Astro/Phys 224   Spring 2012
Origin and Evolution of the Universe

Week 8
Phase Transitions, Cosmic Defects, and Cosmic Inflation

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Thursday, May 24, 12

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Week		Topic
1	4/3	History and Structure of Cosmology, Surveys
2	4/10	General Relativistic Cosmology: Distances, Horizons
3	4/17	Nucleosynthesis, Recombination, Dark Matter
4	4/24	Dark Matter Candidates and Detection
5	5/1	Cosmic Microwave Background
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7	5/15	Galaxy Formation: Simulations, Semi-Analytic Models
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Outline

Grand Unification of Forces
Phase Transitions in the Early Universe
Topological Defects: Strings, Monopoles
Cosmic Inflation
    Motivation: Horizon, Flatness, Dragons, Structure
    How much inflation is needed?

Grand Unification

The basic premise of grand unification is that the known symmetries of the elementary particles result from a larger (and so far unknown) symmetry group G. Whenever a phase transition occurs, part of this symmetry is lost, so the symmetry group changes. This can be represented mathematically as

$$G \rightarrow H \rightarrow ... \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1).$$

Here, each arrow represents a symmetry breaking phase transition where matter changes form and the groups - G, H, SU(3), etc. - represent the different types of matter, specifically the symmetries that the matter exhibits and they are associated with the different fundamental forces of nature.

The liquid phase of water is rotationally symmetric, that is, it looks the same around each point regardless of the direction in which we look. We could represent this large three-dimensional symmetry by the group G (actually SO(3)). The solid form of frozen water, however, is not uniform in all directions; the ice crystal has preferential lattice directions along which the water molecules align. The group describing these different discrete directions H, say, will be smaller than G. Through the process of freezing, therefore, the original symmetry G is broken down to H.

Grand Unified Theory

GUT refers to a theory in physics that unifies the strong interaction and electroweak interaction. Several such theories have been proposed, but none is currently universally accepted. The (future) theory that will also include gravity is termed theory of everything. Some common GUT models are:

- Georgi-Glashow (1974) model -- SU(5)
- SO(10)
- Flipped SU(5) -- SU(5)×U(1)
- Pati-Salam model -- SU(4)×SU(2)×SU(2)
- E_6

GUT models generically predict the existence of topological defects such as monopoles, cosmic strings, domain walls, and others. None have been observed and their absence is known as the monopole problem in cosmology.

There is still no hard evidence nature is described by a GUT theory. In fact, since the Higgs particle hasn't been officially discovered yet (?), it's not even certain that the Standard Model is fully accurate.

Topological Defects

These arise when some n-component scalar field $\phi_i(\mathbf{x}) = 0$ because of topological trapping that occurs as a result of a phase transition in the early universe (as I will explain shortly).

If the ϕ field is complex then n=2, and $\phi_i(\mathbf{x}) = 0$ occurs along a linear locus of points, a **string,** in three dimensional space. This corresponds to a 2-dimensional world-sheet in the 3+1 dimensions of spacetime.

If the ϕ field has three components, then $\phi_i(\mathbf{x}) = 0$ occurs at a point in three dimensional space, a **monopole**. This corresponds to a 1-dimensional world-line in the 3+1 dimensions of spacetime.

If the ϕ field has four components, then $\phi_i(\mathbf{x}) = 0$ occurs at a point in space-time, an **instanton**. A related concept is **texture**.
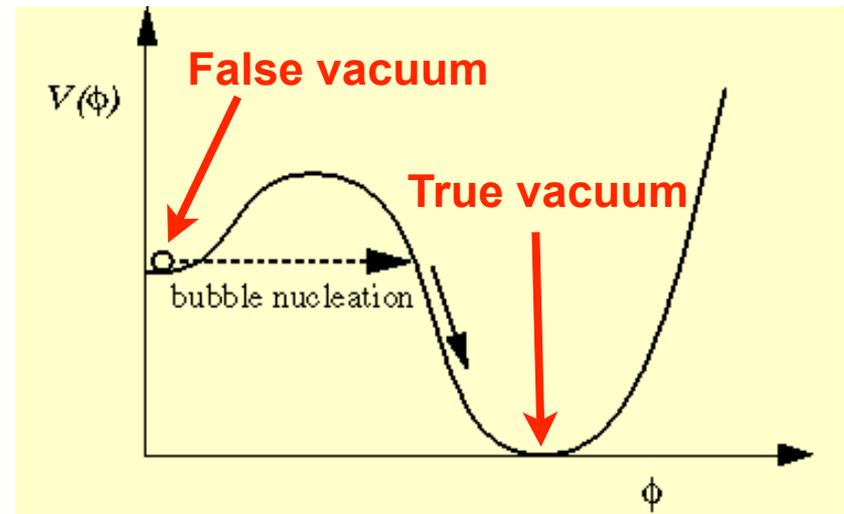
Phase transitions

The cosmological significance of symmetry breaking is due to the fact that symmetries are restored at high temperature (just as it is for liquid water when ice melts). For extremely high temperatures in the early universe, we might even achieve a grand unified state G. Viewed from the moment of creation forward, the universe will pass through a sucession of phase transitions at which the strong nuclear force will become differentiated and then the weak nuclear force and electromagnetism.

Phase transitions can have a wide variety of important implications including the formation of topological defects - cosmic strings, domain walls, monopoles and textures, or it may even trigger a period of exponential expansion (inflation). Phase transitions can be either dramatic - first order, or smooth - second order.

During a first-order phase transition, the matter fields get trapped in a `false vacuum' state from which they can only escape by nucleating bubbles of the new phase, that is, the `true vacuum' state.

False vacuum

True vacuum

$V(\phi)$

bubble nucleation

ϕ

First-order phase transitions (illustrated below) occur through the formation of bubbles of the new phase in the middle of the old phase; these bubbles then expand and collide until the old phase disappears completely and the phase transition is complete.
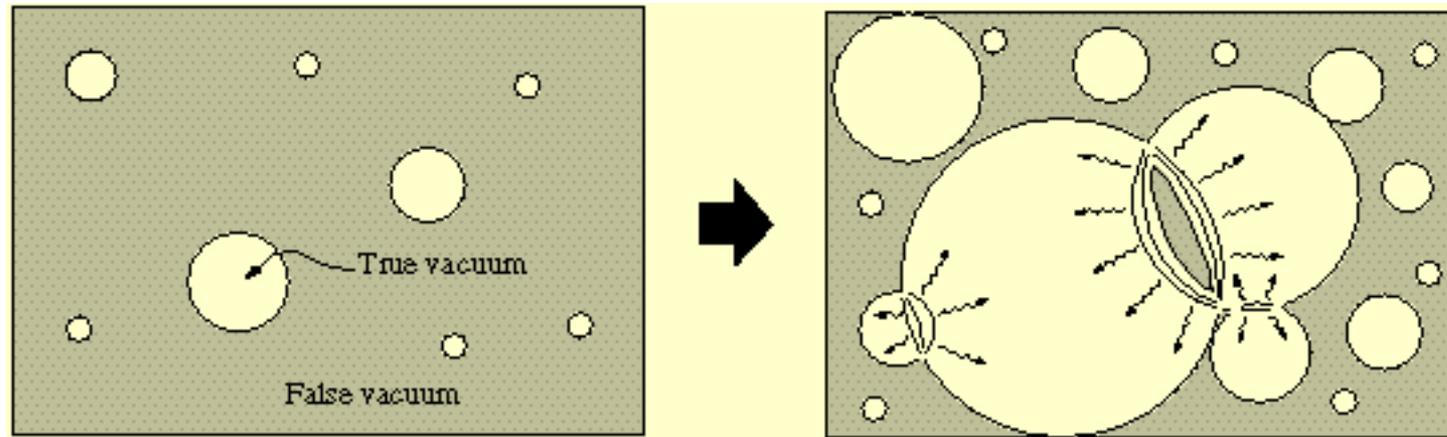
First-order phase transitions proceed by bubble nucleation. A bubble of the new phase (the true vacuum) forms and then expands until the old phase (the false vacuum) disappears. A useful analogue is boiling water in which bubbles of steam form and expand as they rise to the surface.

Second-order phase transitions, on the other hand, proceed smoothly. The old phase transforms itself into the new phase in a continuous manner. There is energy (specific heat of vaporization, for example) associated with a first order phase transition.

Either type of phase transition can produce stable configurations called "topological defects."

Cosmic Strings & Other Topological Defects

Topological defects are stable configurations that are in the original, symmetric or old phase, but nevertheless for topological reasons they persist after a phase transition to the asymmetric or new phase is completed - because to unwind them would require a great deal of energy. There are a number of possible types of defects, such as domain walls, cosmic strings, monopoles, and textures. The type of defect is determined by the symmetry properties of the matter and the nature of the phase transition.

Domain walls: These are two-dimensional objects that form when a discrete symmetry is broken at a phase transition. A network of domain walls effectively partitions the universe into various `cells'. Domain walls have some rather peculiar properties. For example, the gravitational field of a domain wall is repulsive rather than attractive.
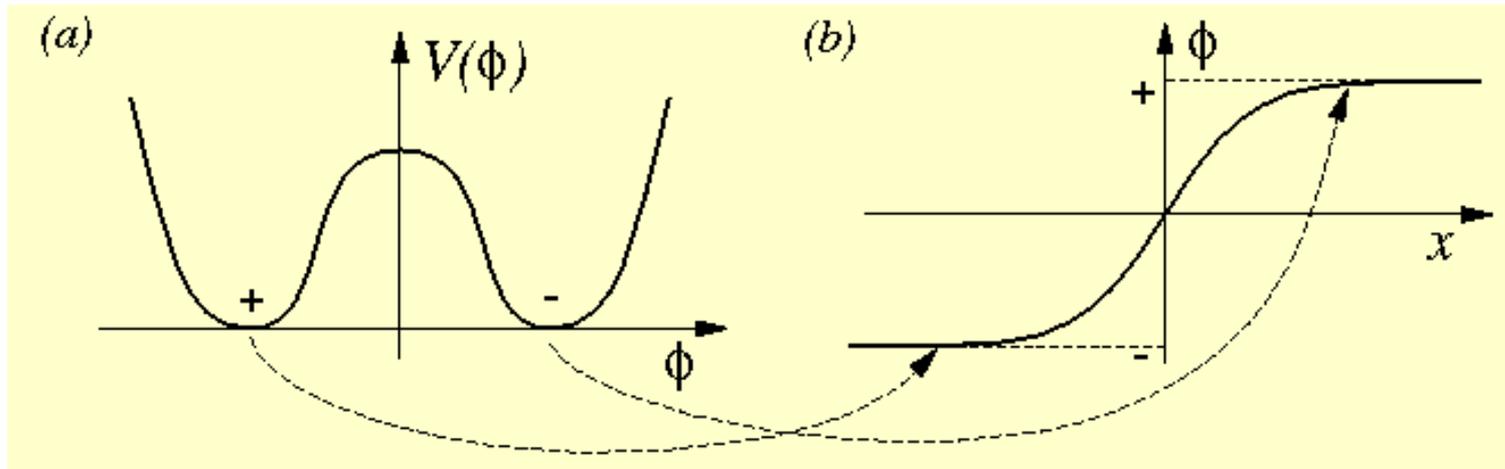
Cosmic strings: These are one-dimensional (that is, line-like) objects which form when an axial or cylindrical symmetry is broken. Strings can be associated with grand unified particle physics models, or they can form at the electroweak scale. They are very thin and may stretch across the visible universe. A typical GUT string has a thickness that is less then a trillion times smaller that the radius of a hydrogen atom, but a 10 km length of one such string would weigh as much as the earth itself!
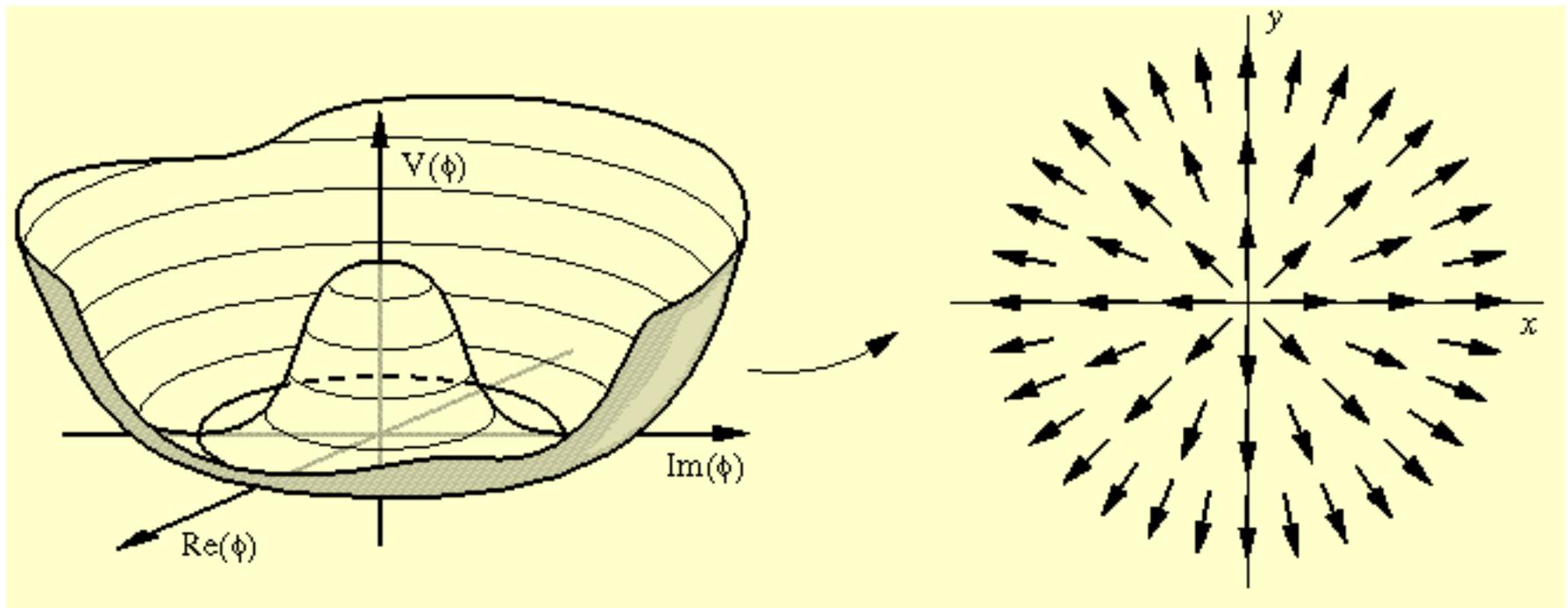

Cosmic strings are associated with models in which the set of minima are not simply-connected, that is, the vacuum manifold has `holes' in it. The minimum energy states on the left form a circle and the string corresponds to a non-trivial winding around this.

Monopoles: These are zero-dimensional (point-like) objects which form when a spherical symmetry is broken. Monopoles are predicted to be supermassive and carry magnetic charge. The existence of monopoles is an inevitable prediction of grand unified theories (GUTs - more on this shortly); why the universe isn't filled with them is one of the puzzles of the standard cosmology.

Monopole
(a)

No Monopole
(b)
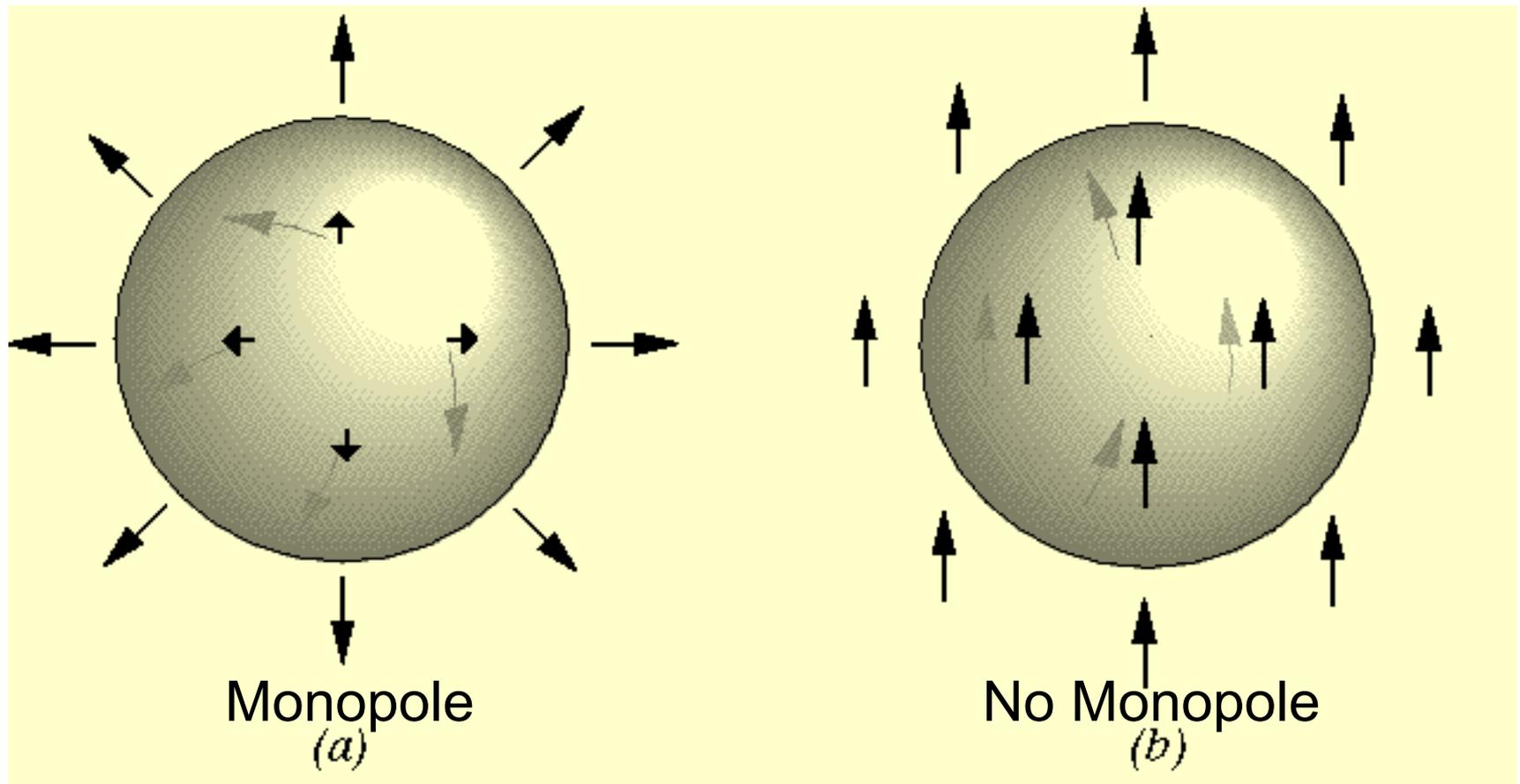
Textures: These form when larger, more complicated symmetry groups are completely broken. Textures are delocalized topological defects which are unstable to collapse. A speculation that the largest "cold spot" in the WMAP CMB data was caused by cosmic textures was published by Cruz et al. (2007, Science 318, 1612), but by 2010 this was shown to be unlikely.

