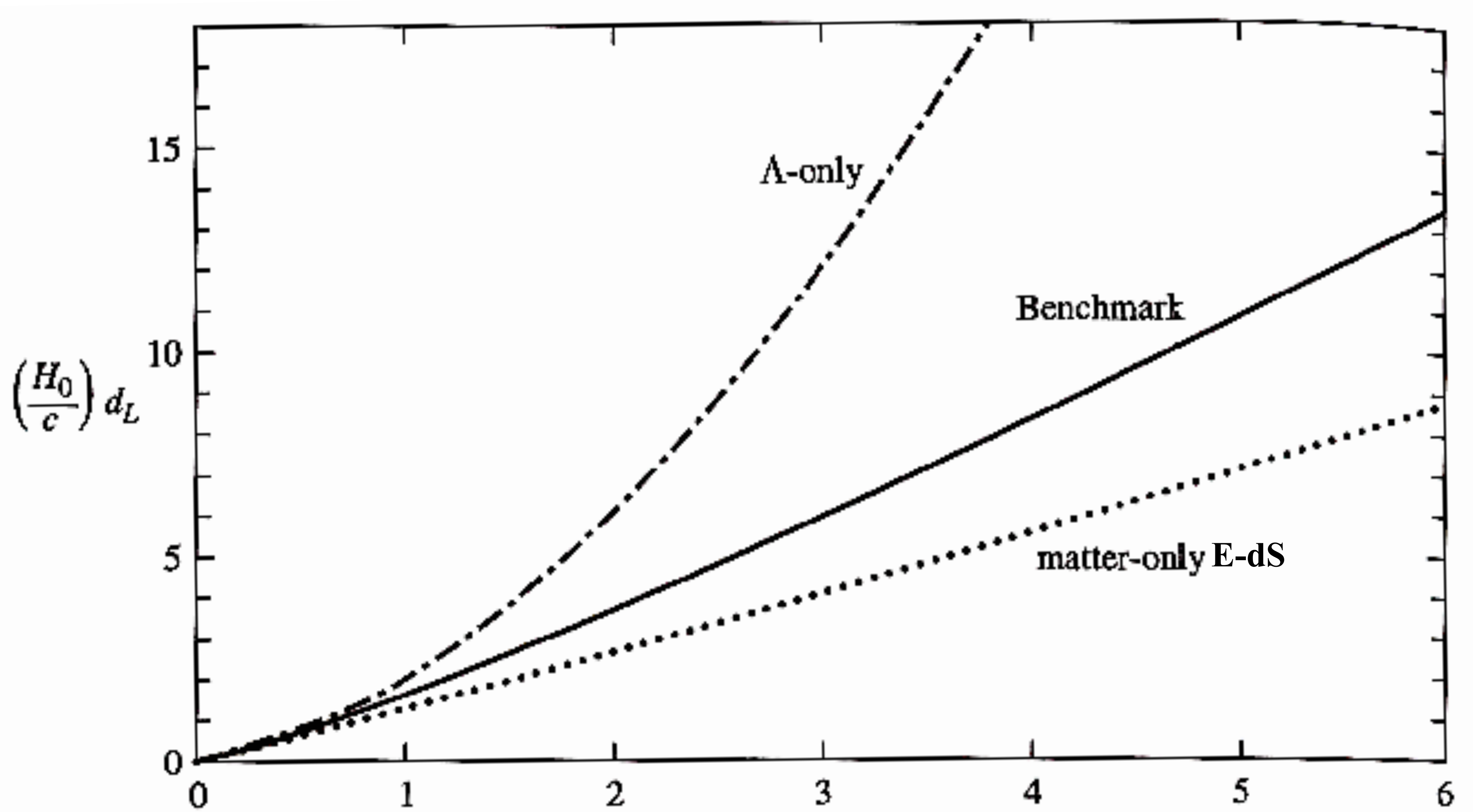
fraction of photons reaching unit area at t_0

$$= L\,[a(t_1)/a(t_0)]^2 / [4\pi d_p(t_0)^2] = L\,a(t_1)^2 / 4\pi r_1^2 = L / 4\pi r_1^2 (1+z_1)^2$$

