

Physics/Astronomy 224 Spring 2012

Origin and Evolution of the Universe

Week 2 - Part 2

Big Bang Nucleosynthesis

Joel Primack

University of California, Santa Cruz

Neutrino Decoupling and Big Bang Nucleosynthesis, Photon Decoupling, and WIMP Annihilation

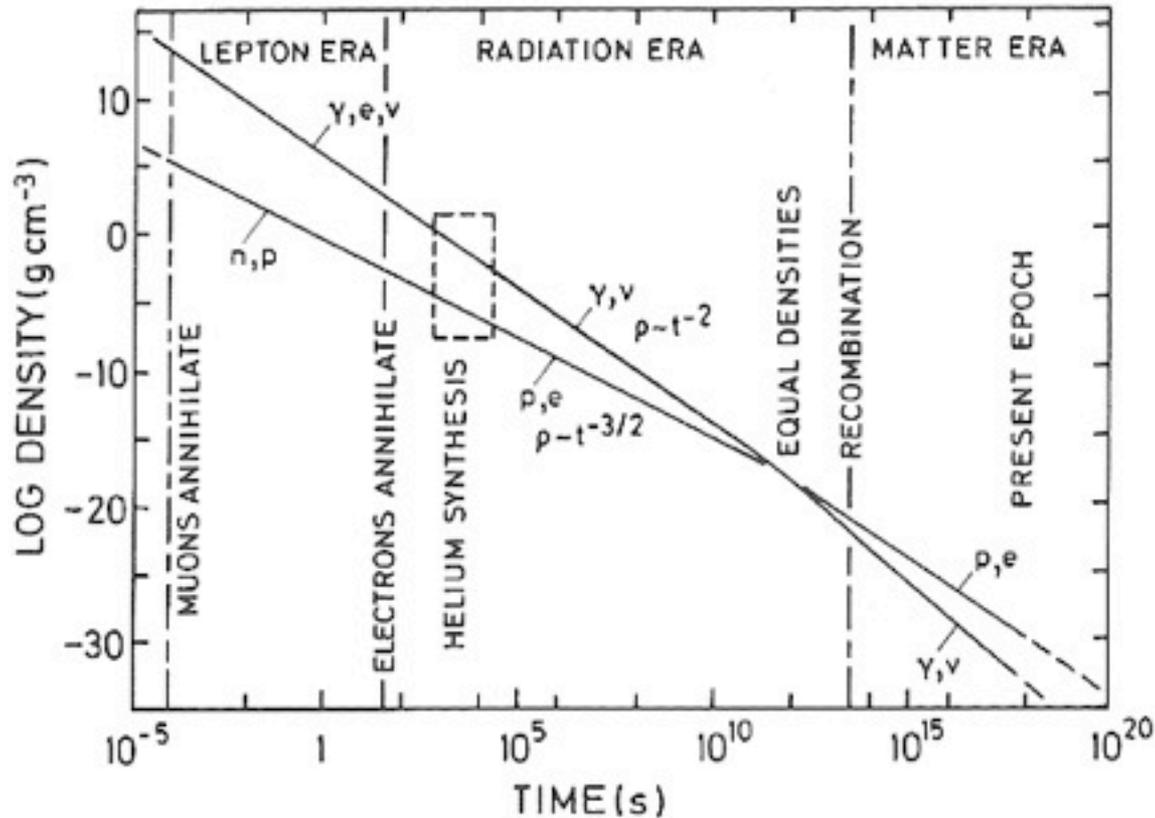
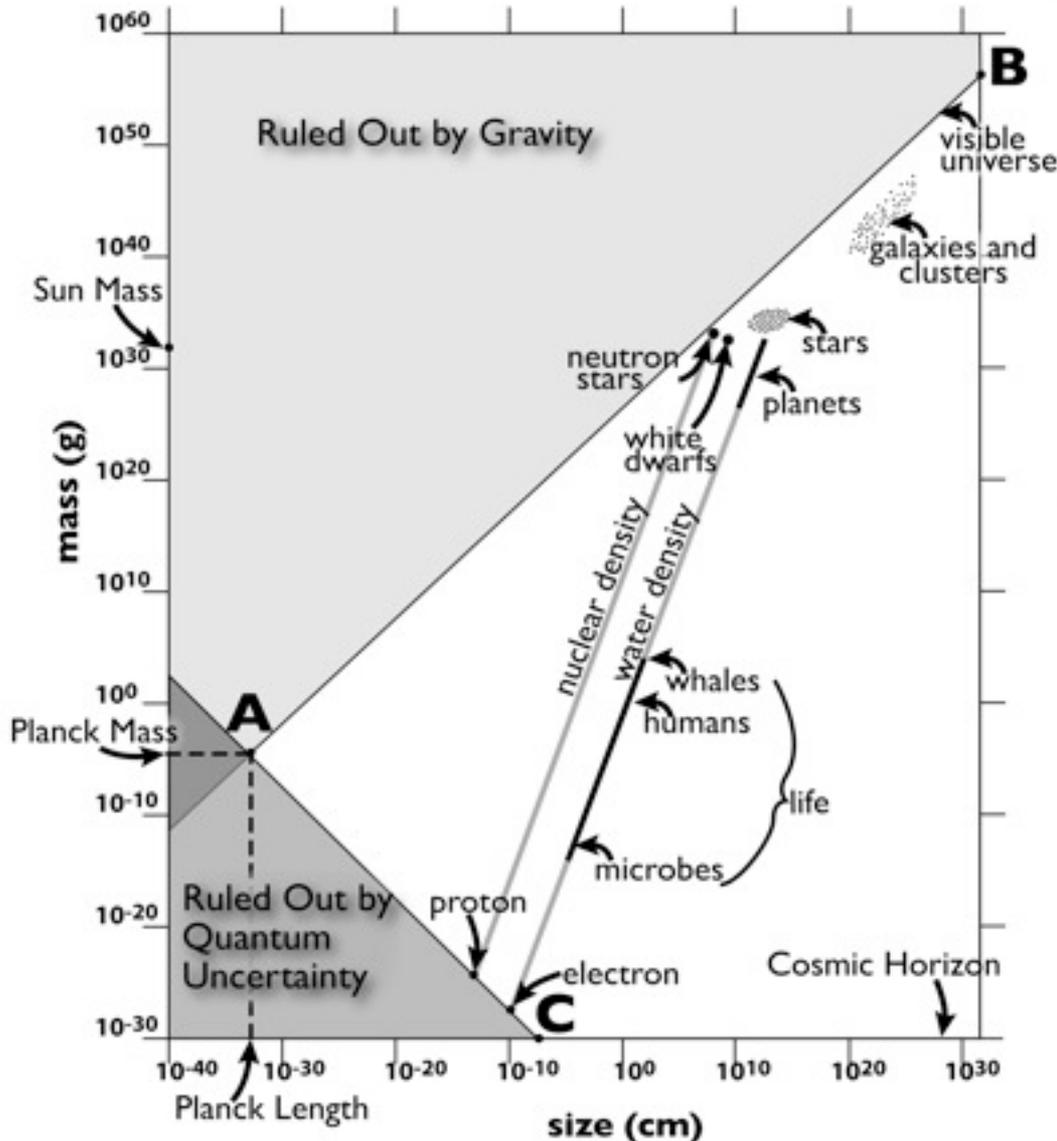