(a)

(b)
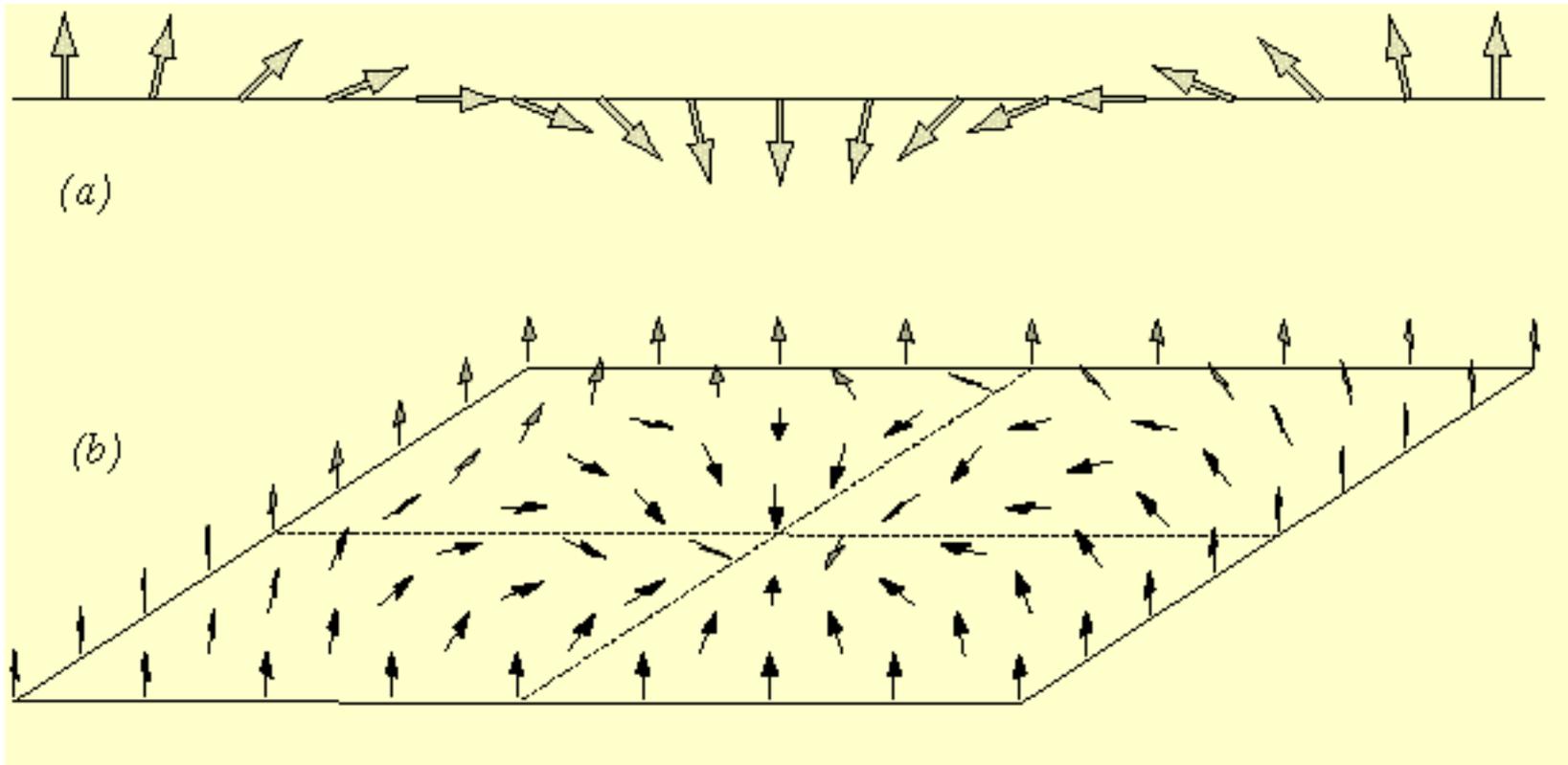
Examples of delocalized texture configurations in one and two dimensions.

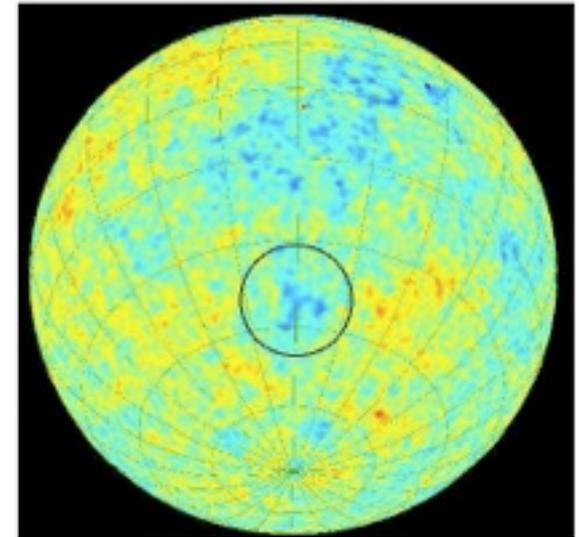# A Cosmic Microwave Background Feature Consistent with a Cosmic Texture

M. Cruz,[1,2]* N. Turok,[3] P. Vielva,[1] E. Martínez-González,[1] M. Hobson[4]

The Cosmic Microwave Background provides our most ancient image of the universe and our best tool for studying its early evolution. Theories of high-energy physics predict the formation of various types of topological defects in the very early universe, including cosmic texture, which would generate hot and cold spots in the Cosmic Microwave Background. We show through a Bayesian statistical analysis that the most prominent 5°-radius cold spot observed in all-sky images, which is otherwise hard to explain, is compatible with having being caused by a texture. From this model, we constrain the fundamental symmetry-breaking energy scale to be $\phi_0 \approx 8.7 \times 10^{15}$ gigaelectron volts. If confirmed, this detection of a cosmic defect will probe physics at energies exceeding any conceivable terrestrial experiment.

The Axis of Evil revisited

Kate Land, Joao Magueijo, 2007 MNRAS, 378, 153

Abstract: In light of the three-year data release from WMAP we re-examine the evidence for the ``Axis of Evil'' (AOE) [anomalous alignment of CMB multipoles in the direction $l \simeq -100$, $b = 60$]. We discover that previous statistics are not robust with respect to the data-sets available and different treatments of the galactic plane. We identify the cause of the instability and implement an alternative ``model selection'' approach. A comparison to Gaussian isotropic simulations find the features significant at the 94-98% level, depending on the particular AOE model. The Bayesian evidence finds lower significance, ranging from ``substantial'' to no evidence for the most general AOE model.

The zone of the CS has been placed at the center of the black circle.

Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Are There Cosmic Microwave Background Anomalies? C. Bennett et al. WMAP7 Jan 2010

A simple six-parameter LCDM model provides a successful fit to WMAP data, both when the data are analyzed alone and in combination with other cosmological data. Even so, it is appropriate to search for any hints of deviations from the now standard model of cosmology, which includes inflation, dark energy, dark matter, baryons, and neutrinos. The cosmological community has subjected the WMAP data to extensive and varied analyses. While there is widespread agreement as to the overall success of the six-parameter LCDM model, various "anomalies" have been reported relative to that model. In this paper we examine potential anomalies and present analyses and assessments of their significance. In most cases we find that claimed anomalies depend on posterior selection of some aspect or subset of the data. Compared with sky simulations based on the best fit model, one can select for low probability features of the WMAP data. Low probability features are expected, but it is not usually straightforward to determine whether any particular low probability feature is the result of the a posteriori selection or of non-standard cosmology. We examine in detail the properties of the power spectrum with respect to the LCDM model. We examine several potential or previously claimed anomalies in the sky maps and power spectra, including cold spots, low quadrupole power, quadrupole-octupole alignment, hemispherical or dipole power asymmetry, and quadrupole power asymmetry. **We conclude that there is no compelling evidence for deviations from the LCDM model, which is generally an acceptable statistical fit to WMAP and other cosmological data.**

Why do cosmic topological defects form?

If cosmic strings or other topological defects *can* form at a cosmological phase transition, then they *will* form. This was first pointed out by Tom Kibble and, in a cosmological context, the defect formation process is known as the **Kibble mechanism**.

The simple fact is that causal effects in the early universe can only propagate (as at any time) at the speed of light c. This means that at a time t, regions of the universe separated by more than a distance $d=ct$ can know nothing about each other. In a symmetry breaking phase transition, different regions of the universe will choose to fall into different minima in the set of possible states (this set is known to mathematicians as the vacuum manifold). Topological defects are precisely the "boundaries" between these regions with different choices of minima, and their formation is therefore an inevitable consequence of the fact that different regions cannot agree on their choices.

For example, in a theory with two minima, plus + and minus -, then neighboring regions separated by more than ct will tend to fall randomly into the different states (as shown below). Interpolating between these different minima will be a **domain wall**.

domain walls
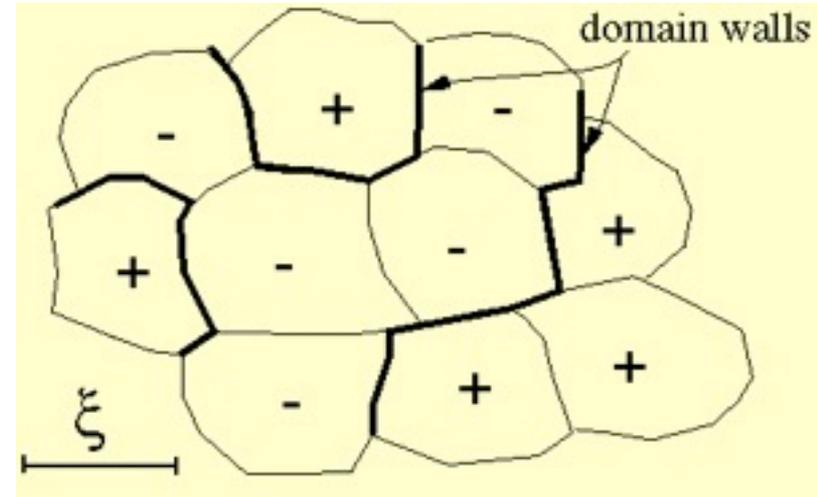
Cosmic strings will arise in slightly more complicated theories in which the minimum energy states possess `holes'. The strings will simply correspond to non-trivial `windings' around these holes (as illustrated at right).

The Kibble mechanism for the formation of cosmic strings.
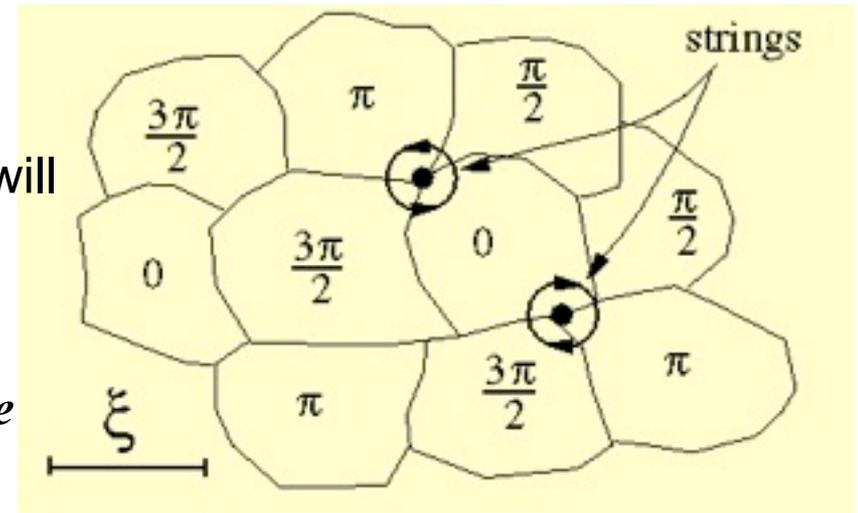
Topological defects can provide a unique link to the physics of the very early universe. Furthermore, they can crucially affect the evolution of the universe, so their study is an unavoidable part of any serious attempt to understand the early universe. The cosmological consequences vary with the type of defect considered. **Domain walls and monopoles are cosmologically catastrophic.** Any cosmological model in which they form will evolve in a way that contradicts the basic observational facts that we know about the universe. Such models must therefore be ruled out! **Cosmic inflation was invented to solve this problem.**

Cosmic strings and textures are (possibly) much more benign. Among other things, they were until recently thought to be a possible source of the fluctuations that led to the formation of the large-scale structures we observe today, as well as the anisotropies in the Cosmic Microwave Background. However, the CMB anisotropies have turned out not to agree with the predictions of this theory.

By 2000, it was clear that **cosmic defects are not the main source of the CMB anisotropies**.
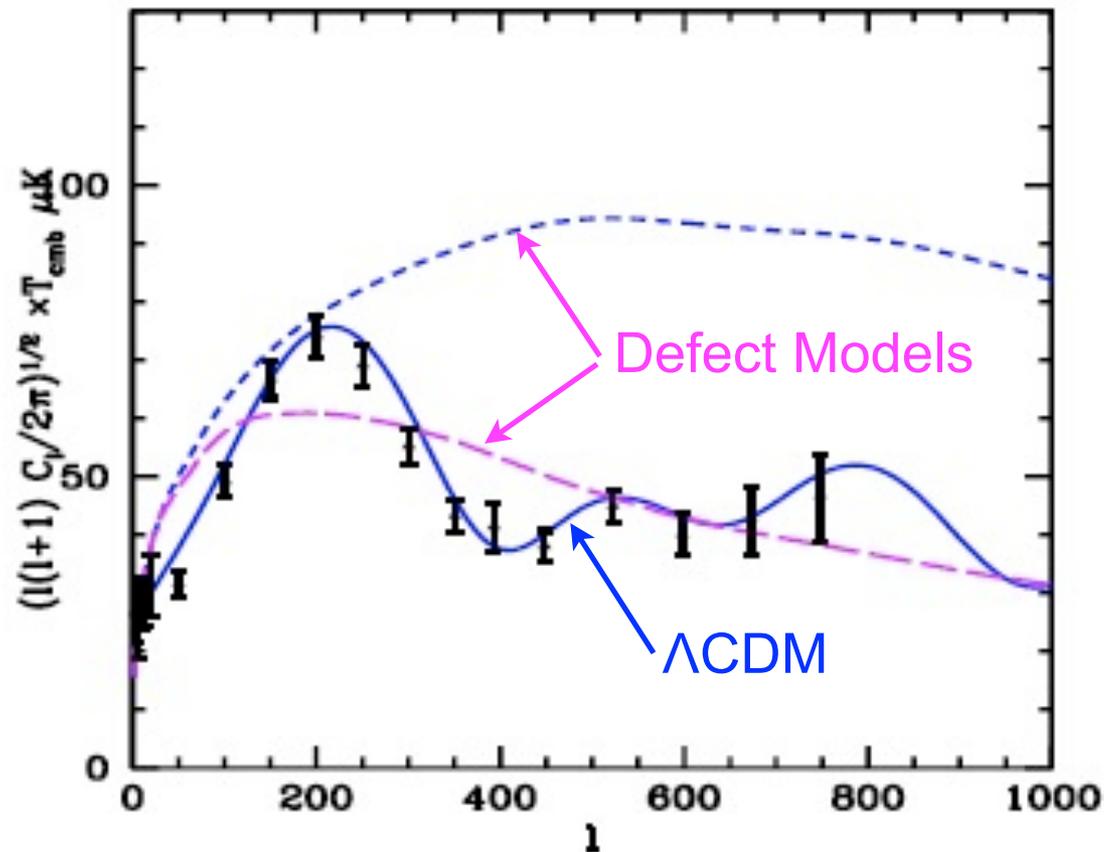

Figure 3: Current data (as complied by Knox[22]) with two defect models (dashed) and an inflation-based model (solid). The upper defect model has a standard ionization history and the lower model has an ionization history specifically designed to produce a sharper, shifted peak.

