Lecture

Structure of Dark-Matter Halos

Universal Halo profile

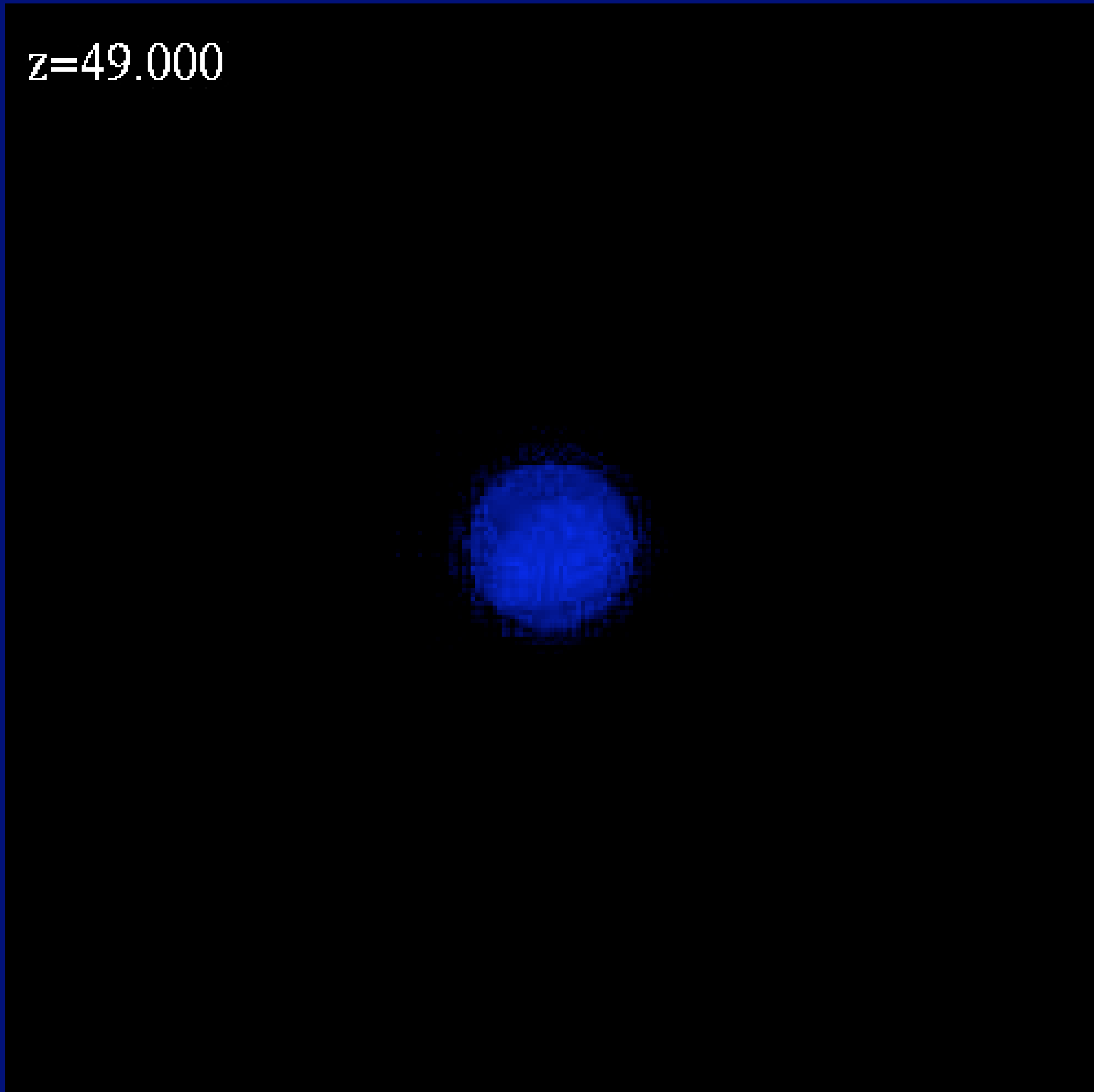
The cusp/core problem

Dynamical friction

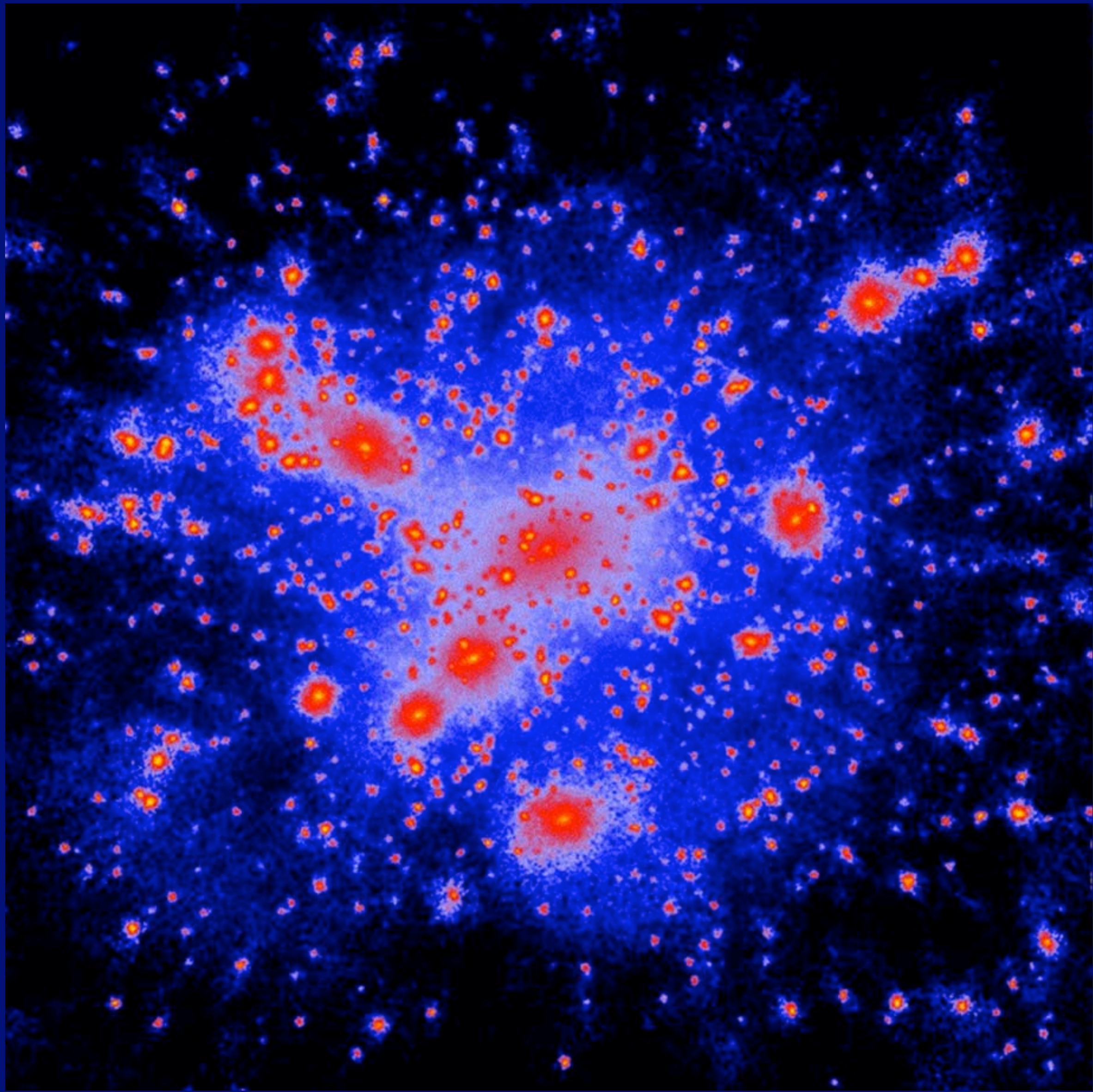
Tidal effects

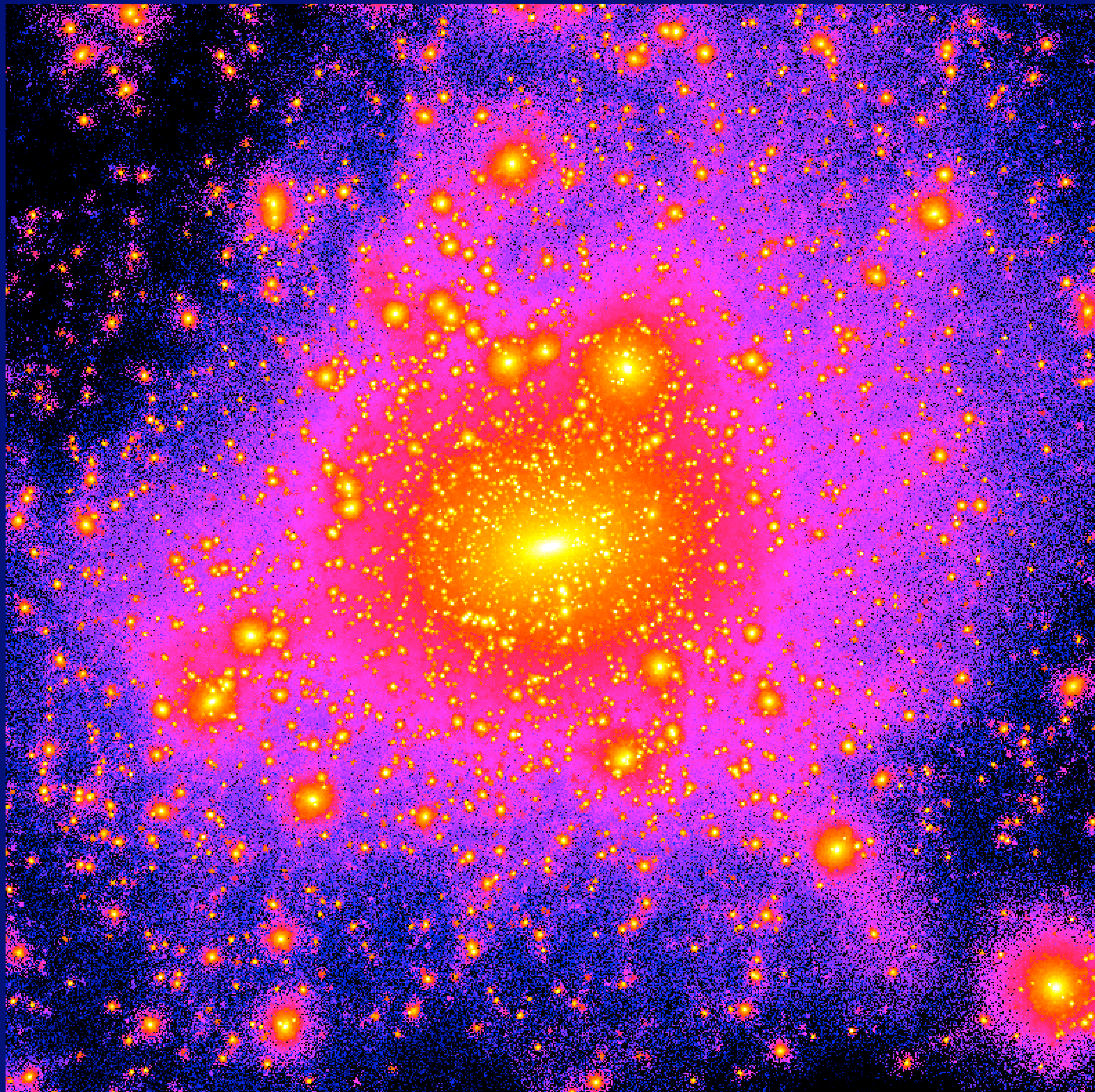
Origin of the cusp in hierarchical clustering

N-body simulation of Halo Formation

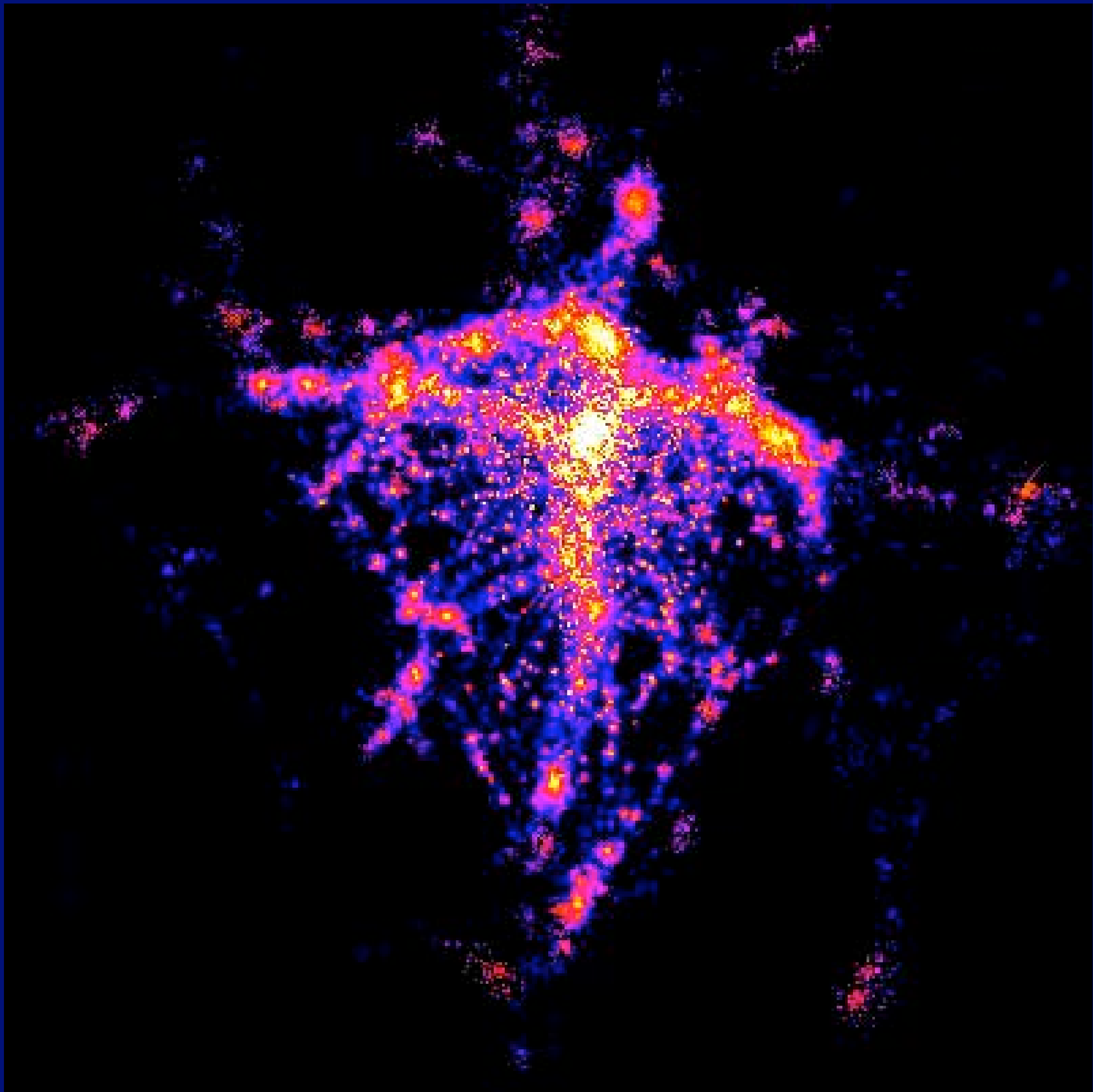


N-body simulation of Halo Formation





Dark Halo (Moore)



CDM halos (simulations)

- Density profiles are universal

  shape independent of mass and cosmology.

- Density profiles are cuspy

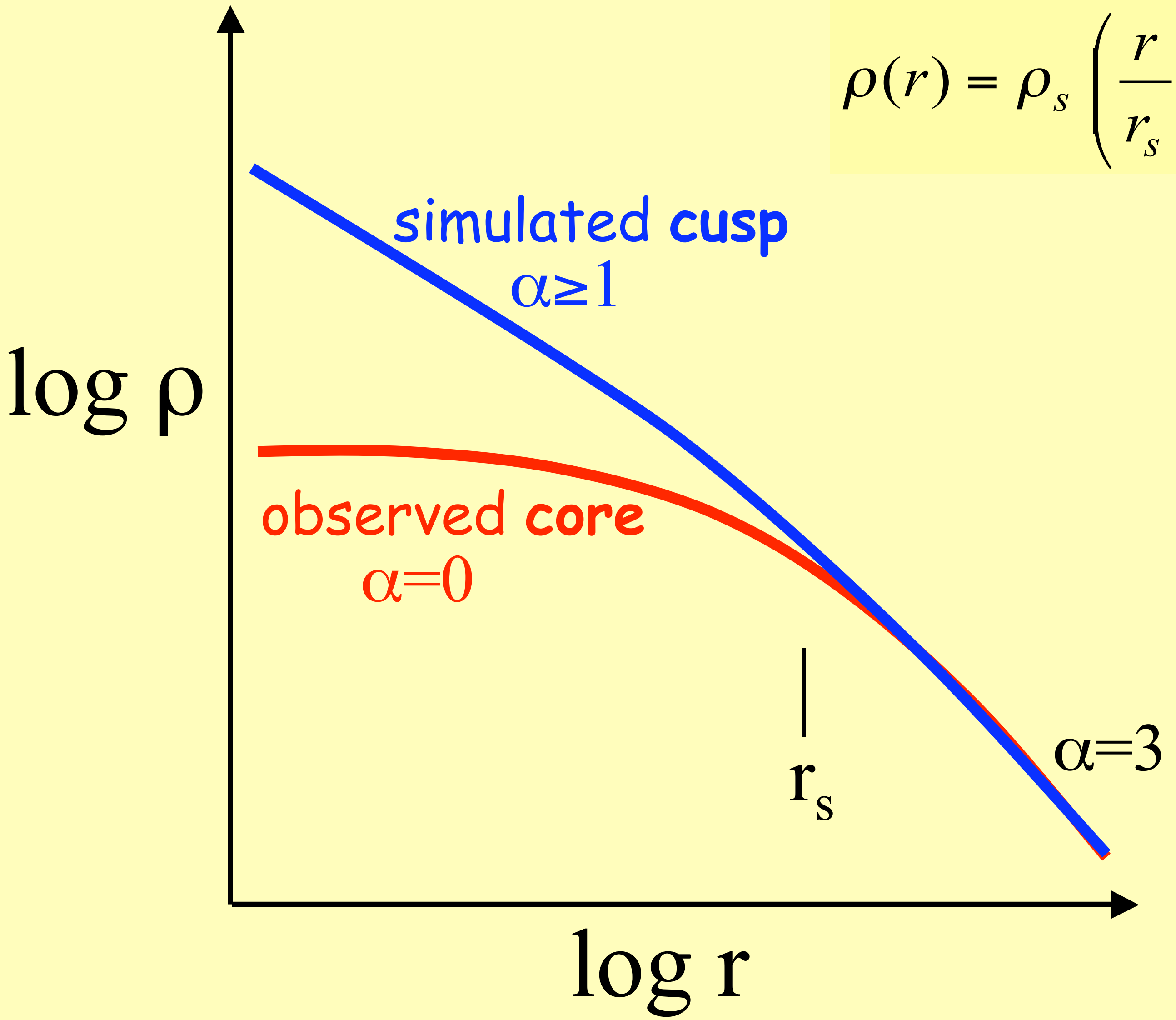
  density increases inward down to the innermost resolved radius. Asymptotic power-law near the center?

- Halos are clumpy

  ~10% of the mass is in self-bound clumps ---
  the surviving cores of accreted satellites.

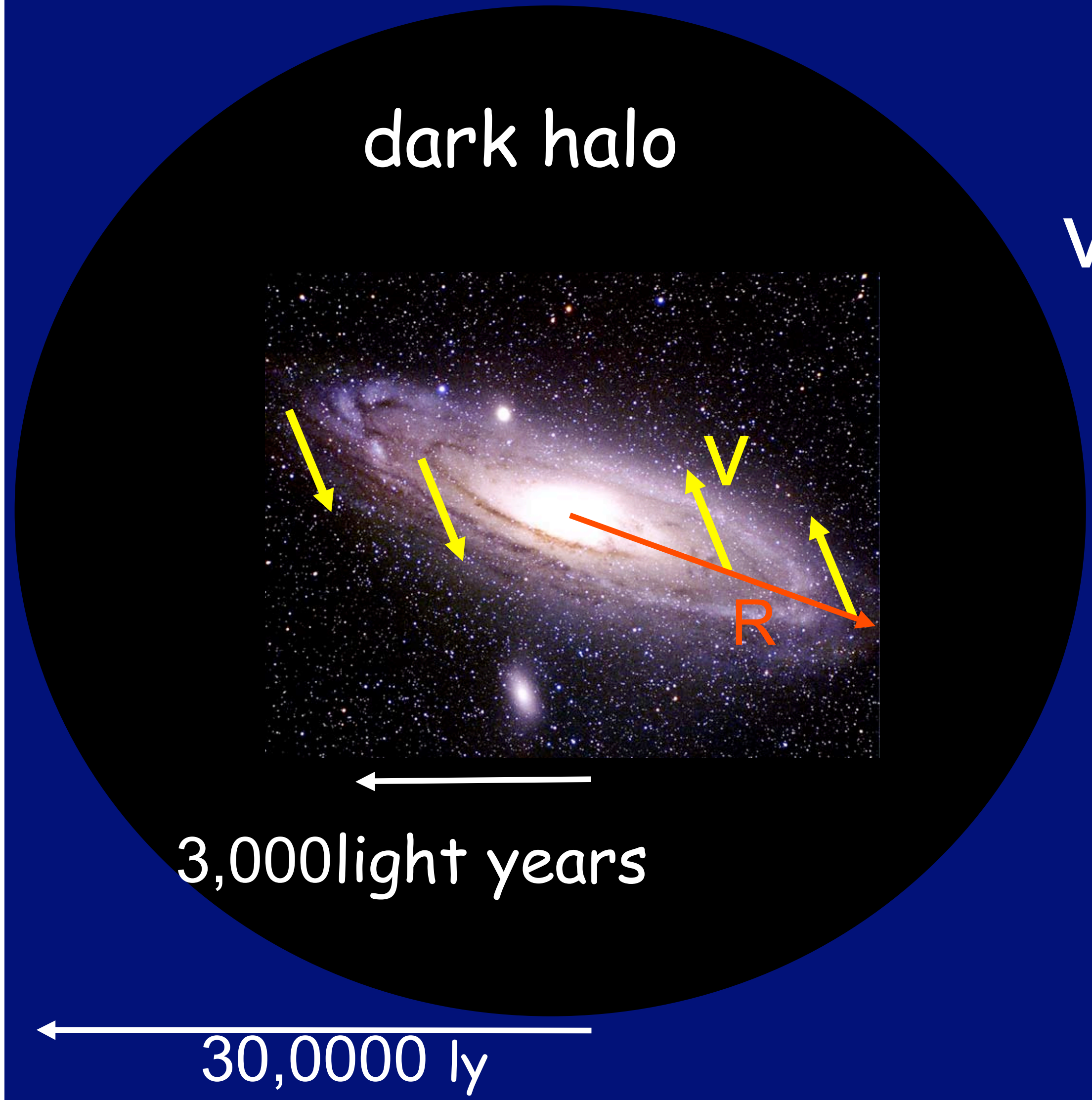
The dark-halo cusp/core problem

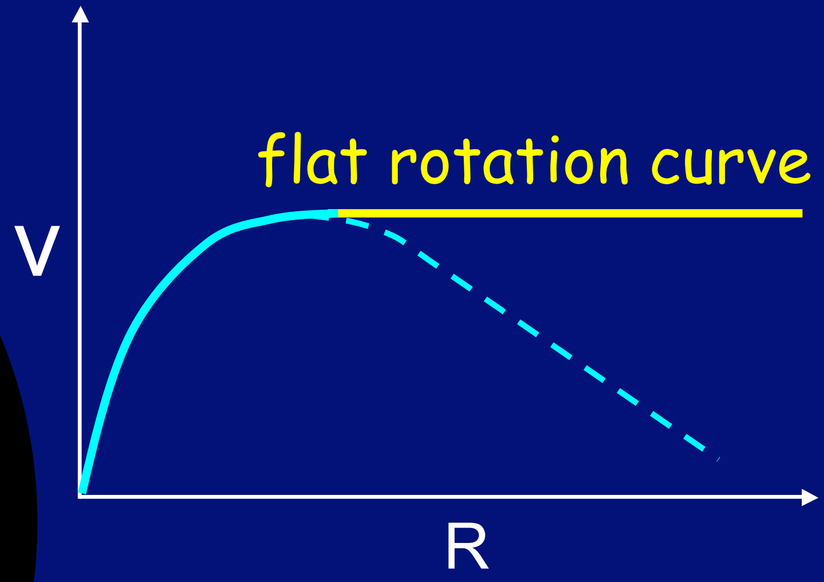
$$\rho(r) = \rho_s \left(\frac{r}{r_s}\right)^{-\alpha_0} \left(1 + \frac{r}{r_s}\right)^{-3+\alpha_0}$$



