Homework Set 2

DUE: Thursday May 10

1. The "tired light" hypothesis states that the universe is not expanding, but that photons lose energy per unit distance

$$\frac{dE}{dr} = -KE\,.$$

Show that this hypothesis gives a distance-redshift relation that is linear in the limit $z \ll 1$. What value of K gives a Hubble constant $h = 0.7$? What are some arguments against the "tired light" hypothesis?

2. Short problems:

(a) If a neutrino has mass m_ν and decouples at $T_{\nu d} \sim 1$ MeV, show that the contribution of this neutrino and its antiparticle to the cosmic density today is (Dodelson Eq. 2.80)

$$\Omega_\nu = \frac{m_\nu}{94h^2\mathrm{eV}} \quad .$$

(b) Verify that $\eta_b \equiv n_b/n_\gamma$ is given by (Dodelson Eq. 3.11)

$$\eta_b = 5.5 \times 10^{-10} \left(\frac{\Omega_b h^2}{0.020}\right) \quad .$$

(c) Verify the time-temperature relation (Dodelson Eq. 3.30)

$$t = 132\,\mathrm{sec}\,(0.1\mathrm{MeV}/T)^2 \quad .$$

3. A recent cosmological speculation is that the universe may contain a quantum field called "quintessence" which has an equation of state parameter $w_Q = p_Q/\rho_Q$ with energy density ρ_Q positive (of course) but pressure p_Q negative. Suppose that the universe contains nothing but pressureless matter, i.e. with $w_m = 0$, and quintessence, with $w_Q = -3/4$. The current density parameter of matter is $\Omega_m \approx 0.3$ and that of quintessence is $\Omega_Q = 1 - \Omega_m$. At what scale factor a_{mQ} will the energy density of quintessence and matter be equal? Solve the Friedmann equation to find $a(t)$ for this universe. What is $a(t)$ in the limit $a \gg a_{mQ}$? What is the current age of the universe, expressed in terms of H_0 and $\Omega_{m,0}$?

4. Suppose that the neutron decay time were $\tau_n = 89$ s instead of $\tau_n = 890$ s, with all other physical parameters unchanged. Estimate Y_p, the primordial mass fraction of nucleons in ^{4}He, assuming that all available neutrons are incorporated into ^{4}He.

5. Suppose that there were no baryon asymmetry so that the number density of baryons exactly equaled that of anti-baryons. Determine the final relic density of (baryons + anti-baryons). At what temperature is this relic density reached?

6. There is a fundamental limitation on the annihilation cross section of a particle χ of mass m: because of unitarity, $< \sigma v >$ must be less than or equal to $\sim m^{-2}$, give or take a factor of order unity. Determine Ω_χ for a particle with $< \sigma v >= m^{-2}$. For what value of m is $\Omega_\chi = 1$? (Assume $x_f = 10$ and $g_*(m) = 100$, the nominal values in Dodelson Eq. 3.60.) Note that if m exceeds this value, $\Omega_\chi > 1$, which is ruled out. This is a strong argument against stable particles (and therefore dark matter candidates) with masses above this critical value. (See Griest and Kamionkowski 1990, Phys. Rev. Lett. 64, 615.)