(redshift of each photon)(delay in arrival)

Thus

$$\mathbf{d_L = r_1/a(t_1) = r_1\,(1+z_1) = d_p(t_0)\,(1+z_1) = d_A\,(1+z_1)^2}$$



Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Summary: Distances in an Expanding Universe

FRW: $ds^2 = -c^2\,dt^2 + a(t)^2\,[dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\;\;d\phi^2]$ for curvature K=0 so $\sqrt{g_{rr}} = a(t)$

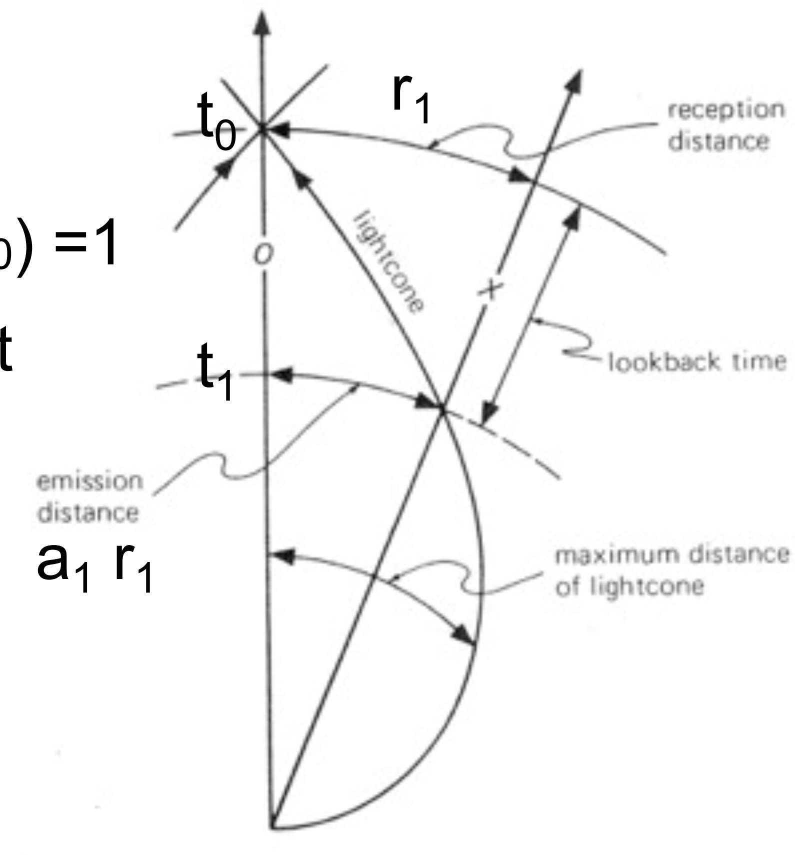
$\chi(t_1)$ = (comoving distance at time t_1) = $\int_0^{r_1} dr = r_1 = \int_{t_1}^{t_0} dt/a$ adding distances at time t_1

$d(t_1)$ = (physical distance at t_1) = $a(t_1)\,\chi(t_1) = a_1 r_1$

$\chi(t_1)$ = (comoving distance at time t_0) = r_1

$\mathbf{d_p}$ = (physical distance at time t_0) = $a(t_0)\,r_p = r_p$

since $a(t_0) = 1$



From the FRW metric above, the distance D across a source at comoving distance r_1 which subtends an angle $d\theta$ is $D = a(t_1)\,r_1\,d\theta$. The **angular diameter distance d_A** is defined by $d_A = D/d\theta$, so

$$\mathbf{d_A = a(t_1)\,r_1 = r_1/(1+z_1)}$$

In Euclidean space, the **luminosity** L of a source at distance d is related to the **apparent luminosity** ℓ by

$$\ell = \text{Power/Area} = L/4\pi d^2$$

χ (t₁) = (comoving distance at tim

so the **luminosity distance d_L** is defined by $d_L = (L/4\pi\ell)^{1/2}$.