Fig. 3.1. The thermal history of the standard model. The densities of protons, electrons, photons, and neutrinos are shown at various stages of cosmological evolution [after Harrison (1973)]

The Wedge of Material Reality



From *The View from the Center of the Universe* © 2006

The Planck Length

$$l_{Pl} = \sqrt{\frac{hG}{2\pi c^3}} = 1.6 \times 10^{-33} \text{ cm}$$

is the smallest possible length.

Here h is Planck's constant

$$h = 6.626068 \times 10^{-34} \text{ m}^2 \text{ kg} / \text{s}$$

The Planck Mass is

$$m_{Pl} = \sqrt{\frac{hc}{2\pi G}} = 2.2 \times 10^{-5} \text{ g}$$

The Compton (i.e. quantum) wavelength

$$l_C = \frac{h}{2\pi mc}$$

equals the Schwarzschild radius

$$l_S \approx \frac{Gm}{c^2}$$

when $m = m_{Pl} = 1.2 \times 10^{19} \text{ GeV}/c^2$

“Natural Units” = High Energy Physics Units $\hbar = c = k_B = 1$

These are especially appropriate for the hot early universe.

There is one fundamental dimension, which we can take to be mass or energy $= mc^2 = m$.

$$[\text{Energy}] = [\text{Mass}] = [\text{Temperature}] = [\text{Length}]^{-1} = [\text{Time}]^{-1}$$

$$1 \text{ GeV} = 1.78 \times 10^{-24} \text{ g} = 1.16 \times 10^{13} \text{ K} = 1.97 \times 10^{-14} \text{ cm}^{-1} = 6.58 \times 10^{-25} \text{ s}^{-1} \\ = 1.602 \times 10^{-3} \text{ erg}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J} = 1.16 \times 10^4 \text{ K} \quad (k_B = 1.38 \times 10^{-16} \text{ erg K}^{-1})$$

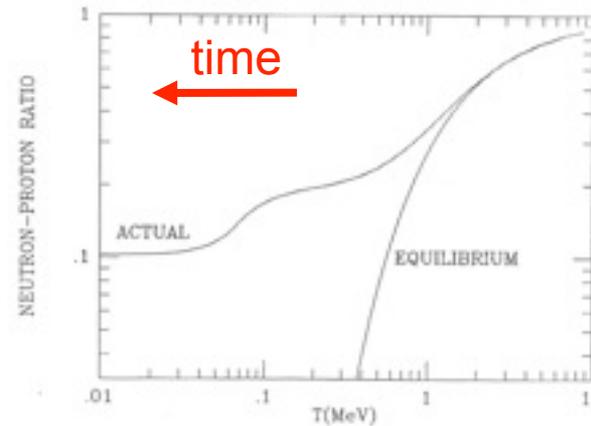
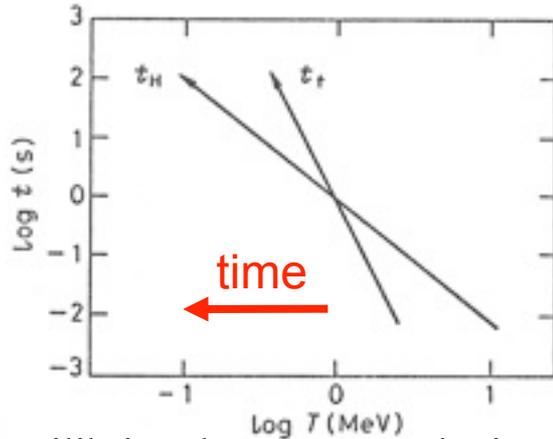
$$1 \text{ pc} = 3.26 \text{ yr} = 3.09 \times 10^{18} \text{ cm}$$

To add gravity to this scheme, we usually express it in terms of the Planck mass $M_{\text{Pl}} = (\hbar c/G)^{1/2} = 1.2 \times 10^{19} \text{ GeV}/c^2$

For more conversion factors, see Appendix A of Kolb & Turner, *The Early Universe*

Big Bang Nucleosynthesis

BBN was conceived by Gamow in 1946 as an explanation for the formation of all the elements, but the absence of any stable nuclei with $A=5,8$ makes it impossible for BBN to proceed past Li. The formation of carbon and heavier elements occurs instead through the triple- α process in the centers of red giants (Burbidge², Fowler, & Hoyle 57). At the BBN baryon density of $2 \times 10^{-29} \Omega_b h^2 (T/T_0)^3 \text{ g cm}^{-3} \approx 2 \times 10^{-5} \text{ g cm}^{-3}$, the probability of the triple- α process is negligible even though $T \approx 10^9 \text{ K}$.



