Physics/Astronomy 224 Spring 2012

Origin and Evolution of the Universe

Week 2 - Part 1

General Relativity - Time and Distances

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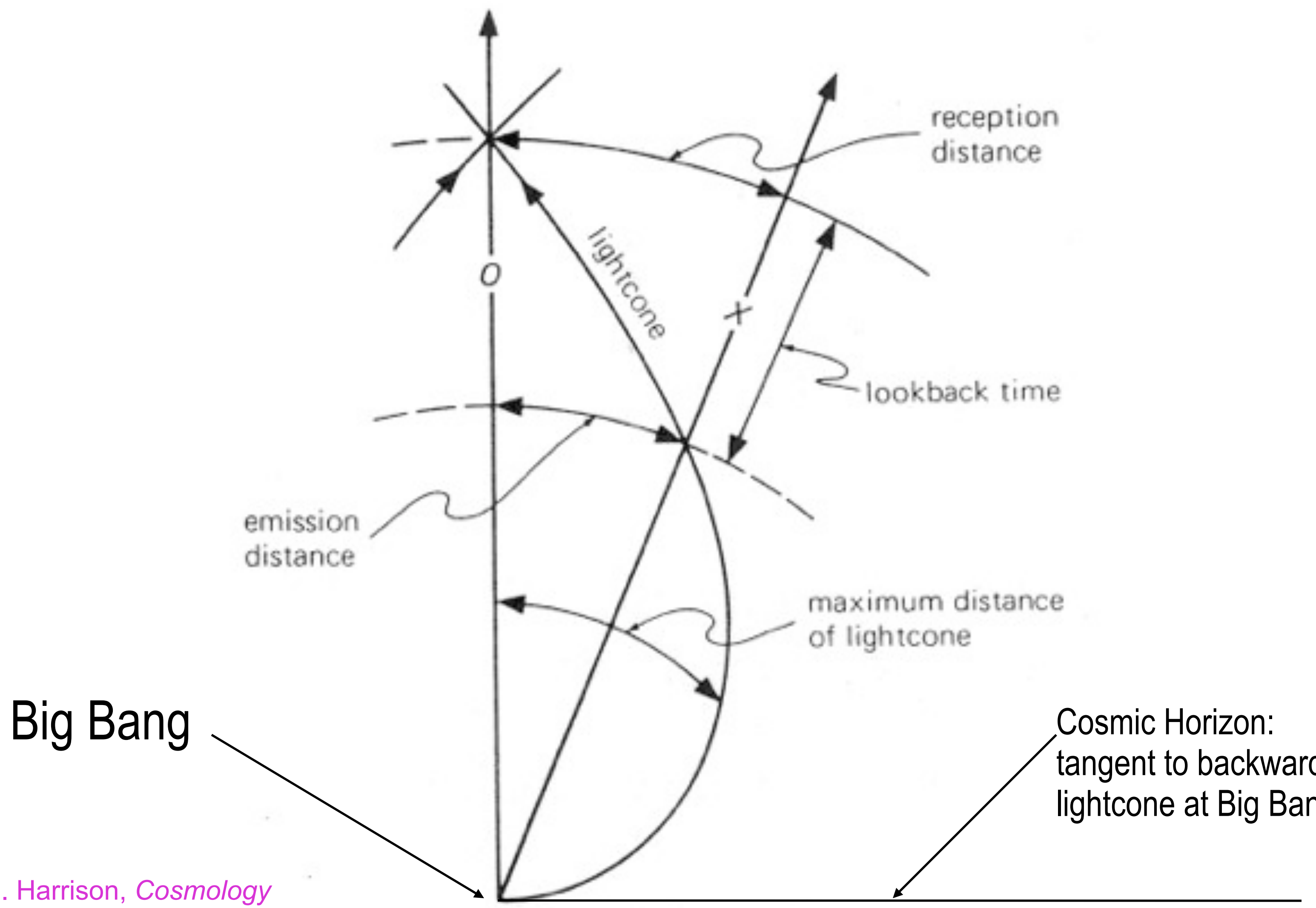
Picturing the History of the Universe: The Backward Lightcone



From E. Harrison, *Cosmology* (Cambridge UP, 2000).

Picturing the History of the Universe: The Backward Lightcone

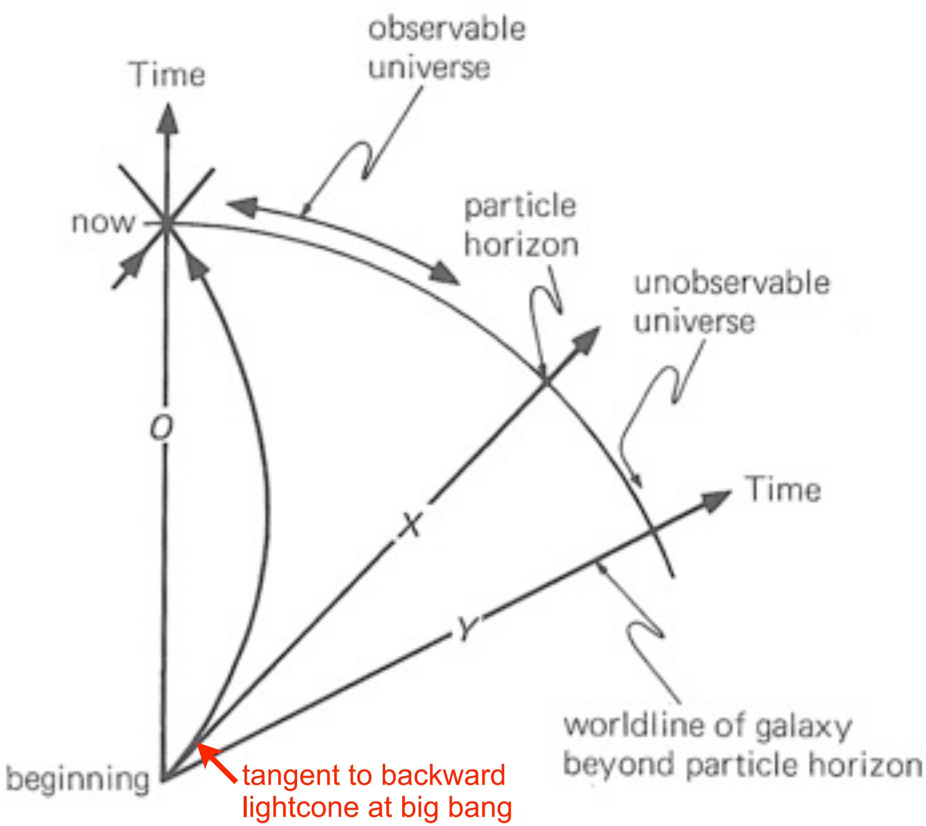


Figure 21.11. At the instant labeled "now" the particle horizon is at worldline X. In a big bang universe, all galaxies at the particle horizon have infinite redshift.

From E. Harrison, *Cosmology* (Cambridge UP, 2000).