Universal Profile

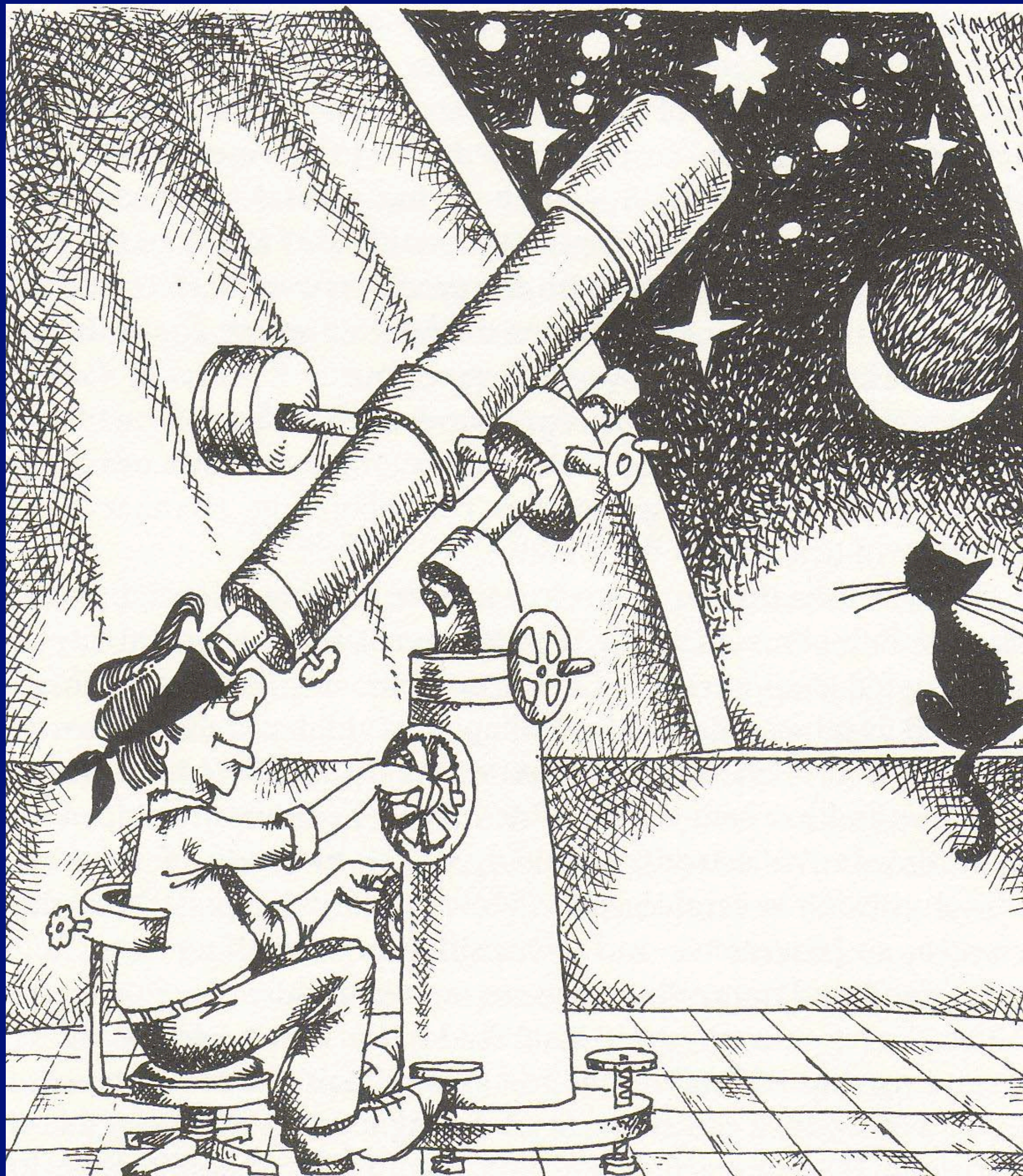
Dark Halos





$$V^2 = \frac{GM(R)}{R}$$

$$\rightarrow M(R) \propto R$$



Isothermal Sphere

Hydrostatic equilibrium:

$$\frac{GM(r)\rho(r)}{r^2} = -\frac{dP}{dr} = \frac{\alpha\sigma^2\rho(r)}{r}$$

$$\rho(r) = \rho_0 r^{-\alpha} \quad \rightarrow \quad M(r) = \frac{4\pi}{(3-\alpha)}\rho_0 r^{(3-\alpha)}$$

$$P = nkT = \rho\frac{kT}{m} = \rho(r)\sigma^2 \quad \rightarrow \quad \frac{dP}{dr} = -\frac{\alpha\rho(r)\sigma^2}{r}$$

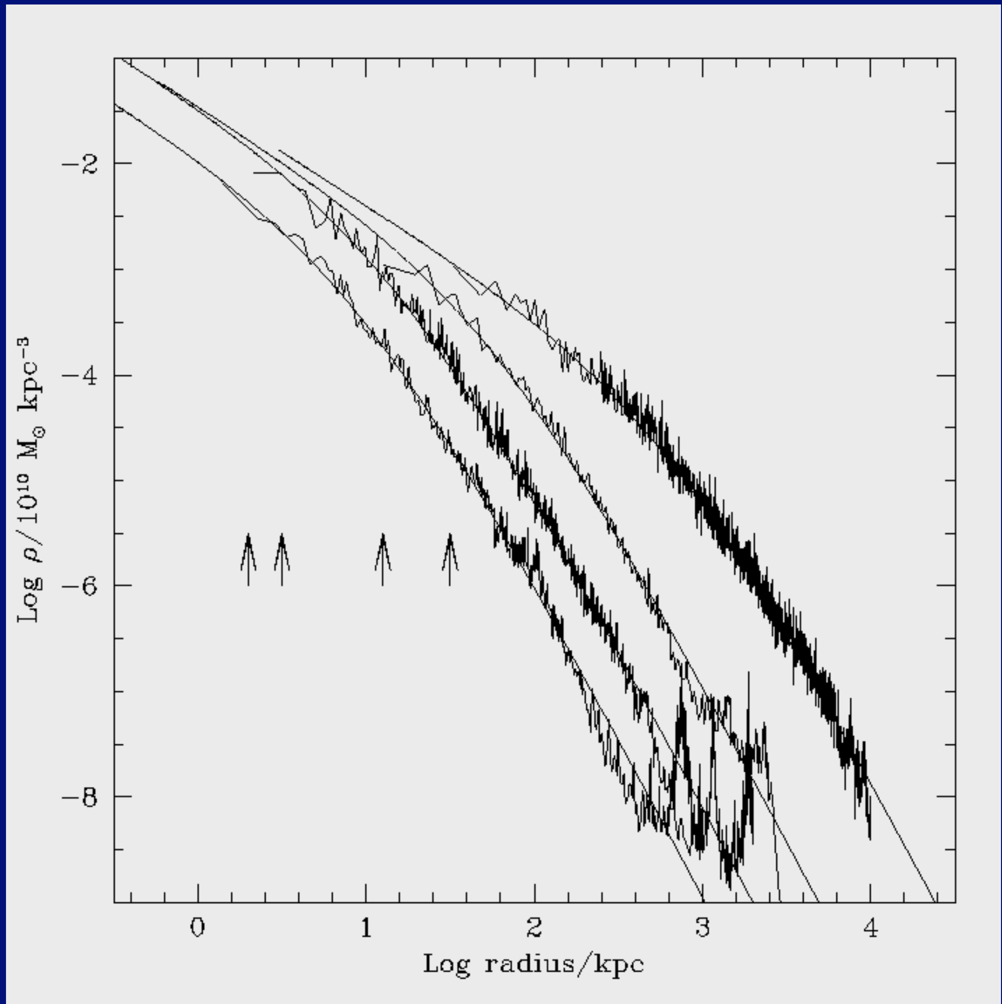
isothermal

$$\rightarrow \quad M(r) = \frac{2\sigma^2}{G}r \quad \rightarrow \quad \rho(r) = \frac{\sigma^2}{2\pi G}r^{-2}$$

$$V^2(r) = \frac{GM(r)}{r} = 2\sigma^2$$

Universal Mass Profile of CDM Halos

Density



Radius

Mass profile general shapes are independent of halo mass & cosmological parameters

Density profiles differ from power law

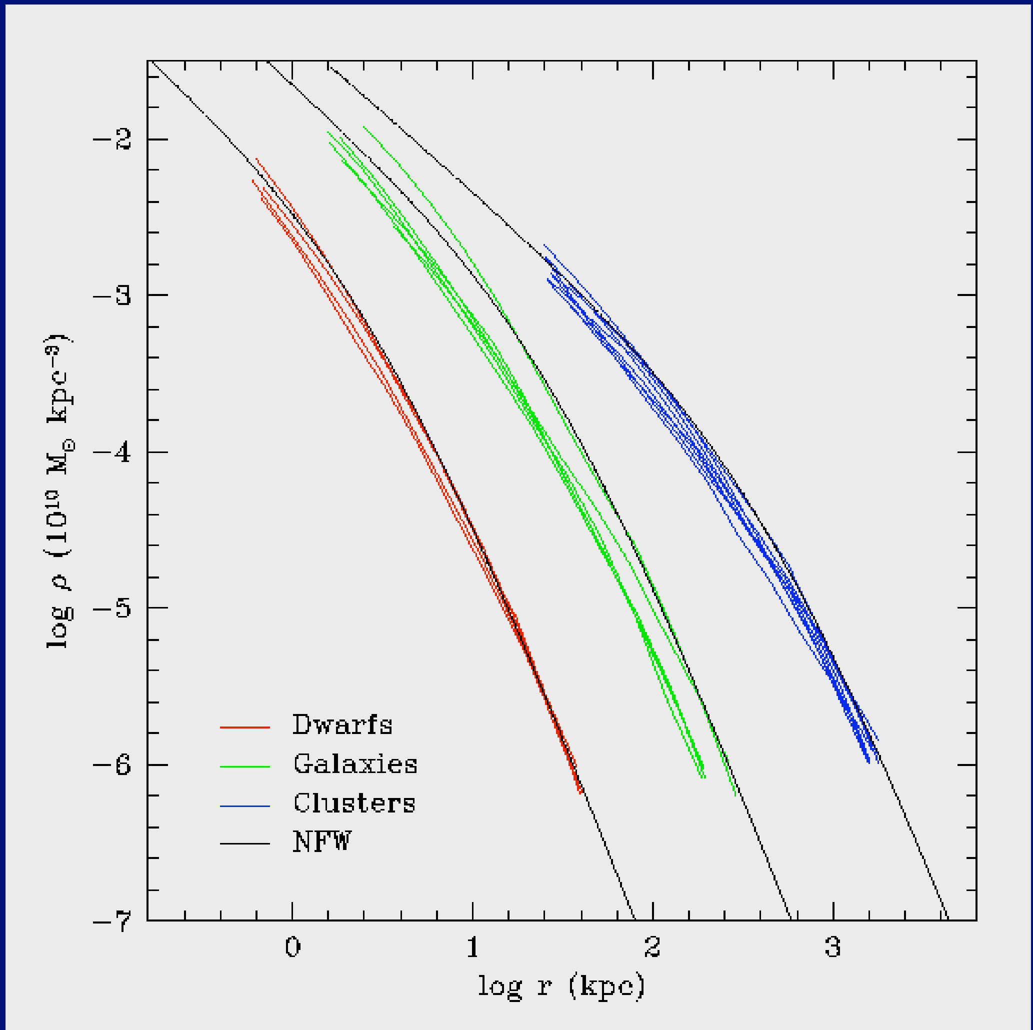
The profile is shallower than isothermal near the center

But no obvious flat-density core near the center

A cusp; some controversy about inner slope

New results for ΛCDM halos

Density



Radius

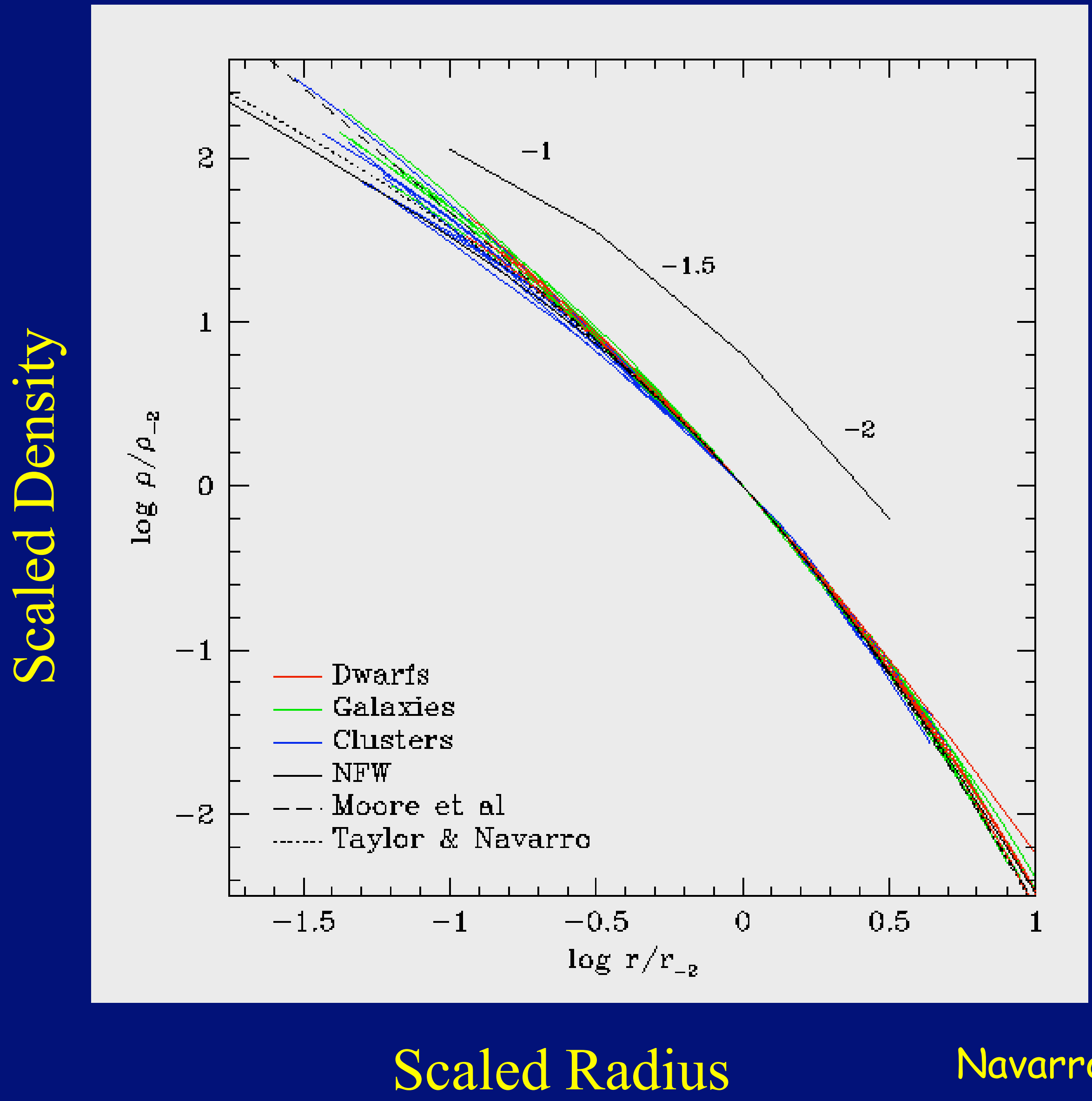
Simulations span ~6 decades in M_{vir}, from dwarf galaxies (V_c~ 50 km/s) to galaxy clusters (V_c~1000 km/s)

~million particles within R_{vir}

Controled numerical effects via convergence studies

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Recent results for ΛCDM halos



Properly scaled, all halos look alike: CDM halo structure appears to be "universal"

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

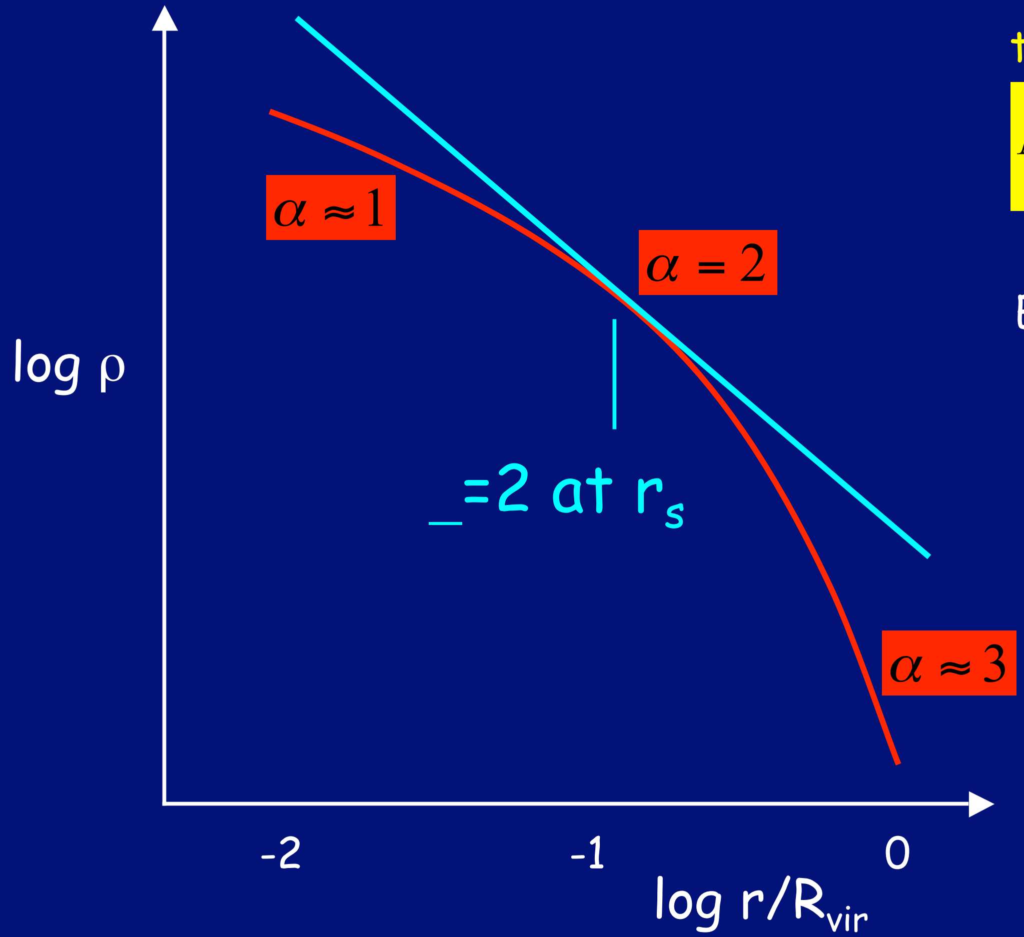
Universal Profile: NFW

$$\rho(r) = \frac{\rho_s}{x(1+x)^2} \qquad x \equiv \frac{r}{r_s} \qquad \text{for } \; 0.01 R_{vir} < r < R_{vir}$$

generalized cusp:

$$\rho(r) = \frac{\rho_s}{x^{\alpha_0}(1+x)^{3-\alpha_0}}$$

slope: $\alpha(r) = -\dfrac{d\ln\rho}{d\ln r}$



two parameters:

$$M_{vir} \qquad C \equiv \frac{R_{vir}}{r_s} \sim 10$$

Ellipsoidal shape: $a_3/a_1 \sim 0.5$

Navarro, Frenk & White 95, 96, 97
Cole & Lacey 96
Moore et al. 98
Ghinga et al. 00
Klypin et al. 01
Power et al. 02
Navarro, Hayashi et al. 03, 04
Stoehr et al. 04, 05

Halo Concentration vs Mass and History

Self-similar Toy model (Bullock et al. 2001):

Define a_c as the time when typically a constant fraction f of M is collapsing:

$$M_*(a_c) \equiv f\,M \qquad (1)$$

Define a characteristic halo density:

$$\tilde{\rho}_s \equiv \frac{M}{(4\pi/3)r_s^3} \qquad = 3\rho_s\left(\ln(1+C) - \frac{C}{1+C}\right)$$

for NFW

Assume additional contraction of inner halo by a constant factor k:

$$\tilde{\rho}_s = k^3\ \Delta(a)\ \rho_u(a_c) = k^3\ \Delta(a)\ \rho_u(a)\frac{a^3}{a_c^3}$$

$$C \equiv \frac{R_{vir}}{r_s} \longrightarrow C(\mu,a) = k\,\frac{a}{a_c} \qquad (2)$$

EdS $P_k \propto k^n$

$$\sigma \propto M^{-\alpha} \rightarrow M_* \propto a^{1/\alpha} \rightarrow^1 \frac{a_c}{a_0} = (\mu f)^\alpha \longrightarrow C(\mu,a) = k(f\mu)^{-\alpha}$$

$$\mu \equiv M(a)/M_*(a)$$

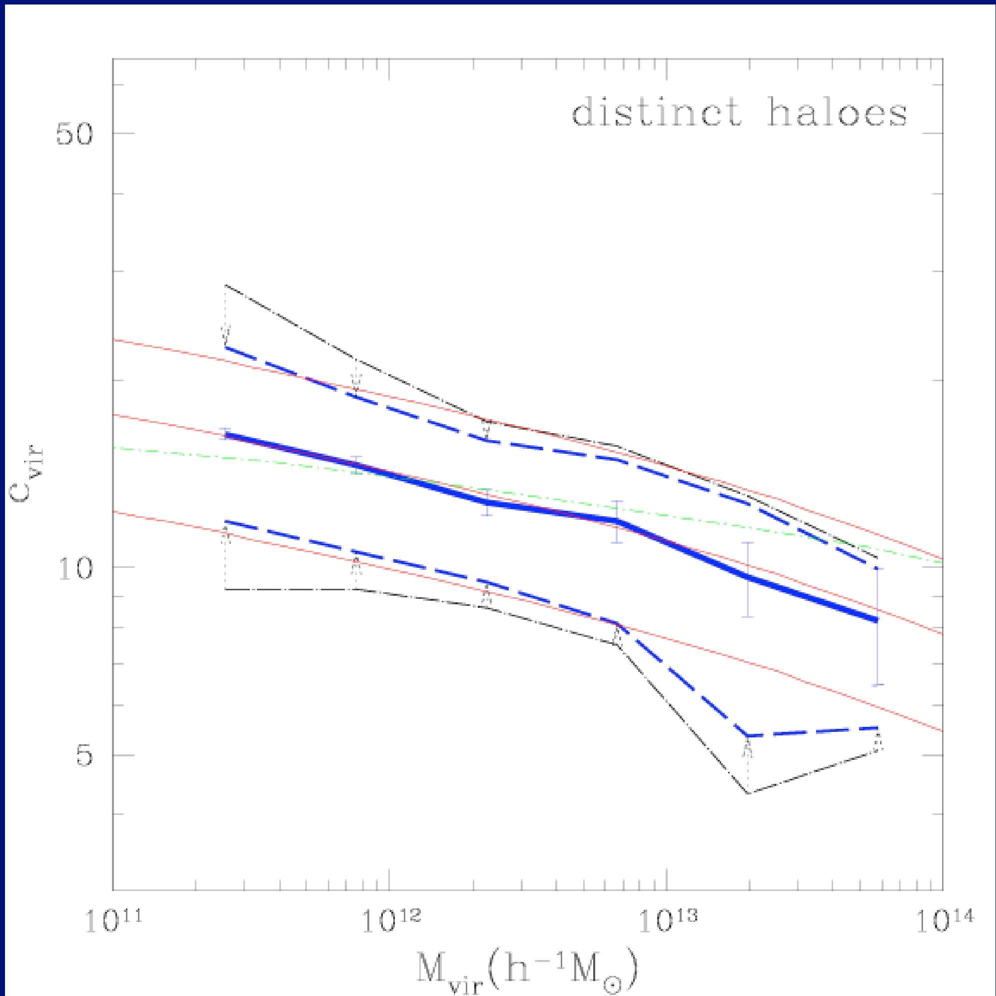
Determine parameters from simulations:

$$f \sim 0.01 \quad k \approx 4 \quad \alpha \approx 0.13$$

Excellent fit!