Kolb & Turner

Thermal equilibrium between n and p is maintained by weak interactions, which keeps $n/p = \exp(-Q/T)$ (where $Q = m_n - m_p = 1.293 \text{ MeV}$) until about $t \approx 1 \text{ s}$. But because the neutrino mean free time $t_\nu^{-1} \approx \sigma_\nu n_{e\pm} \approx (G_F T)^2 (T^3)$ is increasing as $t_\nu \propto T^{-5}$ (here the Fermi constant $G_F \approx 10^{-5} \text{ GeV}^{-2}$), while the horizon size is increasing only as $t_H \approx (G\rho)^{-1/2} \approx M_{\text{Pl}} T^{-2}$, these interactions freeze out when T drops below about 0.8 MeV . This leaves $n/(p+n) \approx 0.14$. The neutrons then decay with a mean lifetime $887 \pm 2 \text{ s}$ until they are mostly fused into D and then ^4He . The higher the baryon density, the higher the final abundance of ^4He and the lower the abundance of D that survives this fusion process. Since D/H is so sensitive to baryon density, David Schramm called deuterium the “baryometer.” He and his colleagues also pointed out that since the horizon size increases more slowly with T^{-1} the larger the number of light neutrino species N_ν contributing to the energy density ρ , BBN predicted that $N_\nu \approx 3$ before N_ν was measured at accelerators by measuring the width of the Z^0 (Cviburk et al 2005: $2.67 < N < 3.85$)

THE NUMBER OF LIGHT NEUTRINO TYPES FROM COLLIDER EXPERIMENTS

<http://pdg.lbl.gov/2010/reviews/rpp2010-rev-light-neutrino-types.pdf>

Revised March 2008 by D. Karlen (University of Victoria and TRIUMF).

The most precise measurements of the number of light neutrino types, N_ν , come from studies of Z production in e^+e^- collisions. The invisible partial width, Γ_{inv} , is determined by subtracting the measured visible partial widths, corresponding to Z decays into quarks and charged leptons, from the total Z width. The invisible width is assumed to be due to N_ν light neutrino species each contributing the neutrino partial width Γ_ν as given by the Standard Model. In order to reduce the model dependence, the Standard Model value for the ratio of the neutrino to charged leptonic partial widths, $(\Gamma_\nu/\Gamma_\ell)_{\text{SM}} = 1.991 \pm 0.001$, is used instead of $(\Gamma_\nu)_{\text{SM}}$ to determine the number of light neutrino types:

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_\ell} \left(\frac{\Gamma_\ell}{\Gamma_\nu} \right)_{\text{SM}} . \quad (1)$$

The combined result from the four LEP experiments is $N_\nu = 2.984 \pm 0.008$ [1].

In the past, when only small samples of Z decays had been recorded by the LEP experiments and by the Mark II at SLC, the uncertainty in N_ν was reduced by using Standard Model fits to the measured hadronic cross sections at several center-of-mass energies near the Z resonance. Since this method is much more dependent on the Standard Model, the approach described above is favored.

1. ALEPH, DELPHI, L3, OPAL, and SLD Collaborations, and LEP Electroweak Working Group, and SLD Electroweak Group, and SLD Heavy Flavour Group, Phys. Reports **427**, 257 (2006).

Cosmological Constraint on the Effective Number of Neutrino Species

<http://arxiv.org/abs/0706.3465v1>

Kazuhide Ichikawa Proceedings for the XIXth Rencontres de Blois, May 2007

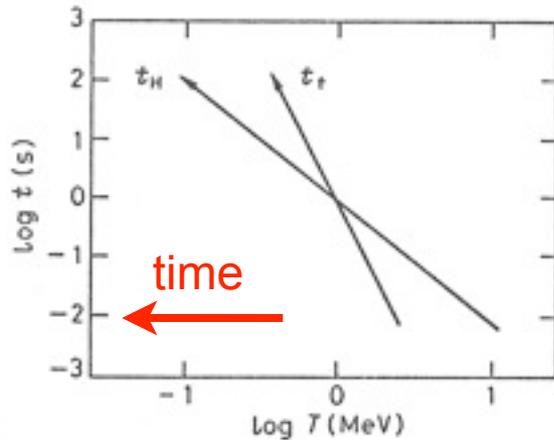
Institute for Cosmic Ray Research, University of Tokyo, Kashiwa 277-8582, Japan

Abstract We discuss constraints on the effective number of neutrino species N_ν from recent cosmological observations such as CMB, LSS, BBN, including our own analysis which uses the WMAP and the Luminous Red Galaxy power spectrum data. We also discuss their implications on some non-standard cosmological scenarios such as the low (MeV-scale) reheating temperature scenario and the scenario with decaying particles between BBN and structure formation.