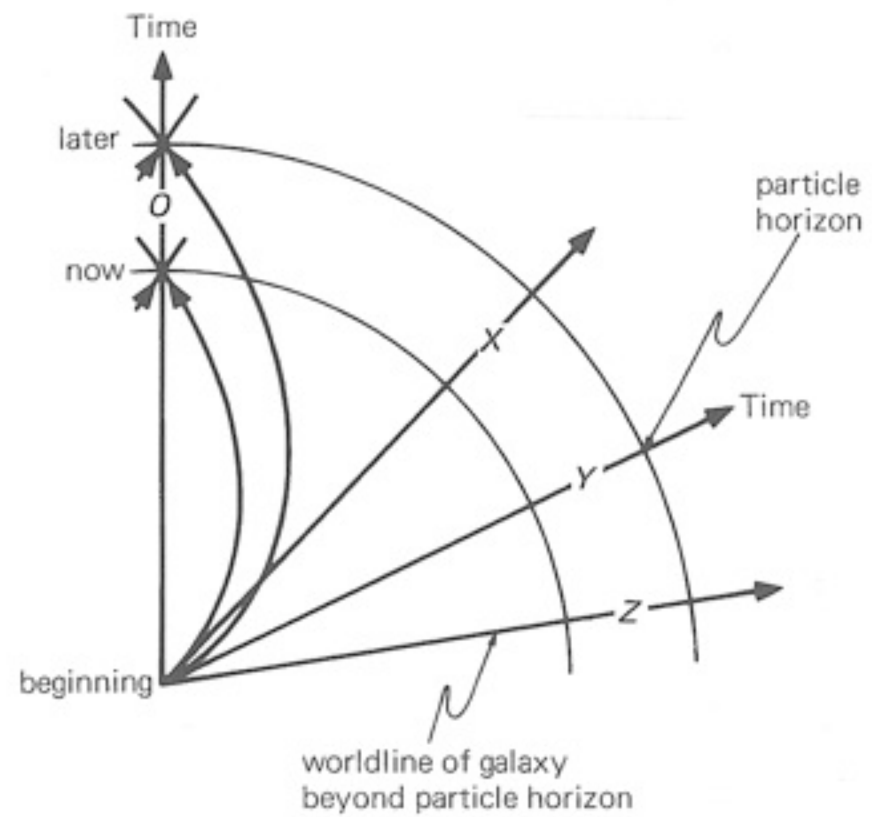


Figure 21.12. At the instant labeled "later" the particle horizon has receded to world line Y. Notice the distance of the particle horizon is always a reception distance, and the particle horizon always overtakes the galaxies and always the fraction of the universe observed increases.

Distances in an Expanding Universe

Proper distance = physical distance = d_p

$d_p(t_0)$ = (physical distance at t_0) = $a(t_0)\, r_e = r_e$

$\chi(t_e)$ = (comoving distance of galaxy emitting at time t_e)

$$\chi(t_e) = \int_0^{r_e} dr = r_e = c\int_{t_e}^{t_0} dt/a = c\int_{a_e}^{1} da/(a^2 H)$$

because

$$dt = (dt/da)\, da = (a\, dt/da)\, da/a = da/(aH)$$

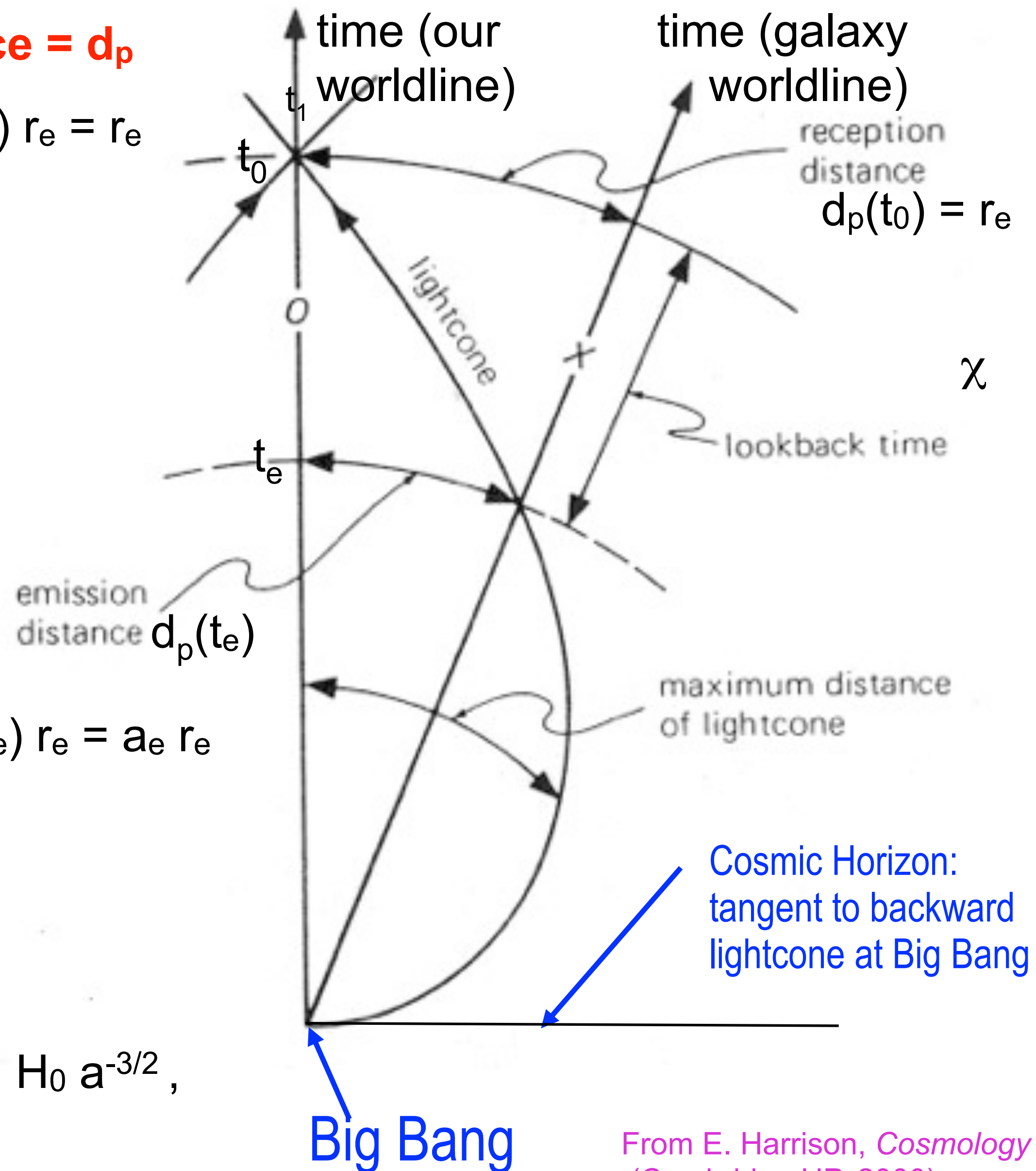
$d_p(t_e)$ = (physical distance at t_e) = $a(t_e)\, r_e = a_e\, r_e$

The Hubble radius $d_H = c\, H_0^{-1}$ = $4.29\, h_{70}^{-1}$ Gpc = $13.97 h_{70}^{-1}$ Glyr

For E-dS ($\Omega_m = 1$, $\Omega_\Lambda = 0$), where $H = H_0\, a^{-3/2}$,

$$\chi(t_e) = r_e = d_p(t_0) = 2 d_H (1 - a_e^{1/2})$$

$$d_p(t_e) = 2 d_H\, a_e (1 - a_e^{1/2})$$



From E. Harrison, *Cosmology* (Cambridge UP, 2000).

Our Particle Horizon

FRW: $ds^2 = -c^2\,dt^2 + a(t)^2\,[dr^2 + r^2\,d\theta^2 + r^2 \sin^2\theta\;\; d\phi^2]$ for curvature $k=0$ so $\sqrt{g_{rr}} = a(t)$

Particle Horizon

$\mathbf{d_p}$(horizon) = (physical distance at time t_0) = $a(t_0)\,r_p = r_p$

$$\mathbf{d_p}(\text{horizon}) = \int_0^{r_{horizon}} dr = r_{horizon} = c\int_0^{t_0} dt/a = c\int_0^{1} da/(a^2 H)$$

For E-dS, where $H = H_0\, a^{-3/2}$,

$$r_{horizon} = \lim_{a_e \to 0} 2d_H\,(1 - a_e^{1/2}) = 2d_H =$$

$= 8.58\; h_{70}^{-1}$ Gpc $= 27.94\; h_{70}^{-1}$ Glyr

For the Benchmark Model with h=0.70,
$r_{horizon}$ = 13.9 Gpc = 45.2 Glyr.