Andreas Albrecht, Defect models of cosmic structure in light of the new CMB data, XXXVth Rencontres de Moriond ``Energy Densities in the Universe'' (2000).

Cosmic String Dynamics and Evolution

The evolution of cosmic string network is the relatively complicated result of only three rather simple and fundamental processes: cosmological expansion, intercommuting & loop production, and radiation.

Cosmological expansion

The overall expansion of the universe will 'stretch' the strings, just like any other object that is not gravitationally bound. You can easily undertand this through the well-known analogy of the expanding balloon. If you draw a line of the surface of the balloon and then blow it up, you will see that the length of your `string' will grow at the same rate as the radius of the balloon.

Intercommuting & loop production

Whenever two long strings cross each other, they exchange ends, or `intercommute' (case (a) in the figure below). In particular, a long string can intercommute with itself, in which case a loop will be produced (this is case (b) below).


(a)   (b)
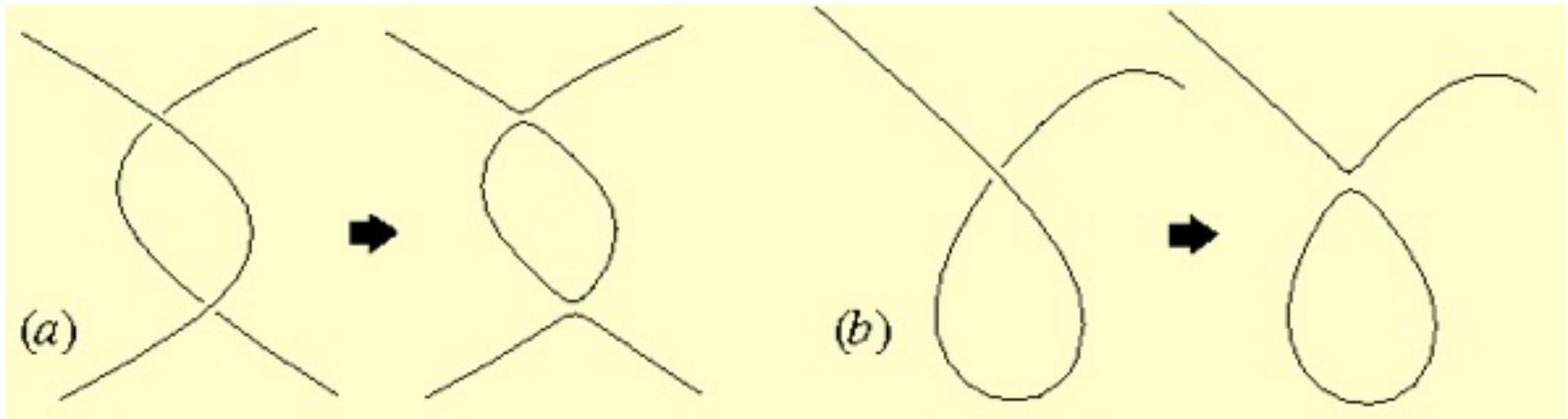
Radiation from strings

Both long cosmic strings and small loops will emit radiation. In most cosmological scenarios this will be **gravitational radiation**, but electromagnetic radiation or axions can also be emitted in some cases (for some specific phase transitions).

The effect of radiation is much more dramatic for loops, since they lose all their energy this way, and eventually disappear. Here you can see what happens in the case of two interlocked loops. This configuration is unlikely to happen in a cosmological setting, but it is nevertheless quite enlightening. Notice the succession of complicated dynamic processes before the loop finally disappears!

After formation, an initially high density string network begins to chop itself up by producing small loops. These loops oscillate rapidly (relativistically) and decay away into gravitational waves.
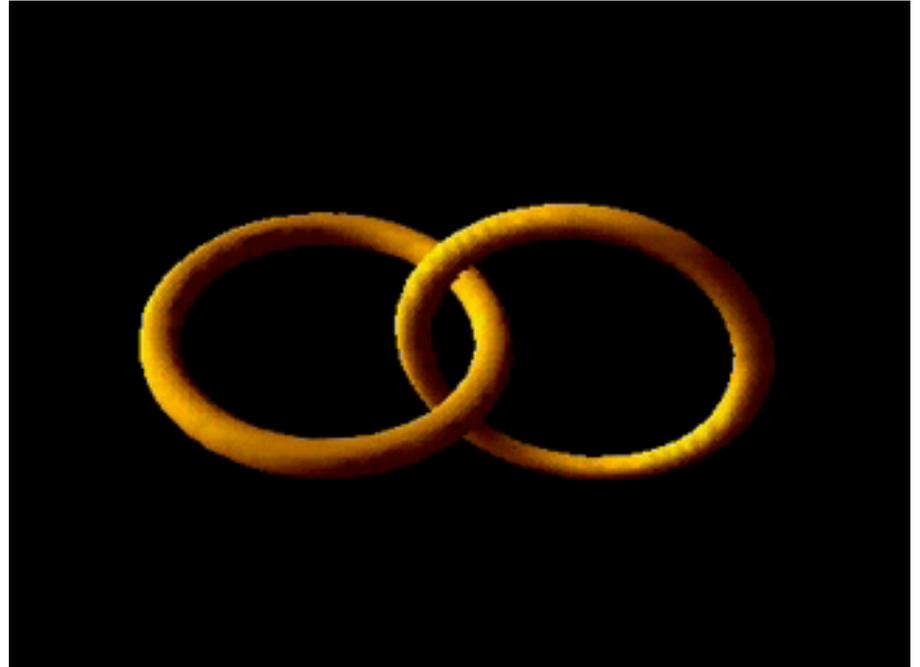
The net result is that the strings become more and more dilute with time as the universe expands. From an enormous density at formation, mathematical modelling suggests that today there would only be about 10 long strings stretching across the observed universe, together with about a thousand small loops!

In fact the network dynamics is such that the string density will eventually stabilize at an exactly constant level relative to the rest of the radiation and matter energy density in the universe. Thus the string evolution is described as `**scaling**' or scale-invariant, that is, the properties of the network look the same at any particular time t if they are scaled (or multiplied) by the change in the time. This is schematically represented below:
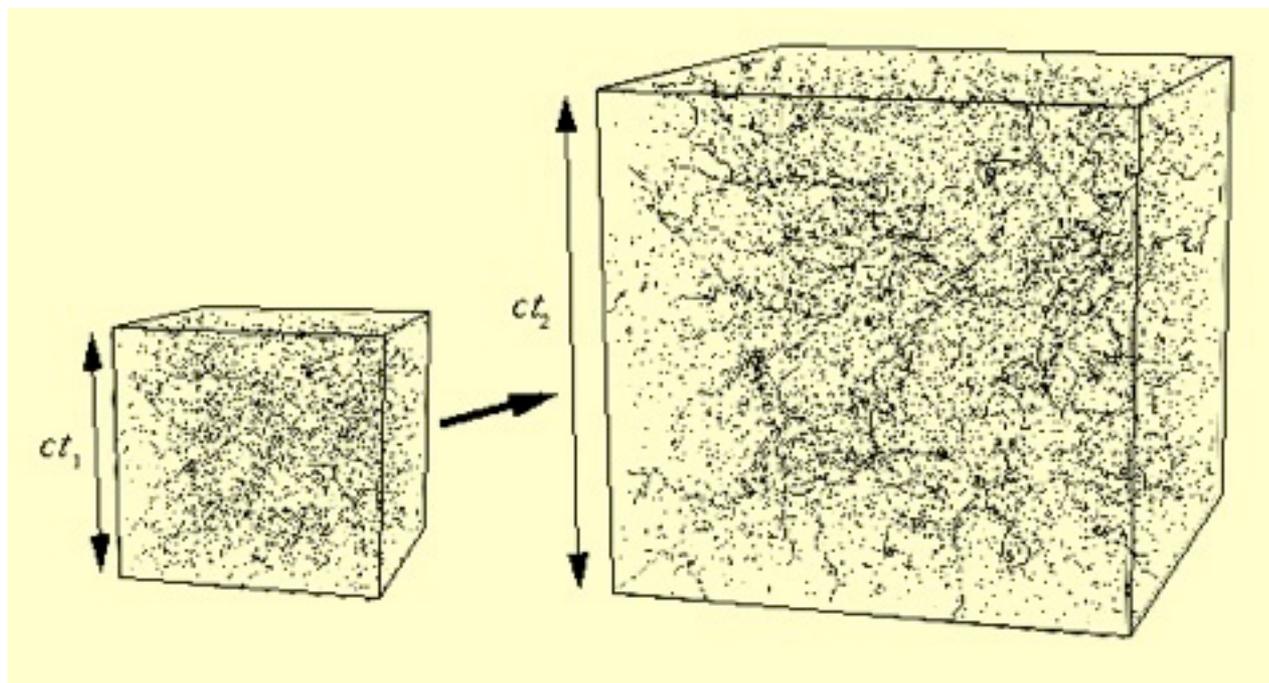
Because strings are extremely complex non-linear objects, the only rigorous way to study their evolution and cosmological consequences is to simulate in on the computer. One of the aims of performing numerical simulations of the evolution of cosmic string networks is to subsequently use the resulting information as an input to build (relatively) simpler semianalytic models that reproduce (in an averaged sense) the crucial properties of these objects. One starts by generating an initial "box of stings" containing a configuration of strings such as one would expect to find after a phase transition in the early universe. Then one evolves this initial box, by using the laws of motion of the strings.
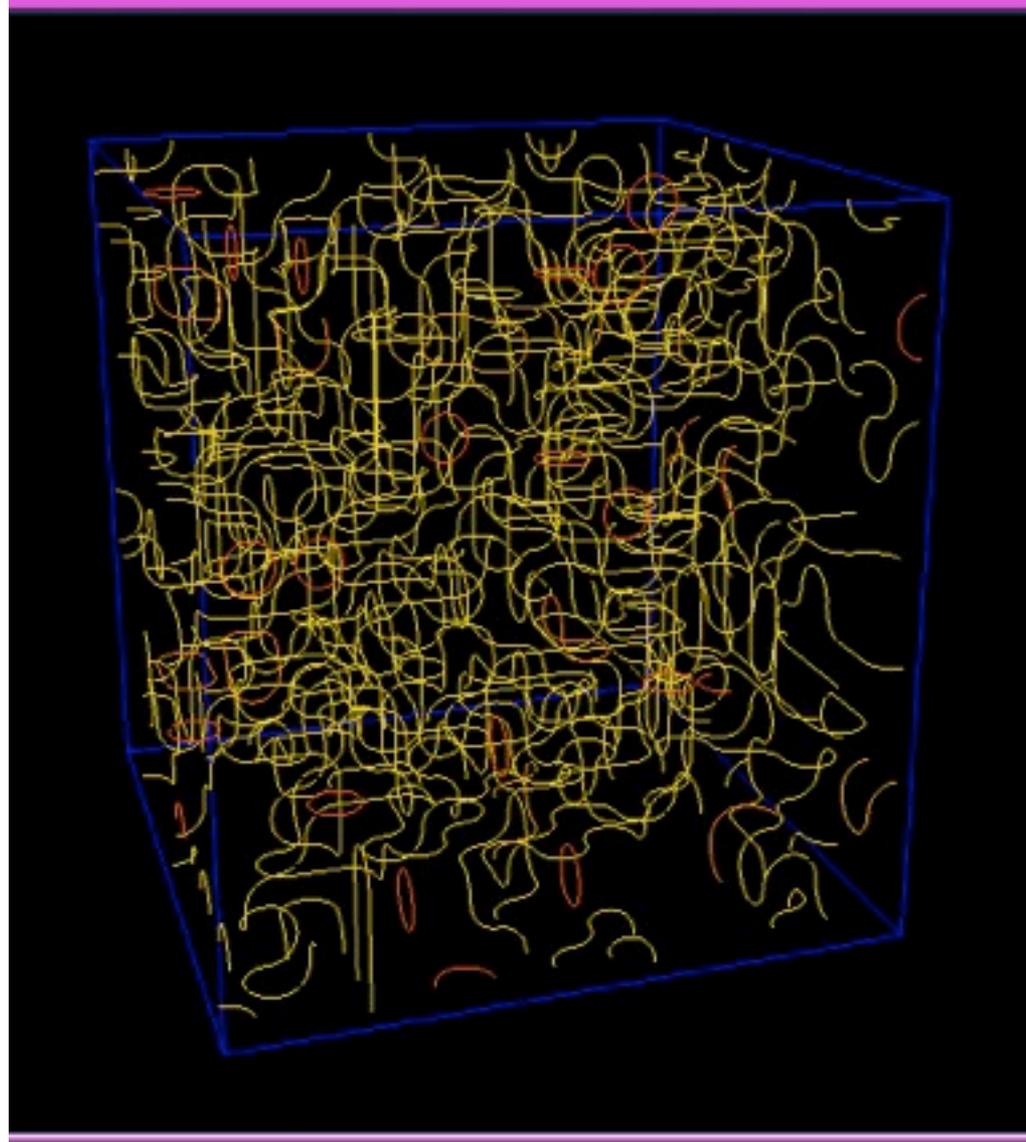
In this and all other pictures and movies below long strings are shown in yellow, while small loops have a color code going from yellow to red according to their size (red loops being the smallest).

Why do the two boxes below look different? Because the rate at which the universe is expanding is different.
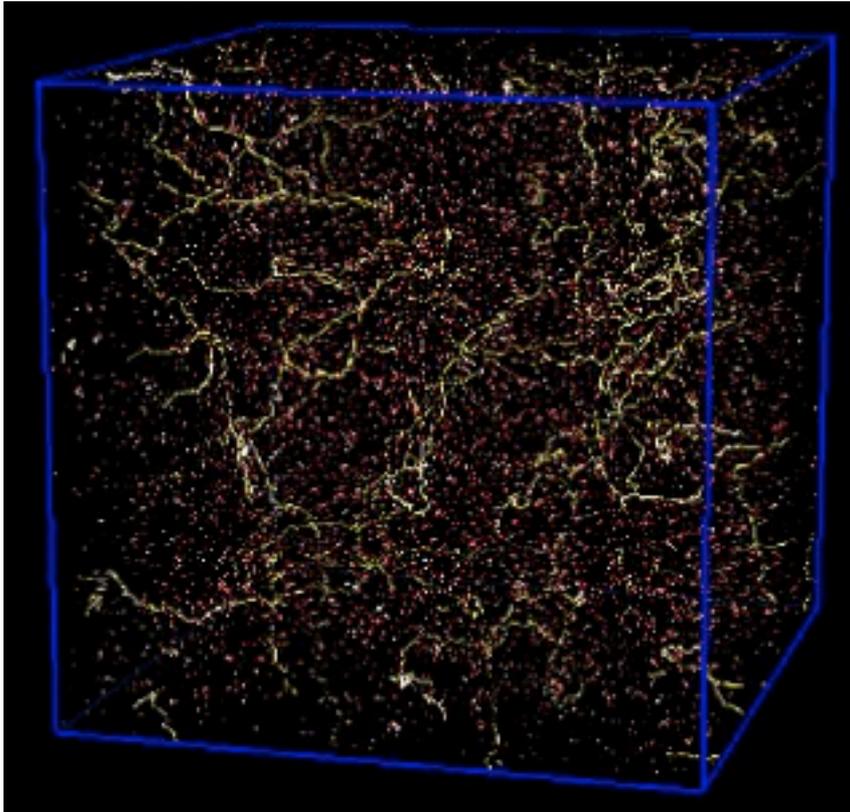
Snapshot of a string network in the radiation era. Note the high density of small loops and the `wiggliness' of the long strings in the network. The box size is about 2ct. (B. Allen & E. P. Shellard)
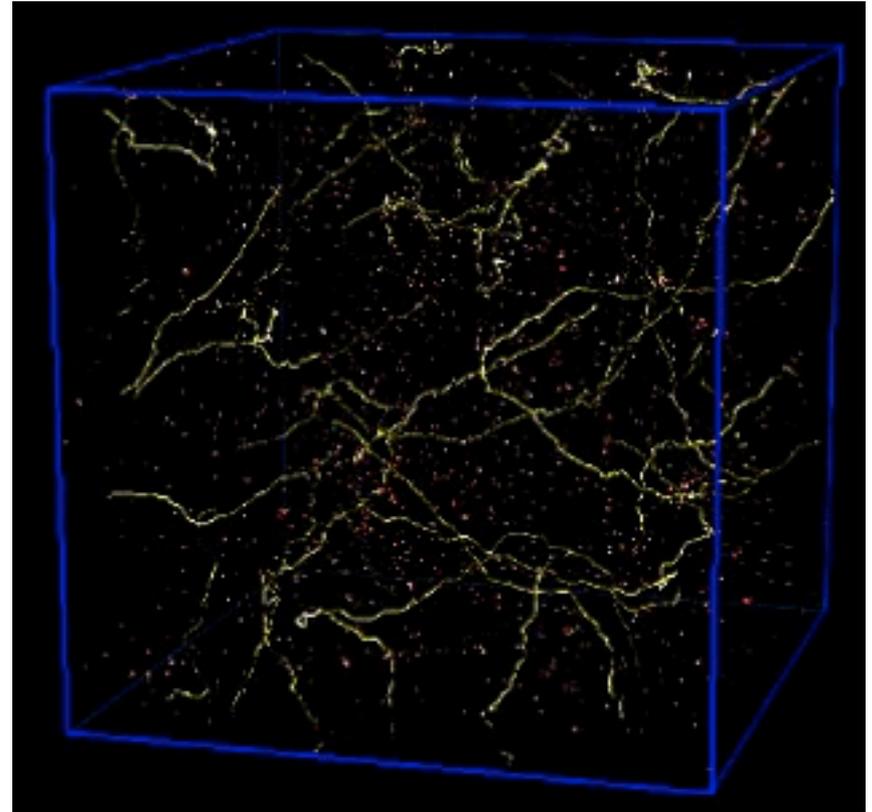
Snapshot of a string network in the matter era. Compare with the radiation case at left. Notice the lower density of both long strings and loops, as well as the lower `wiggliness' of the former. The box size is again about 2ct.