	95% limit	Data set
Seljak, Slosar, McDonald [4]	$N_\nu = 5.3^{+2.1}_{-1.7}$	All
	$N_\nu = 4.8^{+1.6}_{-1.4}$	All + HST
	$N_\nu = 6.0^{+2.9}_{-2.4}$	All - BAO
	$N_\nu = 3.9^{+2.1}_{-1.7}$	All - Ly α
	$N_\nu = 7.8^{+2.3}_{-3.2}$	WMAP3+SN+SDSS(main)
	$N_\nu = 3.2^{+3.6}_{-2.3}$	WMAP3+SN+2dF
	$N_\nu = 5.2^{+2.1}_{-1.8}$	All-2dF-SDSS(main)
Ichikawa, Kawasaki, Takahashi [11]	$N_\nu = 3.1^{+5.1}_{-2.2}$	WMAP3+SDSS(LRG)

Comparison of N_ν constraints using various data set combinations. “All” refers to WMAP3 + other CMB + Ly α + galaxy power spectrum (SDSS main sample + 2dF) + SDSS baryon acoustic oscillation (BAO) + Supernovae Ia (SN). See Ref. [4] for details. SDSS (main) and Ly α favor $N_\nu > 3$.

Neutrinos in the Early Universe



As we discussed, neutrino decoupling occurs at $T \sim 1$ MeV. After decoupling, the neutrino phase space distribution is

$$f_\nu = [1 + \exp(p_\nu c / T_\nu)]^{-1} \quad (\text{note: } \neq [1 + \exp(E_\nu / T_\nu)]^{-1}$$

for NR neutrinos)

After e^+e^- annihilation, $T_\nu = (4/11)^{1/3} T_\gamma = 1.9\text{K}$. Proof :

Number densities of primordial particles

$$n_\gamma(T) = 2 \zeta(3) \pi^{-2} T^3 = 400 \text{ cm}^{-3} (T/2.7\text{K})^3, \quad n_\nu(T) = \left(\frac{3}{4}\right) n_\gamma(T) \text{ including antineutrinos}$$

FermiDirac/BoseEinstein factor

Conservation of entropy s_i of interacting particles per comoving volume

$s_i = g_i(T) N_\gamma(T) = \text{constant}$, where $N_\gamma = n_\gamma V$; we only include neutrinos for $T > 1$ MeV.

Thus for $T > 1$ MeV, $g_i = 2 + 4(7/8) + 6(7/8) = 43/4$ for γ , e^+e^- , and the three ν species, while for $T < 1$ MeV, $g_i = 2 + 4(7/8) = 11/2$. At e^+e^- annihilation, below about $T = 0.5$ MeV, g_i drops to 2, so that $2N_{\gamma 0} = g_i(T < 1 \text{ MeV}) N_\gamma(T < 1 \text{ MeV}) = (11/2) N_\gamma(T < 1 \text{ MeV}) = (11/2)(4/3) N_\nu(T < 1 \text{ MeV})$. Thus $n_{\nu 0} = (3/4)(4/11) n_{\gamma 0} = 109 \text{ cm}^{-3} (T/2.7\text{K})^3$, or

$$T_\nu = (4/11)^{1/3} T = 0.714 T$$

Statistical Thermodynamics

$$n_i = \frac{g_i}{2\pi^2} \left(\frac{kT_i}{hc}\right)^3 I_i''(\pm), \quad \rho_i = \frac{g_i kT_i}{2\pi^2 c^2} \left(\frac{kT_i}{hc}\right)^3 I_i^{2'}(\pm), \quad \text{where}$$

$$I_i^{mn} \equiv \int_{\theta_i}^{\infty} x^m (x^2 - \theta_i^2)^{n/2} (e^{\pm x})^{-1} dx, \quad \theta_i = \frac{kT_i}{m_i c^2}, \quad g_i = \# \text{ spin states}$$