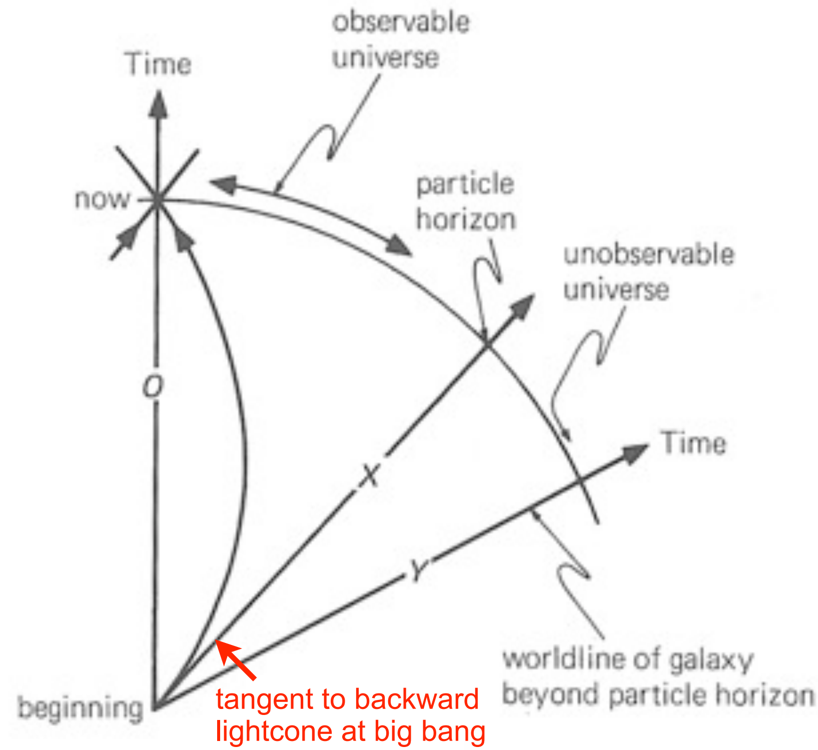


Figure 21.11. At the instant labeled "now" the particle horizon is at worldline X. In a big bang universe, all galaxies at the particle horizon have infinite redshift.

For the parameters of
WMAP5 h = 0.70, Ω_m = 0.28, k = 0, t_0 = 13.7 Gyr, $r_{horizon}$ = 14.3 Gpc = 46.5 Glyr.
WMAP7 h = 0.70, Ω_m = 0.27, k = 0, t_0 = 13.9 Gyr, $r_{horizon}$ = 14.5 Gpc = 47.1 Glyr.

Horizons

$$ds^2 = dt^2 - dX^2 = dt^2 - R^2 dx^2 = R^2(d\eta^2 - dx^2)$$

conformal time $d\eta = dt/R$ comoving coord. $dx = dX/R$

PARTICLE HORIZON

Spherical surface that at time t separates ***worldlines*** into observed vs. unobserved

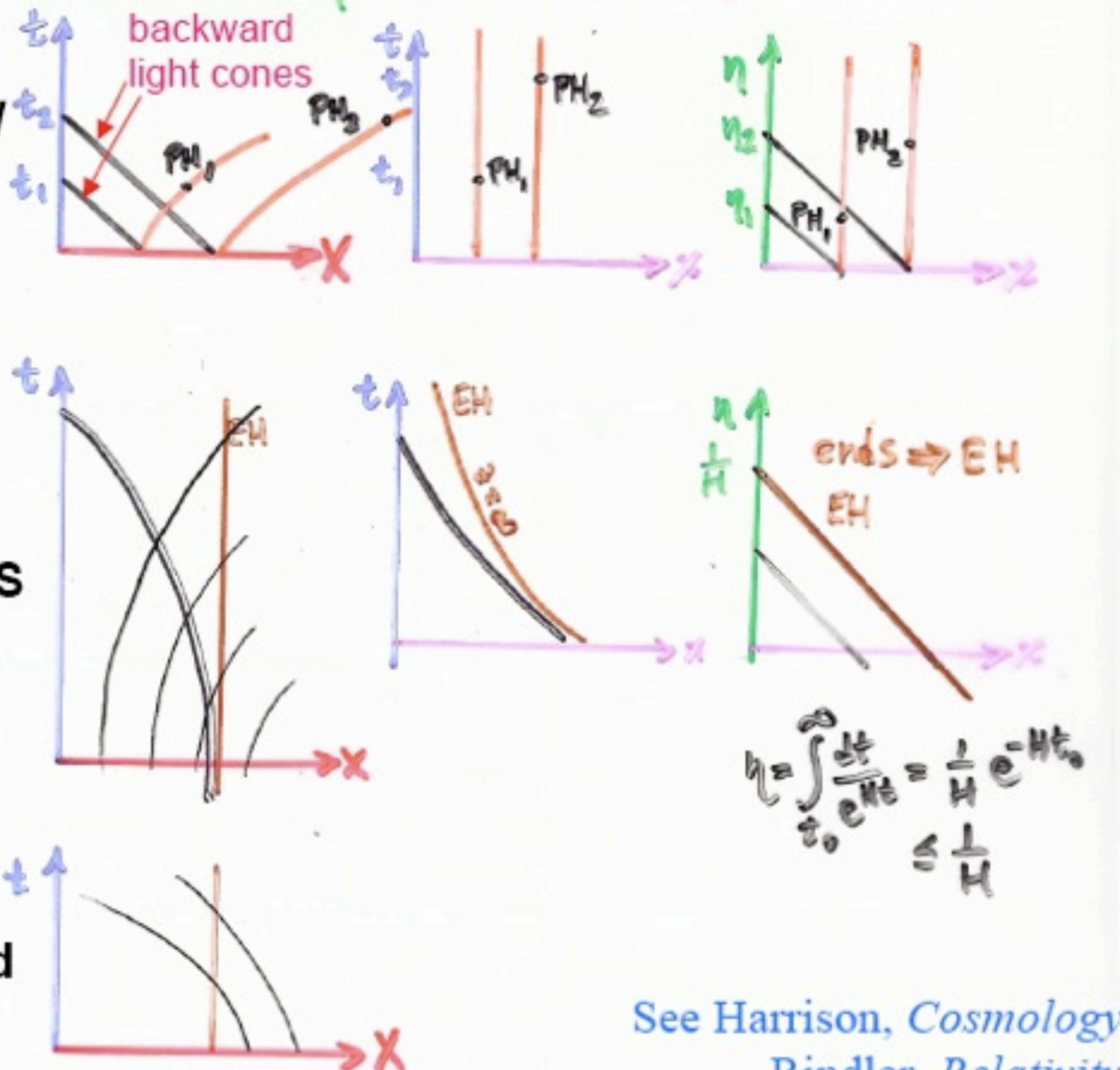
FRW

EVENT HORIZON

Backward lightcone that separates *events* that will someday be observed from those never observed

