Physics 224 **Origin and Evolution of the Universe** **Spring 2010**

Homework Set 2

DUE: Thursday May 20

1. The "tired light" hypothesis states that the universe is not expanding, but that photons lose energy per unit distance

$$\frac{dE}{dr} = -KE\,.$$

Show that this hypothesis gives a distance-redshift relation that is linear in the limit $z \ll 1$. What value of K gives a Hubble constant $h = 0.7$? What are some arguments against the "tired light" hypothesis?

2. Short problems:

(a) If a neutrino has mass m_ν and decouples at $T_{\nu d} \sim 1$ MeV, show that the contribution of this neutrino and its antiparticle to the cosmic density today is (Dodelson Eq. 2.80)

$$\Omega_\nu = \frac{m_\nu}{94 h^2 \mathrm{eV}} \quad .$$

(b) Verify that $\eta_b \equiv n_b/n_\gamma$ is given by (Dodelson Eq. 3.11)

$$\eta_b = 5.5 \times 10^{-10} \left(\frac{\Omega_b h^2}{0.020} \right) \quad .$$

(c) Verify the time-temperature relation (Dodelson Eq. 3.30)

$$t = 132\,\mathrm{sec}\,(0.1\mathrm{MeV}/T)^2 \quad .$$

3. A recent cosmological speculation is that the universe may contain a quantum field called "quintessence" which has an equation of state parameter $w_Q = p_Q/\rho_Q$ with energy density ρ_Q positive (of course) but pressure p_Q negative. Suppose that the universe contains nothing but pressureless matter, i.e. with $w_m = 0$, and quintessence, with $w_Q = -3/4$. The current density parameter of matter is $\Omega_m \approx 0.3$ and that of quintessence is $\Omega_Q = 1 - \Omega_m$. At what scale factor a_{mQ} will the energy density of quintessence and matter be equal? Solve the Friedmann equation to find $a(t)$ for this universe. What is $a(t)$ in the limit $a \gg a_{mQ}$? What is the current age of the universe, expressed in terms of H_0 and $\Omega_{m,0}$?

4. Suppose that the neutron decay time were $\tau_n = 89$ s instead of $\tau_n = 890$ s, with all other physical parameters unchanged. Estimate Y_p, the primordial mass fraction of nucleons in ^{4}He, assuming that all available neutrons are incorporated into ^{4}He.

5. Suppose that there were no baryon asymmetry so that the number density of baryons exactly equaled that of anti-baryons. Determine the final relic density of (baryons + anti-baryons). At what temperature is this relic density reached?

6. There is a fundamental limitation on the annihilation cross section of a particle χ of mass m: because of unitarity, $< \sigma v >$ must be less than or equal to $\sim m^{-2}$, give or take a factor of order unity. Determine Ω_χ for a particle with $< \sigma v >= m^{-2}$. For what value of m is $\Omega_\chi = 1$? (Assume $x_f = 10$ and $g_*(m) = 100$, the nominal values in Dodelson Eq. 3.60.) Note that if m exceeds this value, $\Omega_\chi > 1$, which is ruled out. This is a strong argument against stable particles (and therefore dark matter candidates) with masses above this critical value. (See Griest and Kamionkowski 1990, Phys. Rev. Lett. 64, 615.)

7. Inflation and Horizons.

(a) Find the deviation from flatness, $1 - \Omega(z)$, at $z_{dec} = 1100$ and $z_{ns} = 10^9$ if $\Omega_m = 0.1$, $\Omega_r = 4 \times 10^{-5}$. Find the deviation if $\Omega_{tot} = \Omega_\Lambda = 0.1$ and discuss the difference.

(b) The particle horizon is the physical size of the universe in causal contact,

$$r_h(t) = a(t)\, r_c(t) = a(t) \int_0^t dt'/a(t').$$

Calculate the particle horizon size at z_{eq} and at z_{dec}, using $\Omega_m = 1$. Calculate the comoving wavelengths and masses $M = (\pi/6)\rho\lambda^3$ those correspond to today. Calculate the angle those horizons subtend today, $\theta_h = r_h(z)/r_a(z)$, using the angular distance for an Einstein-de Sitter universe, $H_0 r_a(z) = 2[(1+z)^{-1} - (1+z)^{-3/2}]$.

(c) The following plot is a handy way of understanding the concept of scales leaving and entering the Hubble volume. Why is comoving wavelength a horizontal line? What is the general cosmological interpretation of $(aH)^{-1}$? This is often informally referred to as the horizon (it is the event horizon scale for the deSitter spacetime of inflation). Identify the times at which a perturbation of comoving wavelength λ leaves/enters this "horizon". Identify the time intervals during which inflation takes place and explain what property of the curve shows this.