+ Fermi-Dirac, - Bose-Einstein

$$\theta_i \gg 1 \quad (\text{ER}): \quad I_i''(+)=\frac{3}{2}\zeta(3)=1.803, \quad I_i^{2'}(+)=\frac{7\pi^4}{120}$$

$$I_i''(-)=2\zeta(3)=\frac{4}{3}I_i''(+), \quad I_i^{2'}(-)=\frac{\pi^4}{15}=\frac{8}{7}I_i^{2'}(+)$$

$$\theta_i \ll 1 \quad (\text{NR}): \quad n_i = \frac{\rho_i}{m_i} = \frac{g_i}{(2\pi)^{3/2}} \left(\frac{kT_i}{hc}\right)^3 \theta_i^{-3/2} e^{-\theta_i} \quad (\text{not } v's)$$

$$[\text{Note: } \zeta(3) = 1.2020569\dots = \sum_{k=1}^{\infty} \frac{1}{k^3} = \prod_{\text{primes}} (1 - p^{-3})^{-1}]$$

Boltzmann Equation

$$a^{-3} \frac{d(n_1 a^3)}{dt} = \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} \quad \text{Dodelson (3.1)}$$

In the absence of interactions (rhs=0) n_1 falls as a^{-3}

$$\times (2\pi)^4 \delta^3(p_1 + p_2 - p_3 - p_4) \delta(E_1 + E_2 - E_3 - E_4) |\mathcal{M}|^2$$

$$\times \{f_3 f_4 [1 \pm f_1][1 \pm f_2] - f_1 f_2 [1 \pm f_3][1 \pm f_4]\}.$$

+ bosons
- fermions

We will typically be interested in $T \gg E - \mu$ (where μ is the chemical potential). In this limit, the exponential in the Fermi-Dirac or Bose-Einstein distributions is much larger than the ± 1 in the denominator, so that

$$f(E) \rightarrow e^{\mu/T} e^{-E/T}$$

and the last line of the Boltzmann equation above simplifies to

$$f_3 f_4 [1 \pm f_1][1 \pm f_2] - f_1 f_2 [1 \pm f_3][1 \pm f_4]$$

$$\rightarrow e^{-(E_1+E_2)/T} \left\{ e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T} \right\}.$$

The number densities are given by $n_i = g_i e^{\mu_i/T} \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T}$. For our applications, i's are

Table 3.1. Reactions in This Chapter: $1 + 2 \leftrightarrow 3 + 4$

	1	2	3	4
Neutron-Proton Ratio	n	ν_e or e^+	p	e^- or $\bar{\nu}_e$
Recombination	e	p	H	γ
Dark Matter Production	X	X	l	l

The equilibrium number densities are given by

$$n_i^{(0)} \equiv g_i \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T} = \begin{cases} g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T} & m_i \gg T \\ g_i \frac{T^3}{\pi^2} & m_i \ll T \end{cases}. \quad (3.6)$$

With this definition, $e^{\mu_i/T}$ can be rewritten as $n_i/n_i^{(0)}$, so the last line of Eq. (3.1) is equal to

$$e^{-(E_1+E_2)/T} \left\{ \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right\}. \quad (3.7)$$

With these approximations the Boltzmann equation now simplifies enormously. Define the thermally averaged cross section as

$$\begin{aligned} \langle \sigma v \rangle &\equiv \frac{1}{n_1^{(0)} n_2^{(0)}} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} e^{-(E_1+E_2)/T} \\ &\times (2\pi)^4 \delta^3(p_1 + p_2 - p_3 - p_4) \delta(E_1 + E_2 - E_3 - E_4) |\mathcal{M}|^2. \end{aligned} \quad (3.8)$$

Then, the Boltzmann equation becomes

$$a^{-3} \frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left\{ \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right\}. \quad (3.9)$$

If the reaction rate $n_2 \langle \sigma v \rangle$ is much smaller than the expansion rate ($\sim H$), then the $\{ \}$ on the rhs must vanish. This is called *chemical equilibrium* in the context of the early universe, *nuclear statistical equilibrium* (NSE) in the context of Big Bang nucleosynthesis, and the *Saha equation* when discussing recombination of electrons and protons to form neutral hydrogen.