$$C(\mu,a) \approx 4\,(0.01\mu)^{-0.13} \approx 4\,\frac{a}{a_c}$$

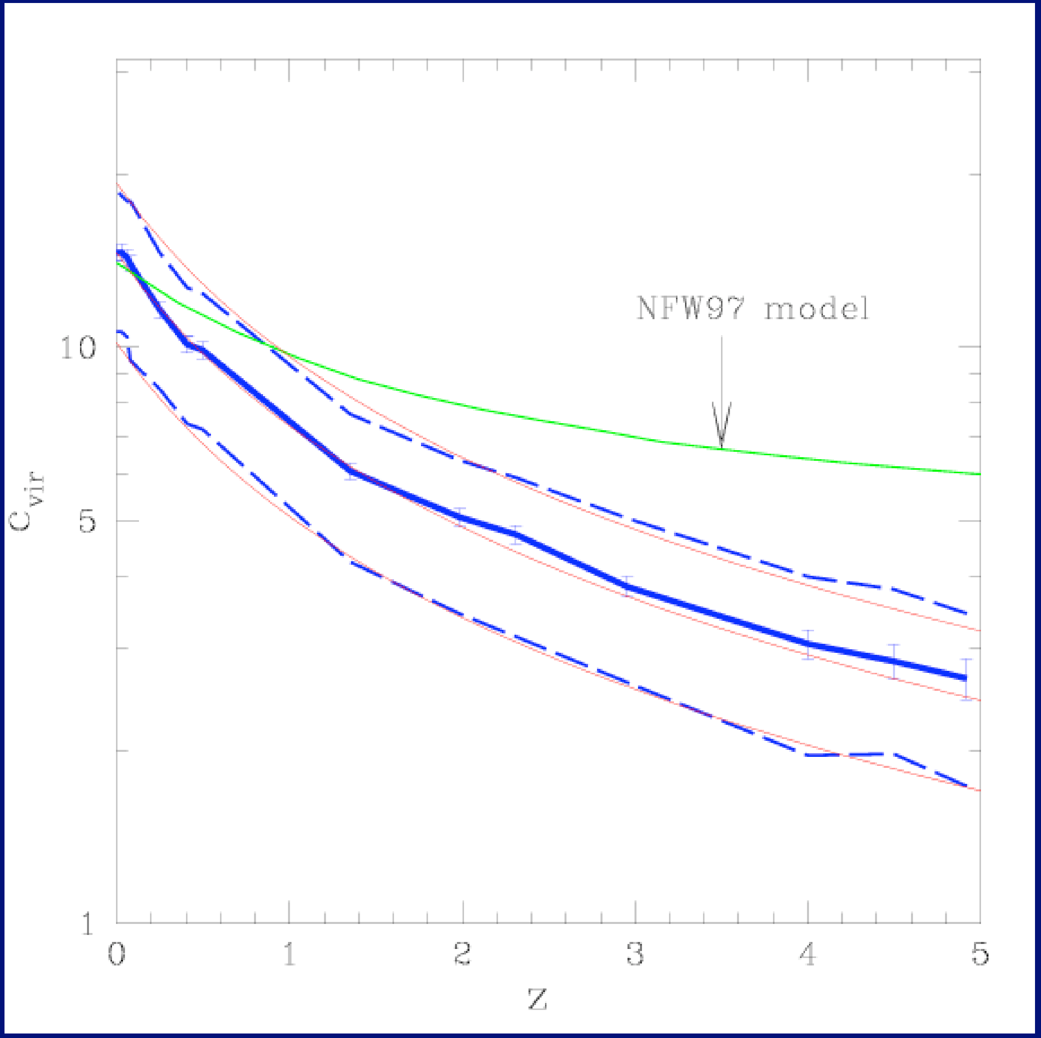
Concentration vs Mass



$$C(\mu, a) \approx 4\,(0.01\mu)^{-0.13} \approx 4\frac{a}{a_c}$$

Bullock et al. 2001

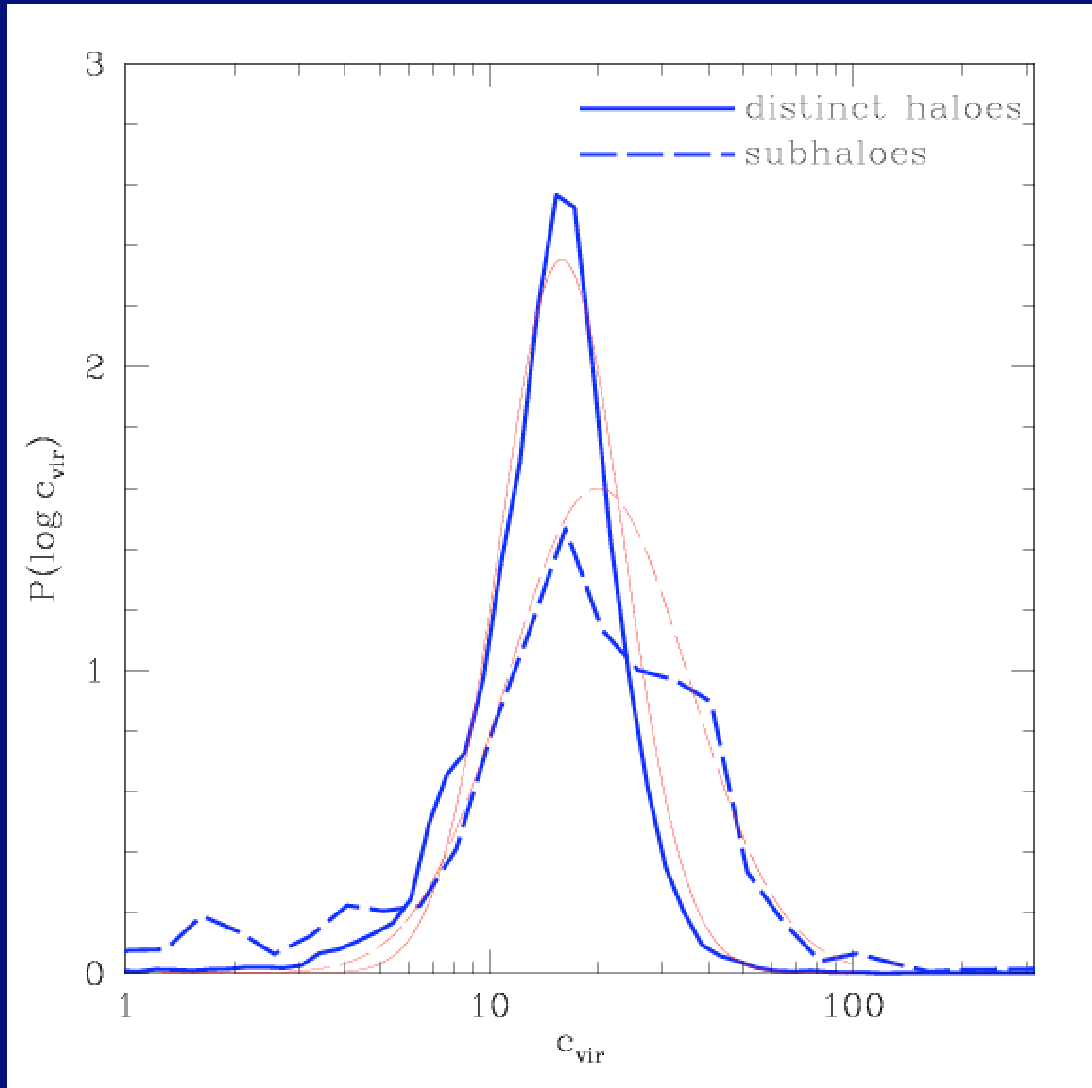
Concentration vs time, given mass



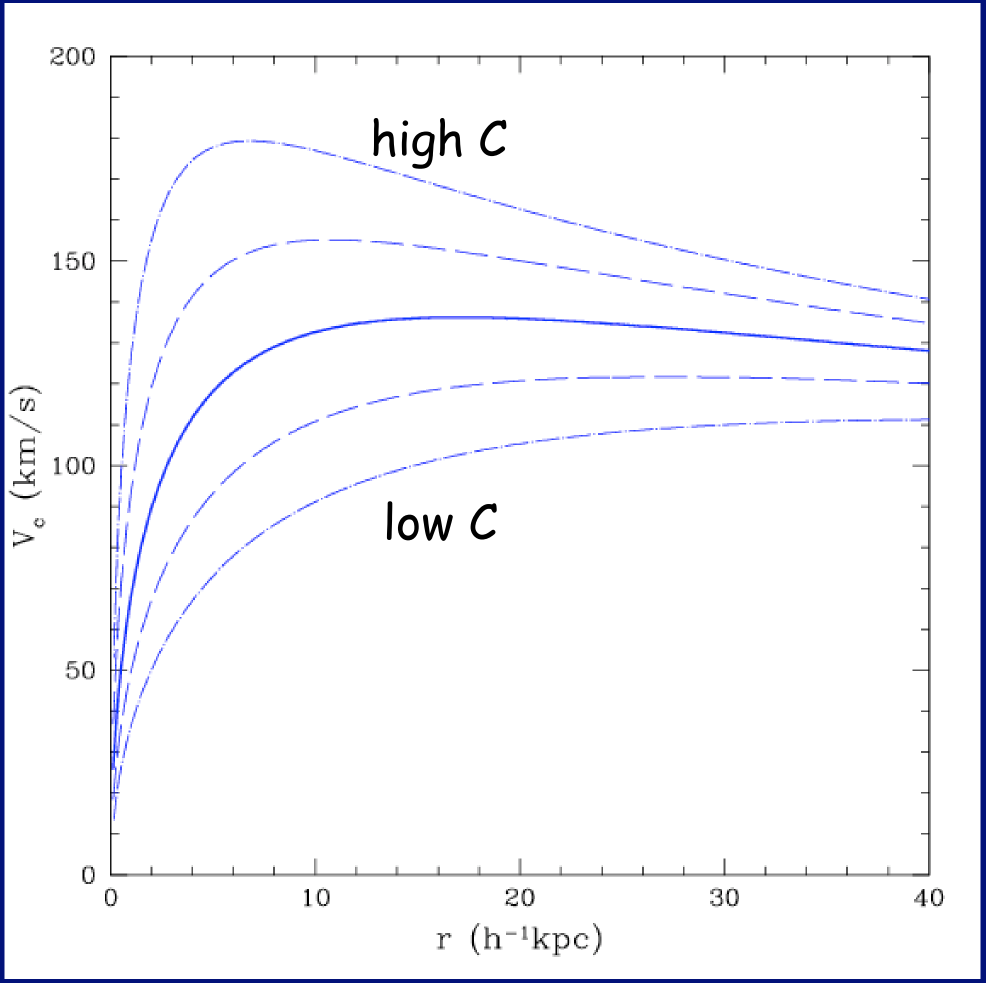
$$C(\mu, a) \approx 4\,(0.01\mu)^{-0.13} \approx 4\,\frac{a}{a_c}$$

Bullock et al. 2001

Distribution of C: log-normal



NFW Rotation Curve



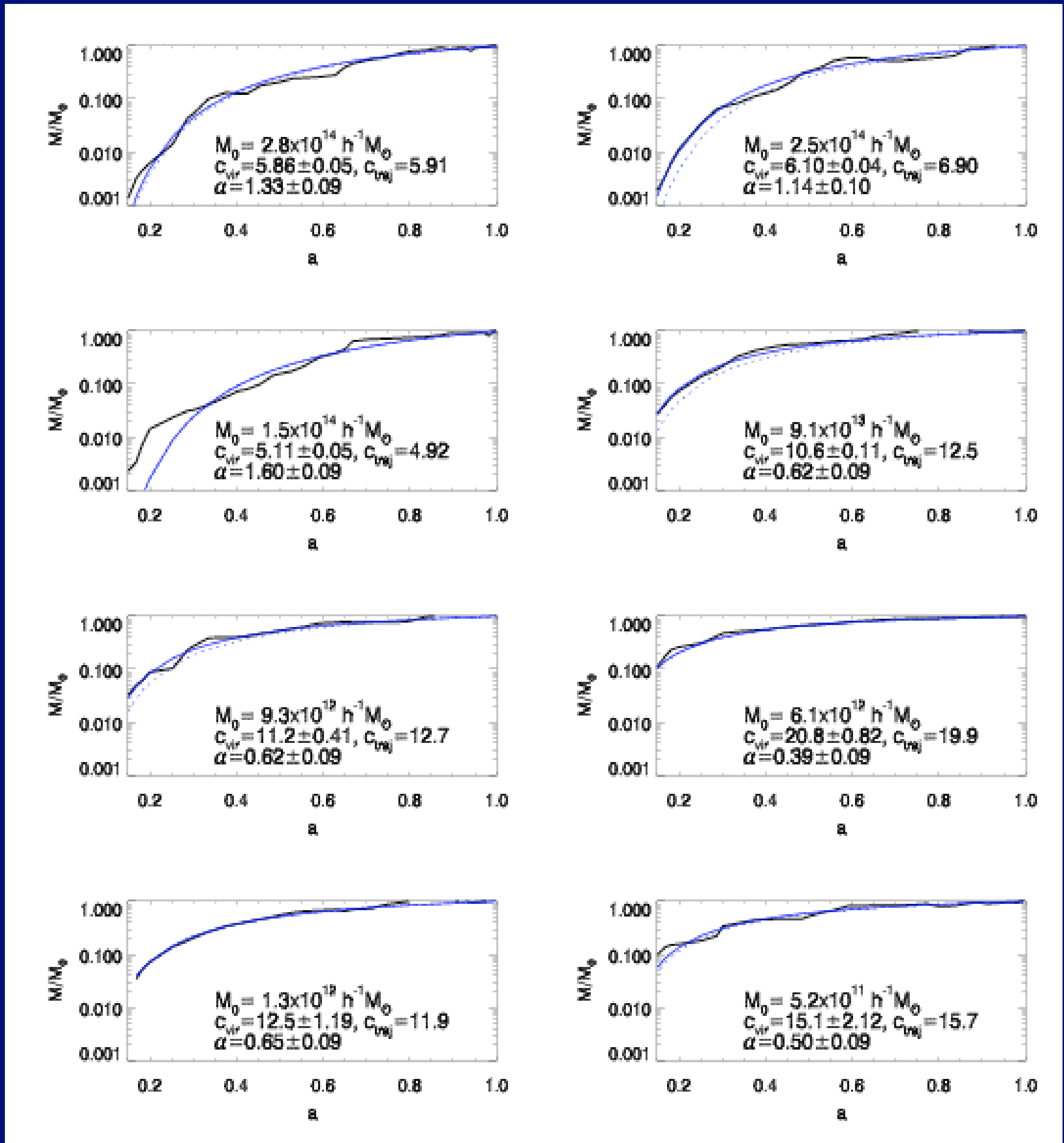
$$M = 4\pi\rho_s r_s^3 A(C) \qquad A(C) \equiv \ln(1+C) - \frac{C}{1+C}$$

$$V^2(x) = V_{vir}^2 \frac{C}{A(C)} \frac{A(x)}{x}$$

$$r_{max} \approx 2.16 r_s \qquad \frac{V_{max}^2}{V_{vir}^2} \approx 0.216 \frac{C}{A(C)}$$

Mass Assembly History

Wechsler et al. 2002

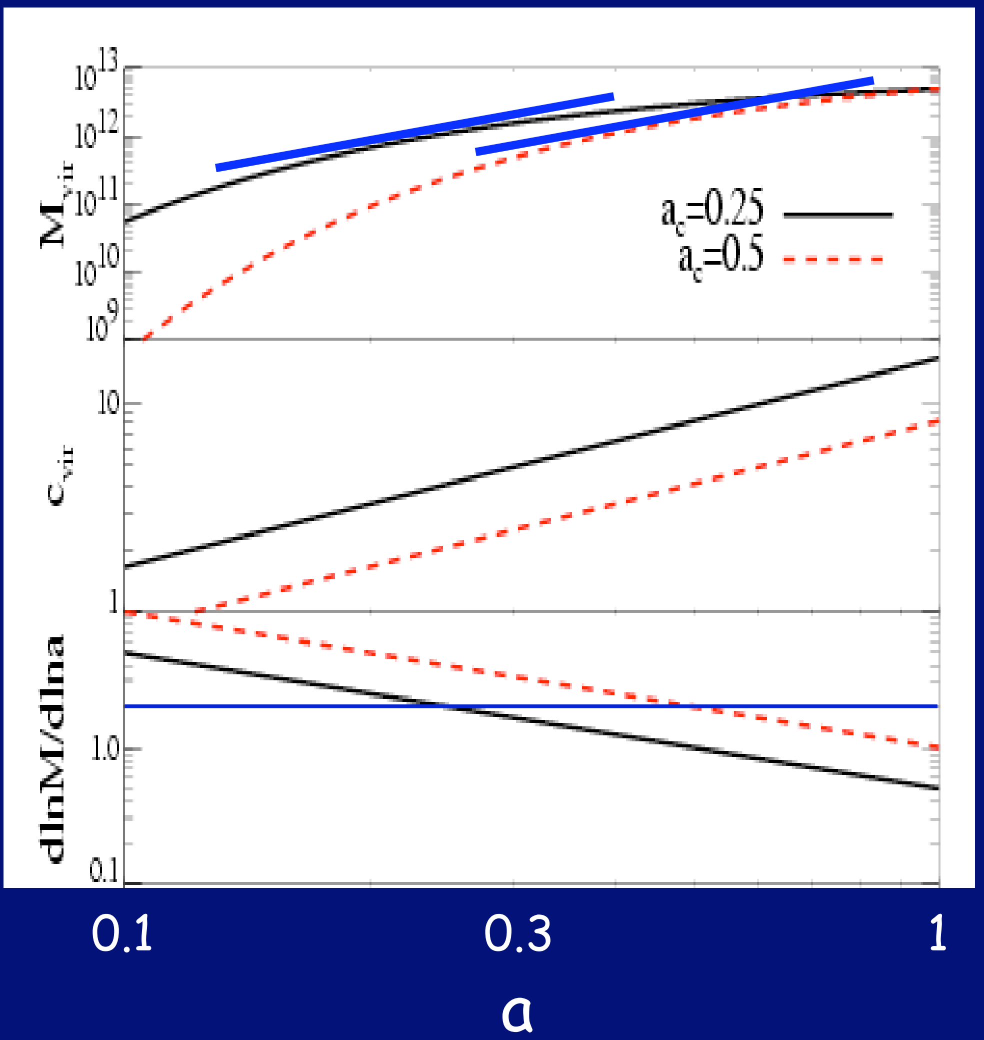


$$M(a) \propto e^{-2a_c z}$$

$$\frac{d\log M}{d\log a} = 2 \quad \text{defines } a_c$$

Mass Assembly History

Wechsler et al. 2002



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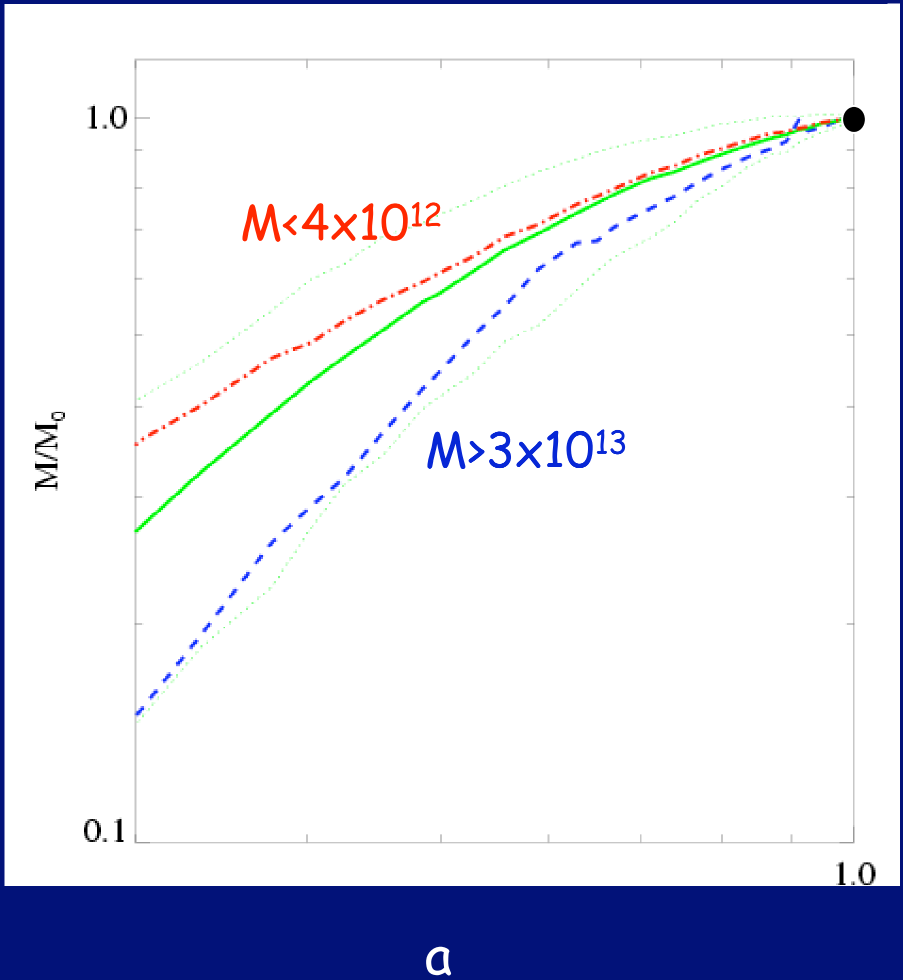
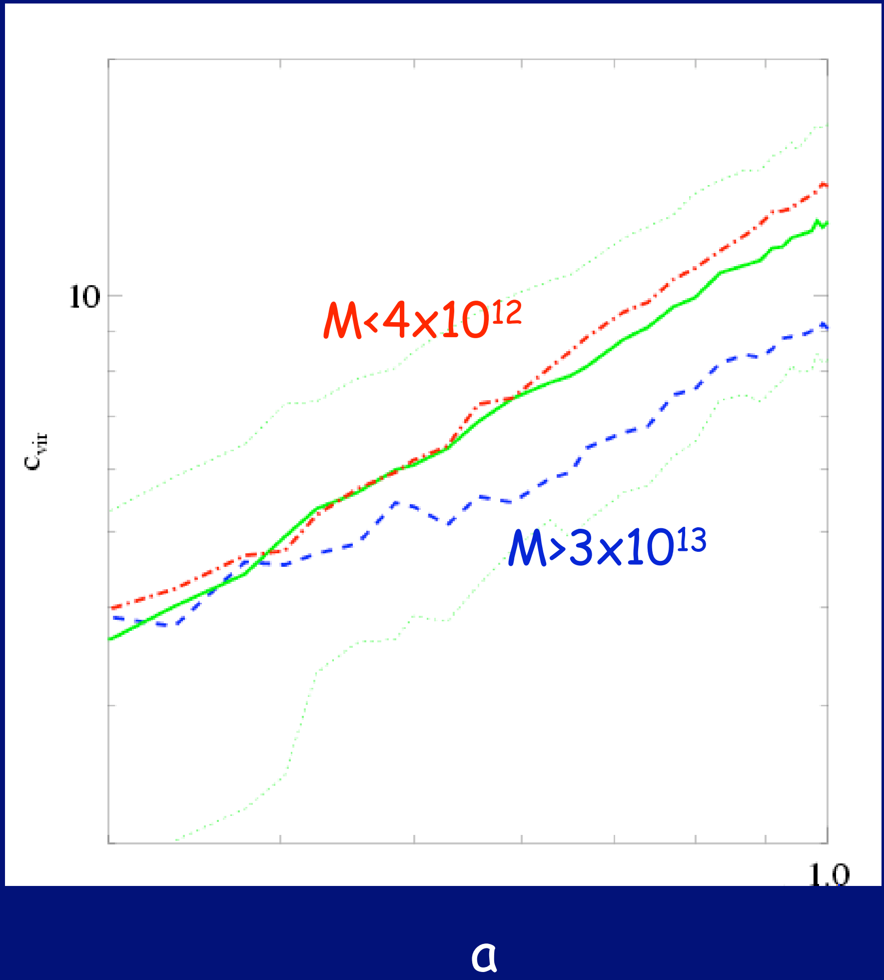
Mass dependence of History and Concentration