deS

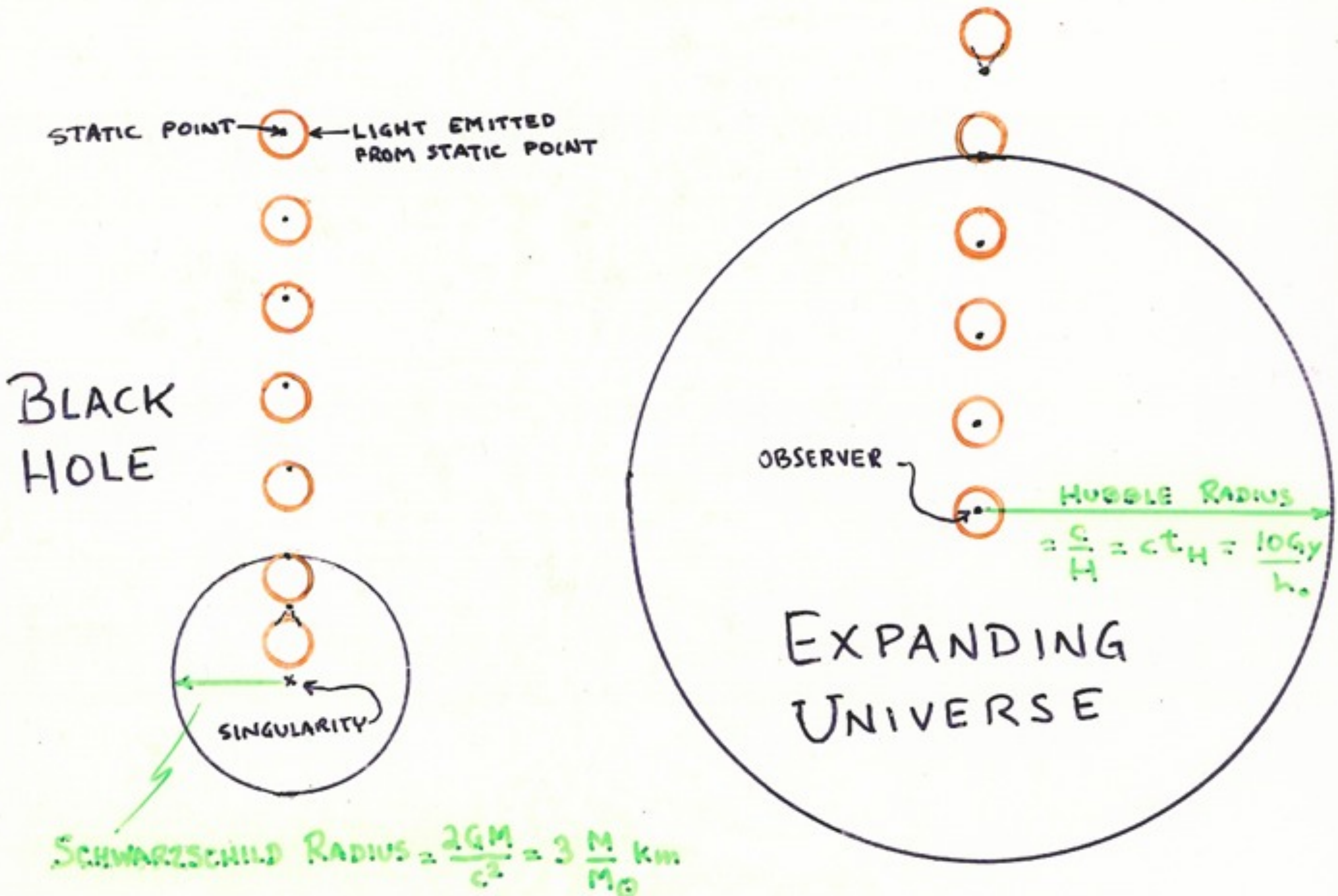
Schwarzschild



$$\eta = \int_{t_0}^{\infty} \frac{dt}{e^{Ht}} = \frac{1}{H} e^{-Ht_0} \leq \frac{1}{H}$$

See Harrison, *Cosmology*
Rindler, *Relativity*

CURVED SPACE-TIME IS NOT JUST AN ARENA IN WHICH THINGS MOVE, IT IS DYNAMIC. CURVATURE CAN CAUSE HORIZONS, BEYOND WHICH INFORMATION CANNOT BE SENT.



Distances in an Expanding Universe

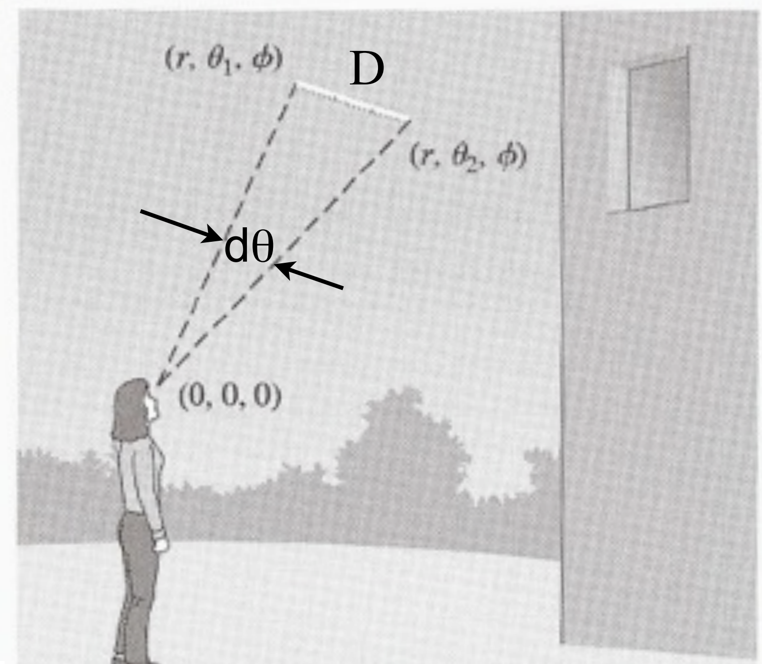
Angular Diameter Distance

From the FRW metric, the distance D across a source at comoving distance $r = r_e$ which subtends an angle $d\theta = \theta_1 - \theta_2$ is
$D = a(t)\, r\, d\theta$, or $d\theta = D/[\, a(t)\, r]$.

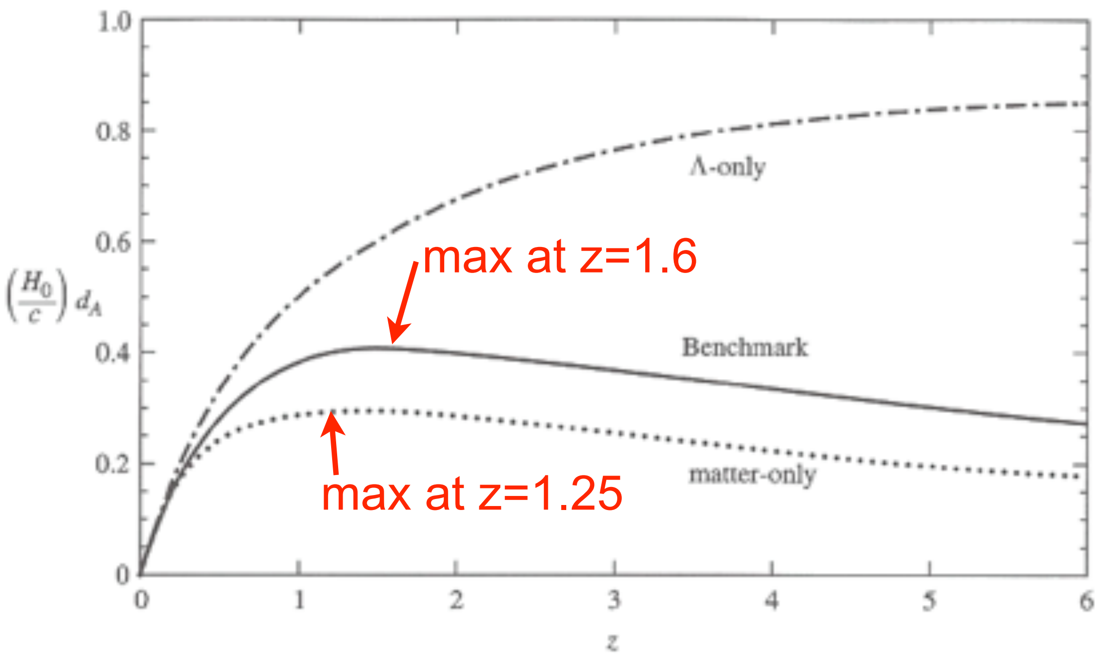
The **angular diameter distance d_A** is defined by $d_A = D/d\theta$, so

$$\mathbf{d_A = a(t_e)\, r_e = r_e/(1+z_e) = d_p(t_e)}\ .$$

This has a maximum, and $d\theta$ a minimum.



Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)



For the Benchmark Model

redshift z	D↔1 arcsec
0.1	1.8 kpc
0.2	3.3
0.5	6.1
1	8.0
2	8.4
3	7.7
4	7.0
6	5.7

Distances in an Expanding Universe

In Euclidean space, the **luminosity** L of a source at distance d is related to the **apparent luminosity** ℓ by ℓ = Power / Area = $L / 4\pi d^2$

The **luminosity distance** $\mathbf{d_L}$ is defined by

$$d_L = (L / 4\pi\ell)^{1/2} .$$

Weinberg, *Cosmology*, pp. 31-32, shows that in FRW

$$\ell = \text{Power/Area} = L / 4\pi d_L^2$$

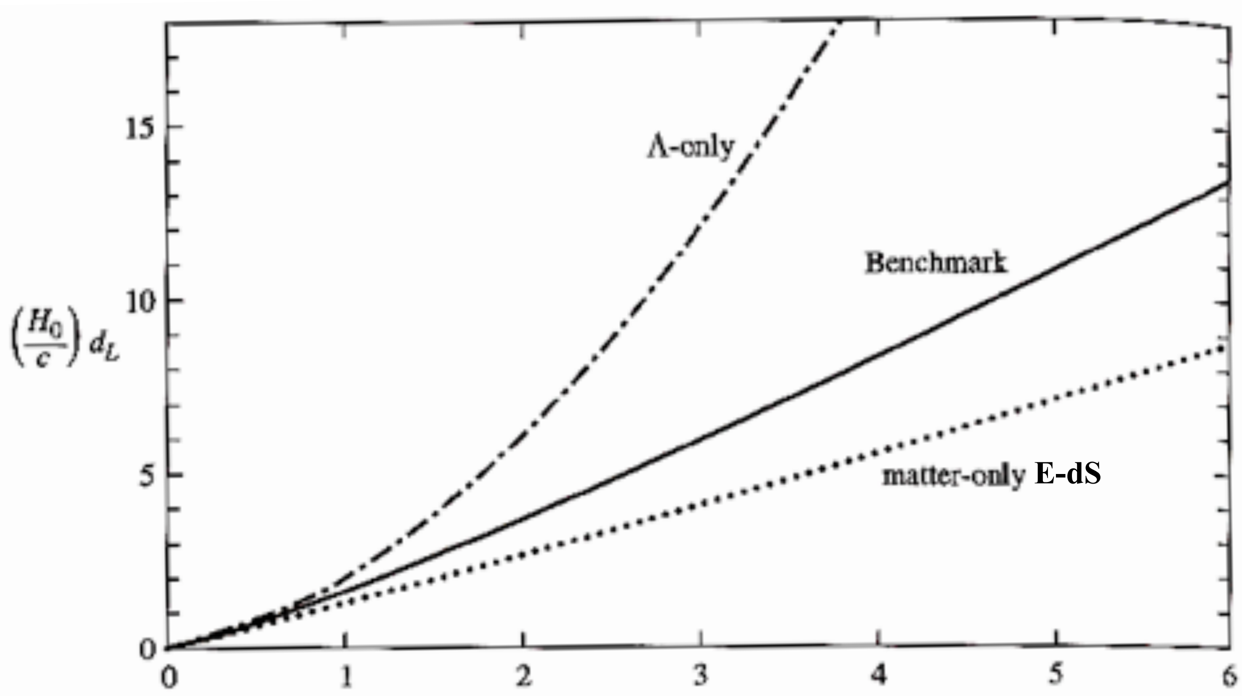
$$= L\,[a(t_1)/a(t_0)]^2 / [4\pi d_p(t_0)^2] = L\,a(t_1)^2 / 4\pi r_1^2 = L / 4\pi r_1^2 (1+z_1)^2$$

fraction of photons reaching unit area at t_0

(redshift of each photon)(delay in arrival)

Thus

$$\mathbf{d_L = r_1/a(t_1) = r_1\,(1+z_1) = d_p(t_0)\,(1+z_1) = d_A\,(1+z_1)^2}$$



redshift z →

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Summary: Distances in an Expanding Universe

FRW: $ds^2 = -c^2\,dt^2 + a(t)^2\,[dr^2 + r^2\,d\theta^2 + r^2 \sin^2\theta\;\; d\phi^2]$ for curvature k=0

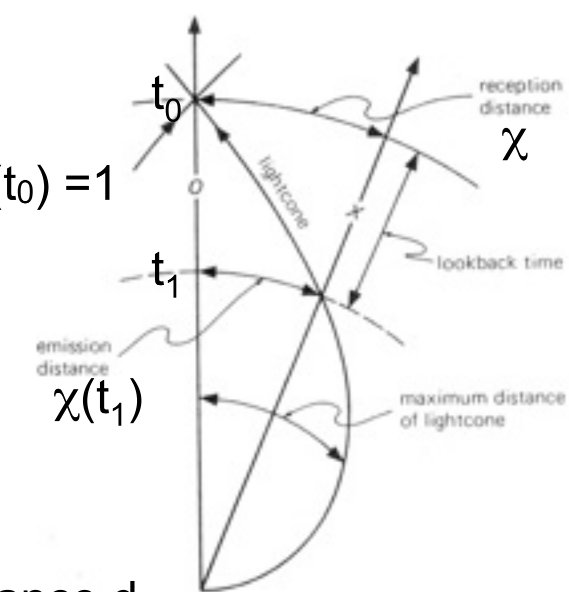
$\chi(t_1)$ = (comoving distance at time t_1) = $\int_0^{t_1} dt/a = r_1$

$d(t_1)$ = (physical distance at t_1) = $a(t_1)\,\chi\,(t_1)$

Particle Horizon

χ_p = (comoving distance at time t_0) = r_p

d_p = (physical distance at time t_0) = $a(t_0)\,r_p = r_p$ since $a(t_0) = 1$



From the FRW metric above, the distance D across a source at distance r_1 which subtends an angle $d\theta$ is $D=a(t_1)\,r_1\,d\theta$. The **angular diameter distance d_A** is defined by $d_A = D/d\theta$, so

$$\mathbf{d_A = a(t_1)\,r_1 = r_1/(1+z_1)}$$

In Euclidean space, the **luminosity** L of a source at distance d is related to the apparent luminosity ℓ by

$$\ell = \text{Power/Area} = L/4\pi d^2$$

so the **luminosity distance d_L** is defined by $d_L = (L/4\pi\ell)^{1/2}$.

Weinberg, *Cosmology*, pp. 31-32, shows that in FRW

$$\ell = \text{Power/Area} = L\,[a(t_1)/a(t_0)]^2\,[4\pi a(t_0)^2\,r_1^{\,2}]^{-1} = L/4\pi d_L^{\,2}$$

fraction of photons reaching unit area at t_0

(redshift of each photon)(delay in arrival)

Thus

$$\mathbf{d_L = r_1/a(t_1) = r_1\,(1+z_1)}$$

Distances in a Flat (k=0) Expanding Universe

$\chi(t_1)$ = (comoving distance at time t_1) = r_1 | $d_A = a(t_1)\, r_1 = r_1/(1+z_1)$ | $d_L = r_1/a(t_1) = r_1\,(1+z_1)$