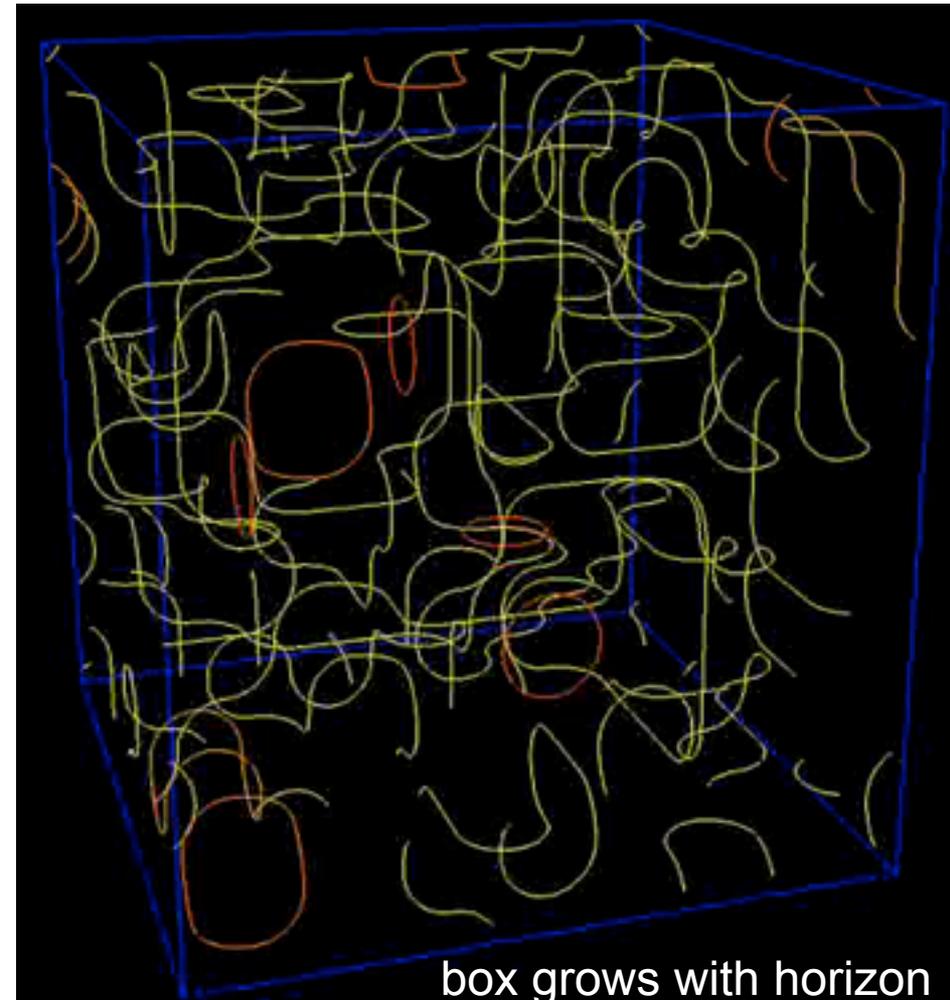**Two movies of the evolution of a cosmic string network in the radiation era. In the movie on the left the box has a fixed size (so you will see fewer and fewer strings as it evolves), while in the one on the right it grows as the comoving horizon. (C. Martins & E. P. Shellard)**

fixed size box

box grows with horizon
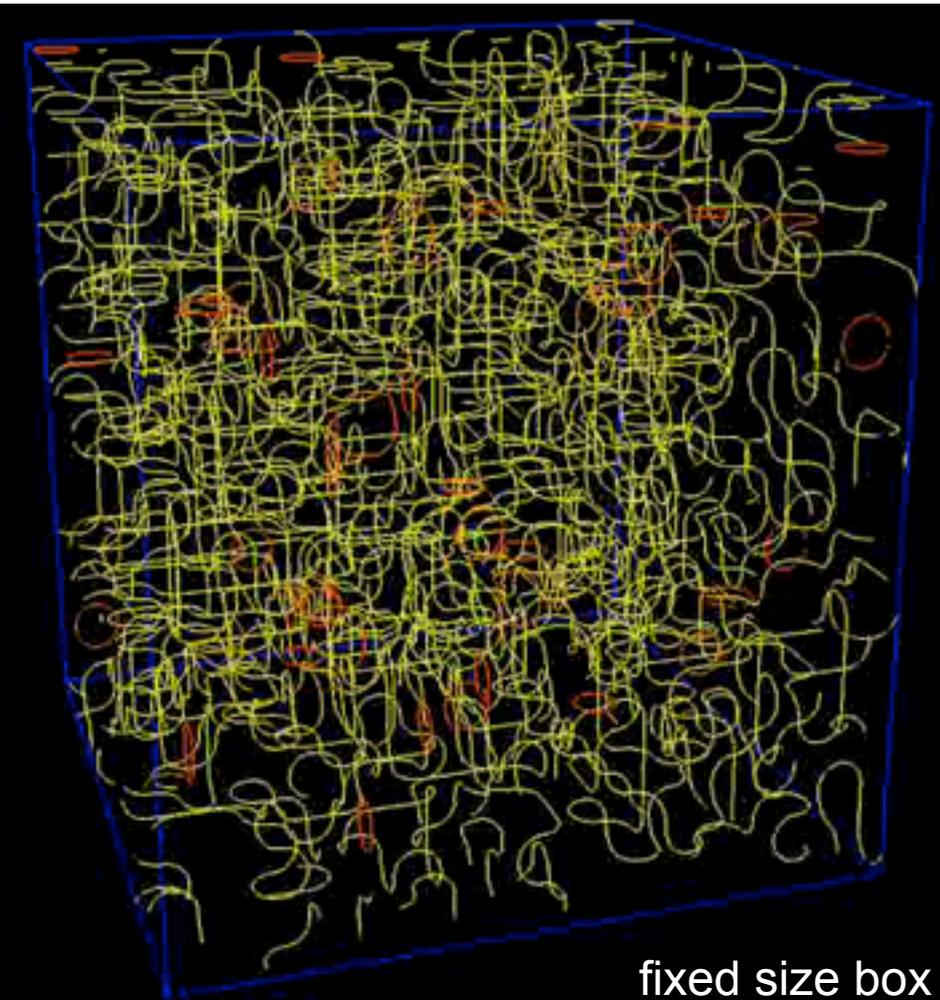
Notice that the number of long strings in the box that grows with the horizon remains roughly constant, in agreement with the scaling hypothesis. This is because the additional length in strings is quickly converted into small loops.

Two movies of the evolution of a cosmic string network in the matter era. In the movie on the left the box has a fixed size (so you will see fewer and fewer strings as it evolves), while in the one on the right it grows as the comoving horizon. (C. Martins & E. P. Shellard)

fixed size box

box grows with horizon
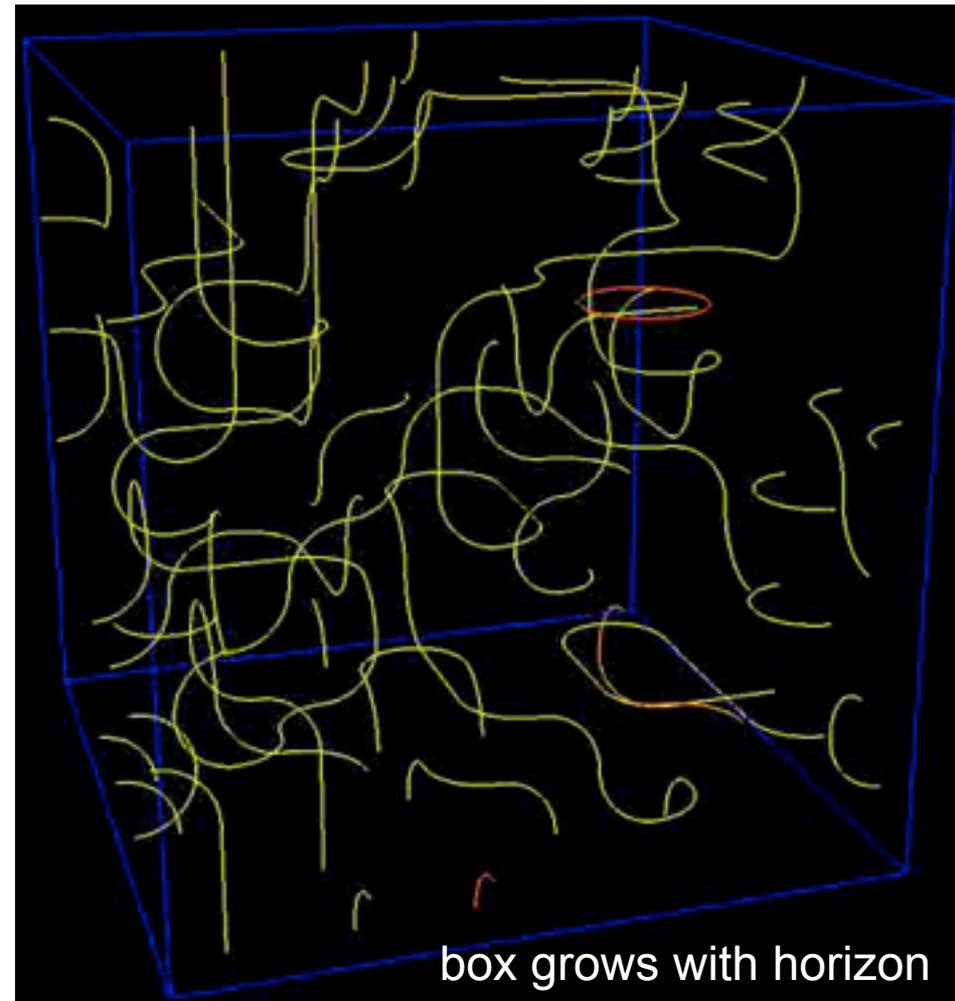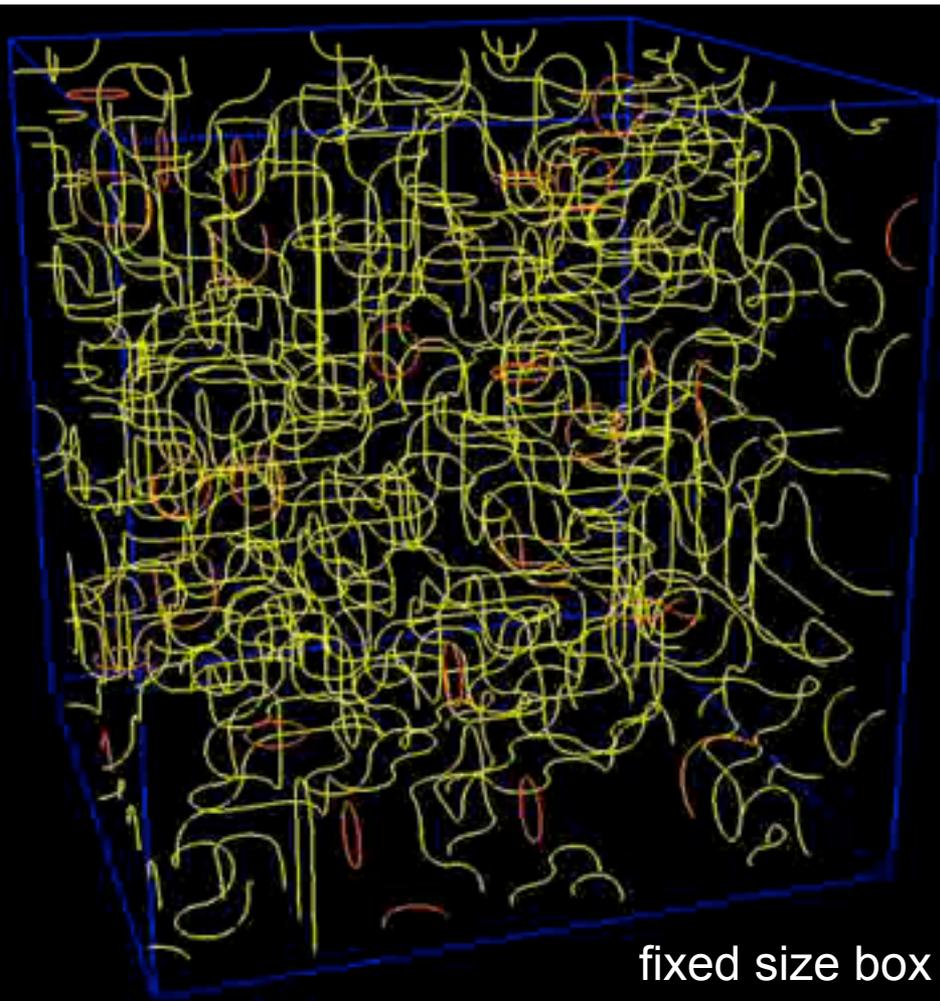
Notice that the number of long strings in the box that grows with the horizon again remains roughly constant, in agreement with the scaling hypothesis. This is because the additional length in strings is quickly converted into

When strings evolve, scaling from smaller scales to larger ones, they create perturbations in the matter energy density of the universe. Because of their tension, cosmic strings pull straight as they come inside the horizon. Although there is no gravitational force from a static string, such moving cosmic strings produce wakes toward which matter falls, thus serving as seeds for structure formation. For a static string along the z axis of mass μ per unit length, the energy momentum tensor is

$$T^{\mu\nu} = \mu \operatorname{diag}(1,0,0,-1)\delta(x)\delta(y)$$

and the metric is

$$ds^2 = dt^2 - dz^2 - dR^2 - (1 - 4G\mu)^2 R^2 d\varphi^2$$

$G\mu \approx (M_{GUT}/M_{Pl})^2 \approx 10^{-6}$ is just the magnitude needed for GUT string structure formation. There is an **angular defect** of $8\pi G\mu = 5.18"$ $(10^6 \, G\mu)$. This implies that the geodesic path of light is curved towards a string when light passes by it. Two copies of a galaxy near a cosmic string will appear to observers on the other side of the string.
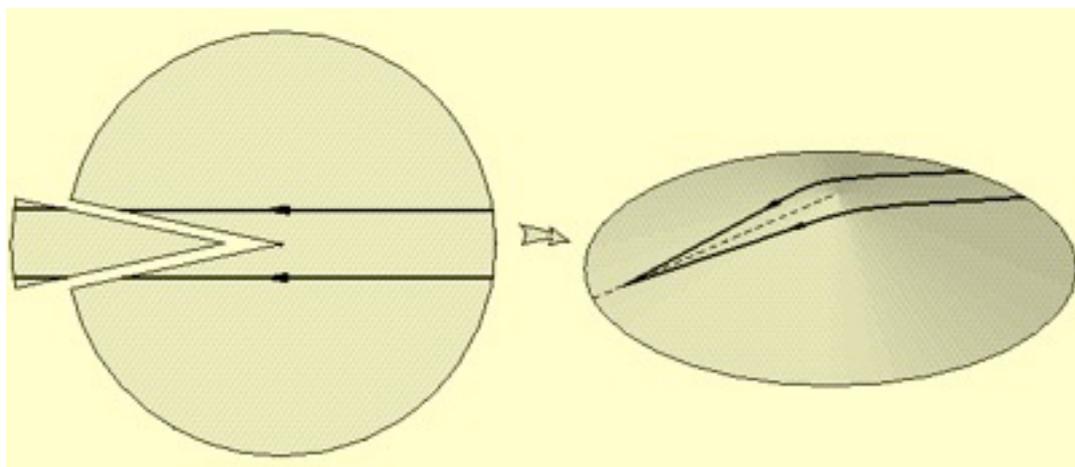
A. Vilenkin, E. P. S. Shellard, **Cosmic Strings and Other Topological Defects** (Cambridge U P, 1994)

Thursday, May 24, 12

Cosmic Strings Summary

Cosmic strings arise in spontaneously broken (SB) gauge theories

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - |\partial_\mu \phi|^2 - \lambda \left(|\phi|^2 - \phi_v^2 \right)^2$$

as a consequence of causality in the expanding universe.
As the temperature T falls, a complex scalar
field ϕ gets a nonzero expectation value

$$\phi(x) = \phi_v \, e^{\,i\theta(x)}$$

The phase θ will inevitably be different in
regions separated by distances greater than
the horizon size when the SB phase transition occurred. If θ runs over $0 \rightarrow$
2π as x goes around a loop in space, the loop encloses a string.
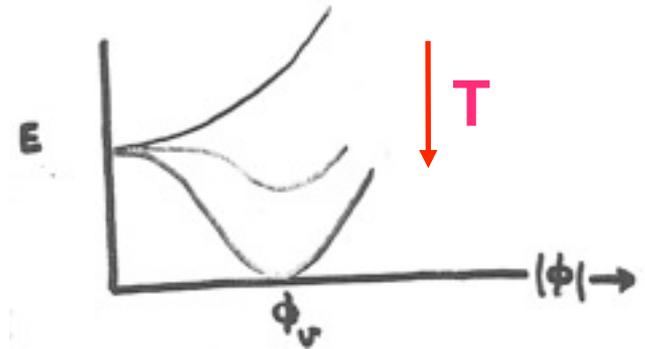
Fitting CMB data with cosmic strings and inflation, by Neil Bevis, Mark Hindmarsh, Martin Kunz, and Jon Urrestilla 2008 PRL100.021301

The inflationary paradigm is successful in providing a match to measurements of the cosmic microwave background (CMB) radiation, and it appears that **any successful theory of high energy physics must be able to incorporate inflation**. While ad hoc single-field inflation can provide a match to the data, **more theoretically motivated models commonly predict the existence of cosmic strings**. These strings are prevalent in supersymmetric inflation models and occur frequently in grand-unified theories (GUTs). String theory can also yield strings of cosmic extent. Hence the observational consequences of cosmic strings are important, including their sourcing of additional anisotropies in the CMB radiation. In this letter we present a multi-parameter fit to CMB data for models incorporating cosmic strings. It is the first such analysis to use simulations of a fully dynamical network of local cosmic strings, and the first to incorporate their microphysics with a field theory. It yields conclusions which differ in significant detail from previous analyses based upon simplified models: **we find that the CMB data moderately favor a 10% contribution from strings to the temperature power spectrum** measured at multipole $\ell = 10$ with a corresponding spectral index of primordial scalar perturbations $n_s \simeq 1$. There are also important implications for models of inflation with blue power spectra ($n_s > 1$). These are disfavored by CMB data under the concordance model (power-law CDM which gives $n_s = 0.951 + 0.015 - 0.019$) and previous work seemed to show that this remains largely the case even if cosmic strings are allowed. However with our more complete CMB calculations, we find that the CMB puts no pressure on such models if they produce cosmic strings. Our conclusions are slightly modified when additional non-CMB data are included, with the preference for strings then reduced.