Wechsler et al. 2002

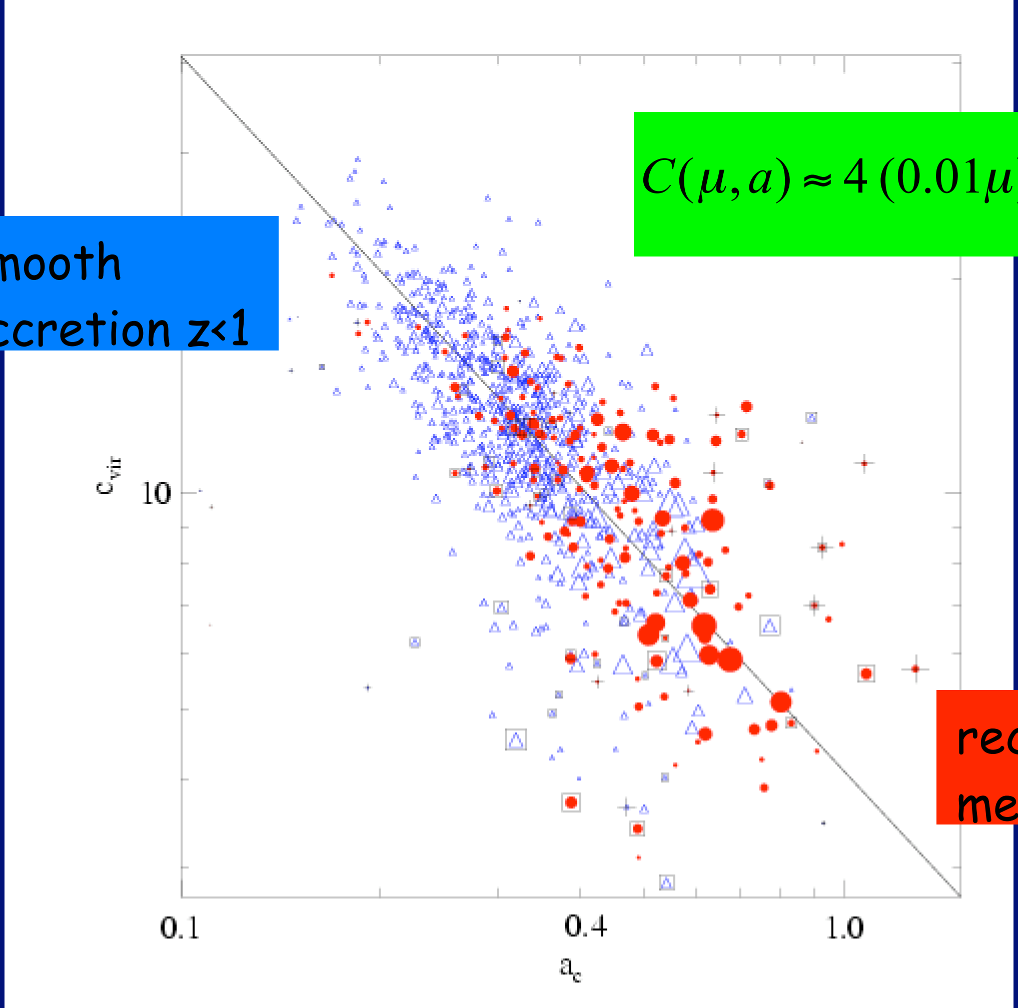




$$C(\mu, a) \approx 4\,(0.01\mu)^{-0.13} \approx 4\,\frac{a}{a_c}$$

Concentration vs History

Wechsler et al. 2002



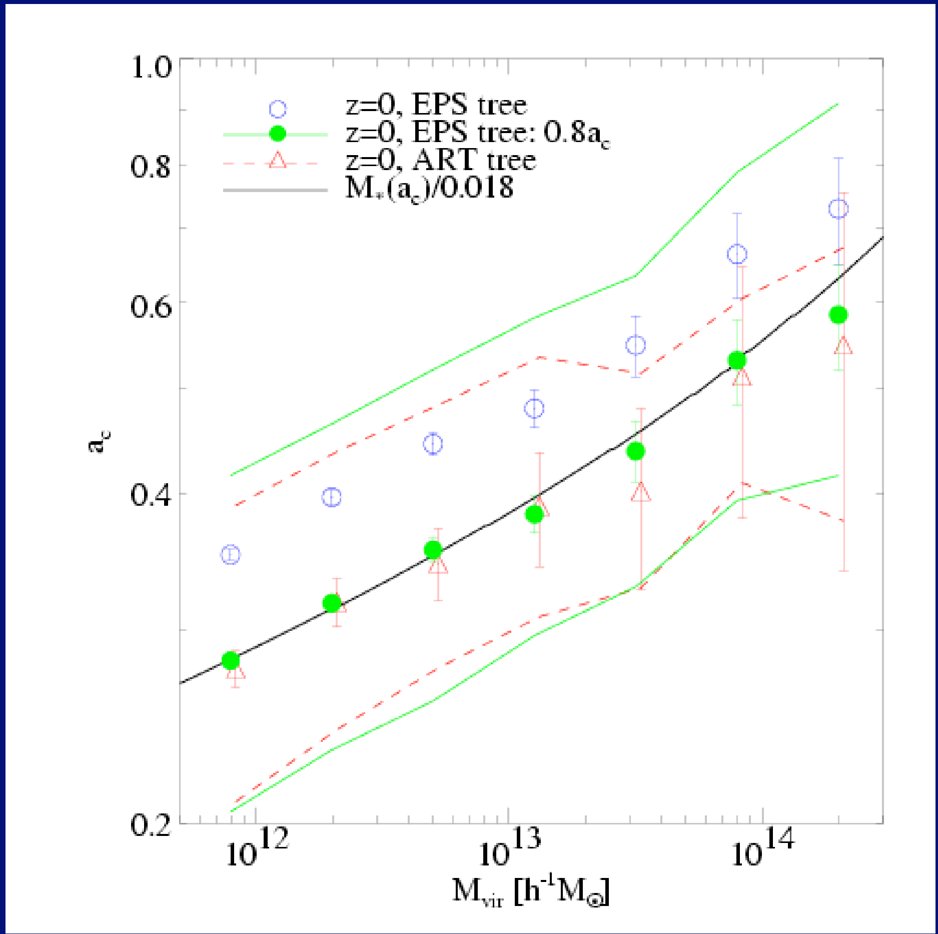
$$C(\mu, a) \approx 4\,(0.01\mu)^{-0.13} \approx 4\frac{a}{a_c}$$

smooth accretion z<1

recent major merger z<1

History vs Mass

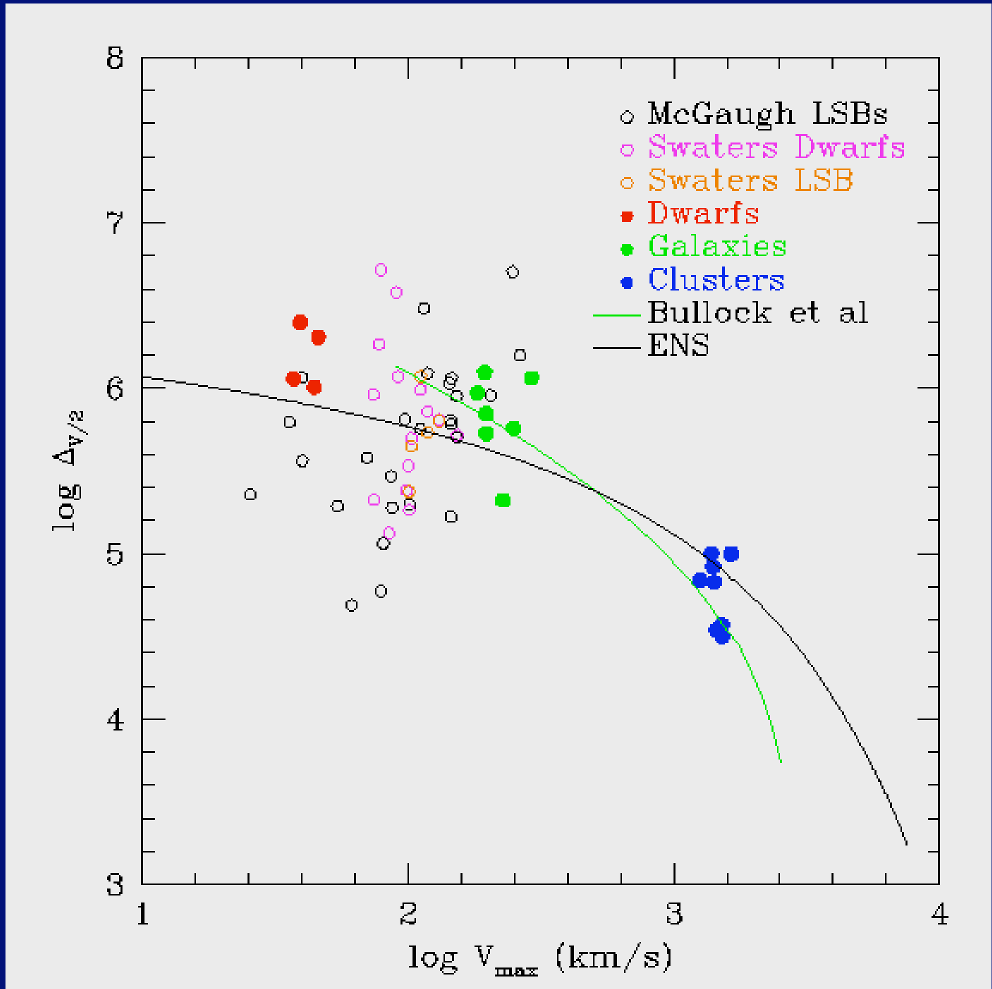
Wechsler et al. 2002



$$C(\mu, a) \approx 4\,(0.01\mu)^{-0.13} \approx 4\,\frac{a}{a_c}$$

Concentration of LSB galaxies and ΛCDM halos

Mean density contrast within $r(V_{max}/2)$



Maximum Rotation Speed

The average intermediate-scale concentration and scatter of ΛCDM halos is roughly consistent with observations of LSB and dwarf galaxies

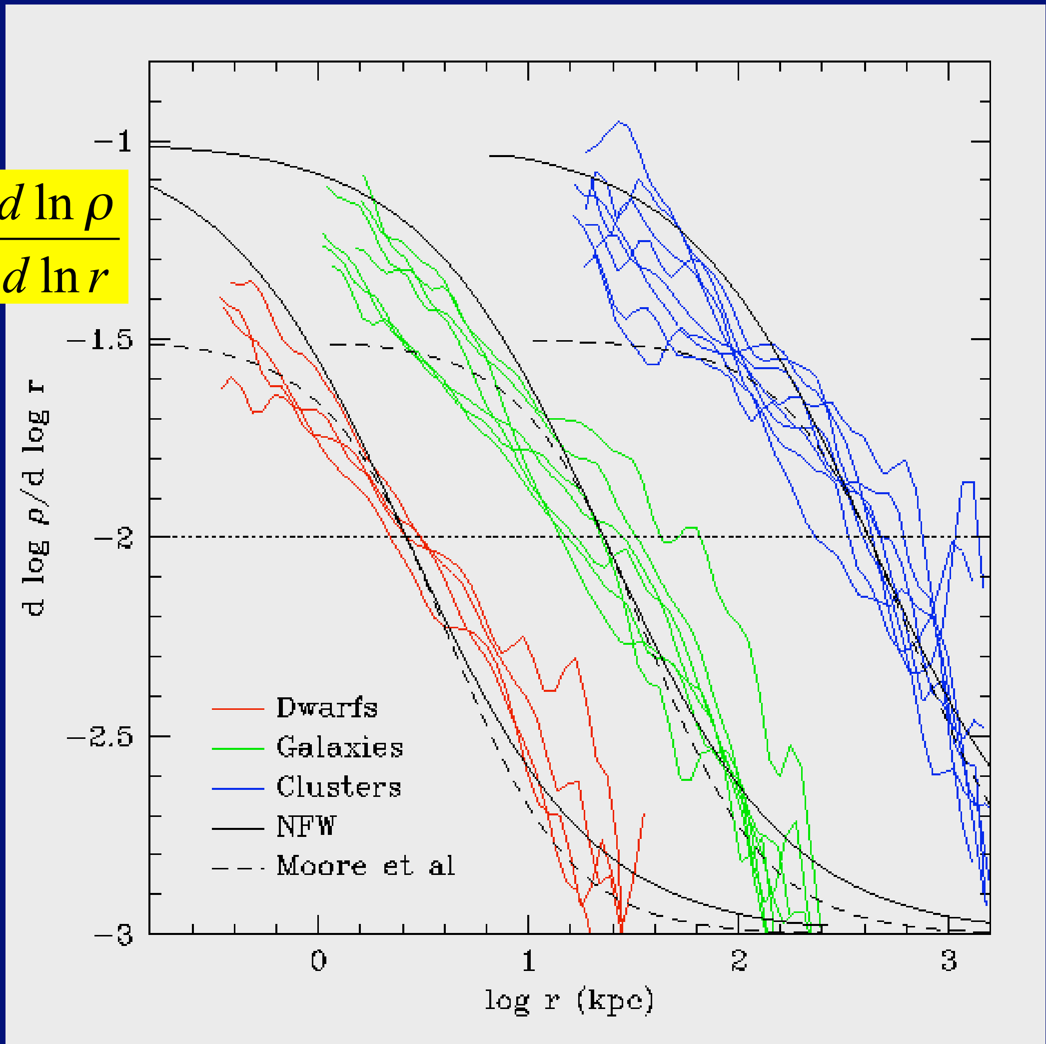
Alam et al 2001
Hayashi et al 2003

Simulated Cusp

Recent results for ΛCDM halos

$$-\alpha(r) = \frac{d\ln\rho}{d\ln r}$$

Logarithmic Slope



Radius

No obvious convergence to a power law: profiles get shallower all the way in.

Innermost slopes are shallower than -1.5

Improved profile:

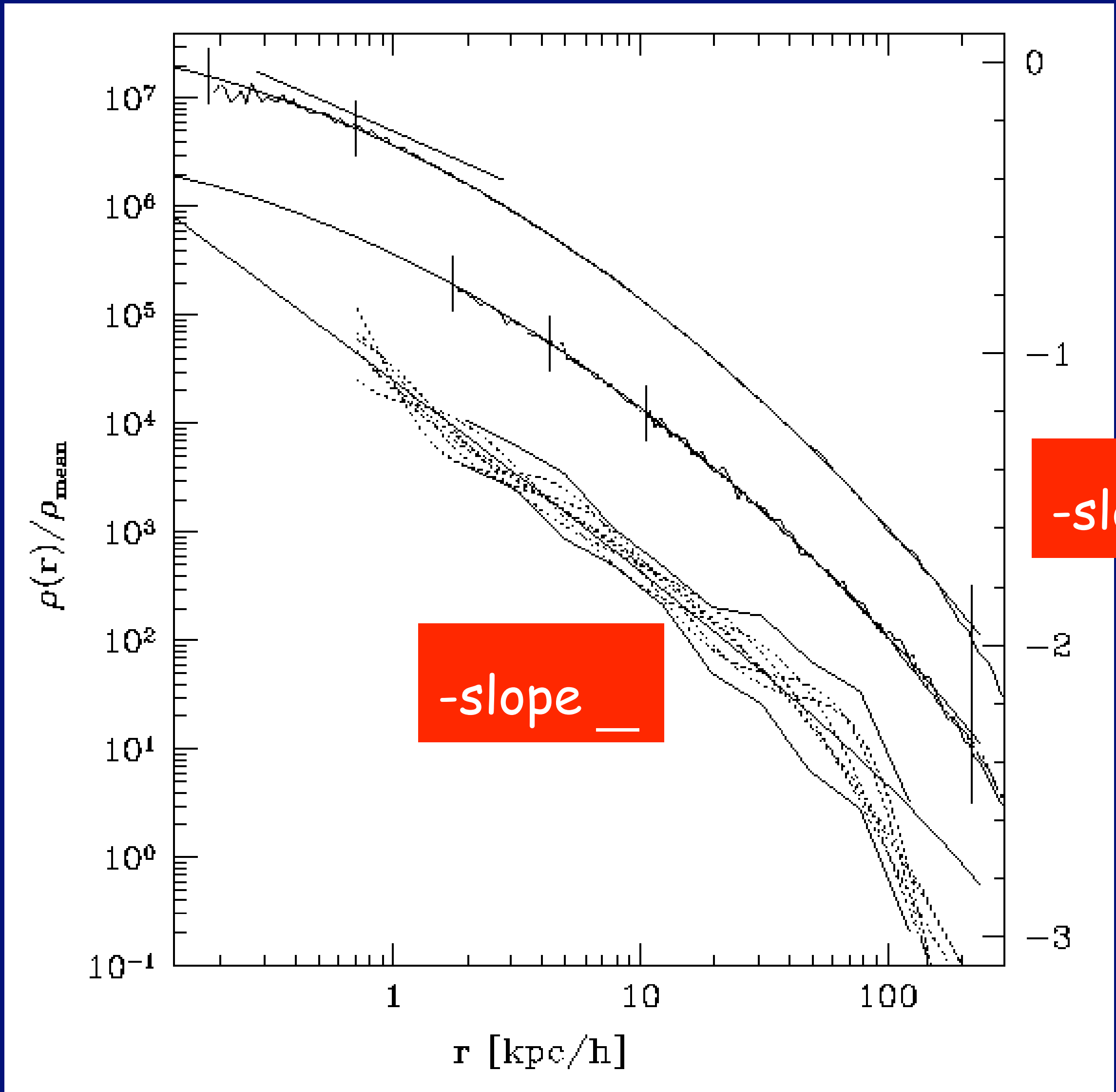
$$\alpha_\beta(r) \equiv -\frac{d\ln\rho}{d\ln r} = 2\left(\frac{r}{r_s}\right)^\beta$$

$$\ln\left(\frac{\rho_\beta}{\rho_s}\right) = -\frac{2}{\beta}\left[\left(\frac{r}{r_s}\right)^\beta - 1\right]$$

$$\beta \sim 0.1 - 0.2$$

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Improved Cusp Profiles



Stoehr et al. 2004

$$\log\left(\frac{V}{V_{\max}}\right) = -a\left[\log\left(\frac{r}{r_{\max}}\right)\right]^2$$

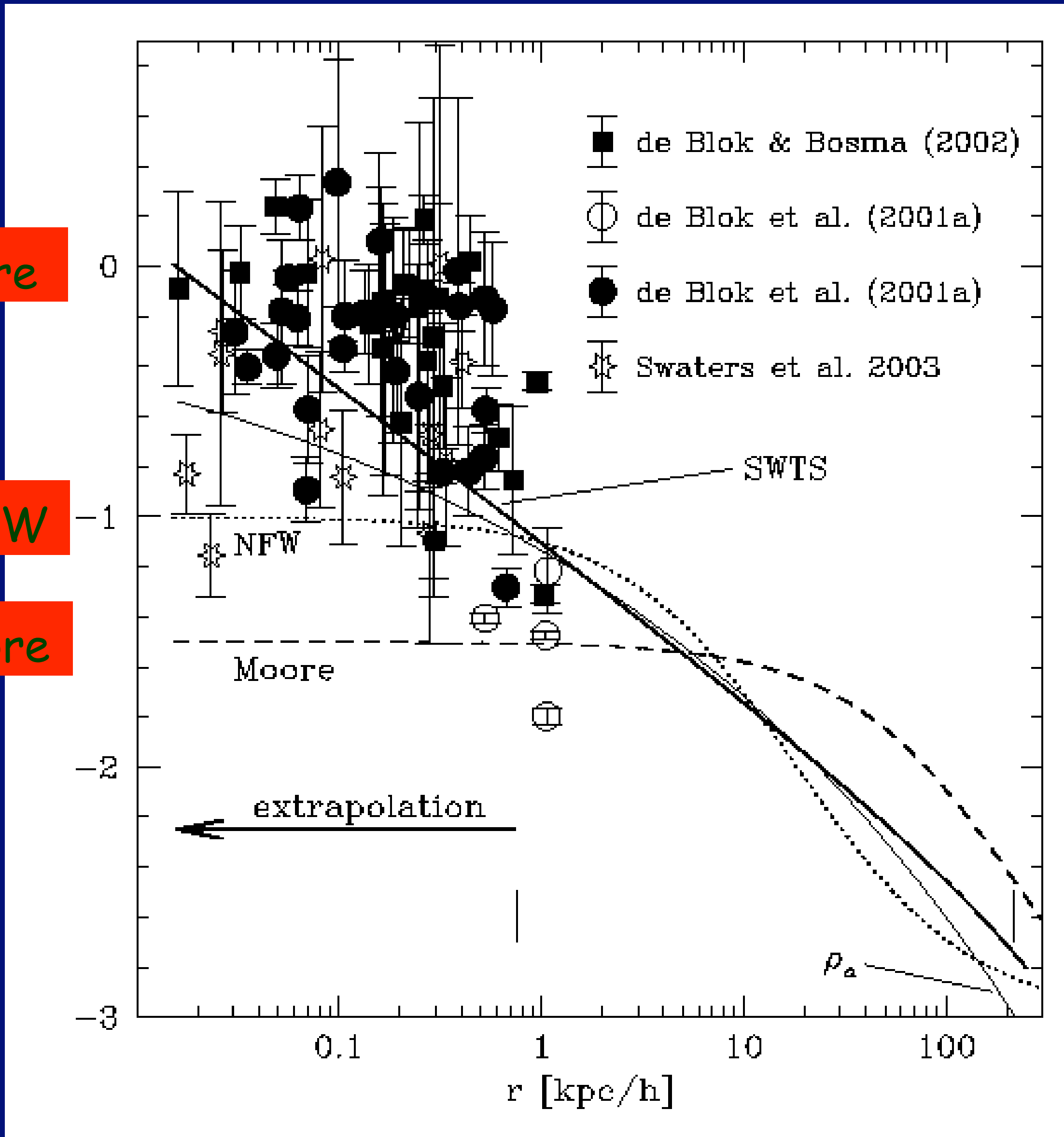
Improved Cusp Profiles: extrapolated to the inner cusp

core

-slope _

NFW

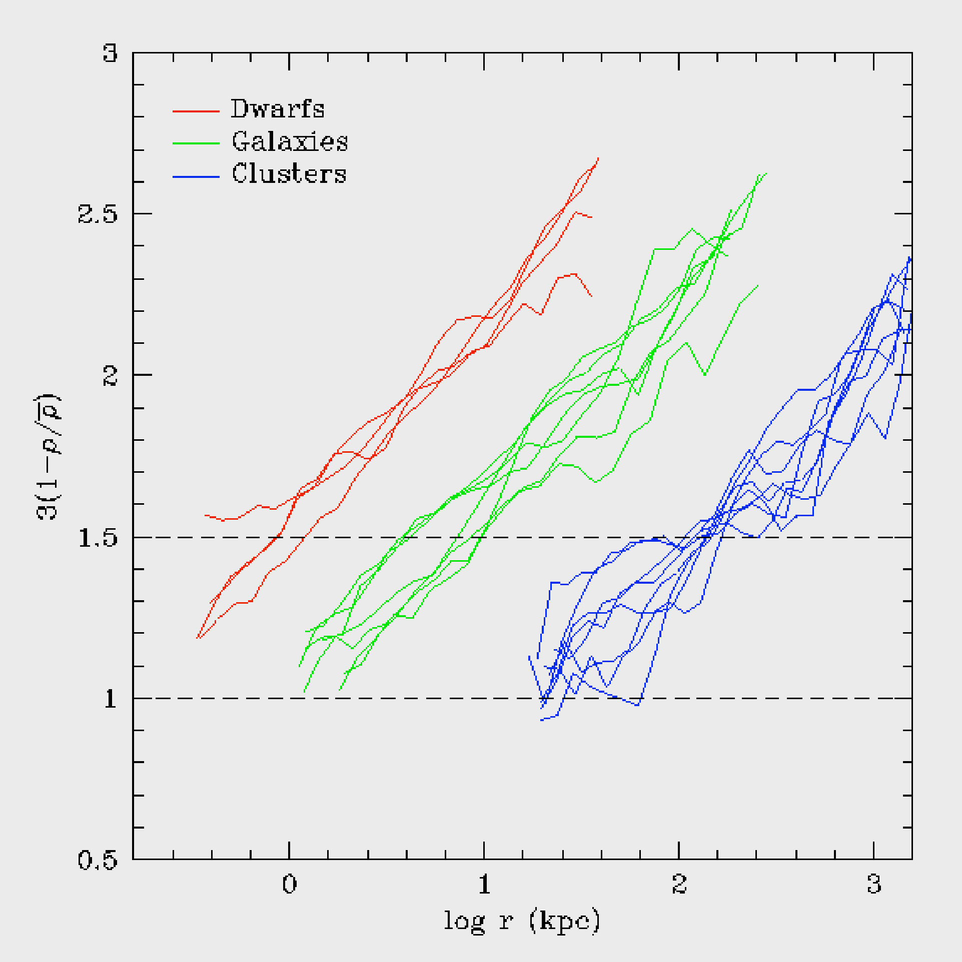
Moore



Maximum Asymptotic Inner Slope

$$\rho = r^{-\alpha} \quad r < r_p \quad \rightarrow \bar{\rho}(r) = \frac{1}{(4\pi/3)r^3}\int_0^r 4\pi r'^2\, dr' \rho(r') = \frac{3}{3-\alpha} r^{-\alpha}$$

$$\rightarrow \alpha = 3[1 - \rho(r)/\bar{\rho}(r)] \quad \text{upper limit for slope in } r < r_p$$



Radius

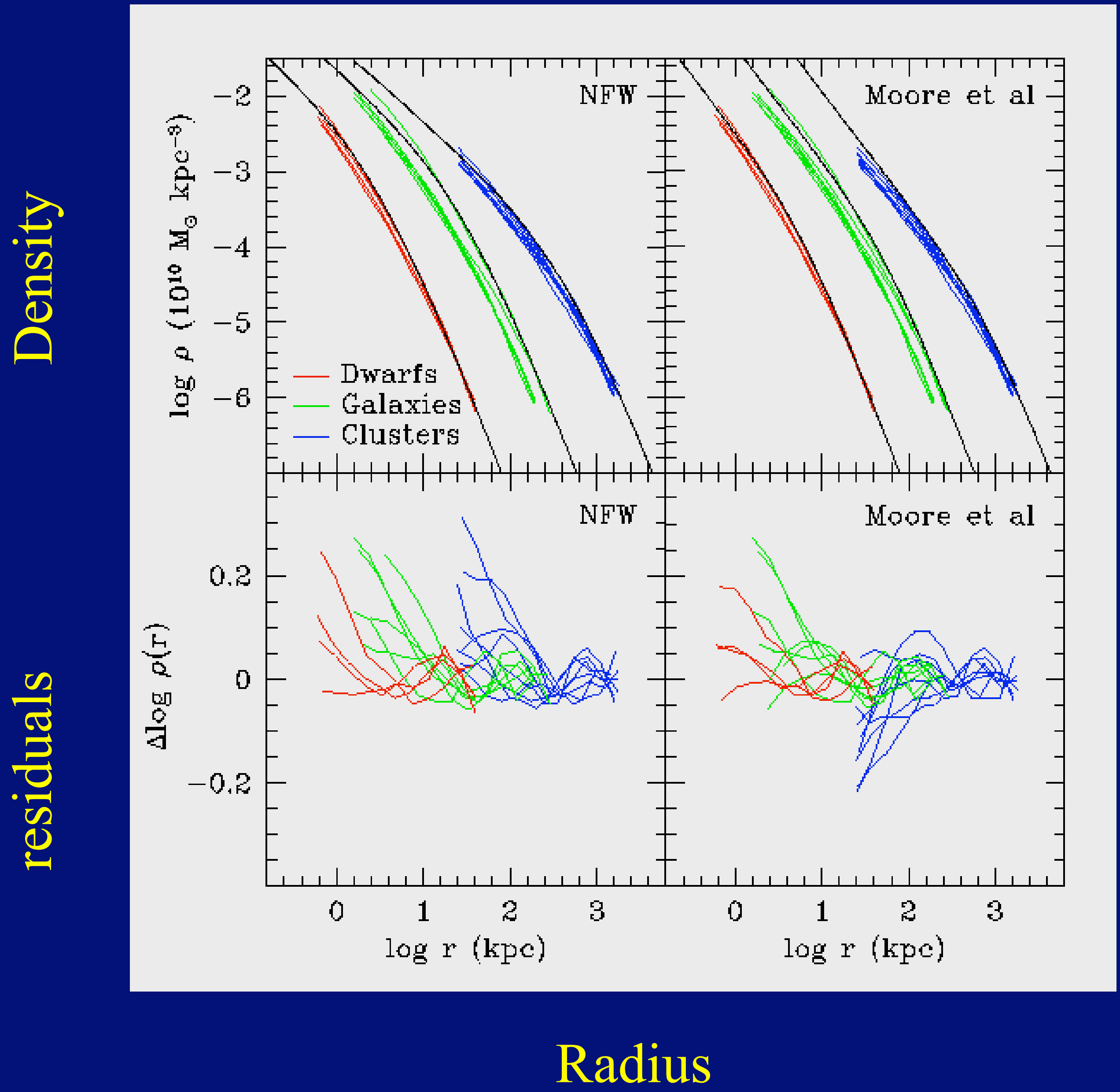
M(r) is robustly measured in the simulations.