Weinberg, *Cosmology*, pp. 31-32, shows that in FRW

$$\ell = \text{Power/Area} = L\,[a(t_1)/a(t_0)]^2\,[4\pi a(t_0)^2\,r_1^2]^{-1} = L/4\pi d_L^2$$

Thus

$$\mathbf{d_L = r_1/a(t_1) = r_1\,(1+z_1)}$$

fraction of photons reaching unit area at t_0

(redshift of each photon)(delay in arrival)

Velocities in an Expanding Universe

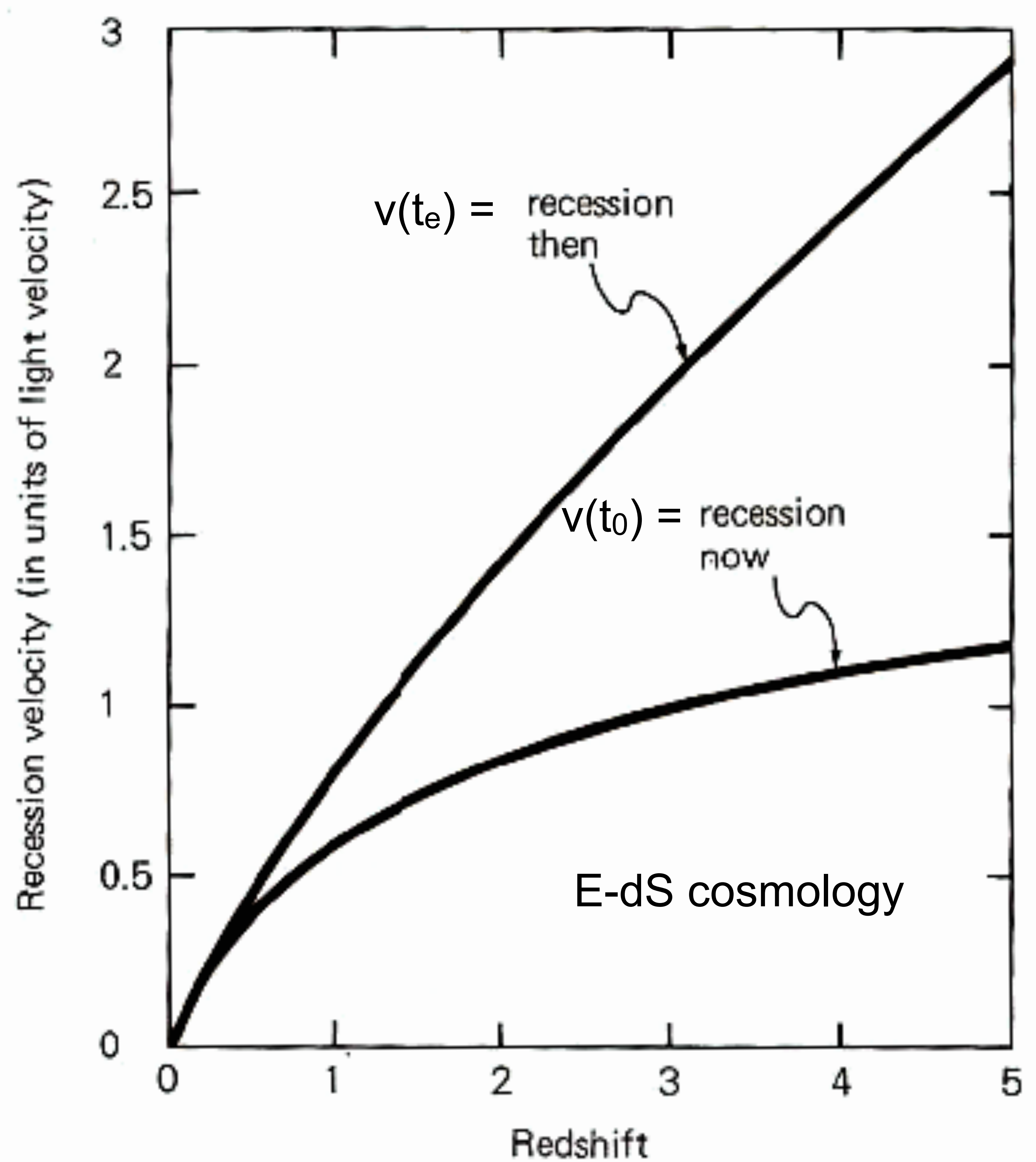
The velocity away from us now of a galaxy whose light we receive with redshift z_e, corresponding to scale factor $a_e = 1/(1 + z_e)$, is given by Hubble's law:

$$v(t_0) = H_0\, d_p(t_0)$$

The velocity away from us that this galaxy had when it emitted the light we receive now is

$$v(t_e) = H_e\, d_p(t_e)$$

The graph at right shows $v(t_0)$ and $v(t_e)$ for the E-dS cosmology.



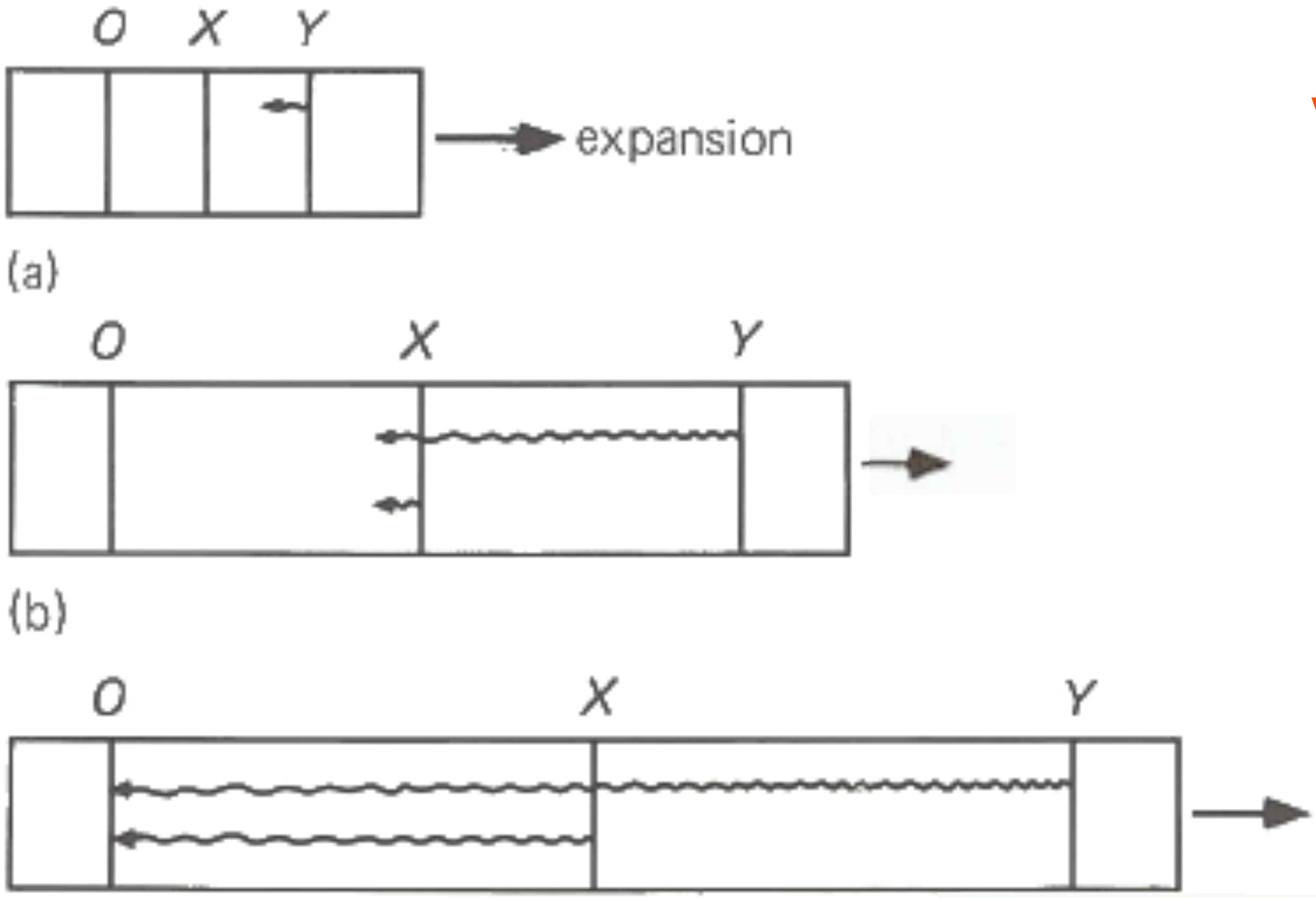
For E-dS, where $H = H_0\, a^{-3/2}$,

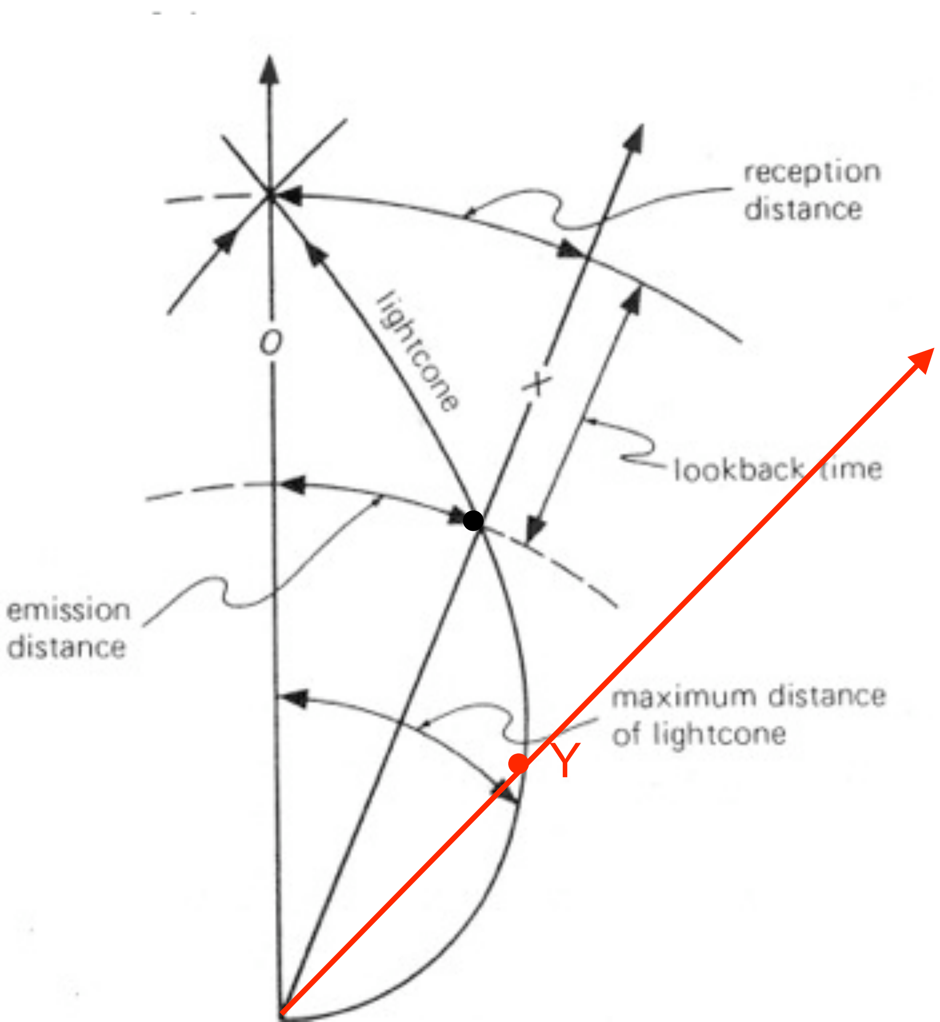
$$v(t_0) = H_0\, d_p(t_0) = 2c\,(1-a^{1/2})$$