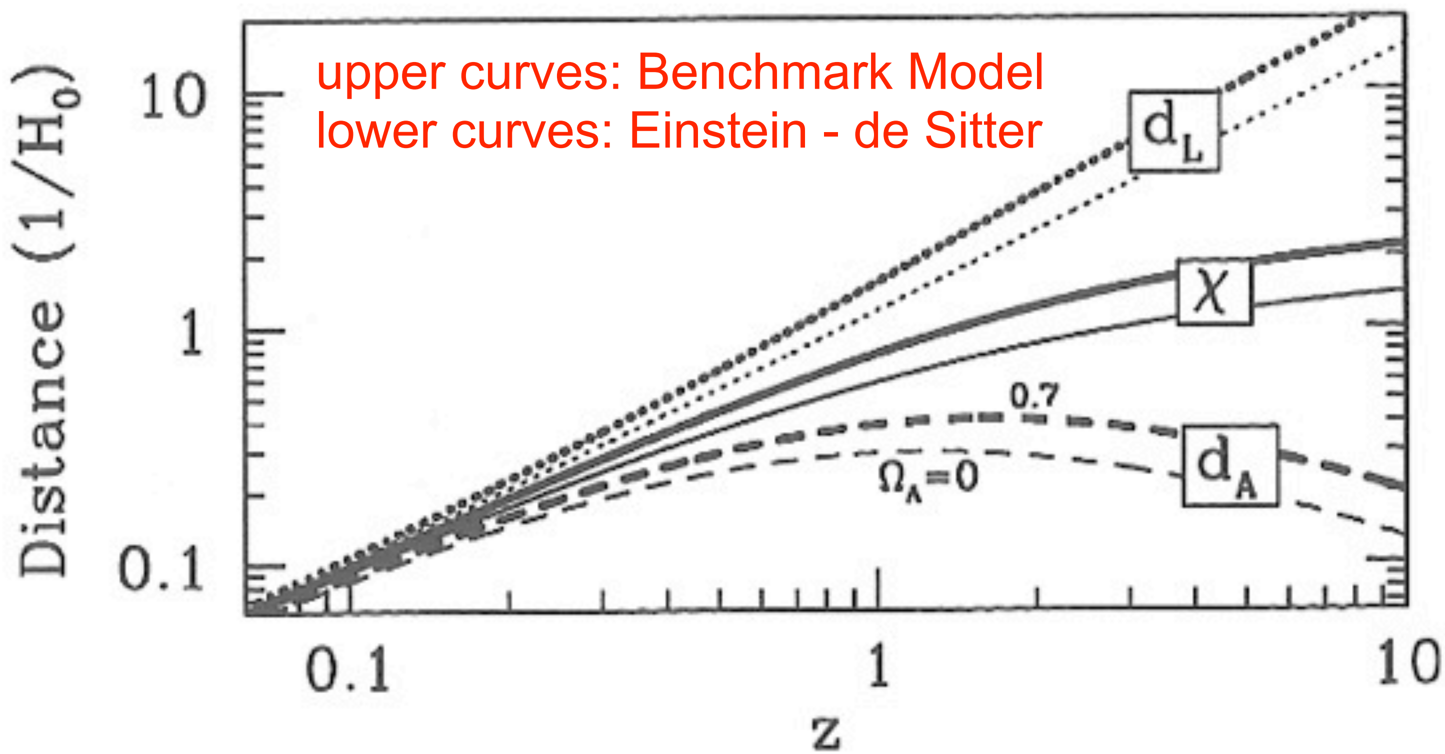
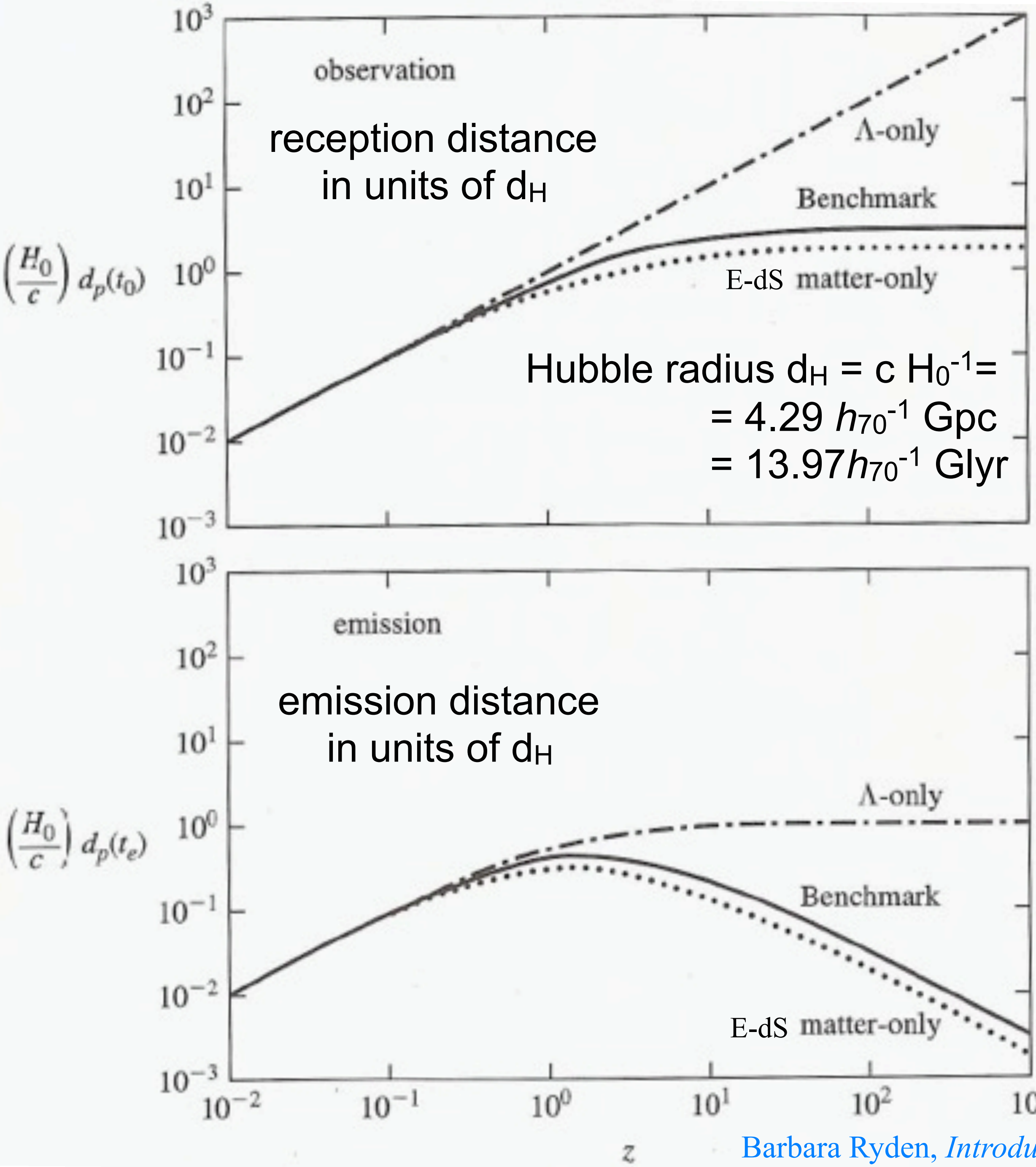


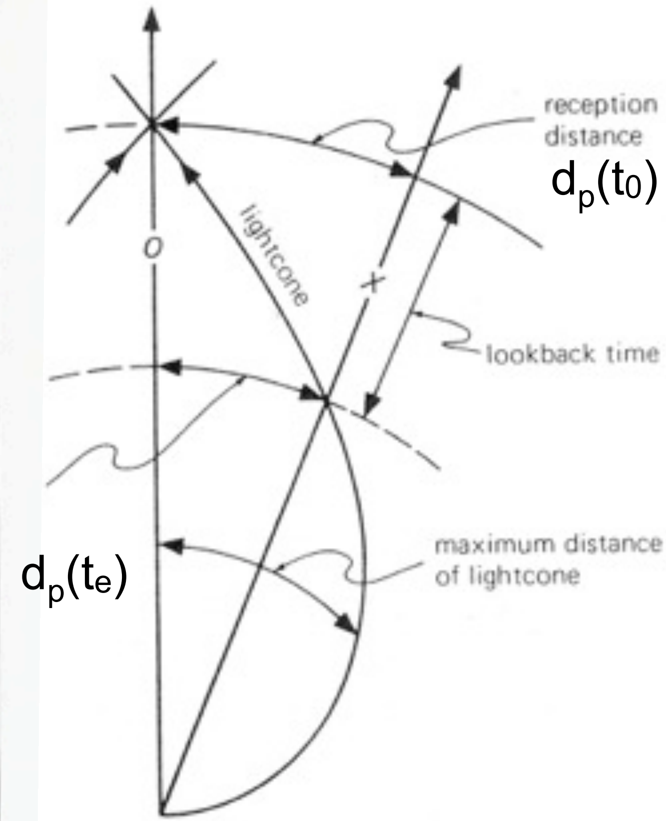
Figure 2.3. Three distance measures in a flat expanding universe. From top to bottom, the luminosity distance, the comoving distance, and the angular diameter distance. The pair of lines in each case is for a flat universe with matter only (light curves) and 70% cosmological constant Λ (heavy curves). In a Λ-dominated universe, distances out to fixed redshift are larger than in a matter-dominated universe.