Improved limits on short-wavelength gravitational waves from the cosmic microwave background, by Irene Sendra, Tristan L. Smith (arXiv:1203.4232)

We present updated observational bounds for both adiabatic and homogeneous initial conditions using the latest CMB data at small scales from the South Pole Telescope (SPT) in combination with Wilkinson Microwave Anisotropy Probe (WMAP), current measurements of the baryon acoustic oscillations, and the Hubble parameter. With the inclusion of the data from SPT the adiabatic bound on the CGWB density is improved by a factor of 1.7 to $\Omega_{gw} < 8.7 \times 10^{-6}$ at the 95% confidence level (C.L.), with weak evidence in favor of an additional radiation component consistent with previous analyses. The homogeneous bound improves by a factor of 3.5 to $\Omega_{gw} < 1.0 \times 10^{-6}$ at 95% C.L., with no evidence for such a component from current data.

Constraints on cosmic string tension imposed by the limit on the stochastic gravitational wave background from the European Pulsar Timing Array, by S. Sanidas + (arXiv:1201.2419)

We investigate the constraints that can be placed on the cosmic string tension by using the current Pulsar Timing Array limits on the stochastic gravitational wave background (SGWB). Taking into account all the possible uncertainties in the parameters we find a conservative upper limit of $G\mu c^2 < 5.3 \times 10^{-7}$. We discuss the prospects for lowering this limit by two orders of magnitude, or even a detection of the SGWB, in the very near future in the context of the Large European Array for Pulsars and the Square Kilometre Array.
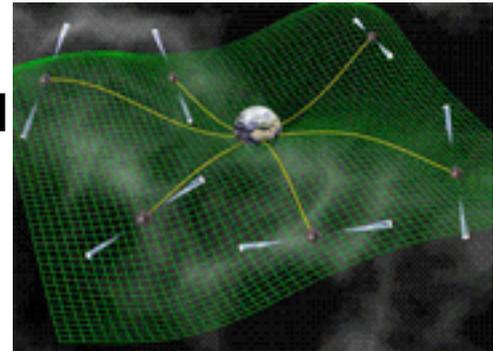
GUT Monopoles

A simple SO(3) GUT illustrates how nonsingular monopoles arise. The Lagragian is

$$\mathcal{L} = \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{8}\lambda(\Phi^a\Phi^a - \sigma^2)^2,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\varepsilon_{abc}A^b_\mu A^c_\nu,$$

$$D_\mu\Phi^a = \partial_\mu\Phi^a - e\varepsilon_{abc}A^b_\mu\Phi^c.$$

The masses of the resulting charged vector and Higgs bosons after spontaneous symmetry breaking are

$$M_V^2 = e^2\sigma^2,$$

$$M_S^2 = \lambda\sigma^2.$$

If the Higgs field Φ^a happens to rotate about a sphere in SO(3) space as one moves around a sphere about any particular point in $\mathbf{x}$-space, then it must vanish at that point. Remarkably, if we identify the massless vector field as the photon, this configuration corresponds to a nonsingular magnetic monopole, as was independently discovered by 'tHooft and Polyakov. The monopole has magnetic charge twice the minimum Dirac value, g = $2\pi/e$ = $(4\pi/e^2)(e/2) \approx$ 67.5 e.

The singular magnetic field is cut off at scale σ, and as a result the GUT monopole has mass $M_{monopole} \approx M_V/\alpha \approx M_{GUT}/\alpha \approx 10^{18}$ GeV.

The first accurate calculation of the mass of the 't Hooft - Polyakov non-singular monopole was Bais & Primack (Phys. Rev. D13:819,1976).

GUT Monopole Problem

The Kibble mechanism produces ~ one GUT monopole per horizon volume when the GUT phase transition occurs. These GUT monopoles have a number density over entropy (using the old $T_{GUT} \sim M_{GUT} \sim 10^{14}$ GeV)

$$n_M/s \sim 10^2 \, (T_{GUT}/M_{Pl})^3 \sim 10^{-13}$$

(compared to $n_B/s \sim 10^{-9}$ for baryons) Their annihilation is inefficient since they are so massive, and as a result they are about as abundant as gold atoms but 10^{16} times more massive, so they "overclose" the universe. This catastrophe must be avoided! **This was Alan Guth's initial motivation for inventing cosmic inflation.**

I will summarize the key ideas of inflation theory, following my lectures at the Jerusalem Winter School, published as the first chapter in Avishai Dekel & Jeremiah Ostriker, eds., *Formation of Structure in the Universe* (Cambridge University Press, 1999), and Dierck-Ekkehard Liebscher, *Cosmology* (Springer, 2005) (available online through the UCSC library).

Motivations for Inflation

PROBLEM SOLVED

Horizon	Homogeneity, Isotropy, Uniform T
Flatness/Age	Expansion and gravity balance
"Dragons"	Monopoles, doman walls,...banished
Structure	Small fluctuations to evolve into galaxies, clusters, voids

Cosmological constant $\Lambda > 0 \Rightarrow$ space repels space, so the more space the more repulsion, $\Rightarrow$ de Sitter exponential expansion $a \propto e^{\sqrt{\Lambda}t}$.

Inflation is exponentially accelerating expansion caused by effective cosmological constant ("false vacuum" energy) associated with hypothetical scalar field ("inflaton").

	FORCES OF NATURE	Spin
Known {	Gravity	2
	Strong, weak, and electromagnetic	1
Goal of LHC	Mass (Higgs Boson)	0
Early universe	Inflation (Inflaton)	0

Inflation lasting only $\sim 10^{-32}$s suffices to solve all the problems listed above. Universe must then convert to ordinary expansion through conversion of false to true vacuum ("re-"heating).

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Inflation Basics

The basic idea of inflation is that before the universe entered the present adiabatically expanding Friedmann era, it underwent a period of de Sitter exponential expansion of the scale factor, termed *inflation* (Guth 1981). Actually, inflation is never precisely de Sitter, and any superluminal (faster-than-light) expansion is now called inflation. Inflation was originally invented to solve the problem of too many GUT monopoles, which, as mentioned in the previous section, would otherwise be disastrous for cosmology.

The de Sitter cosmology corresponds to the solution of Friedmann's equation in an empty universe (i.e., with $\rho = 0$) with vanishing curvature ($k = 0$) and positive cosmological constant ($\Lambda > 0$). The solution is $a = a_o e^{Ht}$, with constant Hubble parameter $H = (\Lambda/3)^{1/2}$. There are analogous solutions for $k = +1$ and $k = -1$ with $a \propto \cosh Ht$ and $a \propto \sinh Ht$ respectively. The scale factor expands exponentially because the positive cosmological constant corresponds effectively to a negative pressure. de Sitter space is discussed in textbooks on general relativity (for example, Rindler 1977, Hawking & Ellis 1973) mainly for its geometrical interest. Until cosmological inflation was considered, the chief significance of the de Sitter solution in cosmology was that it is a limit to which all indefinitely expanding models with $\Lambda > 0$ must tend, since as $a \to \infty$, the cosmological constant term ultimately dominates the right hand side of the Friedmann equation.

Joel Primack, in *Formation of Structure in the Universe*, (Cambridge Univ Press, 1999)

As Guth (1981) emphasized, the de Sitter solution might also have been important in the very early universe because the vacuum energy that plays such an important role in spontaneously broken gauge theories also acts as an effective cosmological constant. A period of de Sitter inflation preceding ordinary radiation-dominated Friedmann expansion could explain several features of the observed universe that otherwise appear to require very special initial conditions: the horizon, flatness/age, monopole, and structure formation problems. (See Table 1.6.)

Let us illustrate how inflation can help with the horizon problem. At recombination $(p^+ + e^- \rightarrow H)$, which occurs at $a/a_o \approx 10^{-3}$, the mass encompassed by the horizon was $M_H \approx 10^{18} M_\odot$, compared to $M_{H,o} \approx 10^{22} M_\odot$ today. Equivalently, the angular size today of the causally connected regions at recombination is only $\Delta\theta \sim 3°$. Yet the fluctuation in temperature of the cosmic background radiation from different regions is very small: $\Delta T/T \sim 10^{-5}$. How could regions far out of causal contact have come to temperatures that are so precisely equal? This is the "horizon problem". With inflation, it is no problem because the entire observable universe initially lay inside a single causally connected region that subsequently inflated to a gigantic scale. Similarly, inflation exponentially dilutes any preceeding density of monopoles or other unwanted relics (a modern version of the "dragons" that decorated the unexplored borders of old maps). Joel Primack, in *Formation of Structure in the Universe*, (Cambridge Univ Press, 1999)

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In the first inflationary models, the dynamics of the very early universe was typically controlled by the self-energy of the Higgs field associated with the breaking of a Grand Unified Theory (GUT) into the standard 3-2-1 model: $GUT \rightarrow SU(3)_{color} \otimes [SU(2) \otimes U(1)]_{electroweak}$. This occurs when the cosmological temperature drops to the unification scale $T_{GUT} \sim 10^{14}$ GeV at about 10^{-35} s after the Big Bang. Guth (1981) initially considered a scheme in which inflation occurs while the universe is trapped in an unstable state (with the GUT unbroken) on the wrong side of a maximum in the Higgs potential. This turns out not to work: the transition from a de Sitter to a Friedmann universe never finishes (Guth & Weinberg 1981). The solution in the "new inflation" scheme (Linde 1982; Albrecht and Steinhardt 1982) is for inflation to occur *after* barrier penetration (if any). It is necessary that the potential of the scalar field controlling inflation ("*inflaton*") be nearly flat (i.e., decrease very slowly with increasing inflaton field) for the inflationary period to last long enough. This nearly flat part of the potential must then be followed by a very steep minimum, in order that the energy contained in the Higgs potential be rapidly shared with the other degrees of freedom ("reheating"). A more general approach, "chaotic" inflation, has been worked out by Linde (1983, 1990) and others; this works for a wide range of inflationary potentials, including simple power laws such as $\lambda\phi^4$. However, for the amplitude of the fluctuations to be small enough for consistency with observations, it is necessary that the inflaton self-coupling be very small, for example $\lambda \sim 10^{-14}$ for the ϕ^4 model. This requirement prevents a Higgs field from being the inflaton, since Higgs fields by definition have gauge couplings to the gauge field (which are expected to be of order unity), and these would generate self-couplings of similar magnitude even if none were present.
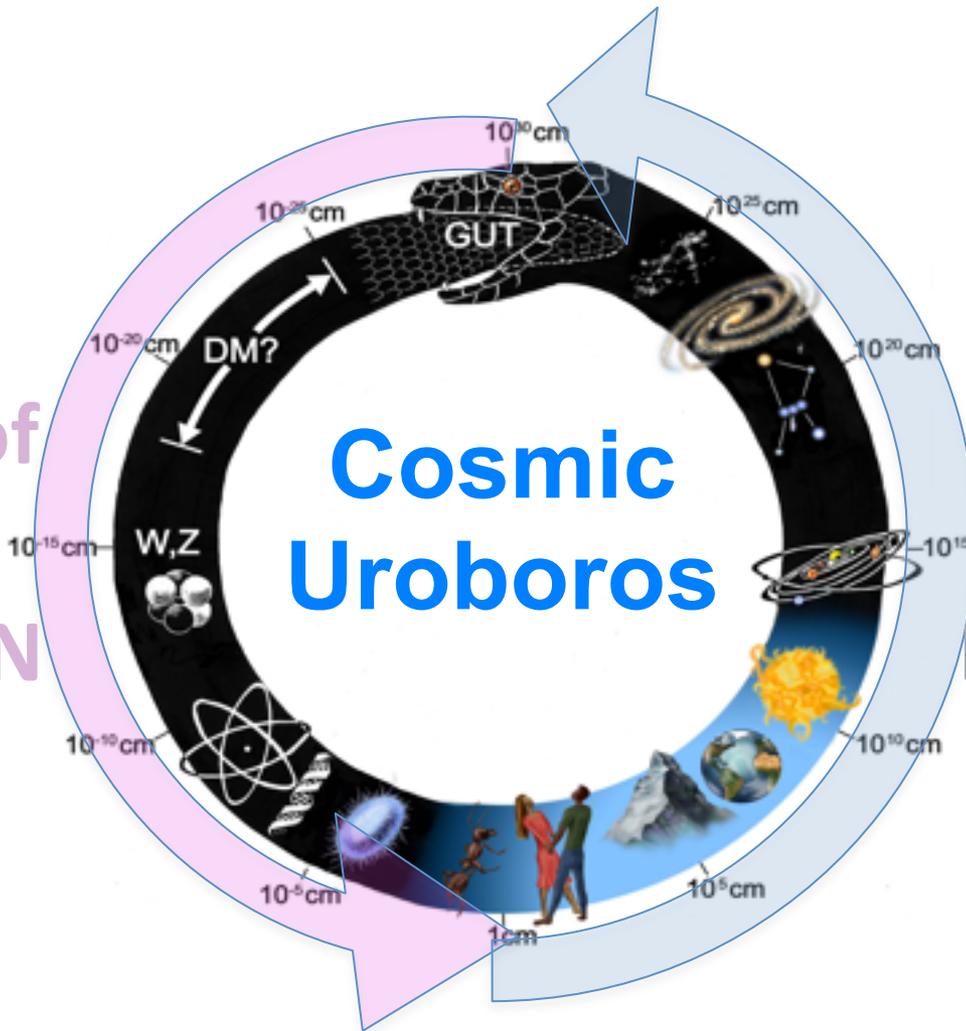
It turns out to be necessary to inflate by a factor $\gtrsim e^{66}$ in order to solve $e^{66}=$ the flatness problem, i.e., that $\Omega_0 \sim 1$. (With $H^{-1} \sim 10^{-34}$ s during the de 4×10^{28} Sitter phase, this implies that the inflationary period needs to last for only a relatively small time $\tau \gtrsim 10^{-32}$ s.) The "flatness problem" is essentially the question why the universe did not become curvature dominated long ago. Neglecting the cosmological constant on the assumption that it is unimportant after the inflationary epoch, the Friedmann equation can be written

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \frac{\pi^2}{30} g(T) T^4 - \frac{kT^2}{(aT)^2}$$

where the first term on the right hand side is the contribution of the energy density in relativistic particles and $g(T)$ is the effective number of degrees of freedom. The second term on the right hand side is the curvature term. Since $aT \approx$ constant for adiabatic expansion, it is clear that as the temperature T drops, the curvature term becomes increasingly important. The quantity $K \equiv k/(aT)^2$ is a dimensionless measure of the curvature. Today, $|K| = |\Omega - 1| H_o^2/T_o^2 \leq 2 \times 10^{-58}$. Unless the curvature exactly vanishes, the most "natural" value for K is perhaps $K \sim 1$. Since inflation increases a by a tremendous factor $e^{H\tau}$ at essentially constant T (after reheating), it increases aT by the same tremendous factor and thereby decreases the curvature by that factor squared. Setting $e^{-2H\tau} \lesssim 2 \times 10^{-58}$ gives the needed amount of inflation: $H\tau \gtrsim 66$. This much inflation turns out to be enough to take care of the other cosmological problems mentioned above as well.

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10⁻³² seconds of COSMIC INFLATION

10^{-32} seconds of **COSMIC INFLATION**

Cosmic Uroboros

13.7 billion years of COSMIC EXPANSION

GUT

DM?