$$v(t_e) = H_e\, d_p(t_e)$$

$$= H_0\, a_e^{-3/2}\, a_e\, 2c\,(1-a_e^{1/2})/H_0 = 2c\,(a_e^{-1/2} - 1)$$

Velocities in an Expanding Universe





From E. Harrison, *Cosmology* (Cambridge UP, 2000).

Figure 15.12. On an elastic strip let O represent our position, and X and Y the positions of two galaxies. If signals from X and Y are to reach us at the same instant, then Y, which is farther away, must emit before X. In (a), Y emits a signal. In (b), X emits a signal at a later instant when it is farther away than Y was when it emitted its signal. In (c), both signals arrive simultaneously at O. Y's signal has the greater redshift (it has been stretched more) although Y was closer than X at the time of emission. This odd situation occurs at large redshifts in all big bang universes.

Distances in a Flat (k=0) Expanding Universe

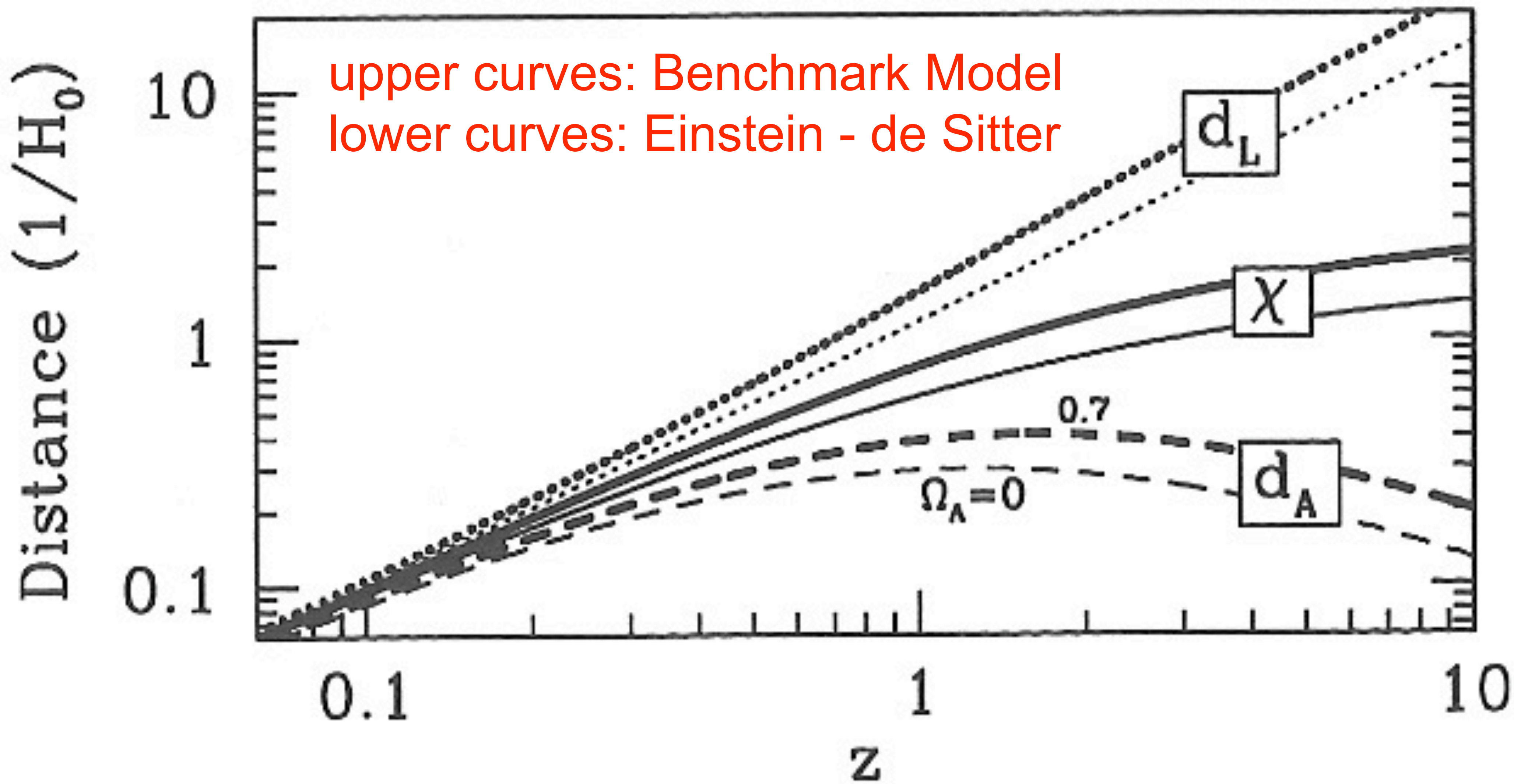


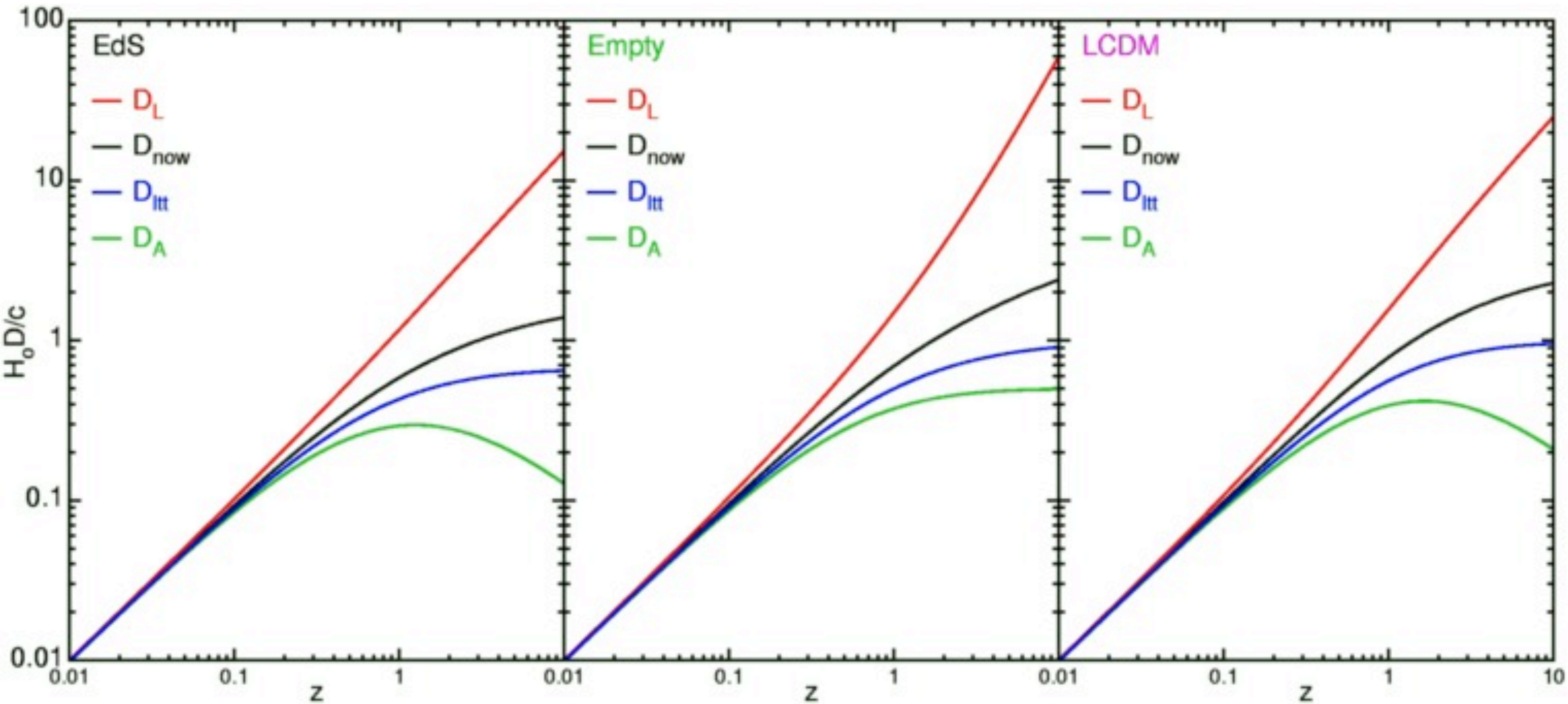
Figure 2.3. Three distance measures in a flat expanding universe. From top to bottom, the luminosity distance, the comoving distance, and the angular diameter distance. The pair of lines in each case is for a flat universe with matter only (light curves) and 70% cosmological constant Λ (heavy curves). In a Λ-dominated universe, distances out to fixed redshift are larger than in a matter-dominated universe.