With the local density, it provides an upper limit to the inner asymptotic log slope

°There is not enough mass in cusp to sustain a power-law as steep as $\rho \sim r^{-1.5}$

Navarro, Hayashi, Frenk, Jenkins, White, Power, Springel, Quinn, Stadel

How good or bad are simple fits?



Over the well resolved regions, both NFW and Moore functions exhibit comparable systematic deviations when fitted to simulated CDM halos.

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

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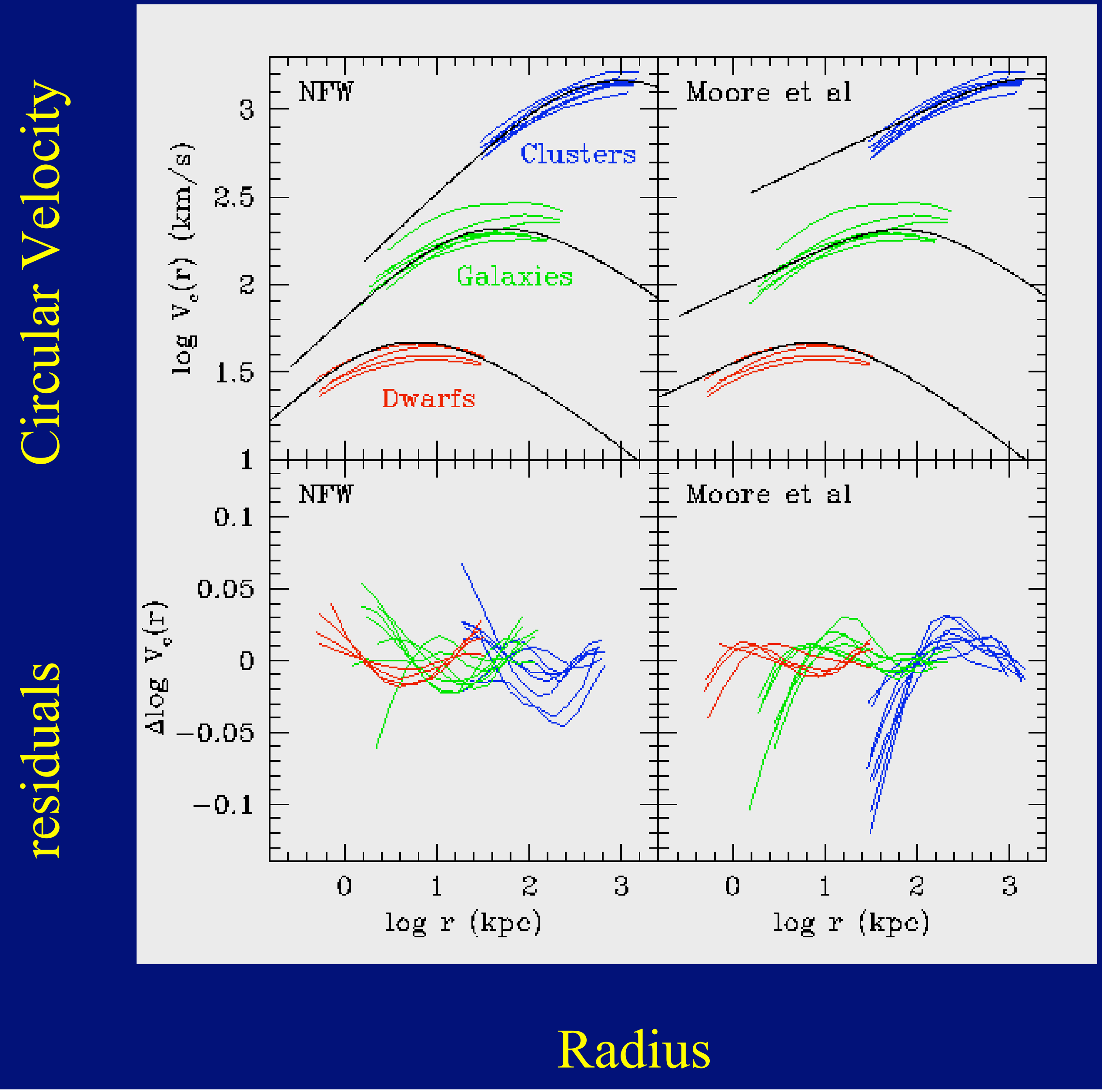

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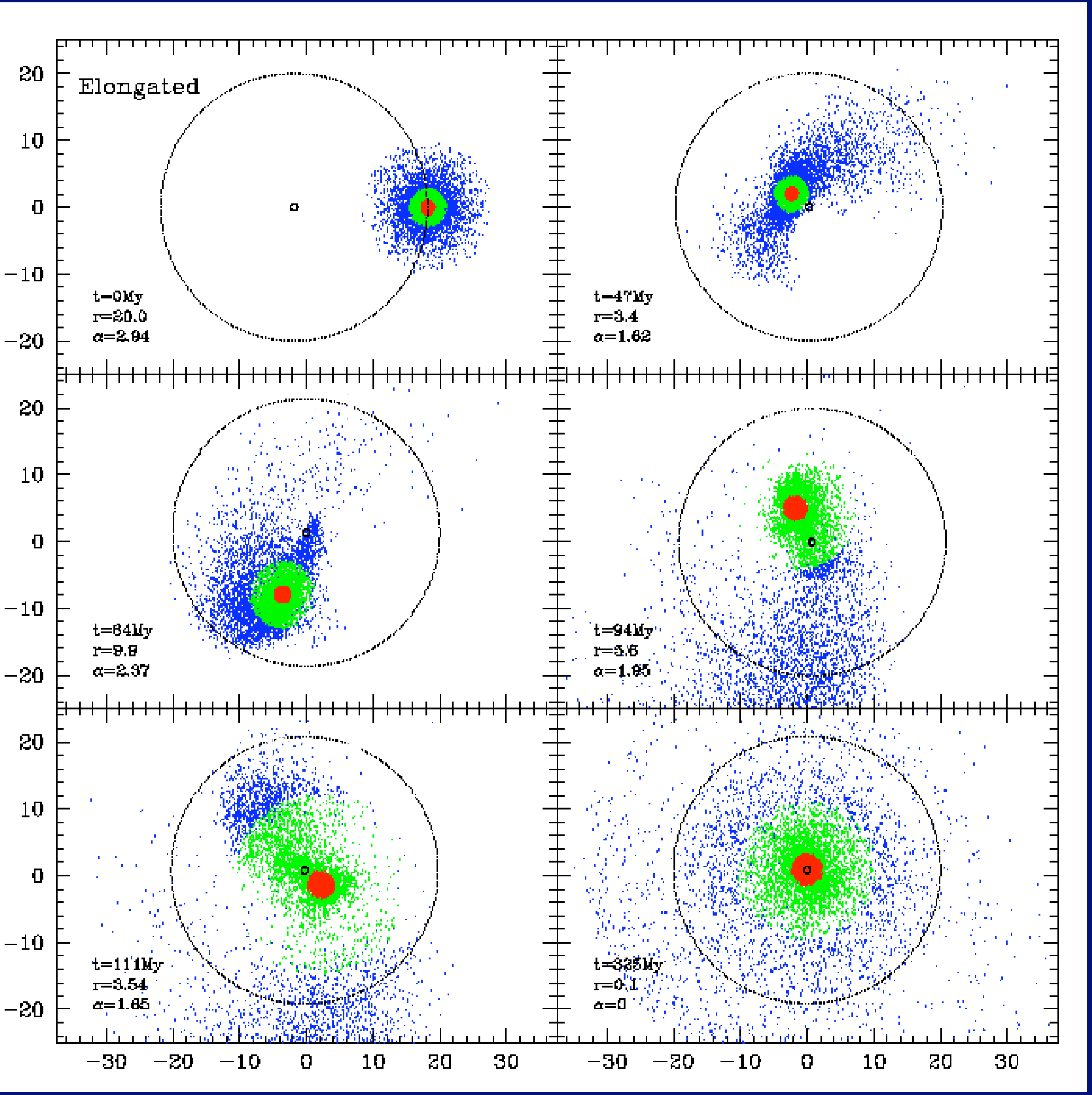
Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Origin of the Halo inner Cusp?

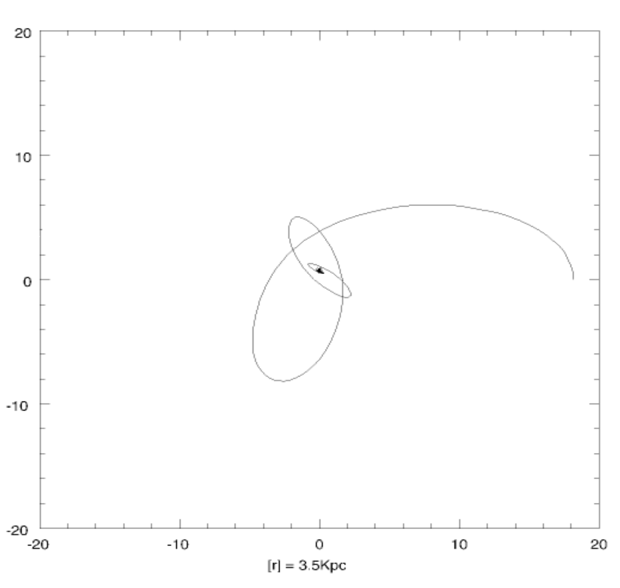
Dynamical Friction and Tidal Effects

Dekel, Arad, Devor, et al. 2003

Halo Bulidup by Mergers

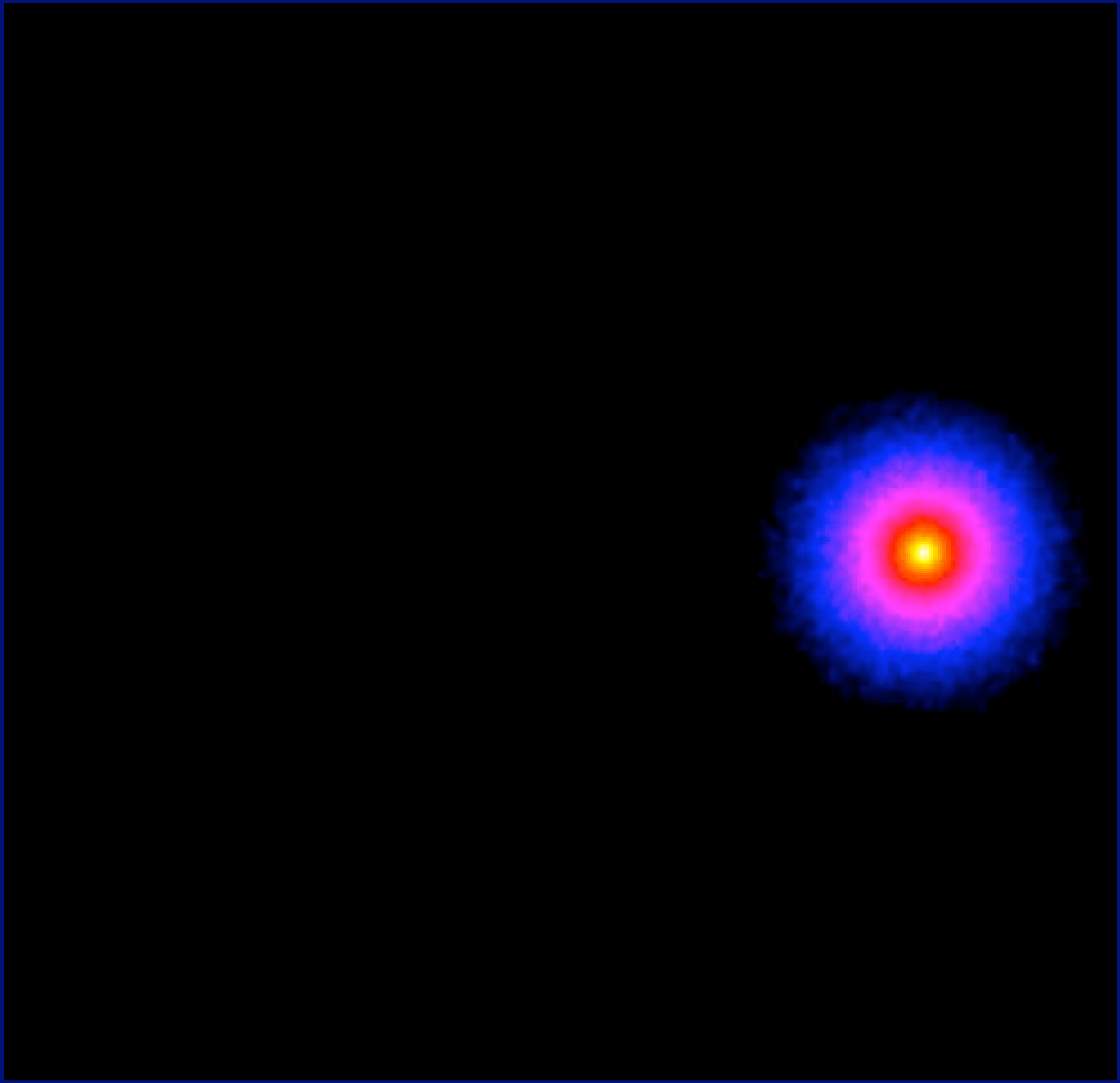


tidal stripping & dynamical friction



Dekel, Devor & Hetzroni 2003

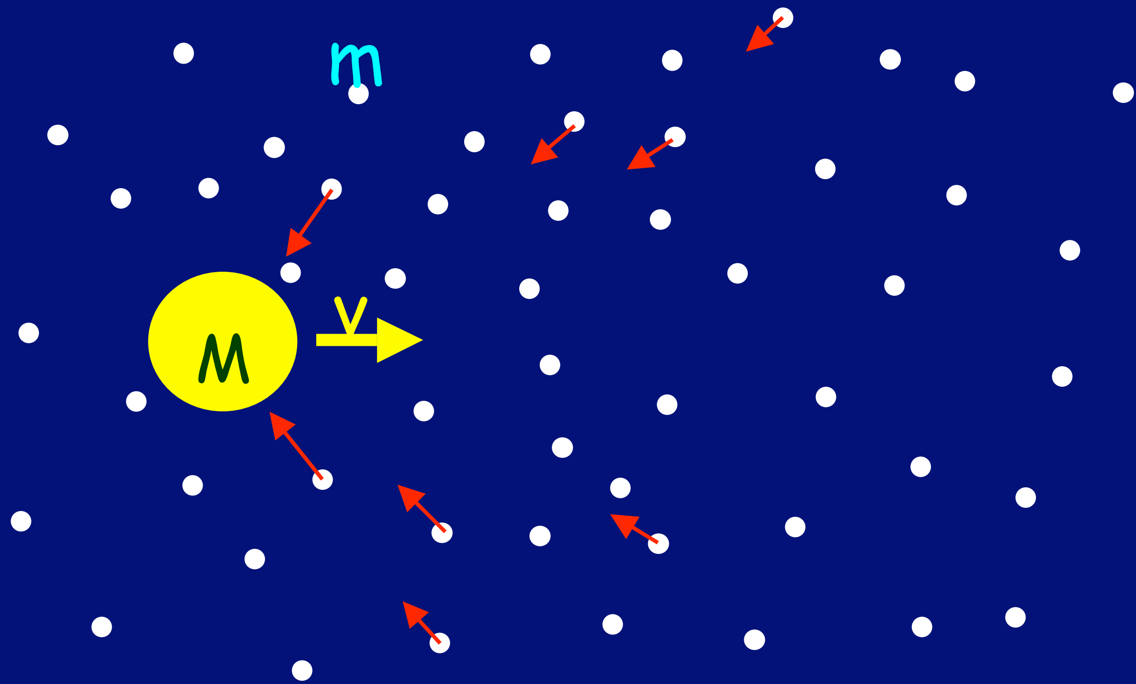
Dynamical Friction and Tidal stripping



Moore et al.

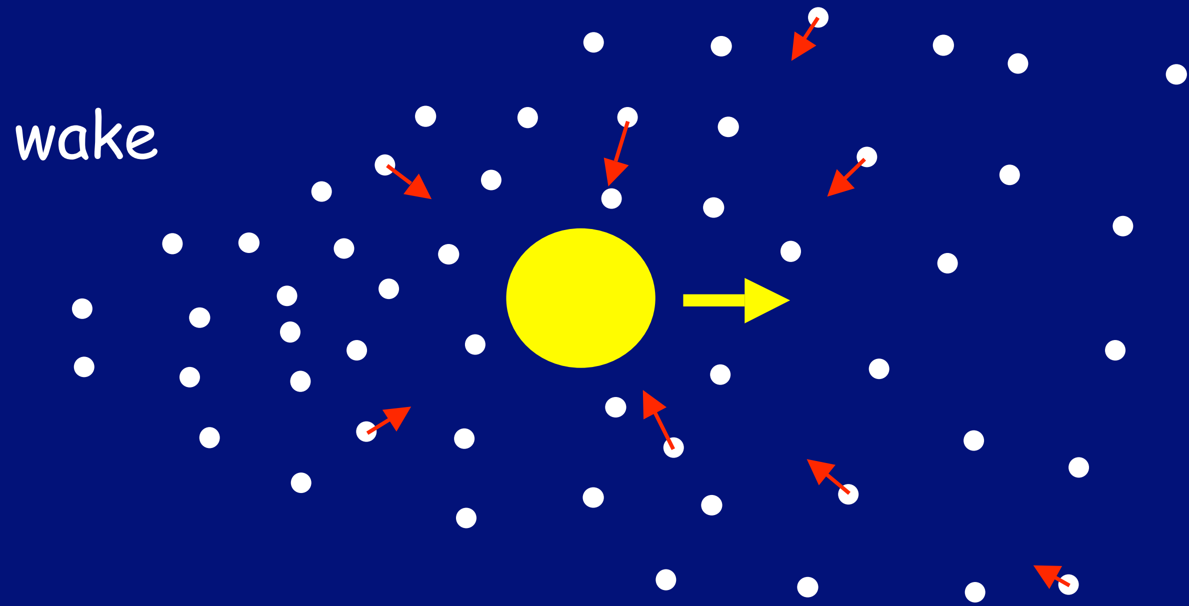
Dynamical Friction

Dynamical Friction



m<<M

Dynamical Friction



Dynamical Friction

Chandrasekhar formula:

$$\frac{d\vec{v}}{dt} = -4\pi G^2 \ln\Lambda \, \rho(<v)\, M_{sat} \frac{\vec{v}}{v^3}\left[erf(X) - \frac{2X}{\pi^{1/2}} e^{-X^2}\right] \qquad m << M_{sat}$$

$$X \equiv \frac{v}{\sqrt{2}\sigma}$$

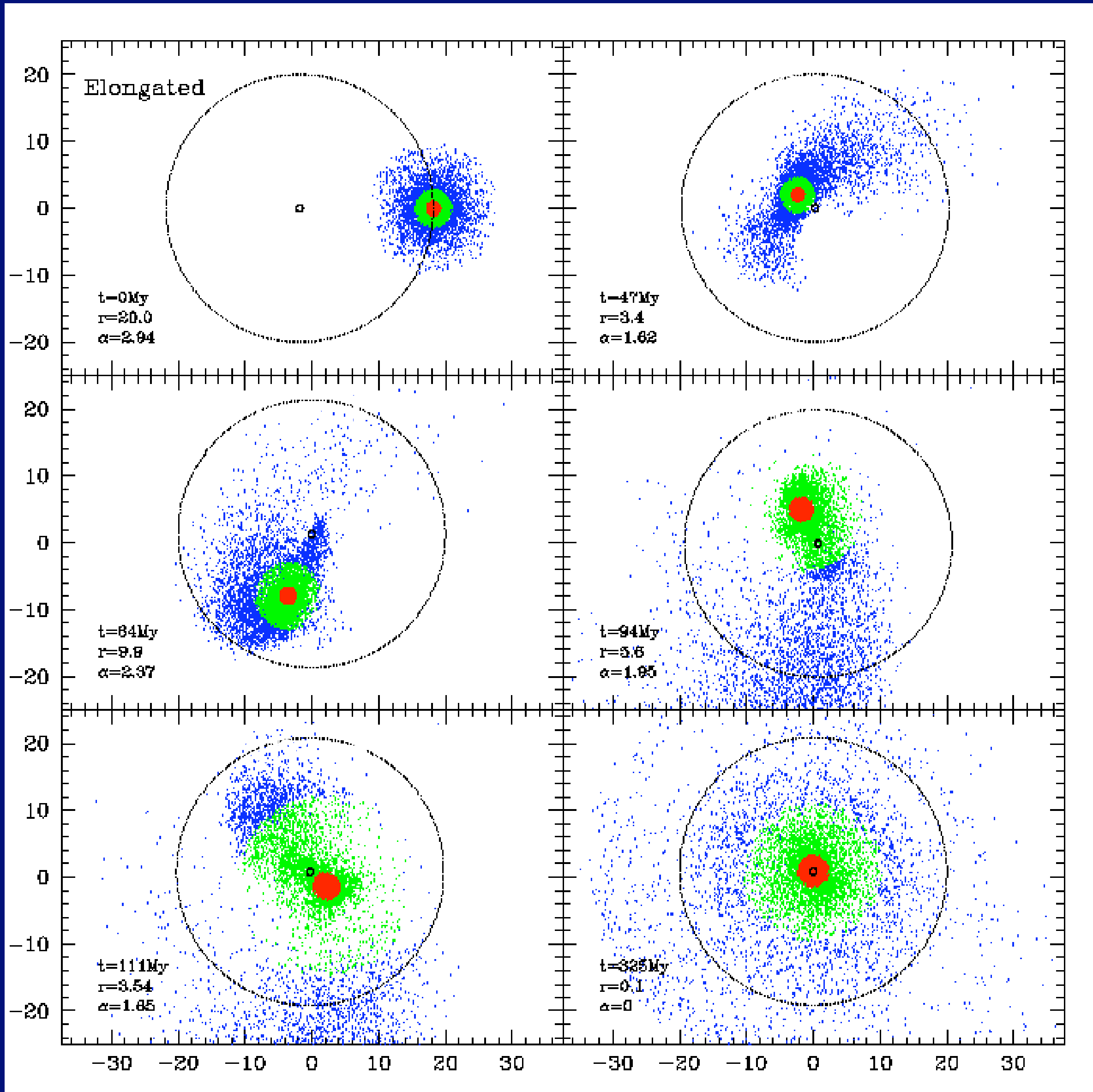
Coulomb logarithm:

$$\Lambda = \frac{b_{max} v_0^{\ 2}}{GM_{sat}} \approx \frac{M_{halo}}{M_{sat}}$$