W,Z

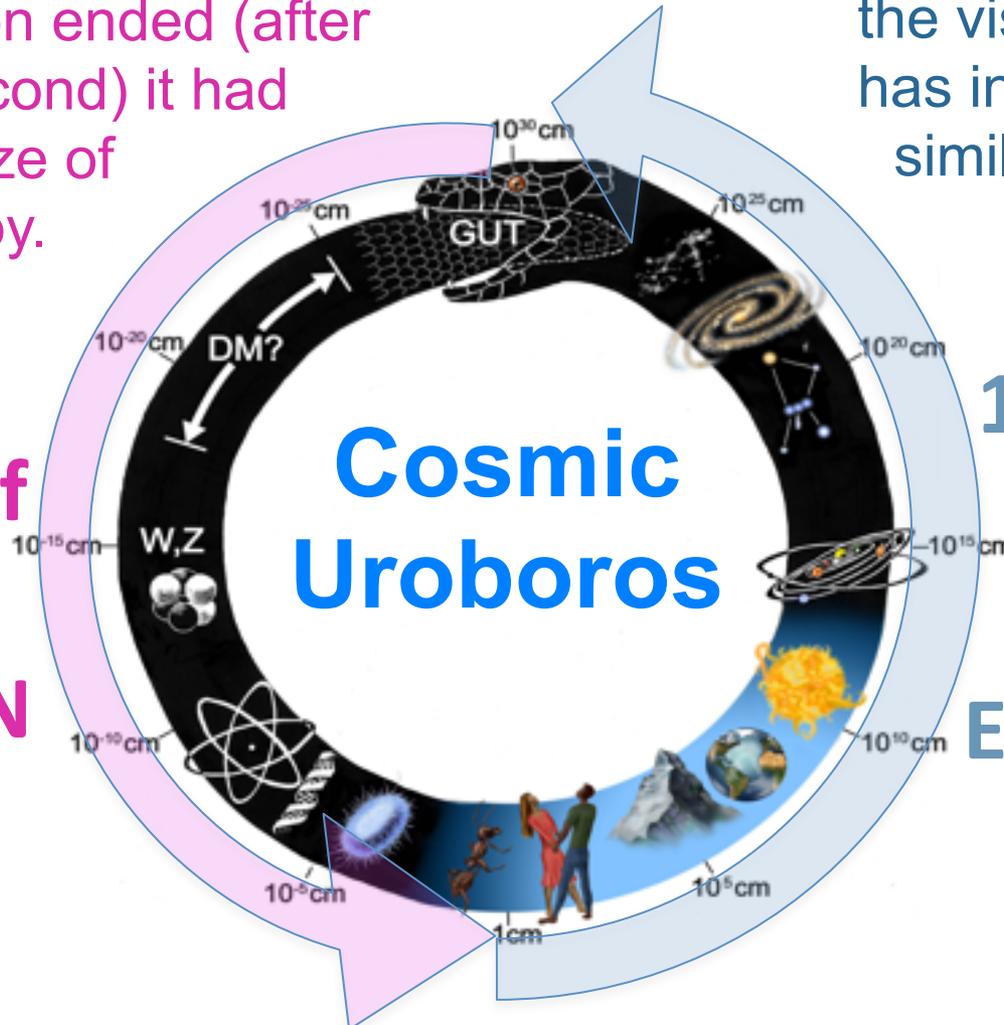10^{-30} cm 10^{-25} cm 10^{-20} cm 10^{-15} cm 10^{-10} cm 10^{-5} cm 1 cm 10^{5} cm 10^{10} cm 10^{15} cm 10^{20} cm 10^{25} cm 10^{30} cm

According to Cosmic Inflation theory, the entire visible universe was once about 10^{-29} cm in size. Its size then inflated by a factor of about 10^{30} so that when Cosmic Inflation ended (after about 10^{-32} second) it had reached the size of a newborn baby.

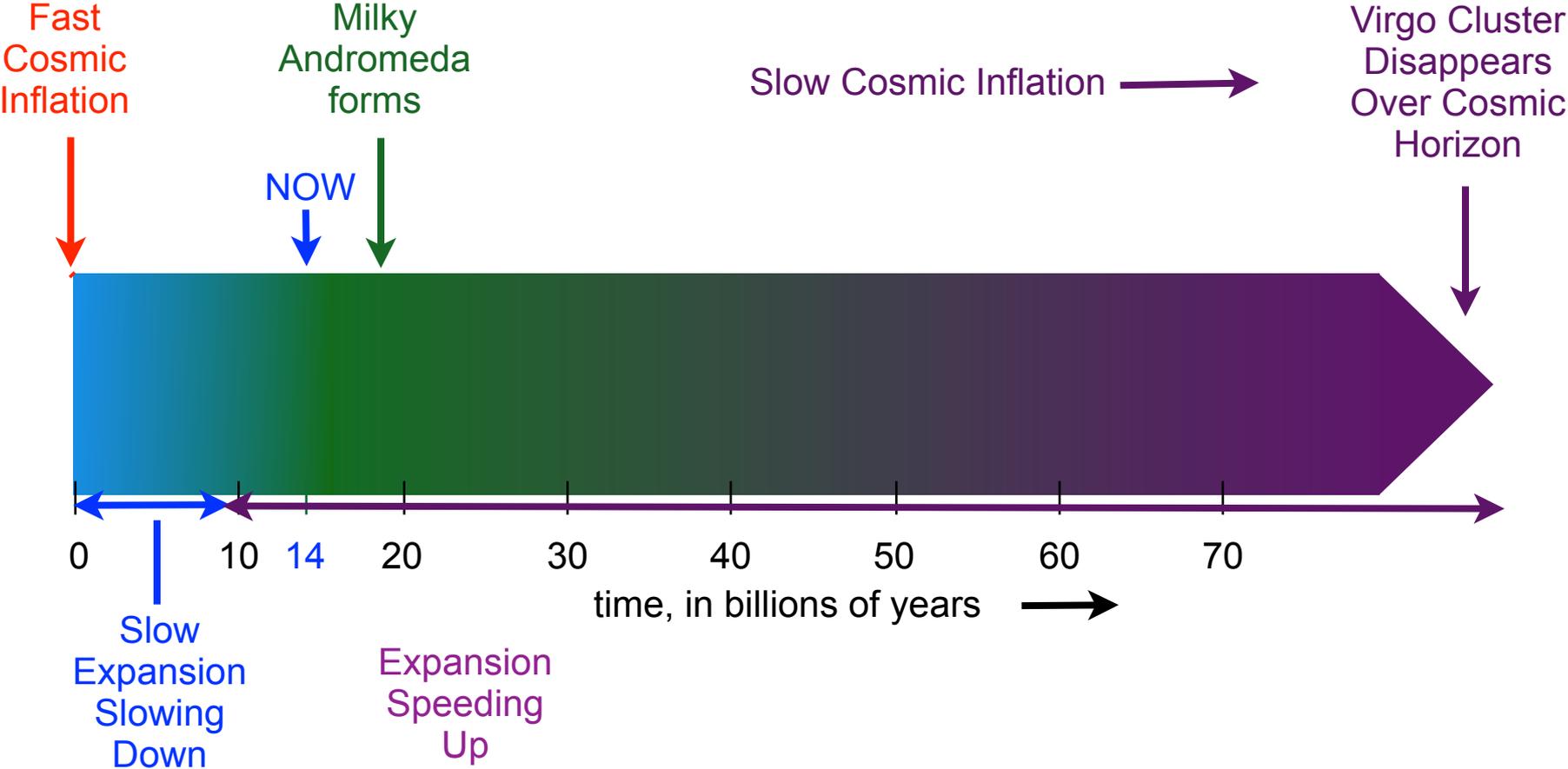During its entire subsequent evolution, the size of the visible universe has increased by a similar factor of 10^{29}.

10^{-32} seconds of COSMIC INFLATION

13.7 billion years of COSMIC EXPANSION

Cosmic Uroboros

10^{30} cm
10^{25} cm
10^{20} cm
10^{15} cm
10^{10} cm
10^{5} cm
1 cm
10^{-5} cm
10^{-10} cm
10^{-15} cm
10^{-20} cm
10^{-29} cm

GUT
DM?
W, Z

COSMIC TIME

Fast Cosmic Inflation

Milky Andromeda forms

NOW

Slow Cosmic Inflation

Virgo Cluster Disappears Over Cosmic Horizon

Slow Expansion Slowing Down

Expansion Speeding Up

0   10   14   20   30   40   50   60   70

time, in billions of years →

Generating the Primordial Density Fluctuations

Early Development of the Universe

BIG BANG

BIG BANG PLUS TINIEST FRACTION OF A SECOND (10^{-43})

INFLATION

COBE SKY MAP — BIG BANG PLUS 300,000 YEARS

LIGHT FROM FIRST GALAXIES

BIG BANG PLUS 15 BILLION YEARS
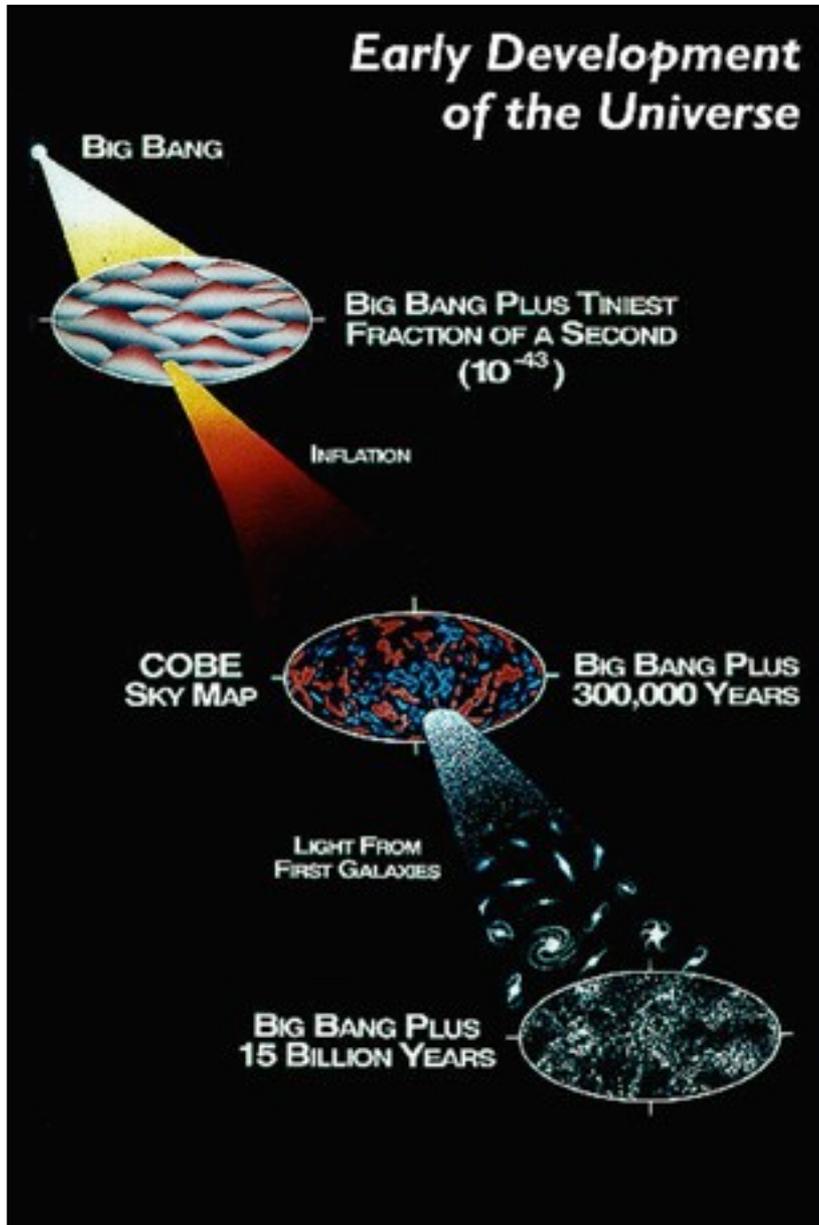
Early phase of exponential expansion (Inflationary epoch)

↓
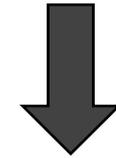
Zero-point fluctuations of quantum fields are stretched and frozen

↓

Cosmic density fluctuations are frozen quantum fluctuations

Inflationary Fluctuations

Thus far, it has been sketched how inflation stretches, flattens, and smooths out the universe, thus greatly increasing the domain of initial conditions that could correspond to the universe that we observe today. But inflation also can explain the origin of the fluctuations necessary in the gravitional instability picture of galaxy and cluster formation. Recall that the very existence of these fluctuations is a problem in the standard Big Bang picture, since these fluctuations are much larger than the horizon at early times. How could they have arisen?

The answer in the inflationary universe scenario is that they arise from quantum fluctuations in the inflaton field ϕ whose vacuum energy drives inflation. The scalar fluctuations $\delta\phi$ during the de Sitter phase are of the order of the Hawking temperature $H/2\pi$. Because of these fluctuations, there is a time spread $\Delta t \approx \delta\phi/\dot\phi$ during which different regions of the same size complete the transition to the Friedmann phase. The result is that the density fluctuations when a region of a particular size re-enters the horizon are equal to (Guth & Pi 1982; see Linde 1990 for alternative approaches) $\delta_H \equiv (\delta\rho/\rho)_H \sim \Delta t/t_H = H\Delta t$. The time spread Δt can be estimated from the equation of motion of ϕ (the free Klein-Gordon equation in an expanding universe): $\ddot\phi + 3H\dot\phi = -(\partial V/\partial\phi)$. Neglecting the $\ddot\phi$ term, since the scalar potential V must be very flat in order for enough inflation to occur (this is called the "slow roll" approximation), $\dot\phi \approx -V'/(3H)$, so $\delta_H \sim H^3/V' \sim V^{3/2}/V'$. Unless there is a special feature in the potential $V(\phi)$ as ϕ rolls through the scales of importance in cosmology (producing such "designer inflation" features generally requires fine tuning — see e.g. Hodges et al. 1990), V and V' will hardly vary there and hence δ_H will be essentially constant. These are fluctuations of all the contents of the universe, so they are adiabatic fluctuations.

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Thus *inflationary models typically predict a nearly constant curvature spectrum δ_H = constant of adiabatic fluctuations.* Some time ago Harrison (1970), Zel'dovich (1972), and others had emphasized that this is the only scale-invariant (i.e., power-law) fluctuation spectrum that avoids trouble at both large and small scales. If $\delta_H \propto M_H^{-\alpha}$, where M_H is the mass inside the horizon, then if $-\alpha$ is too large the universe will be less homogeneous on large than small scales, contrary to observation; and if α is too large, fluctuations on sufficiently small scales will enter the horizon with $\delta_H \gg 1$ and collapse to black holes (see e.g. Carr, Gilbert, & Lidsey 1995, Bullock & Primack 1996); thus $\alpha \approx 0$. The $\alpha = 0$ case has come to be known as the Zel'dovich spectrum.

Inflation predicts more: it allows the calculation of the value of the constant δ_H in terms of the properties of the scalar potential $V(\phi)$. Indeed, this proved to be embarrassing, at least initially, since the Coleman-Weinberg potential, the first potential studied in the context of the new inflation scenario, results in $\delta_H \sim 10^2$ (Guth & Pi 1982) some six orders of magnitude too large. But this does not seem to be an insurmountable difficulty; as was mentioned above, chaotic inflation works, with a sufficiently small self-coupling. Thus inflation at present appears to be a plausible solution to the problem of providing reasonable cosmological initial conditions (although it sheds no light at all on the fundamental question why the cosmological constant is so small now). Many variations of the basic idea of inflation have been worked out

Many Inflation Models

following Andrei Linde's classification

How Inflation Begins

Old Inflation — T_{initial} high, $\phi_{\text{in}} \approx 0$ is false vacuum until phase transition Ends by bubble creation; Reheat by bubble collisions

New Inflation — Slow roll down $V(\phi)$, no phase transition

Chaotic Inflation — Similar to New Inflation, but ϕ_{in} essentially arbitrary: any region with $\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i \phi)^2 \lesssim V(\phi)$ inflates

Extended Inflation — Like Old Inflation, but slower (e.g., power $a \propto t^p$), so phase transition can finish

Potential $V(\phi)$ During Inflation

Chaotic typically $V(\phi) = \Lambda \phi^n$, can also use $V = V_0 e^{\alpha \phi}$, etc.
$$\Rightarrow a \propto t^p, \; p = 16\pi/\alpha^2 \gg 1$$

How Inflation Ends

First-order phase transition — e.g., Old or Extended inflation
Faster rolling $\rightarrow$ oscillation — e.g., Chaotic $V(\phi)^2 \Lambda \phi^n$
"Waterfall" — rapid roll of σ triggered by slow roll of ϕ

(Re)heating

Decay of inflatons
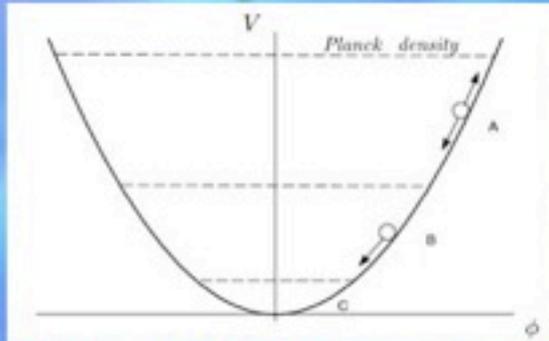"Preheating" by parametric resonance, then decay

Before Inflation?