Scott Dodelson, *Modern Cosmology* (Academic Press, 2003)

Distances in an Expanding Universe



Hubble radius $d_H = c\,H_0^{-1} =$
$= 4.29\ h_{70}^{-1}$ Gpc
$= 13.97 h_{70}^{-1}$ Glyr



For E-dS, where $H = H_0\, a^{-3/2}$,

$$\chi(t_e) = r_e = d_p(t_0) = 2d_H\,(1-a_e^{1/2})$$

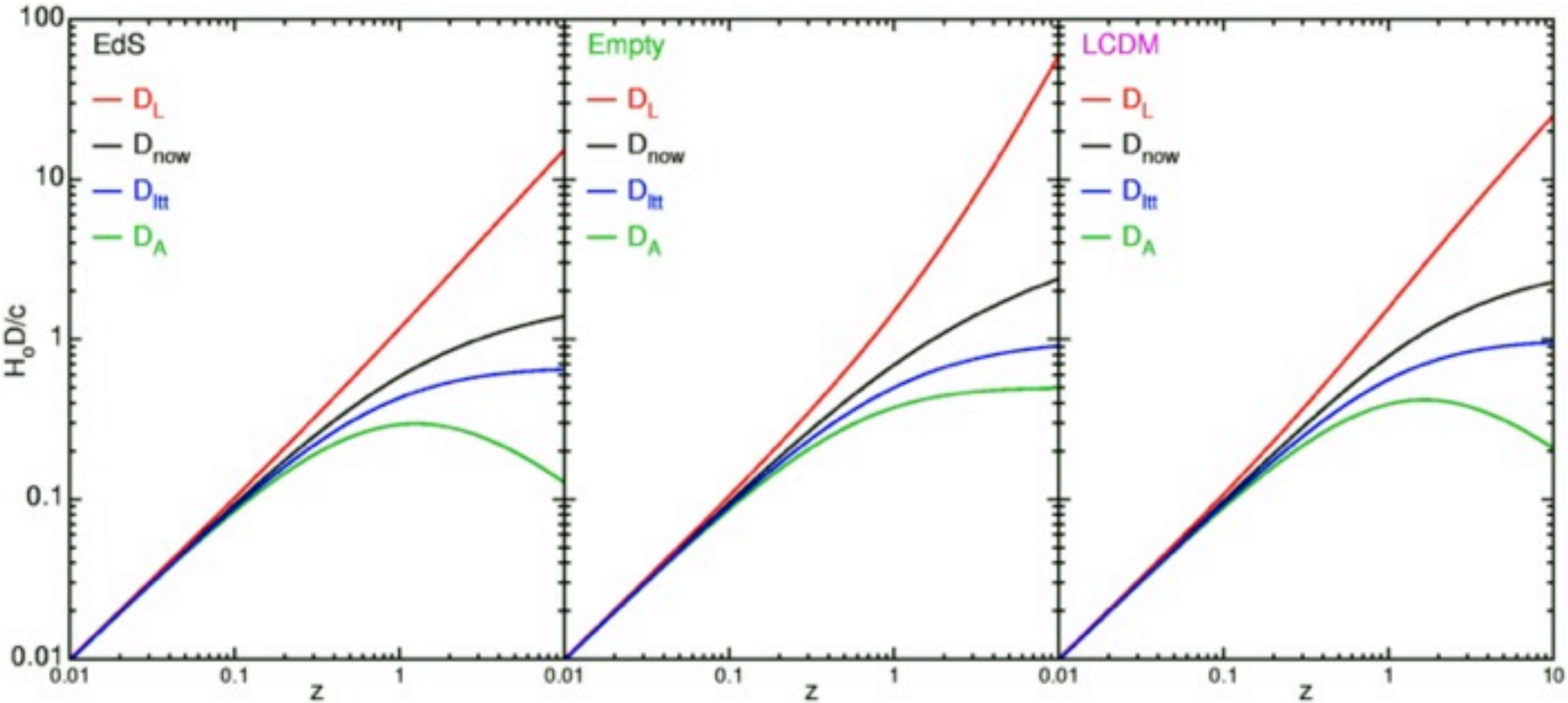
$$d_p(t_e) = 2d_H\, a_e\,(1-a_e^{1/2})$$

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Distances in the Expanding Universe

D_{now} = proper distance, D_L = luminosity distance,

D_A = angular diameter distance, $D_{ltt} = c(t_0 - t_z)$



http://www.astro.ucla.edu/~wright/cosmo_02.htm#DH

Distances in the Expanding Universe:
Ned Wright's Javascript Calculator

Enter values, hit a button

70 H_o

0.30 Omega_M

0.83 z

Open Flat

0.70 Omega_{vac}

General

Open sets $\text{Omega}_{vac} = 0$ giving an open Universe [if you entered $\text{Omega}_M < 1$]

Flat sets $\text{Omega}_{vac} = 1 - \text{Omega}_M$ giving a flat Universe.

General uses the Omega_{vac} that you entered.

For $H_o = 70$, $\text{Omega}_M = 0.300$, $\text{Omega}_{vac} = 0.700$, $z = 0.830$

- It is now 13.462 Gyr since the Big Bang.
- The age at redshift z was 6.489 Gyr.
- The light travel time was 6.974 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 2868.9 Mpc or 9.357 Gly.
- The comoving volume within redshift z is 98.906 Gpc^3.
- The angular size distance D_A is 1567.7 Mpc or 5.1131 Gly.
- This gives a scale of 7.600 kpc/".
- The luminosity distance D_L is 5250.0 Mpc or 17.123 Gly.

$H_0 D_L(z=0.83)$
$=17.123/13.97$
$=1.23$

1 Gly = 1,000,000,000 light years or 9.461×10^{26} cm.
1 Gyr = 1,000,000,000 years.
1 Mpc = 1,000,000 parsecs = 3.08568×10^{24} cm, or 3,261,566 light years.

Tutorial: Part 1 | Part 2 | Part 3 | Part 4
FAQ | Age | Distances | Bibliography | Relativity

Ned Wright's home page

Web app http://www.astro.ucla.edu/~wright/CosmoCalc.html

iPhone app http://itunes.apple.com/us/app/cosmocalc/id334569654?mt=8

Velocities in an Expanding Universe

The velocity away from us now of a galaxy whose light we receive with redshift z_e, corresponding to scale factor $a_e = 1/(1 + z_e)$, is

$$v(t_0) = H_0\, d_p(t_0)$$

The velocity away from us that this galaxy had when it emitted the light we receive now is