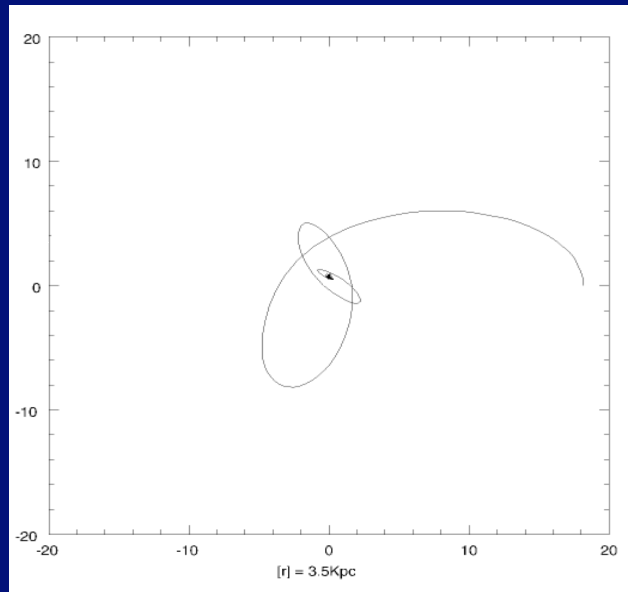
drag proportional to _ but independent of m

acceleration propto M (because wake density propto M)

Halo Bulidup by Mergers



tidal stripping & dynamical friction



Dekel, Devor & Hetzroni 2003

Tidal Effects

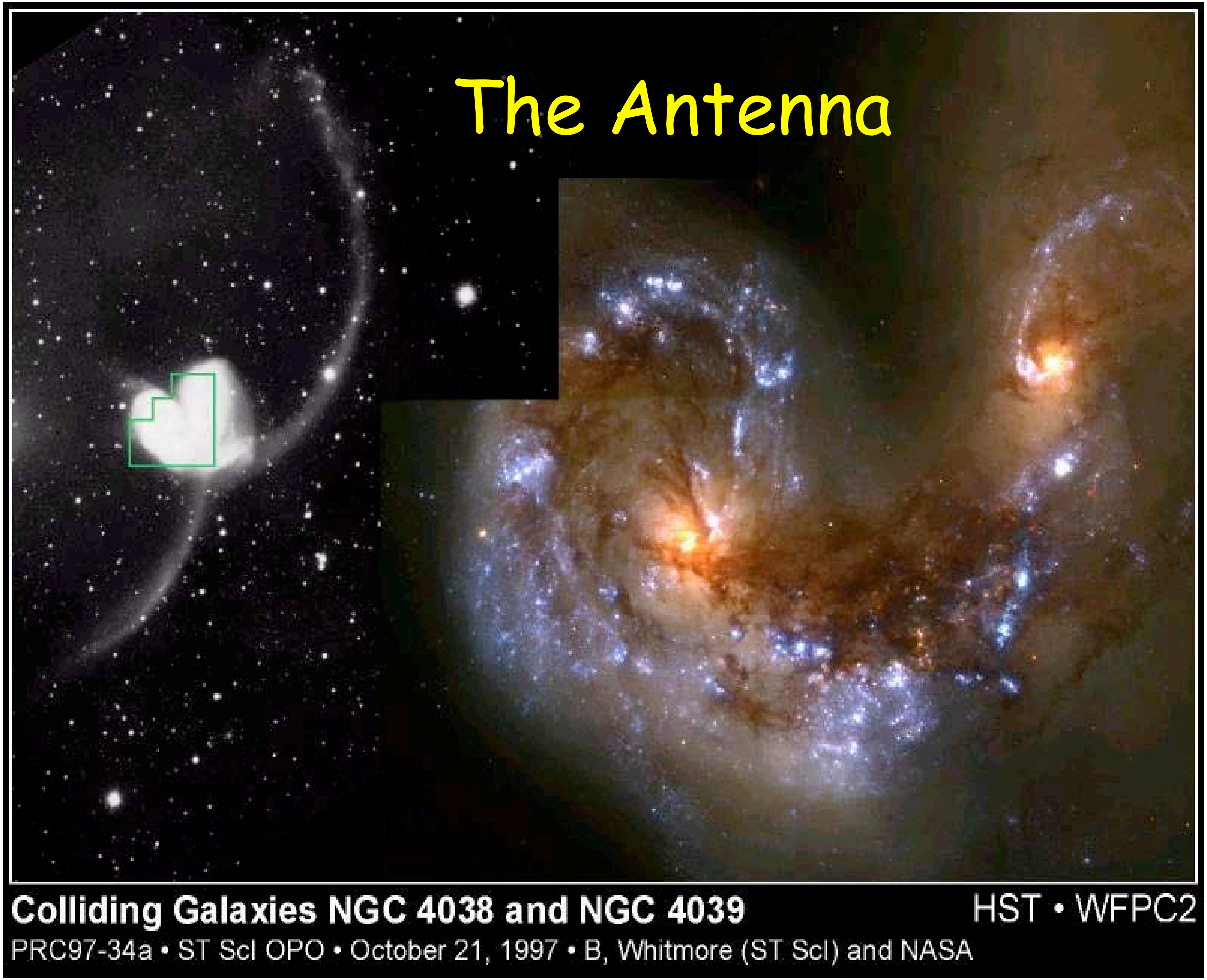
12-hour period

Tidal interaction & Merger

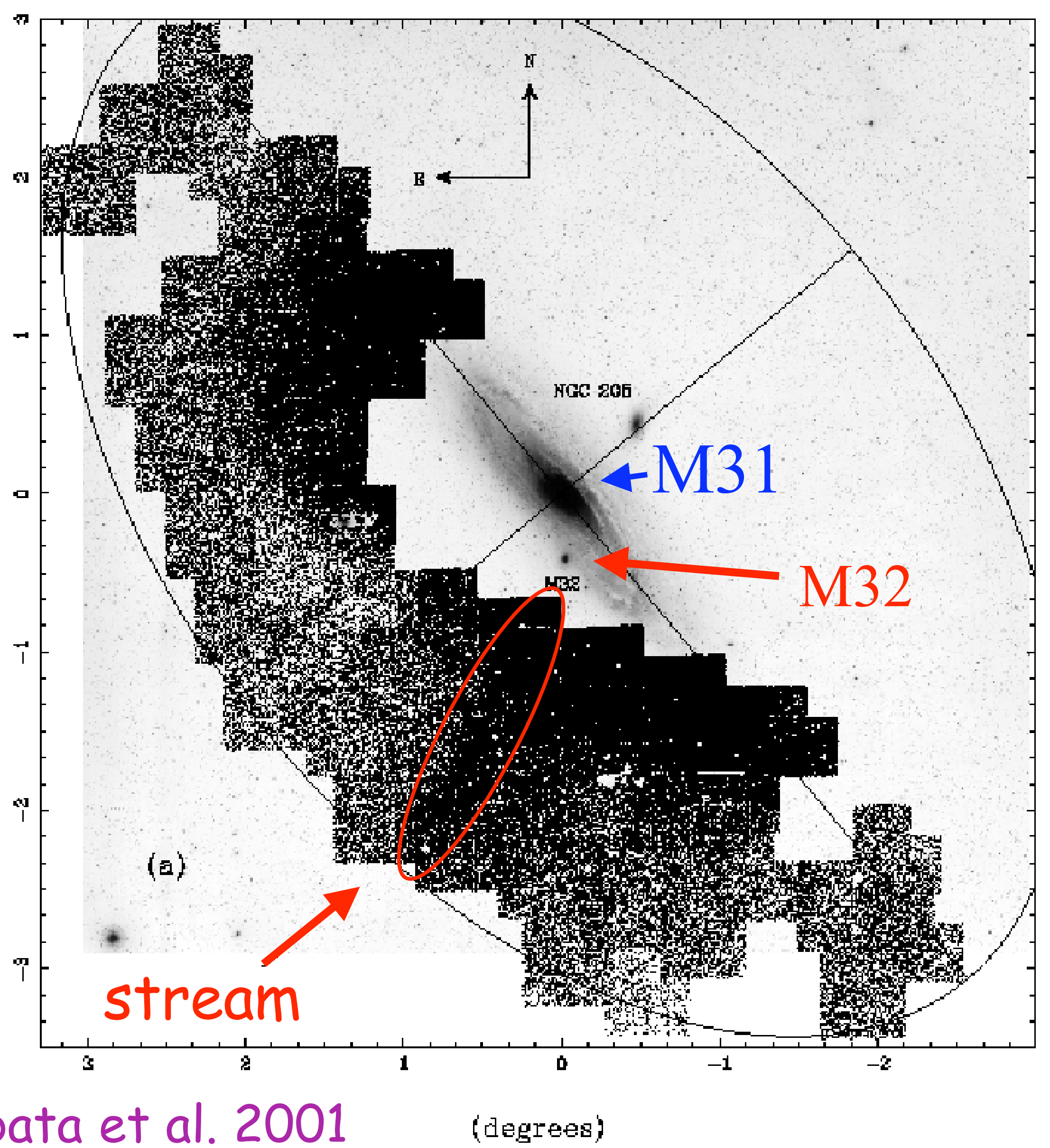
The Mice • Interacting Galaxies NGC 4676
Hubble Space Telescope • Advanced Camera for Surveys

NASA, H. Ford (JHU), G. Illingworth (UCSC/LO), M. Clampin (STScI), G. Hartig (STScI) and the ACS Science Team • STScI-PRC02-11d

The Antenna

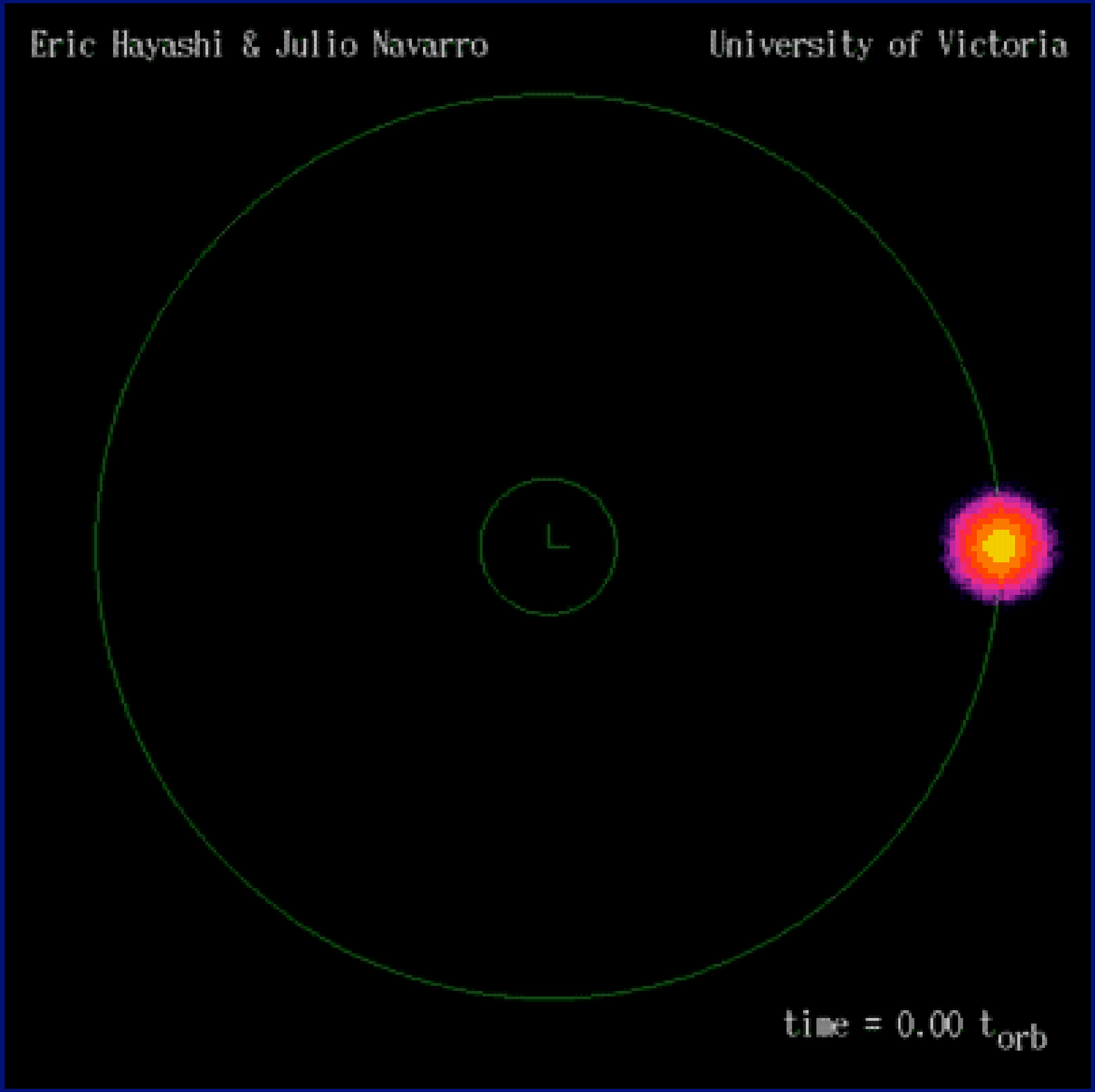

Colliding Galaxies NGC 4038 and NGC 4039
HST • WFPC2
PRC97-34a • ST ScI OPO • October 21, 1997 • B, Whitmore (ST ScI) and NASA

Tidal stripping of a satellite?

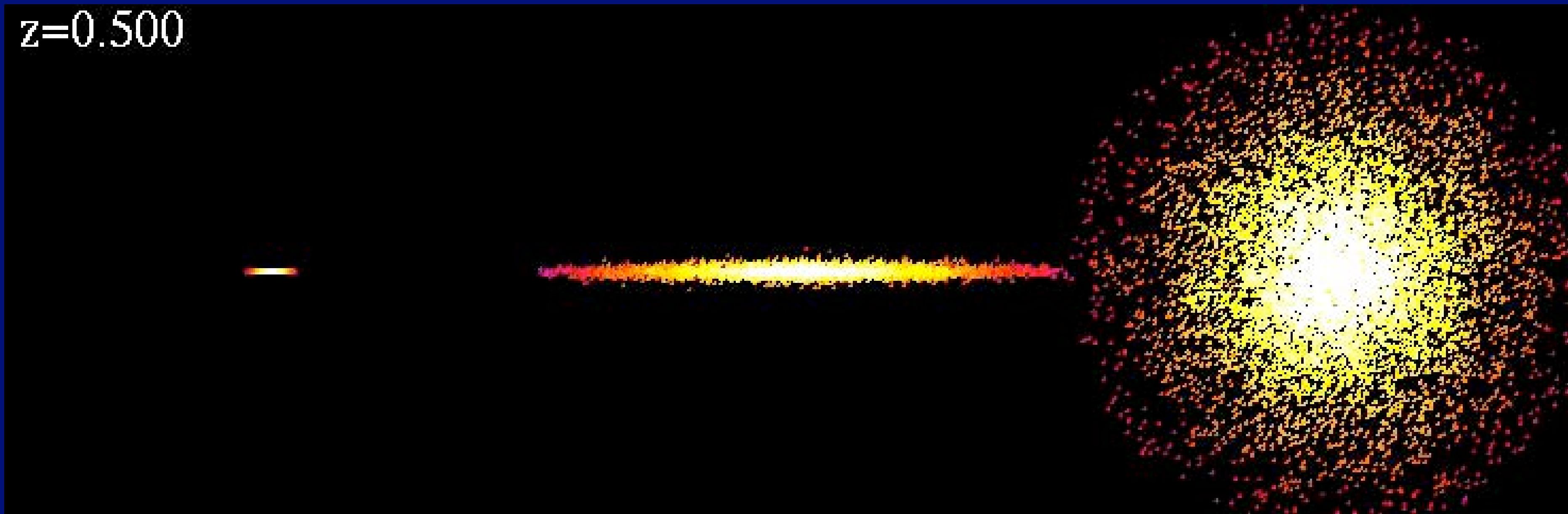


Ibata et al. 2001

The tidal disruption of an NFW Satellite halo

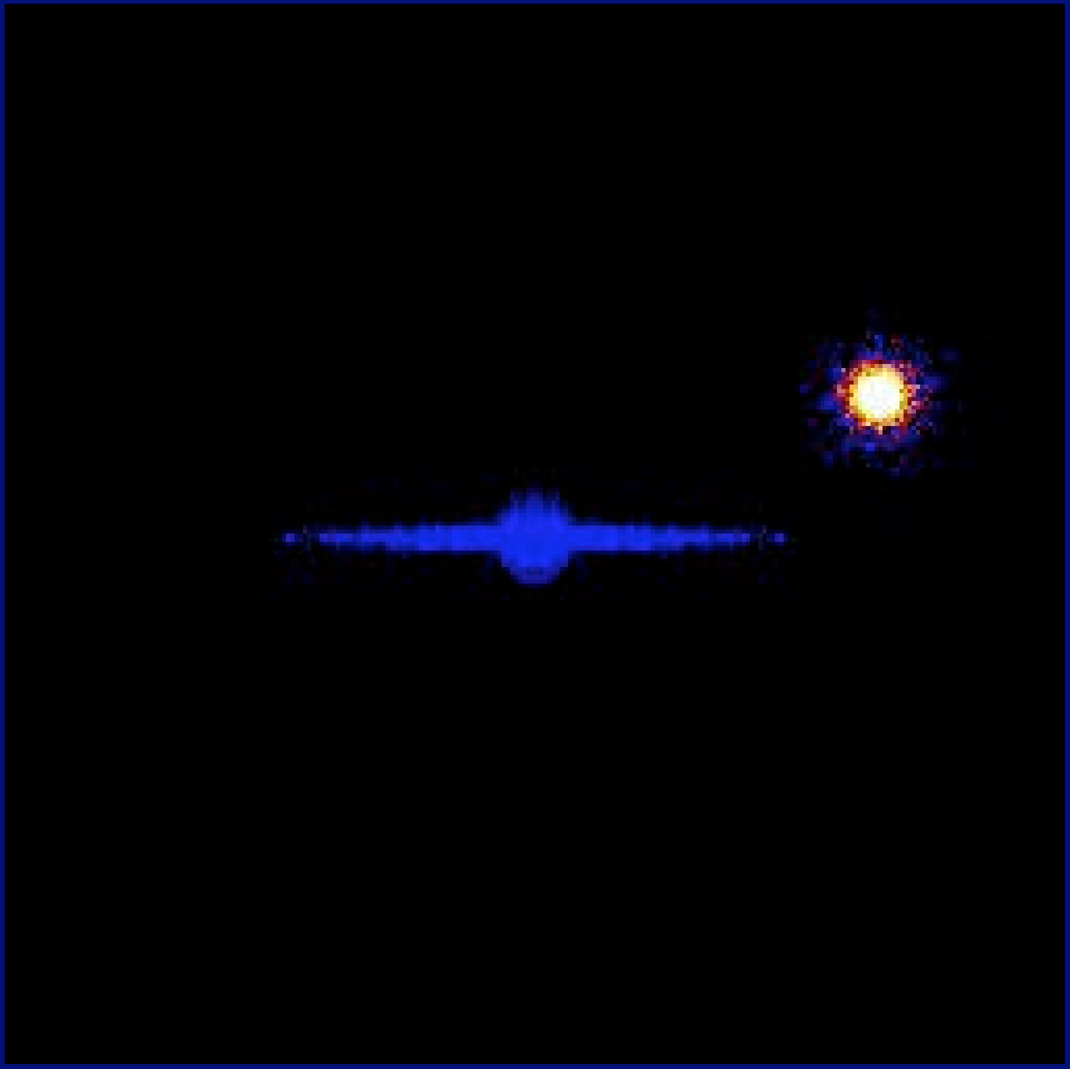


Harrasment of a satellite



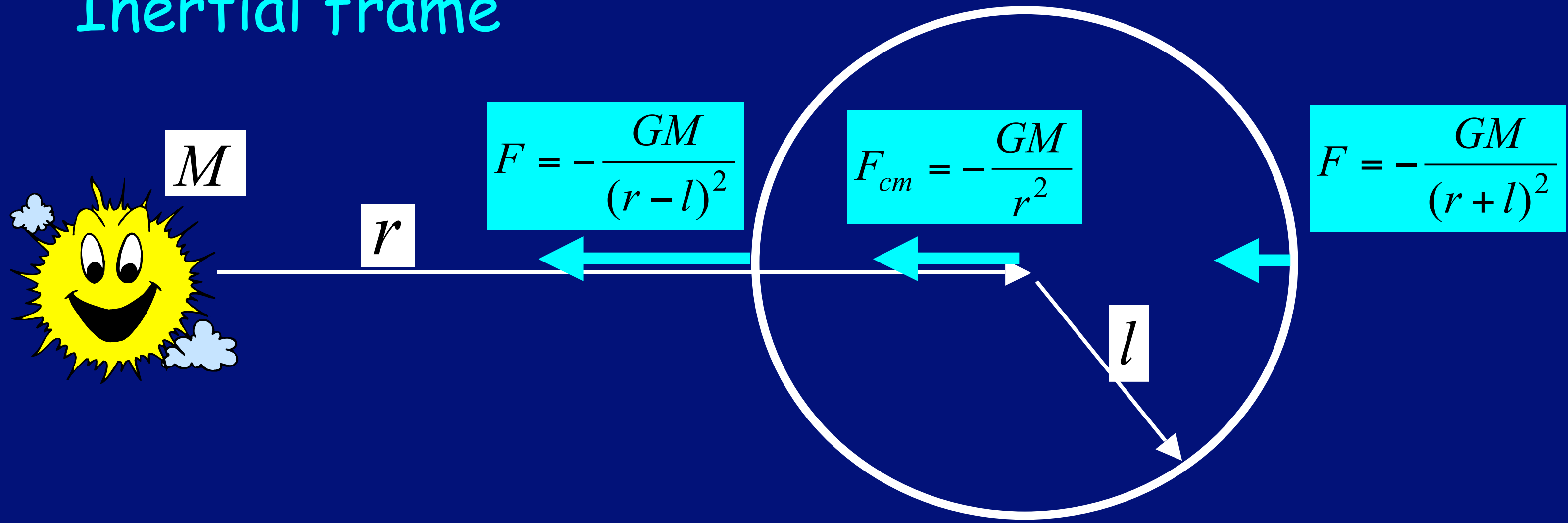
Moore et al.