Eternal Inflation? Can be caused by
- Quantum $\delta\phi \sim H/2\pi >$ rolling $\Delta\phi = \phi \Delta t = \phi H^{-1} \approx V'/V$
- Monopoles or other topological defects

Comparing different inflationary models:

- **Chaotic inflation** can start in the smallest domain of size 10^{-33} cm with total mass $\sim M_p$ (less than a milligram) and entropy $O(1)$
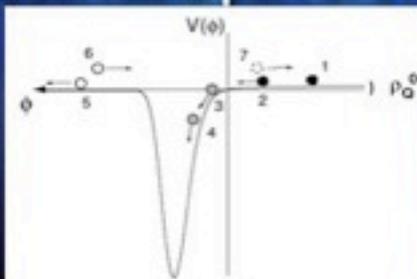

Solves flatness, mass and entropy problem

- **New inflation** can start only in a domain with mass 6 orders of magnitude greater than M_p and entropy greater than 10^9
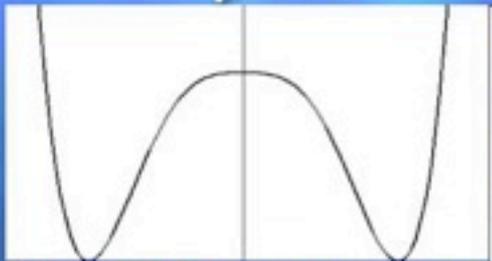
Not very good with solving flatness, mass and entropy problem

- **Cyclic inflation** can occur only in the domain of size greater than the size of the observable part of the universe, with mass $> 10^{55}$ g and entropy $> 10^{87}$

Does not solve flatness, mass and entropy problem

Andrei Linde

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Is the simplest chaotic inflation natural?

Andrei Linde

- Often repeated (but incorrect) argument:

$$V = V_0 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum C_n \frac{\phi^n}{M_p^n}$$

Thus one could expect that the theory is ill-defined at $\phi > M_p$

However, quantum corrections are in fact proportional to

$$\left(\frac{V}{M_p^4}\right)^n \quad \text{and to} \quad \left(\frac{m^2(\phi)}{M_p^2}\right)^n$$

These terms are harmless for sub-Planckian masses and densities, even if the scalar field itself is very large.

Inflaton Theory in More Detail

The action of gravity + scalar inflaton field is

Lagrangian for Scalar Field ϕ

$$S = -\frac{c^3}{16\pi G} \int d^4x \sqrt{-\det g_{mn}} \, R$$

$$+ \int d^4x \sqrt{-\det g_{mn}} \, \hbar \left(\frac{1}{2} \frac{\partial \phi}{\partial x^i} \frac{\partial \phi}{\partial x^k} g^{ik} - V[\phi] \right)$$

The simplest V is just quadratic $\quad V[\phi] = \frac{1}{2} \frac{m^2 c^2}{\hbar^2} \phi^2$

which just gives the inflaton field a mass m.

The Einstein equation $G_{ij} = -(8\pi G/c^4) \, T_{ij}$ follows by varying the metric g_{ij}; the first term in the action S leads to the Einstein tensor G_{ij} and the second term leads to the energy-momentum tensor T_{ij}.

Dierck-Ekkehard Liebscher, *Cosmology* (Springer, 2005)

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The energy–momentum tensor is given by

$$T_{ik} = \hbar c \left(\frac{\partial \phi}{\partial x^i} \frac{\partial \phi}{\partial x^k} - g_{ik} \left(\frac{1}{2} g^{lm} \frac{\partial \phi}{\partial x^l} \frac{\partial \phi}{\partial x^m} - V[\phi] \right) \right)$$

which implies that the energy density and pressure are given by

$$\varepsilon = \hbar c \left(V + \frac{1}{2c^2} \dot{\phi}^2 + \frac{1}{2} \frac{1}{a^2[t]} (\nabla \phi)^2 \right)$$

and

$$p = \hbar c \left(-V + \frac{1}{2c^2} \dot{\phi}^2 - \frac{1}{6} \frac{1}{a^2[t]} (\nabla \phi)^2 \right) .$$

Thus a scalar field with a nearly constant potential V corresponds to

$$\varrho c^2 = \varepsilon = -p \ (= \hbar c V[\phi]).$$

Since w = p/ε = -1, this is effectively a cosmological constant. More generally, a scalar field that is not at the minimum of its potential generates generates "*dark energy*".

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The field equation for the inflaton in expanding space is

$$\frac{\partial^2 \phi}{c^2 \partial t^2} - \frac{1}{a^2} \nabla^2 \phi + \frac{3 \dot{a} \dot{\phi}}{c^2 a} + \frac{dV}{d\phi} \ \ (+3H\Gamma\dot{\phi}^2) = 0 \ .$$

This becomes the following equation if the spatial variations of ϕ (and the last term, which allows the inflaton to decay into other fields at the end of inflation, thus *reheating* the universe) can be neglected

$$\ddot{\phi} + 3H[t]\dot{\phi} = -c^2 \frac{dV[\phi]}{d\phi} \ .$$

This equation must be solved along with the Einstein equations:

$$H^2 = \frac{8\pi G}{3} \frac{\hbar}{c} \left(V + \frac{1}{2c^2} \dot{\phi}^2 \right) \ \ \text{and} \ \ \dot{H} = -4\pi \frac{\hbar G}{c^3} \dot{\phi}^2$$

With a suitably chosen potential V, the inflaton will quickly reach its ground state and inflation will end.

The last equation leads to

$$H' = \frac{dH[\phi]}{d\phi} = -4\pi \frac{\hbar G}{c^3} \dot{\phi}$$

which allows us to write the Friedmann equation as

$$\left(\frac{dH}{d\phi}\right)^2 = 12\pi \frac{\hbar G}{c^3} H^2 - 32\pi^2 \frac{\hbar^2 G^2}{c^4} V[\phi] .$$

When the inflaton is rolling slowly, the evolution of the inflaton is governed by the "slow roll" equations

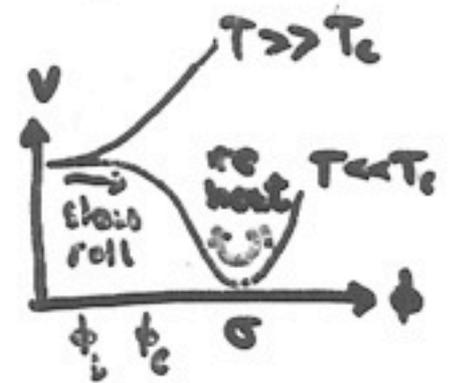$$\dot{\phi} = -\frac{c^2}{3H} \frac{dV}{d\phi} , \quad H^2 = \frac{8\pi\hbar G}{3c} V .$$

Then the number N of e-folds of the scale factor a is given by

$$N = \ln \frac{a}{a_1} = \int_{t_1}^{t} H \, dt = \int_{\phi_1}^{\phi} d\phi \frac{H}{\dot{\phi}} = 4\pi \frac{\hbar G}{c^3} \int_{\phi}^{\phi_1} d\phi \frac{H}{H'} \approx 8\pi \frac{\hbar G}{c^3} \int_{\phi}^{\phi_1} d\phi \frac{V}{V'} .$$

The last approximate equality used the slow roll approximation.

Inflationary Models in More Detail

NEW

PROTOTYPE MODEL FOR $\wedge$ INFLATION

$$V = \lambda(\phi^2 - \sigma^2)^2$$

Assume that in some region R_s, $\phi \approx 0$. Transition to $\phi \to \sigma$ is governed by KG eqn

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} - \frac{1}{R^2}\cancel{\nabla^2\phi} = -V'$$

neglect

where $H^2 = \frac{8\pi}{3m_{P\ell}^2}\rho$, $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_R$, $p = \frac{1}{2}\dot{\phi}^2 - V(\phi) + \frac{1}{3}\rho_R$

If $V(\phi)$ is large and flat enough, $V(\phi)$ will be $\gg \dot{\phi}^2$ and ρ_R, $\ddot{\phi}$ will be $\ll 3H\dot{\phi}$, and

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 \approx \frac{8\pi V}{3m_{P\ell}^2} \implies R = R_0 e^N, \quad N = \int H\,dt$$

For example, take $\sigma = 10^{14}$ GeV, $\lambda = 1$, $R_i = H^{-1} = \dfrac{M_{Pl}}{\sigma^2} = \dfrac{10^{19}\,GeV}{(10^{14}GeV)^2}$

$= R_i = 10^{-9} GeV^{-1} = 10^{-23}$ cm, $\Rightarrow S_i = (R_i T_i)^3 = \left(\dfrac{M_{Pl}}{\sigma^2}\sigma\right)^3 = 10^{15}$

Then $S_F = (R_f T_{RH})^3 = (e^N R_i T_{RH})^3$

If $N = 100$, $e^{300} = 10^{130}$, & this gives $S_F = 10^{145}$

Requiring $e^{3N}\dfrac{M_{Pl}}{\sigma} = S_F \gtrsim 10^{88} \Rightarrow 3N \gtrsim \underbrace{\ell_n 10^{88}}_{202.6} + \ell_n \,{}^{\sigma}/_{M_{Pl}}$

$\Rightarrow N \gtrsim 67.5 - 9.2 + \ell_n \,{}^{\sigma}/_{10^{15}GeV} = 58 + \ell_n\,\sigma/10^{15}GeV$
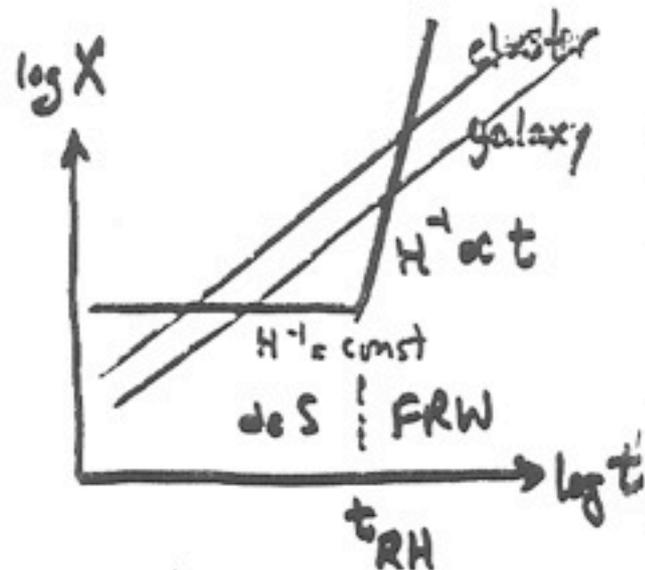
Solves : Horizon

Flatness / Age : k/R^2 decreases by e^{-2N}

Relics : density $\sim R^{-3}$ " " e^{-3N}

Fluctuations ?

FLUCTUATIONS IN INFLATION

LOFI — last scales to cross outside horizon in de S

are first to cross inside in FRW

If present horizon crossed outside 60 e-folds before end of inflation, galaxies crossed 52 efolds before end, and any mass M_H crossed at $60 + \frac{1}{3} \ln M_H / 10^{22} M\odot = N_\lambda$.

There are quantum fluctuations in a deSitter universe corresponding to the Hawking radiation temperature $T_H = \frac{H}{2\pi}$: $\langle (\Delta\phi)^2 \rangle = \left(\frac{H}{2\pi}\right)^2$

[Gath + P: PRL 49, 1110 (1982) got the same answer using deS Green's fens.]

These lead to density fluctuations $\delta\rho = \frac{\partial V}{\partial \phi} \Delta\phi = -3H\dot\phi \Delta\phi$ slow roll!

$V' = -3H\dot\phi$

In deS phase, $\rho = \frac{1}{2}\dot\phi^2 + V(\phi) + \underset{\text{negligible}}{\rho_R}$, $p = \frac{1}{2}\dot\phi^2 - V(\phi) + \underset{\text{negligible}}{\frac{1}{3}\rho_R}$.

Bardeen's gauge invariant parameter $\zeta = \frac{\delta\rho}{\rho+p}$ is constant outside horizon.

deS: $\zeta = \frac{\delta\rho}{\dot\phi^2} = \frac{V'\Delta\phi}{\dot\phi^2} = \frac{-3H\dot\phi\Delta\phi}{\dot\phi^2} = -\frac{3H^2}{2\pi\dot\phi}$

$\parallel$

FRW: $\zeta = \frac{\delta\rho}{\frac{4}{3}\rho}$ $\Rightarrow$ $\left(\frac{\delta\rho}{\rho}\right)_H = -\frac{2H^2}{\pi\dot\phi} = \frac{6H^3}{\pi V'(\phi)}$

when given scale crosses inside H in FRW era

when scale crossed outside H in dS era

CHAOTIC INFLATION $\quad V(\phi) = \lambda\phi^4 \quad\quad \phi = $ inflaton $\quad$ Linde 1983

initial condition: ϕ large and (in inflating patch) homogeneous $\Rightarrow$

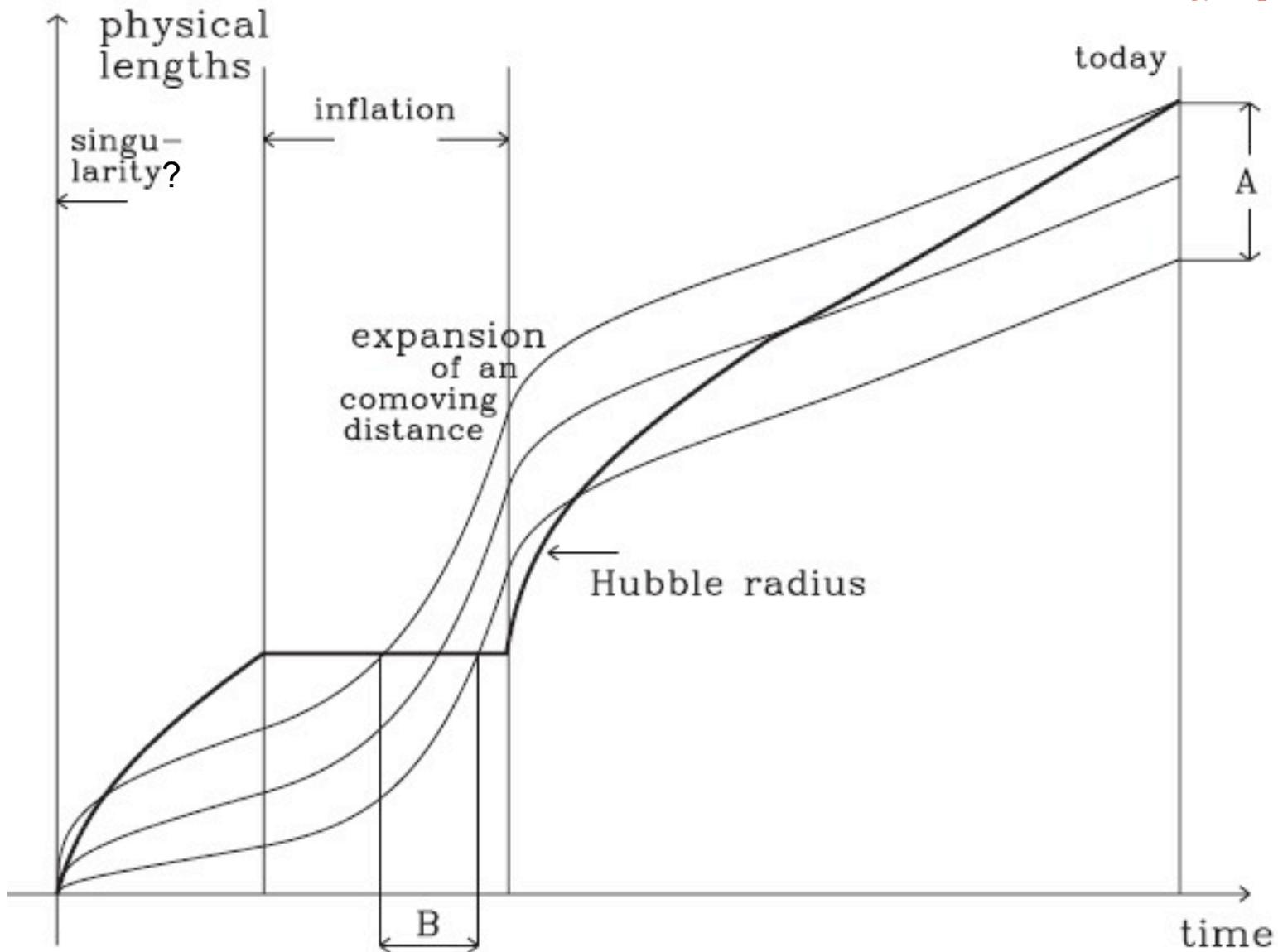$N_\lambda(\phi\to 0) = \int H dt = \int_\phi^0 H \dot\phi^{-1} \, d\phi = \int_\phi^0 \frac{-3H^2}{9\lambda\phi^3} d\phi = \int_0^\phi \frac{2\pi}{m_{P\ell}^2}\phi d\phi = \pi\left(\frac{\phi}{m_{P\ell}}\right)^2$

$\overbrace{\frac{8}{3}\pi G \lambda\phi^4}$

$\Rightarrow \left(\frac{\delta\rho}{\rho}\right)_H = \frac{6H^3}{\pi V'(\phi)} = \frac{6}{\pi}\left(\frac{8\pi}{3m_{P\ell}^2}\lambda\phi^4\right)^{3/2}\frac{1}{4\lambda\phi^3} = 11.6\,\lambda^{1/2}\left(\frac{\phi}{m_{P\ell}}\right)^3 = 2.1\,\lambda^{1/2}N_\lambda^{3/2}$

$\therefore \left(\frac{\delta\rho}{\rho}\right)_H \simeq 10^{-4}$ and $N_\lambda \simeq 50$ for galaxies $\Rightarrow \underline{\lambda \simeq 10^{-12}}$

$N_{tot} = N_\lambda(\phi_i\to 0) > 60 \Rightarrow \frac{\phi_i}{m_{P\ell}} > \left(\frac{60}{\pi}\right)^{1/2} = 4.3 \quad \left(\begin{array}{l}\text{Is classical}\\\text{gravity ok ?}\end{array}\right)$

Again, T_{RH} depends on ϕ coupling to other fields, ϕ is low but b. asym. could be generated by decay.