$$v(t_e) = H_e\, d_p(t_e)$$

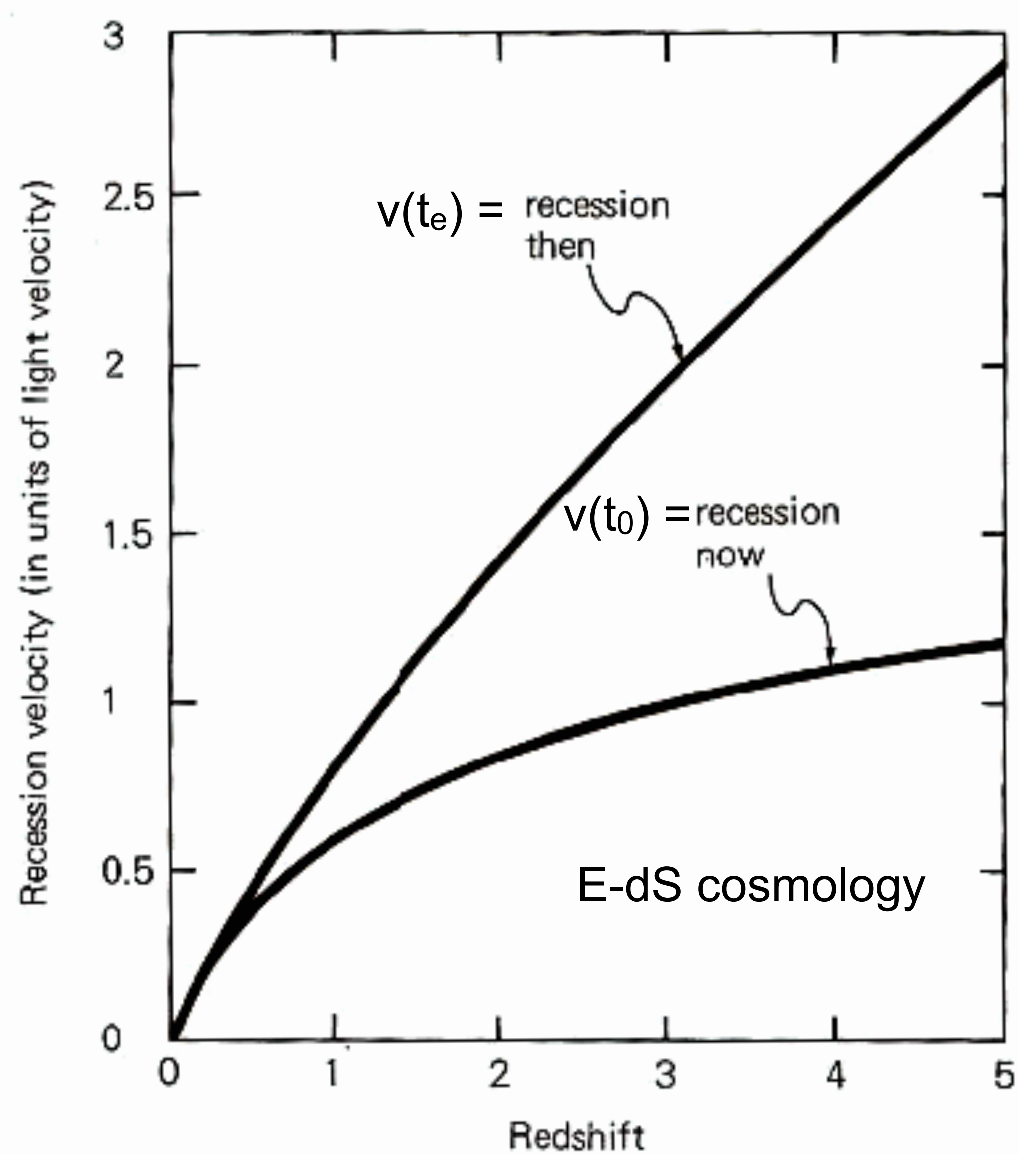
The graph at right shows $v(t_0)$ and $v(t_e)$ for the E-dS cosmology.

For E-dS, where $H = H_0\, a^{-3/2}$,

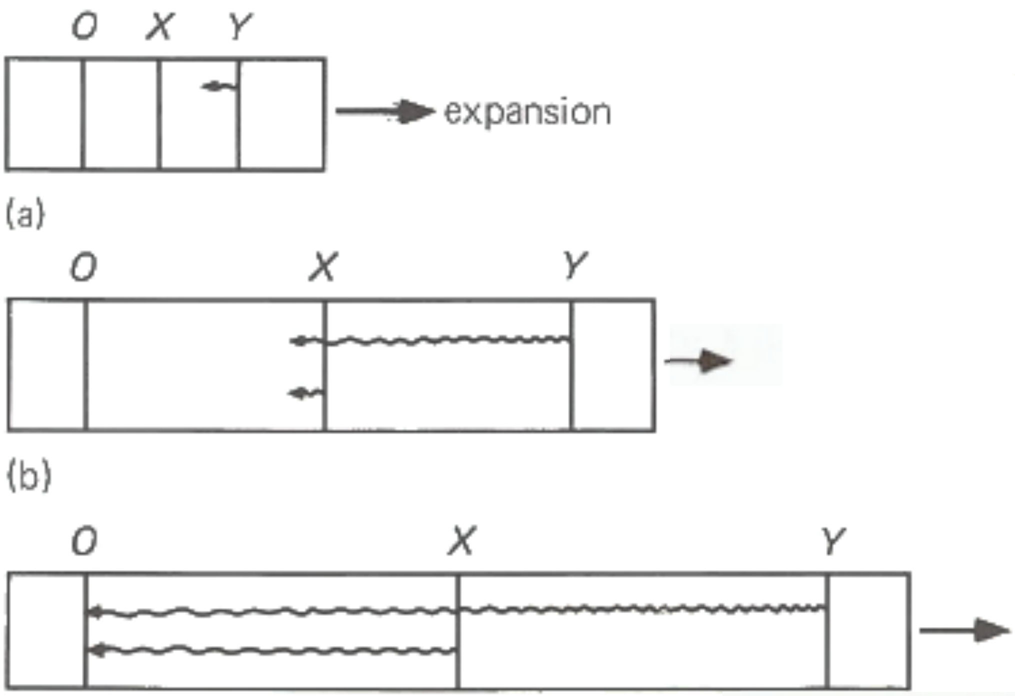
$$v(t_0) = H_0\, d_p(t_0) = 2c\,(1-a^{1/2})$$

$$v(t_e) = H_e\, d_p(t_e)$$

$$= H_0\, a_e^{-3/2}\, a_e\, 2c\,(1-a^{1/2})/H_0 = 2c\,(a^{-1/2} - 1)$$



Velocities in an Expanding Universe



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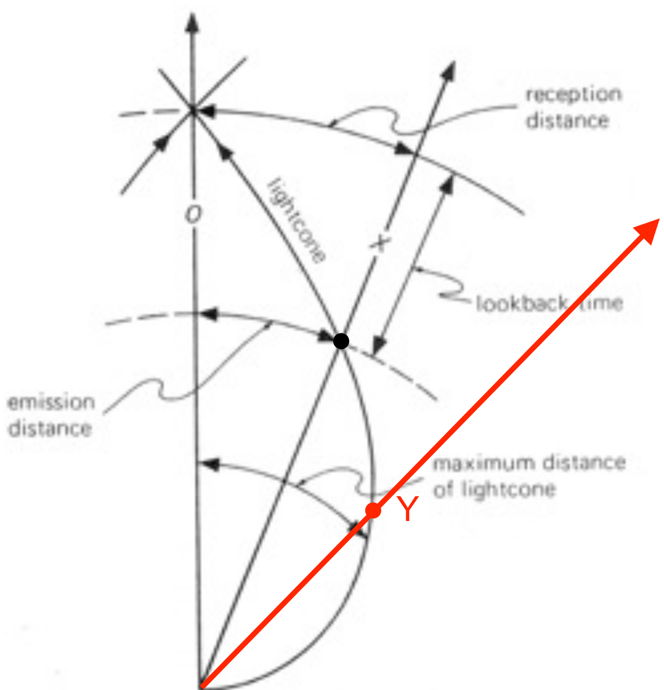

Figure 15.12. On an elastic strip let O represent our position, and X and Y the positions of two galaxies. If signals from X and Y are to reach us at the same instant, then Y, which is farther away, must emit before X. In (a), Y emits a signal. In (b), X emits a signal at a later instant when it is farther away than Y was when it emitted its signal. In (c), both signals arrive simultaneously at O. Y's signal has the greater redshift (it has been stretched more) although Y was closer than X at the time of emission. This odd situation occurs at large redshifts in all big bang universes.