Sagitarius Dwarf

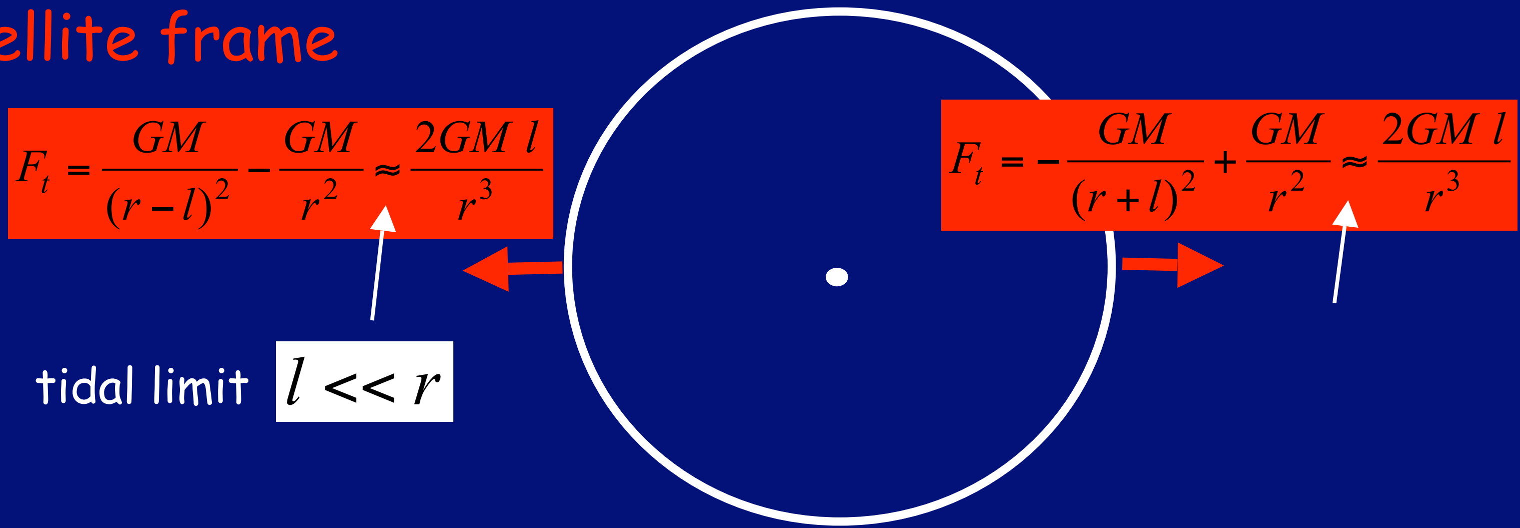


Tidal Force by a Point Mass

Inertial frame

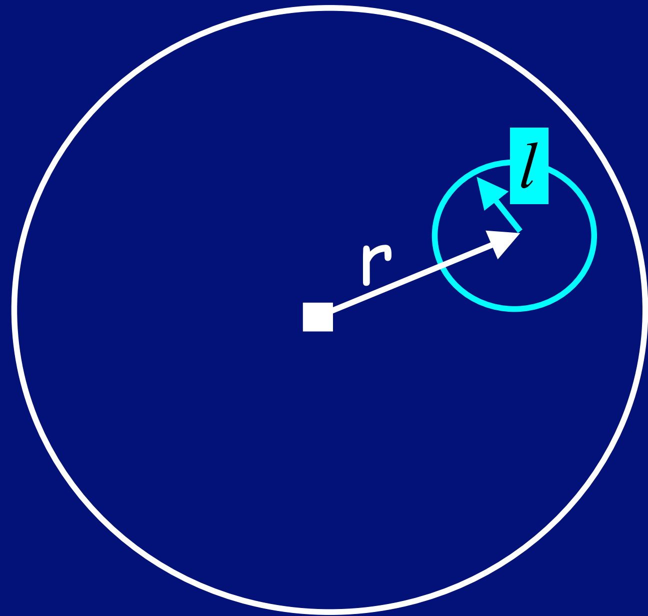


Satellite frame



Tidal Radius of a Satellite

self-gravity force $$\frac{Gm(l_t)}{l_t^2} = \frac{2GM(r)l_t}{r^3}$$ tidal force

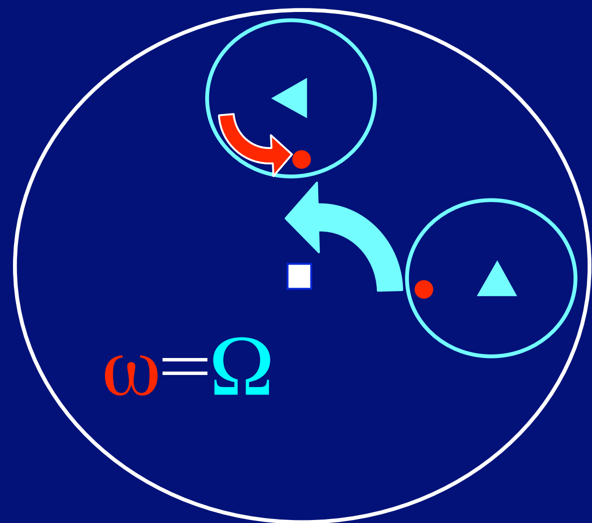


$$\rightarrow \quad \bar{\rho}_{sat}(l_t) \sim \frac{m(l_t)}{l_t^3} \sim \frac{M(r)}{r^3} \sim \bar{\rho}_{halo}(r)$$

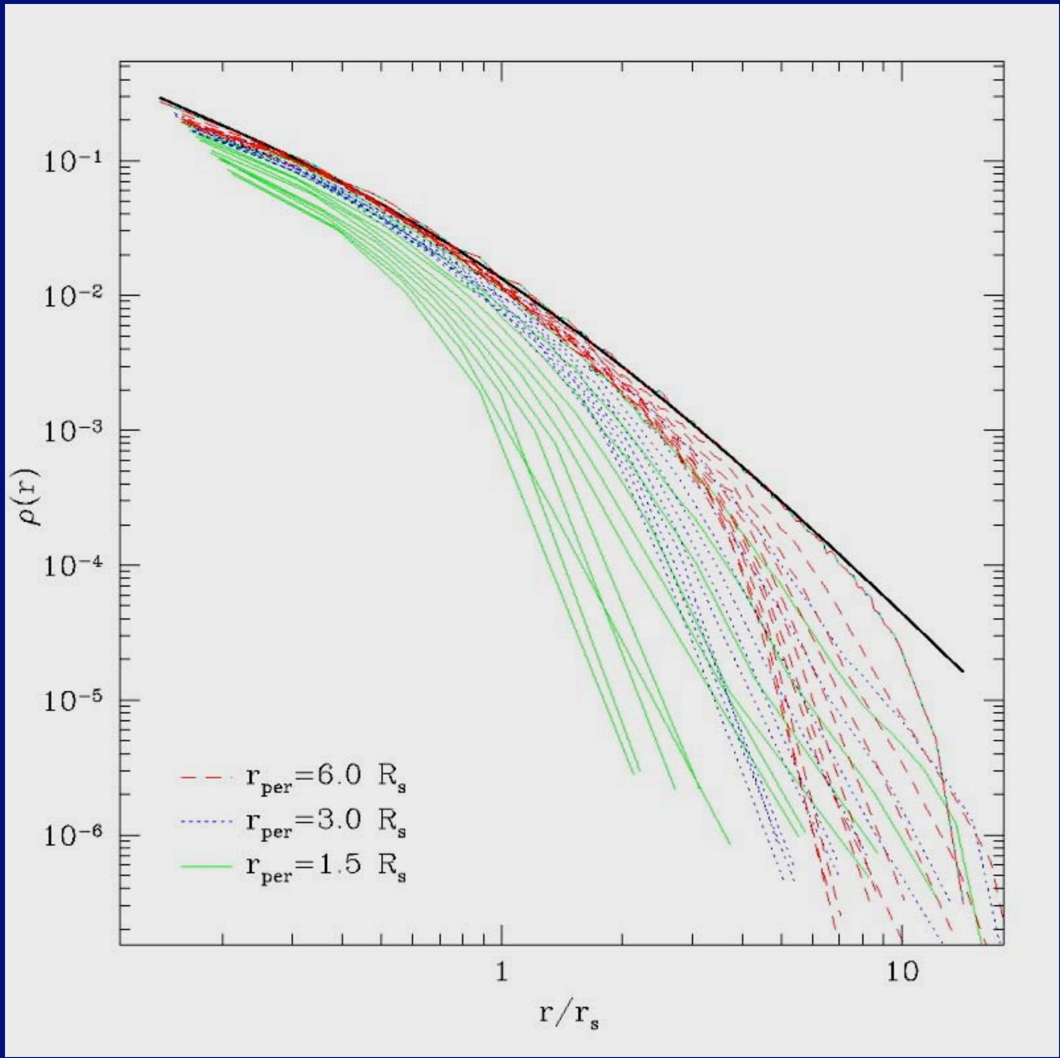
$$t \propto \frac{R}{V} \propto \frac{R}{(GM/R)^{1/2}} \propto \left(\frac{R^3}{M}\right)^{1/2} \propto \rho^{-1/2}$$

$$\rightarrow \quad t_{sat}(l_t) \sim t_{halo}(r)$$

resonance



Density Profiles of stripped NFW halos



Profiles of sub-halos
Stoehr et al 2004:

$$\log\left(\frac{V}{V_{\max}}\right) = -a\left[\log\left(\frac{r}{r_{\max}}\right)\right]^2$$

$$a \approx 0.45 \leftrightarrow \beta \approx 0.7$$

Origin of a cusp:
tidal effects in mergers

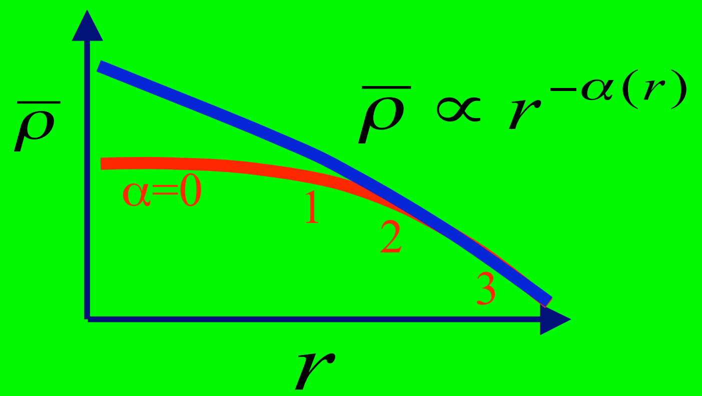
Dekel, Devor, Arad et al.

a. If satellites settle in halo core →
steepening to a cusp $\alpha \geq 1$

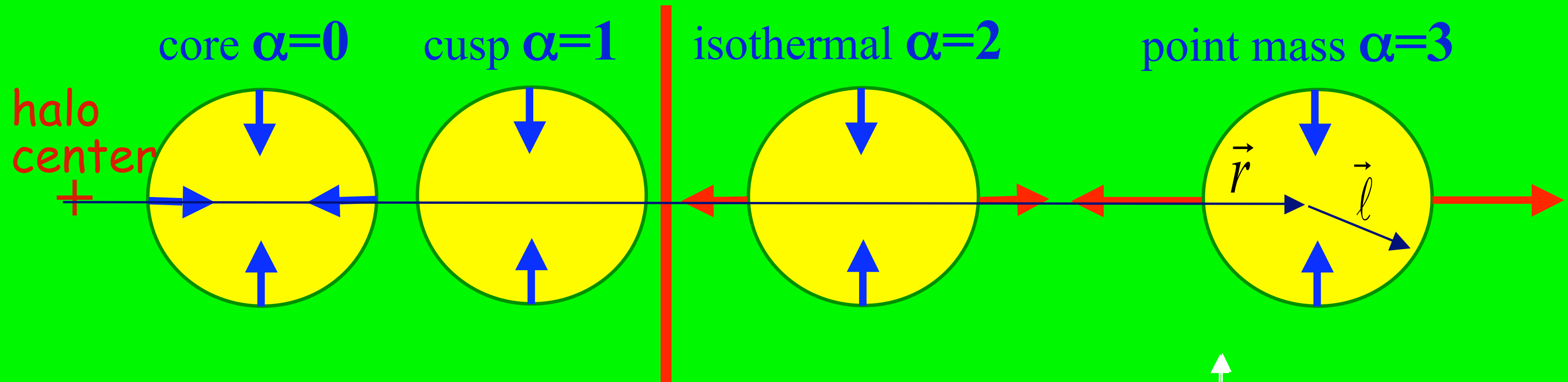
b. Mass-transfer recipe →
convergence to a universal slope $\alpha > 1$

c. Flat-density core? Only if satellites are
puffed up, e.g. by gas blowout

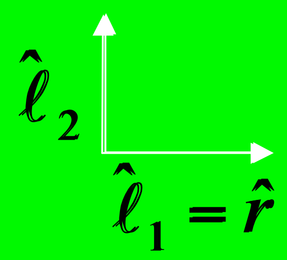
Tidal force on a satellite



$$\alpha(r) \equiv -\frac{d \ln \bar{\rho}(r)}{d \ln r}$$

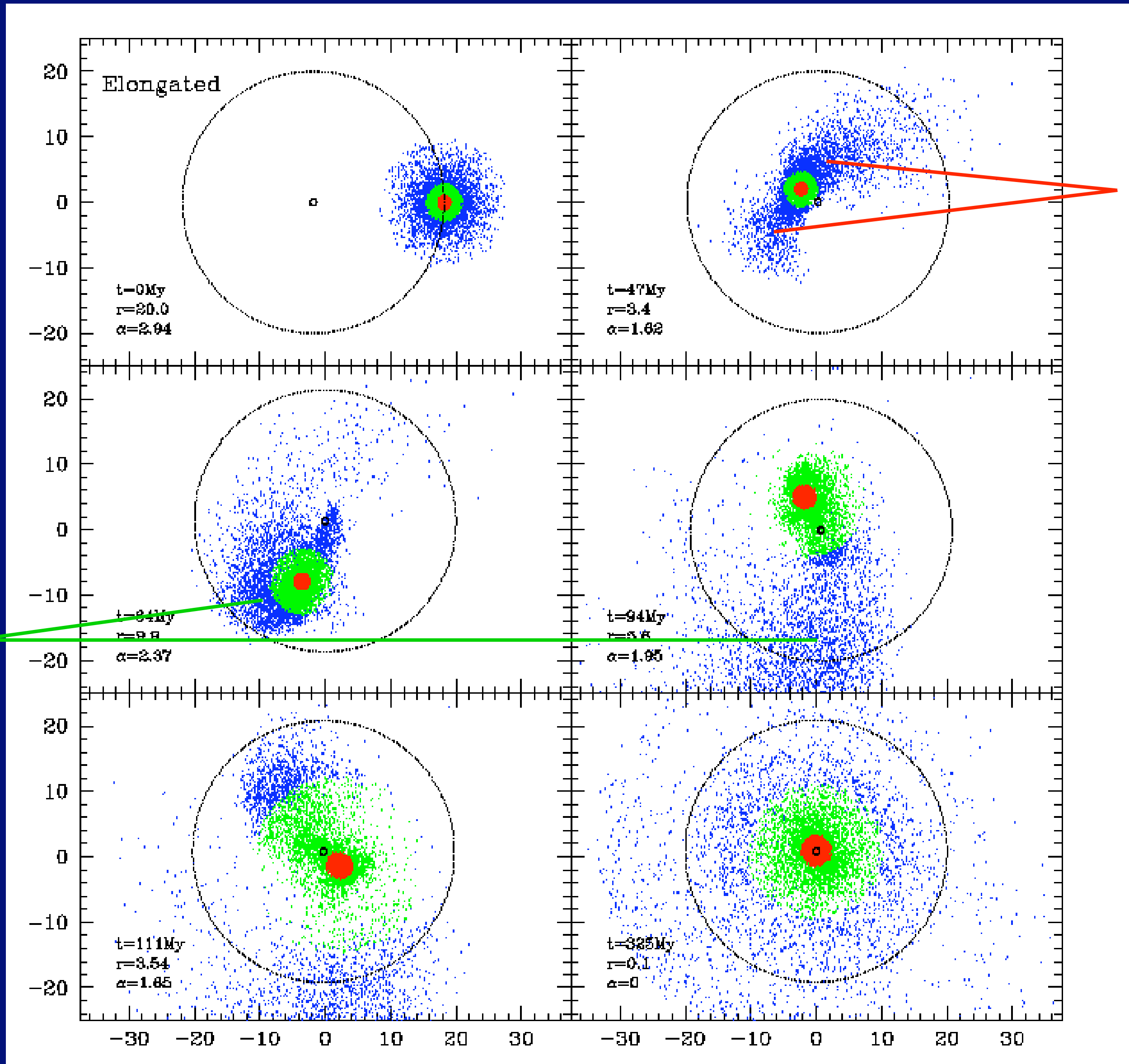


$$\vec{F}_{\text{tidal}} \propto \frac{GM(r)}{r^3} \left([\alpha(r) - 1]\, \vec{\ell}_1 - \vec{\ell}_2 - \vec{\ell}_3 \right)$$



→ no mass transfer where α<1

Impulsive stripping and deposit

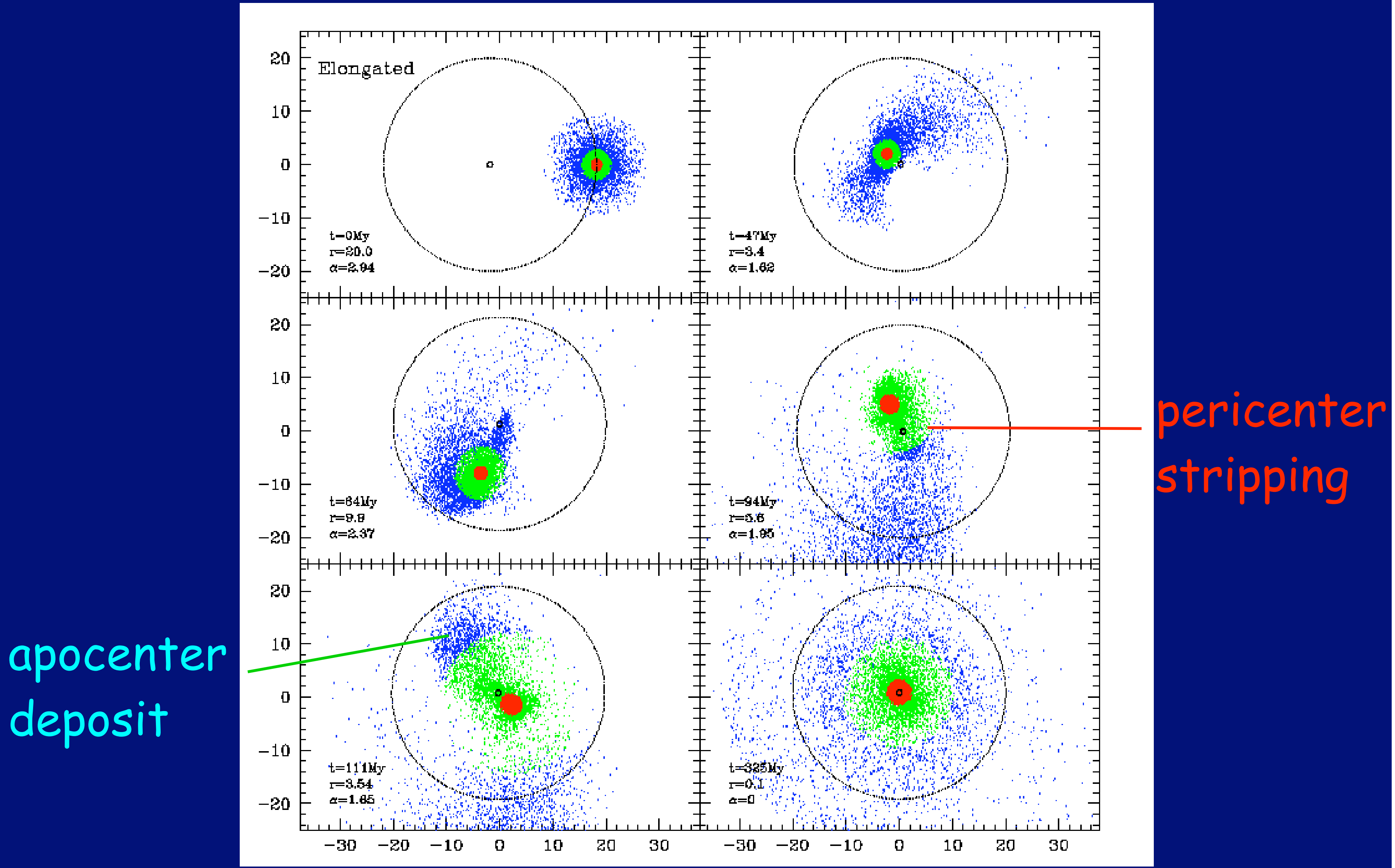


pericenter
stripping

apocenter
deposit

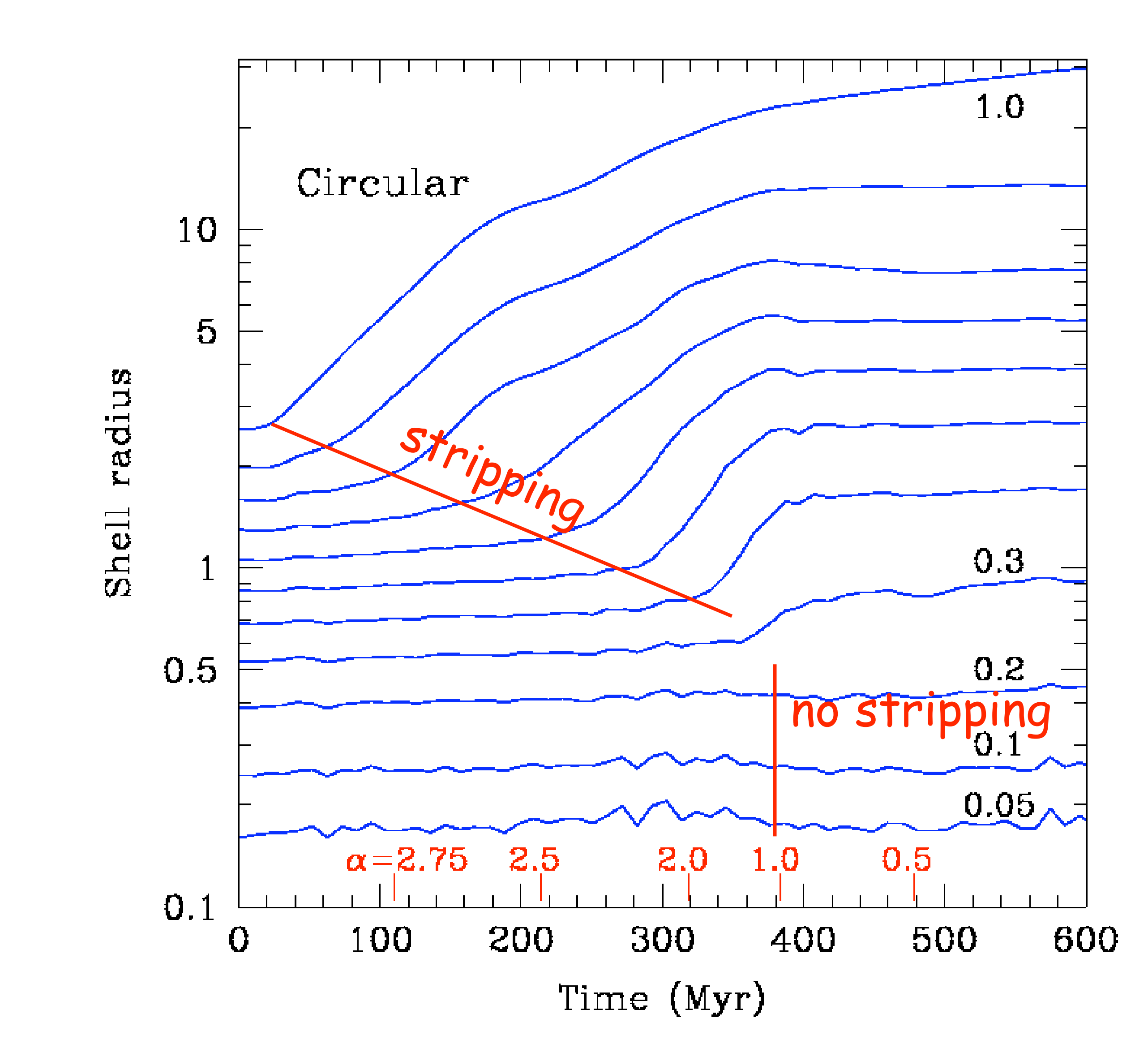
Dekel, Devor & Hetzroni 2003

Impulsive stripping and deposit



Dekel, Devor & Hetzroni 2003

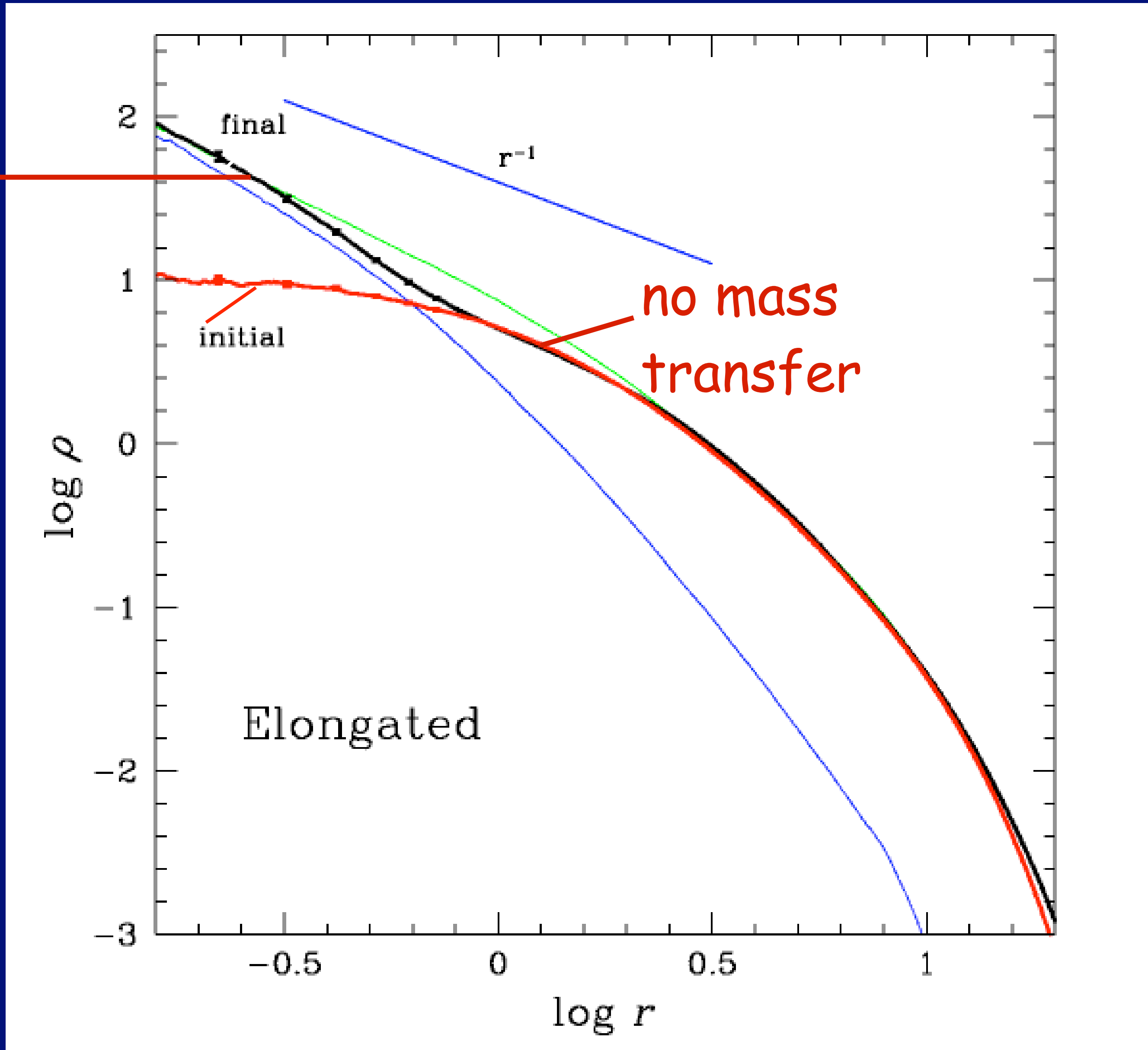
Adiabatic evolution of satellite profile