The evolution of the scales of perturbations. The larger scales overtake the Hubble radius at an early time and fall below it again later. They measure the inflation at an earlier time than do the smaller scales, which overtake the Hubble radius during inflation later and fall below it again earlier. The region *A* of scales that are accessible to evaluation today corresponds to a time span *B* of the inflation and related values of the inflaton field; for this time span, we can tell something – at least in principle – about the potential of the inflaton.

Eternal Inflation

Vilenkin (1983) and Linde (1986, 1990) pointed out that if one extrapolates inflation backward to try to imagine what might have preceeded it, in many versions of inflation the answer is "eternal inflation": in most of the volume of the universe inflation is still happening, and our part of the expanding universe (a region encompassing far more than our entire cosmic horizon) arose from a tiny part of such a region. To see how eternal inflation works, consider the simple chaotic model with $V(\phi) = (m^2/2)\phi^2$. During the de Sitter Hubble time H^{-1}, where as usual $H^2 = (8\pi G/3)V$, the slow rolling of ϕ down the potential will reduce it by

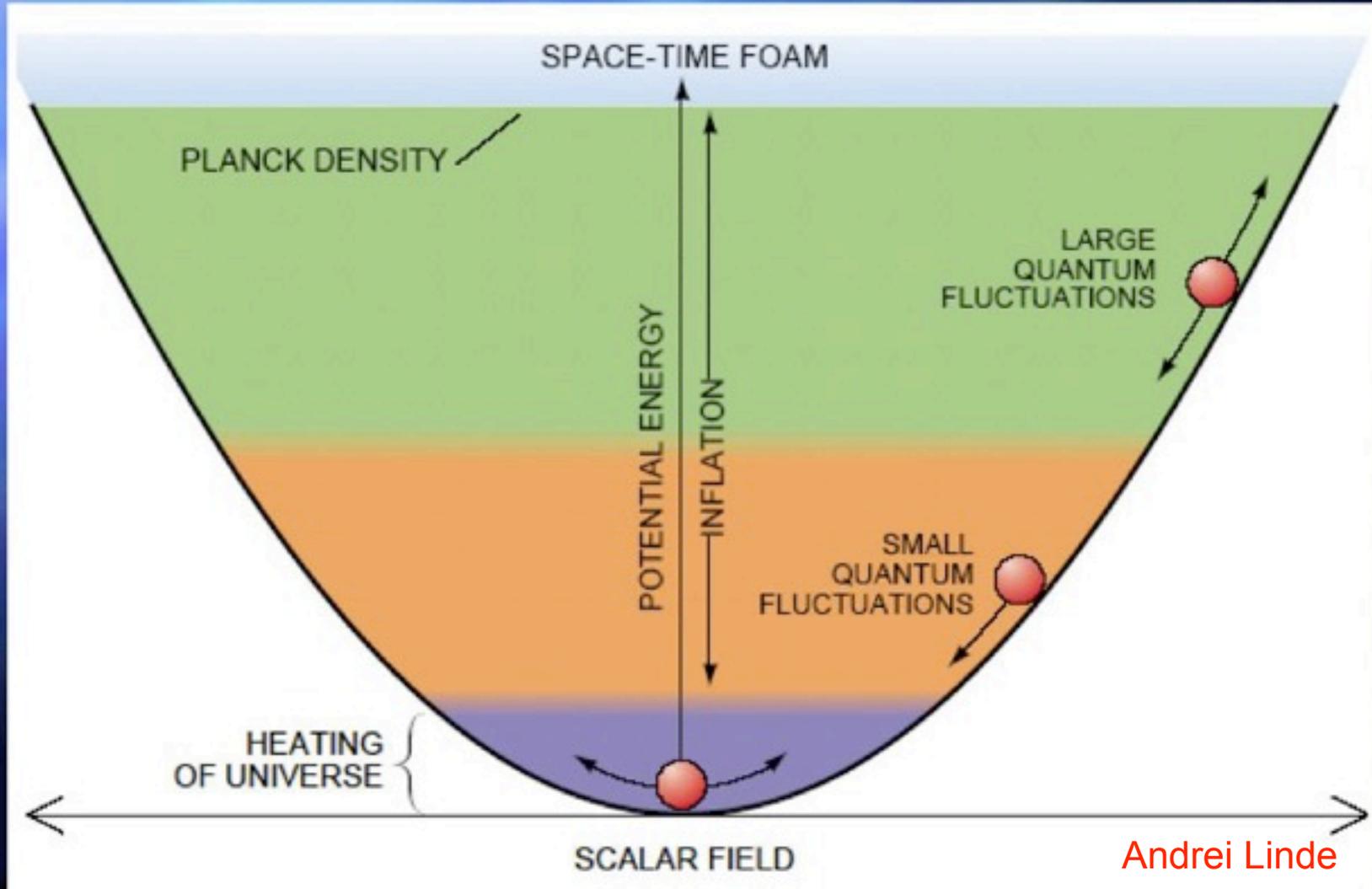$$\Delta\phi = \dot{\phi}\Delta t = -\frac{V'}{3H}\Delta t = \frac{m_{pl}^2}{4\pi\phi}. \tag{1.7}$$

Here m_{Pl} is the Planck mass $(m_{Planck} = 1/G^{1/2})$. But there will also be quantum fluctuations that will change ϕ up or down by

$$\delta\phi = \frac{H}{2\pi} = \frac{m\phi}{\sqrt{3\pi}\,m_{Pl}} \tag{1.8}$$

These will be equal for $\phi_* = m_{pl}^{3/2}/2m^{1/2}$, $V(\phi_*) = (m/8m_{Pl})m_{Pl}^4$. If $\phi \gtrsim \phi_*$, *positive quantum fluctuations dominate* the evolution: after $\Delta t \sim H^{-1}$, an initial region becomes $\sim e^3$ regions of size $\sim H^{-1}$, in half of which ϕ increases to $\phi + \delta\phi$. Since $H \propto \phi$, this drives inflation faster in these regions.
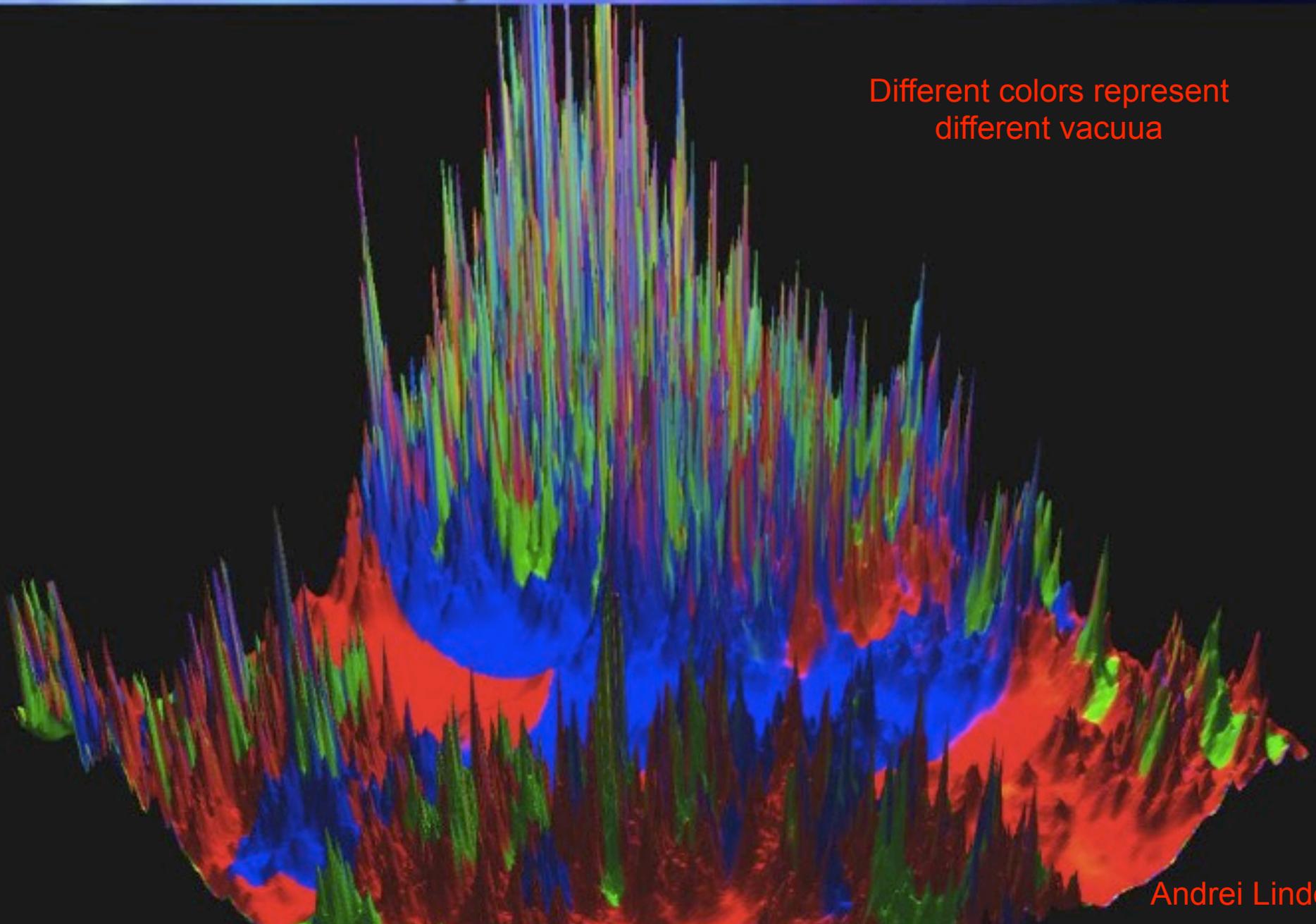
Inflation as a theory of a harmonic oscillator

$$V(\phi) = \frac{m^2}{2}\phi^2$$

SPACE-TIME FOAM

PLANCK DENSITY

LARGE QUANTUM FLUCTUATIONS

POTENTIAL ENERGY

INFLATION

SMALL QUANTUM FLUCTUATIONS

HEATING OF UNIVERSE

SCALAR FIELD

Andrei Linde

Thursday, May 24, 12

Landscape of eternal inflation

Different colors represent different vacuua

Andrei Linde

THE COSMIC LAS VEGAS

Coins constantly flip. Heads, and the coin is twice the size and there are two of them. Tails, and a coin is half the size.

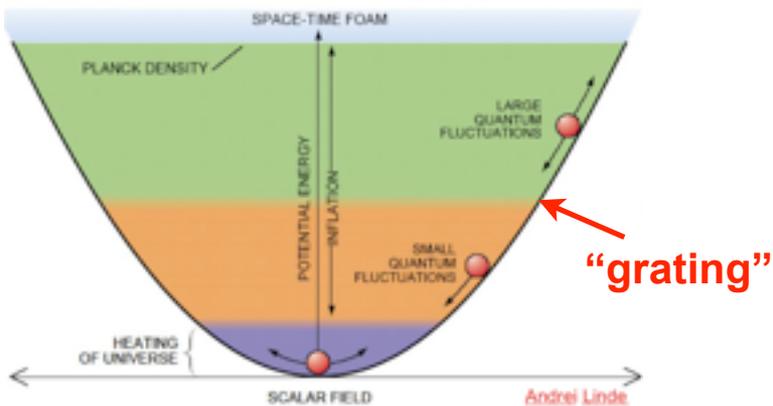**Consider a coin that has a run of tails. It becomes so small it can pass through the grating on the floor.**

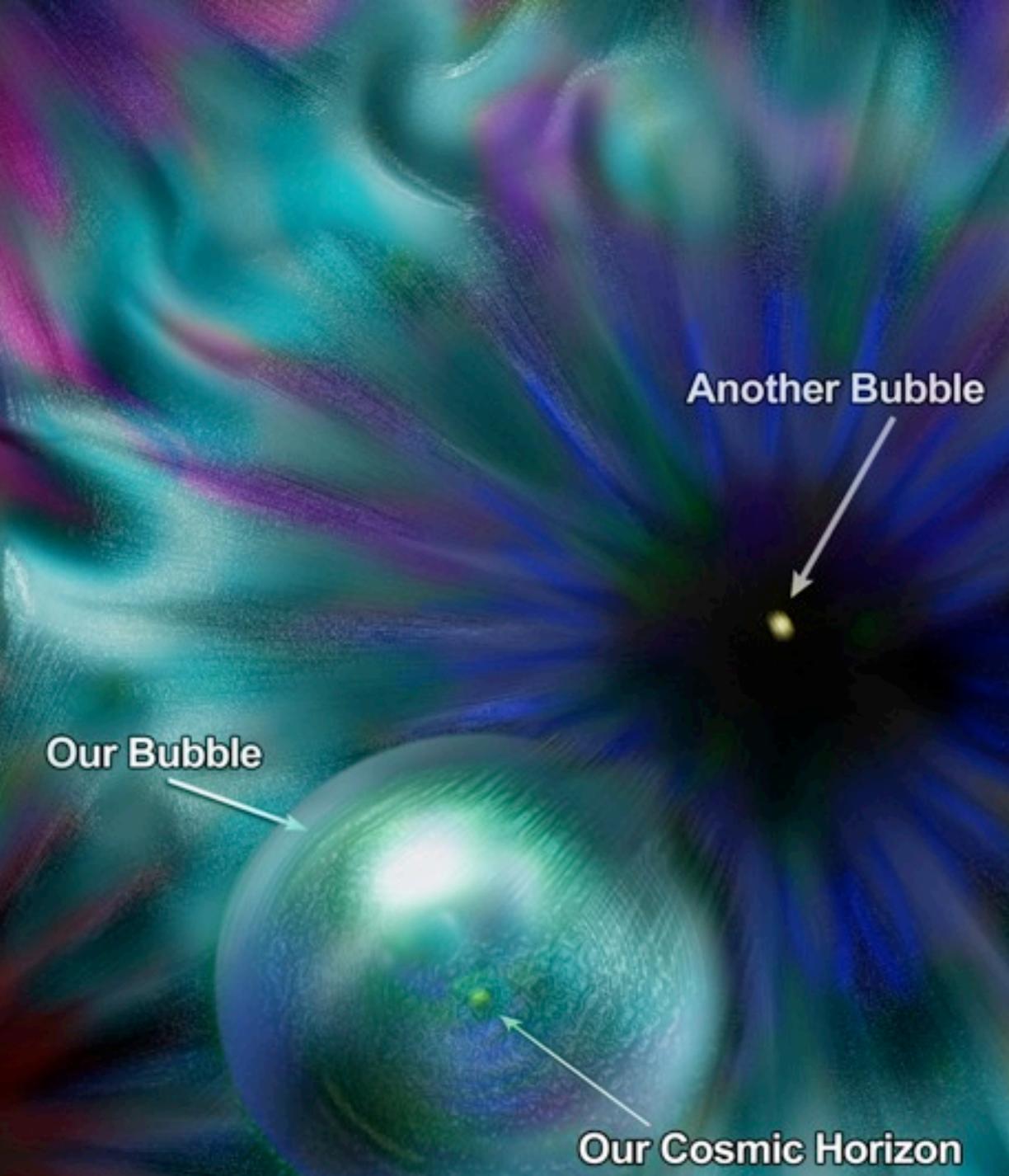**At the instant it passes through the floor, it exits eternity.**

Time begins with a Big Bang, and it becomes a universe and starts evolving.

The Multiverse

"grating"

OUR COSMIC BUBBLE IN ETERNAL INFLATION

Supersymmetric Inflation

When Pagels and I (1982) first suggested that the lightest supersymmetric partner particle (LSP), stable because of R-parity, might be the dark matter particle, that particle was the gravitino in the early version of supersymmetry then in fashion. Weinberg (1982) immediately pointed out that if the gravitino were not the LSP, it could be a source of real trouble because of its long lifetime $\sim M_{\rm Pl}^2/m_{3/2}^3 \sim (m_{3/2}/{\rm TeV})^{-3}10^3$ s, a consequence of its gravitational-strength coupling to other fields. Subsequently, it was realized that supersymmetric theories can naturally solve the gauge hierarchy problem, explaining why the electroweak scale $M_{\rm EW} \sim 10^2$ GeV is so much smaller than the GUT or Planck scales. In this version of supersymmetry, which has now become the standard one, the gravitino mass will typically be $m_{3/2} \sim$ TeV; and the late decay of even a relatively small number of such massive particles can wreck BBN and/or the thermal spectrum of the CBR. The only way to prevent this is to make sure that the reheating temperature after inflation is sufficiently low: $T_{\rm RH} \lesssim 2 \times 10^9$ GeV (for $m_{3/2} = $ TeV) (Ellis, Kim, & Nanopoulos 1984, Ellis et al. 1992).