Scott Dodelson, *Modern Cosmology* (Academic Press, 2003)

Distances in the Expanding Universe

D_{now} = proper distance, D_L = luminosity distance,

D_A = angular diameter distance, $D_{ltt} = c(t_0 - t_z)$



http://www.astro.ucla.edu/~wright/cosmo_02.htm#DH

Distances in the Expanding Universe:
Ned Wright's Javascript Calculator

Enter values, hit a button

70 H_o

0.30 $Omega_M$

0.83 z

Open Flat

0.70 $Omega_{vac}$

General

Open sets $Omega_{vac}$ = 0 giving an open Universe [if you entered $Omega_M$ < 1]

Flat sets $Omega_{vac}$ = 1-$Omega_M$ giving a flat Universe.

General uses the $Omega_{vac}$ that you entered.

For H_o = 70, $Omega_M$ = 0.300, $Omega_{vac}$ = 0.700, z = 0.830

- It is now 13.462 Gyr since the Big Bang.
- The age at redshift z was 6.489 Gyr.
- The light travel time was 6.974 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 2868.9 Mpc or 9.357 Gly.
- The comoving volume within redshift z is 98.906 Gpc^3.
- The angular size distance D_A is 1567.7 Mpc or 5.1131 Gly.
- This gives a scale of 7.600 kpc/".
- The luminosity distance D_L is 5250.0 Mpc or 17.123 Gly.

$H_0D_L(z=0.83)$
$=17.123/13.97$
$=1.23$

1 Gly = 1,000,000,000 light years or $9.461*10^{26}$ cm.
1 Gyr = 1,000,000,000 years.
1 Mpc = 1,000,000 parsecs = $3.08568*10^{24}$ cm, or 3,261,566 light years.

Tutorial: Part 1 | Part 2 | Part 3 | Part 4
FAQ | Age | Distances | Bibliography | Relativity

Ned Wright's home page

http://www.astro.ucla.edu/~wright/CosmoCalc.html

See also David W. Hogg, "Distance Measures in Cosmology" http://arxiv.org/abs/astro-ph/9905116