tidal compression in halo core

Merger of a compact satellite

satellite
decays intact
to halo center

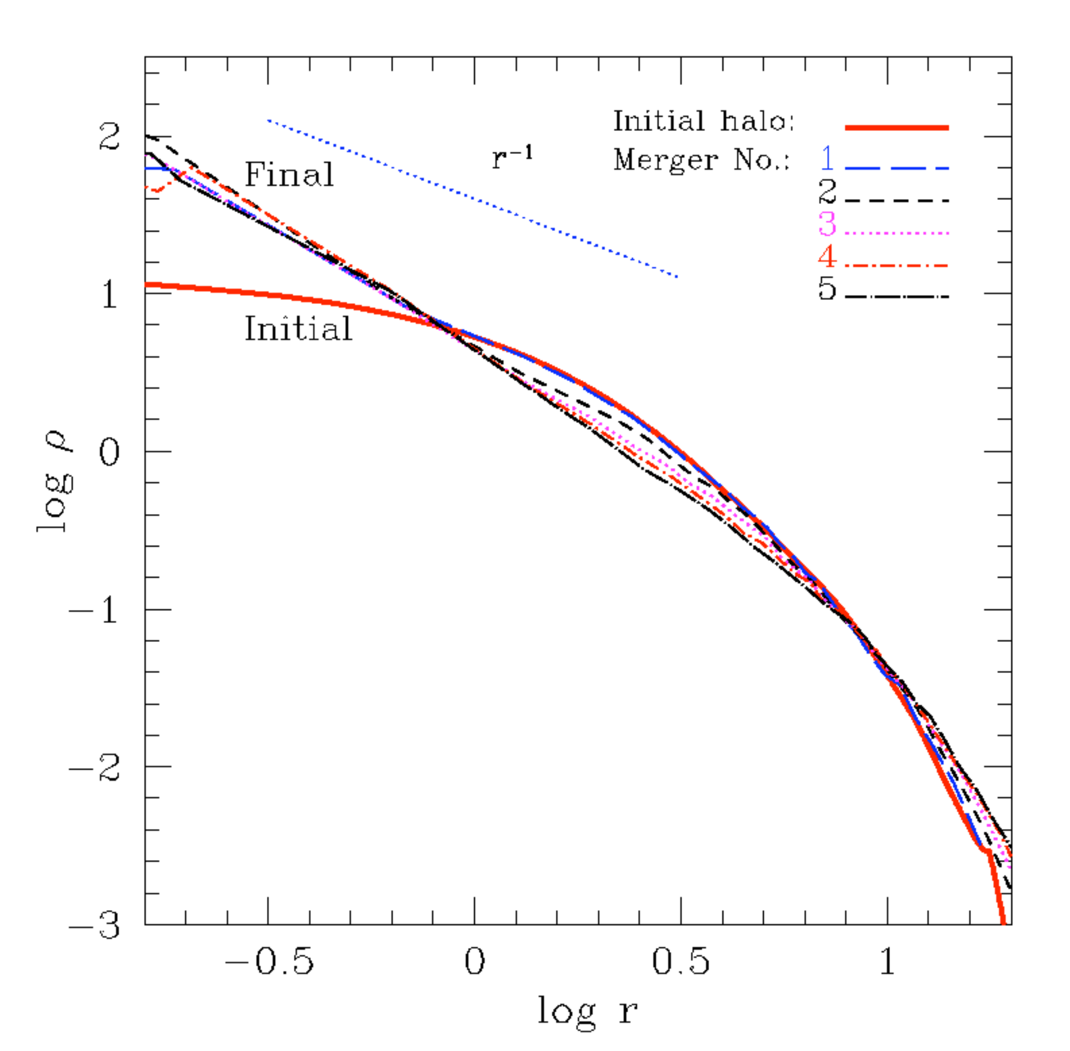


no mass
transfer

N-body
simulation

Dekel, Devor &
Hetzroni 03

Tandem mergers with compact satellites

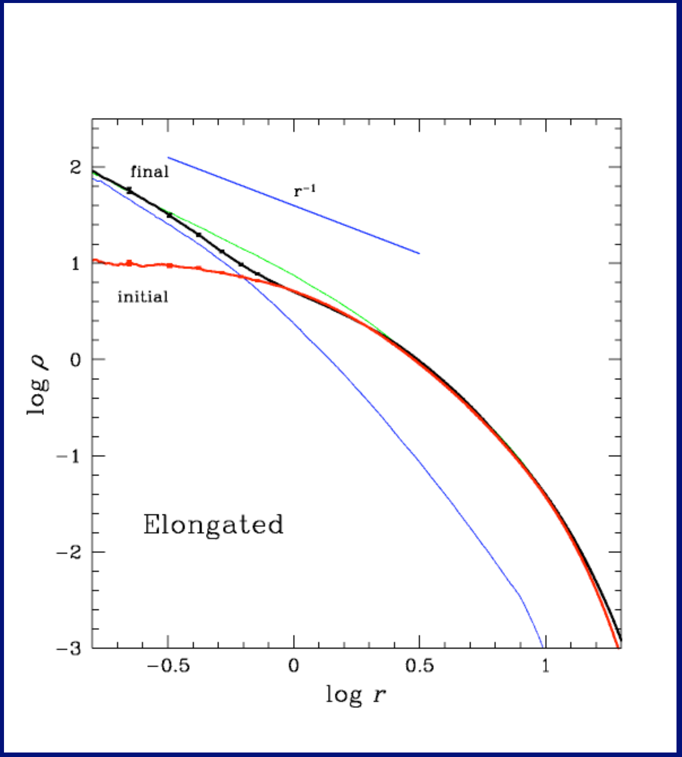


→ The cusp is stable!

Result:

No mass transfer in core →

rapid steepening to a cusp of $\alpha \geq 1$

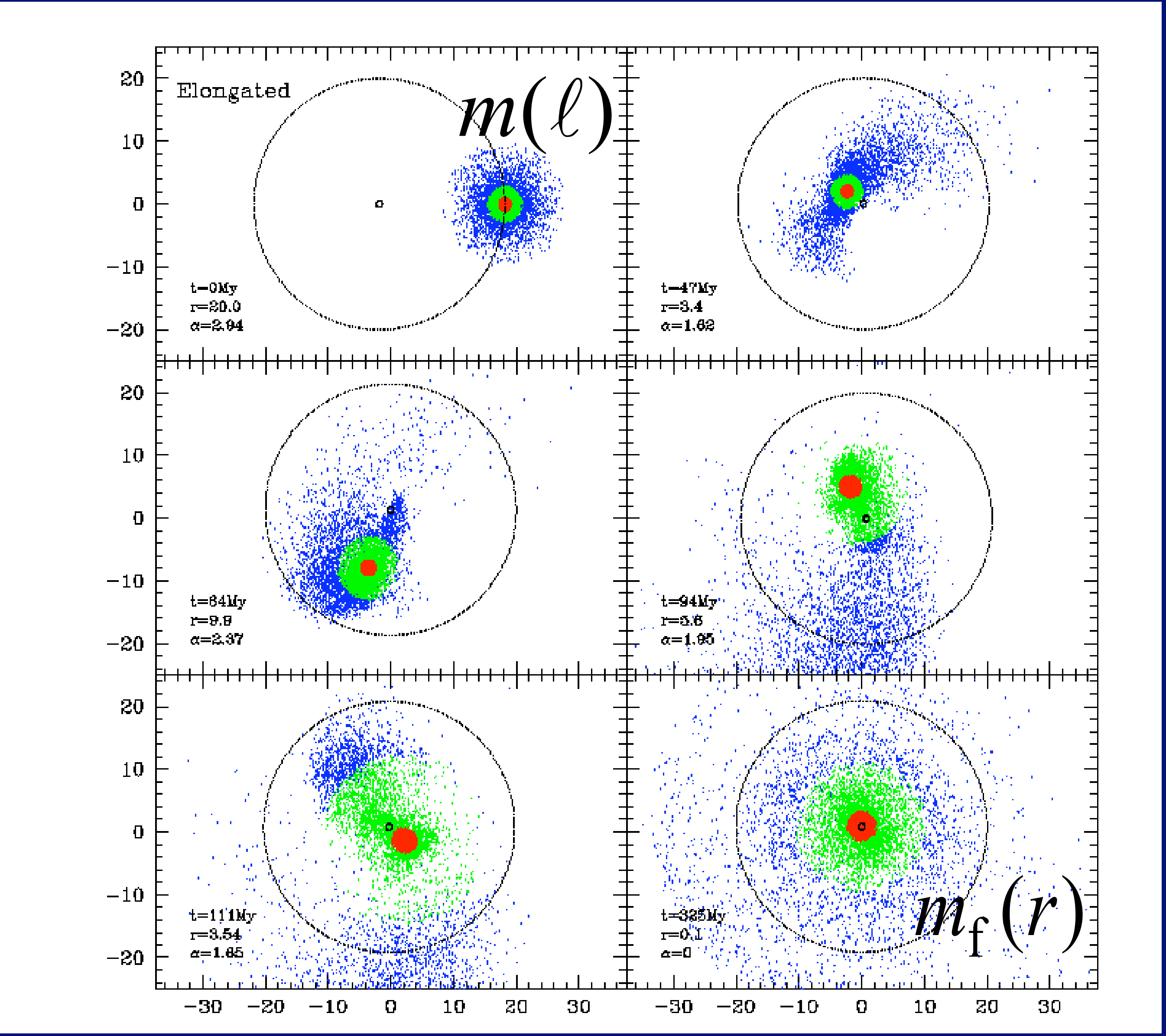


Tidal mass-transfer recipe at α>1

final — initial satellite profile

$$m_{\mathrm{f}}(r) = m(\ell) \quad \rightarrow \quad \ell(r)$$

Deposit radius



Dekel & Devor 2003

Tidal mass-transfer recipe at α>1

final — initial satellite profile

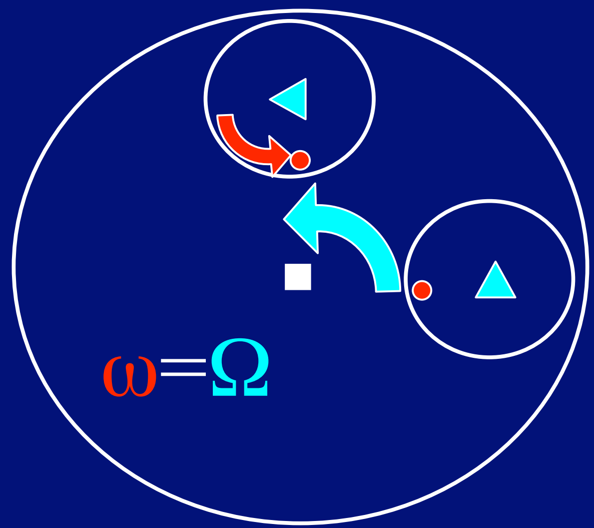
$$m_{\rm f}(r) = m(\ell) \;\longrightarrow\; \ell(r)$$

halo

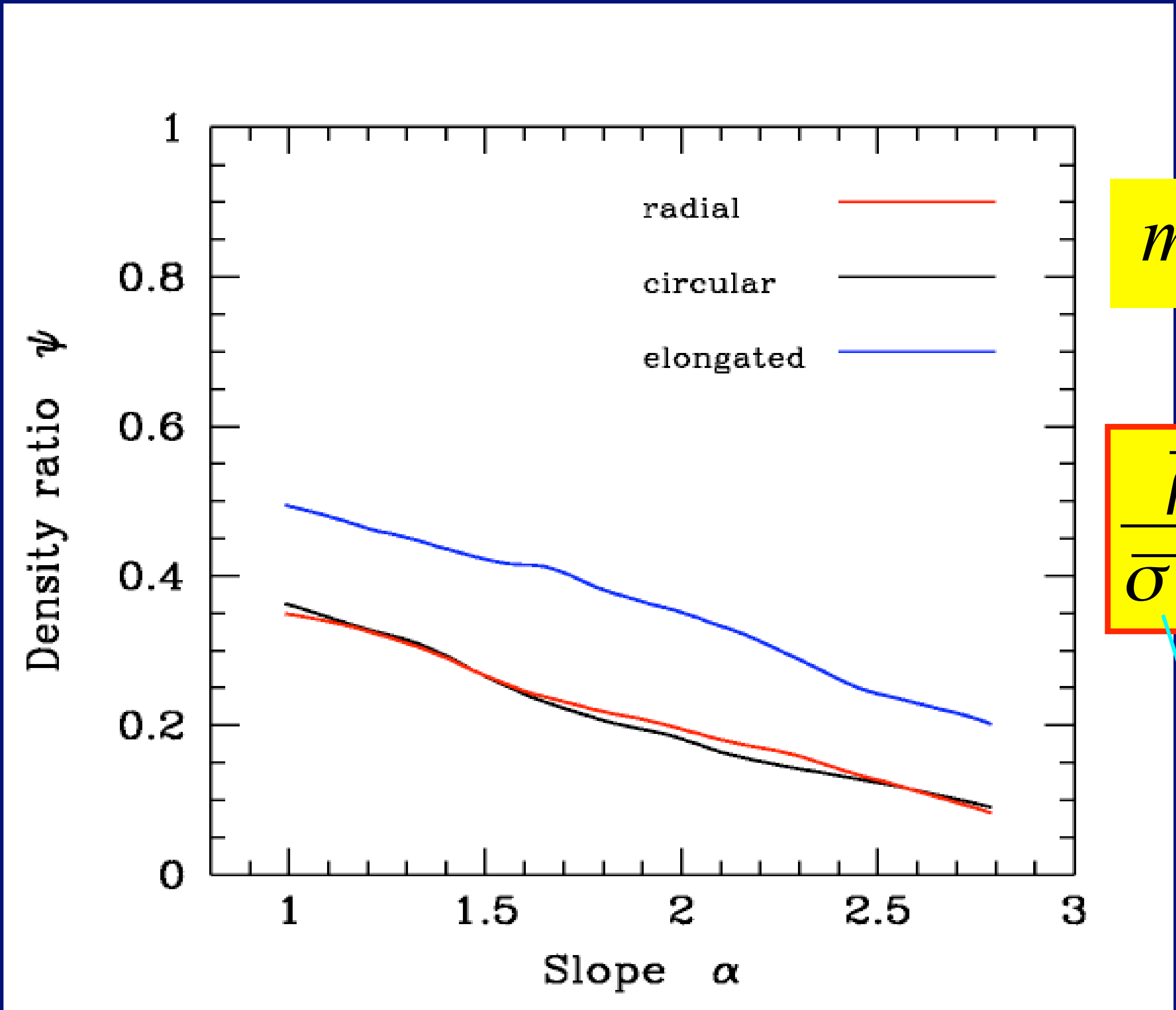
$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)]$$

initial satellite

=1? resonance



Tidal mass-transfer recipe at α>1



final — initial sat. profile

$$m_f(r) = m(\ell) \quad \rightarrow \quad \ell(r)$$

halo — satellite

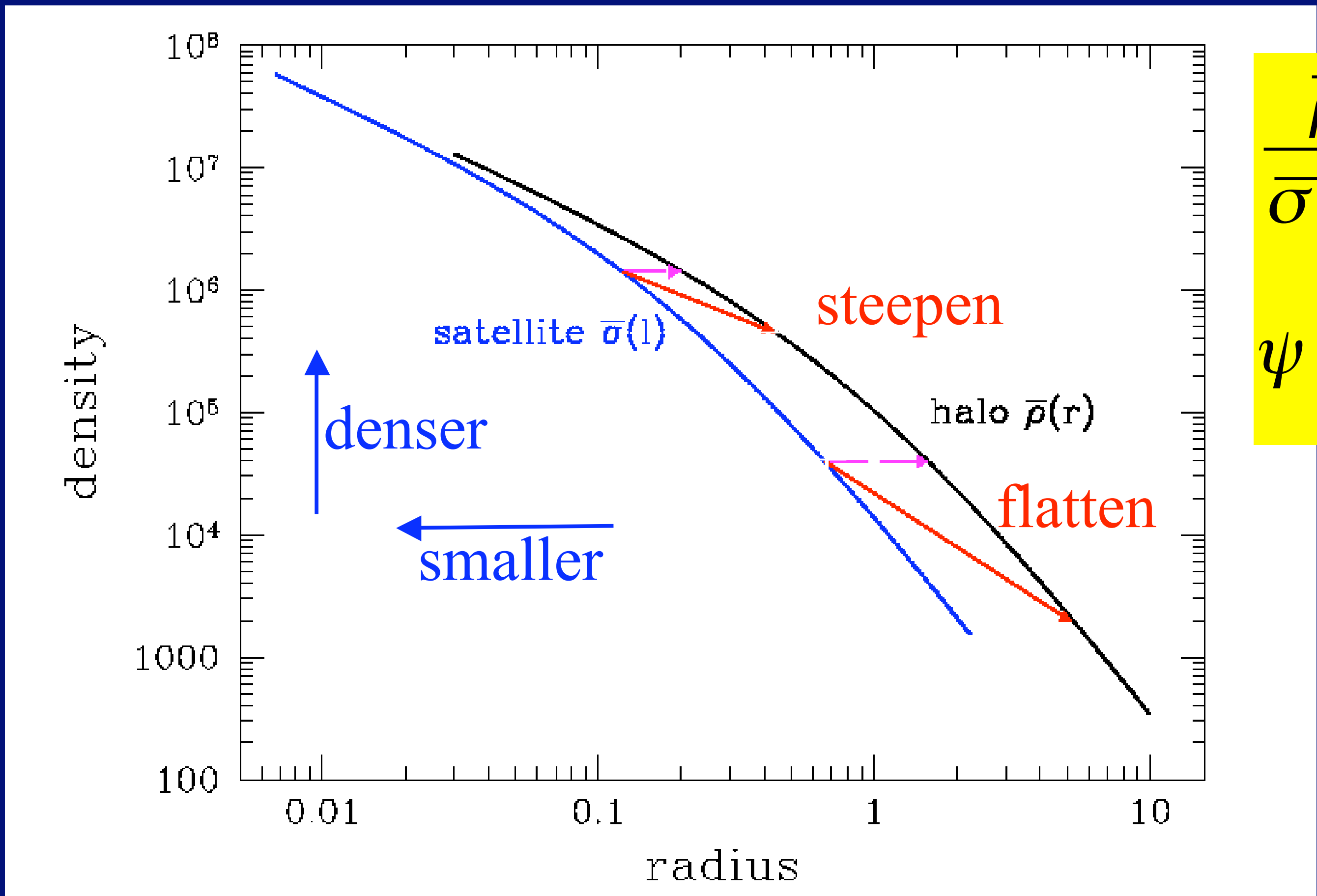
$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)] \quad \approx \frac{1}{2\alpha}$$

→ stripping efficiency grows with α

Steepening / flattening

homologous halo and satellite

scaling : $\rho_s \propto m^{-(3+n)/2} \quad r_s \propto m^{(5+n)/6}$



$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)]$$

$$\psi \approx \frac{1}{2\alpha}$$

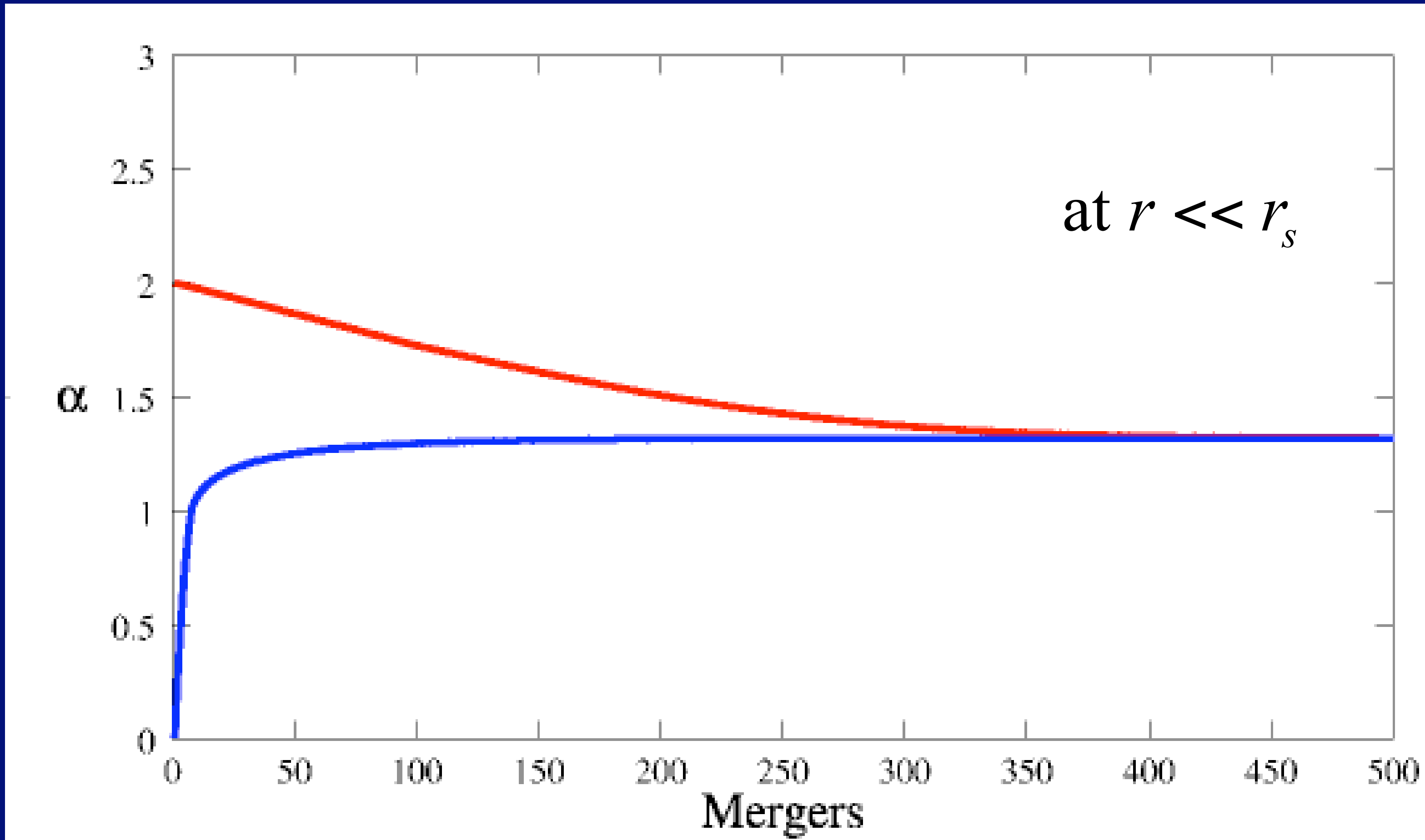
Adding satellite to halo profile

$$\bar{\rho}_{\rm new}(r) = \bar{\rho}_{\rm old}(r) + \bar{\sigma}(\ell)\frac{\ell^3}{r^3}$$

$$\Rightarrow \qquad \Delta\alpha(r) \propto -\frac{d}{dr}\left[\frac{\bar{\sigma}(\ell)}{\bar{\rho}(r)}\frac{\ell^3}{r^3}\right]$$

linear perturbation analysis $\Rightarrow \alpha \rightarrow \alpha_{\rm asymptotic}$

Convergence to an asymptotic slope



__1.3

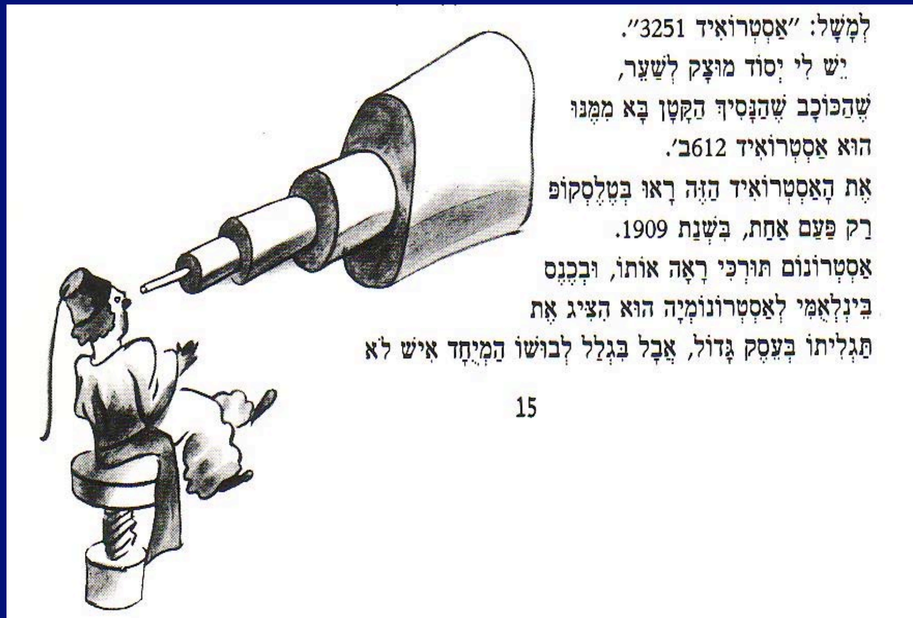
Dekel, Arad, Devor, Birnboim 03

Summary: Cusp

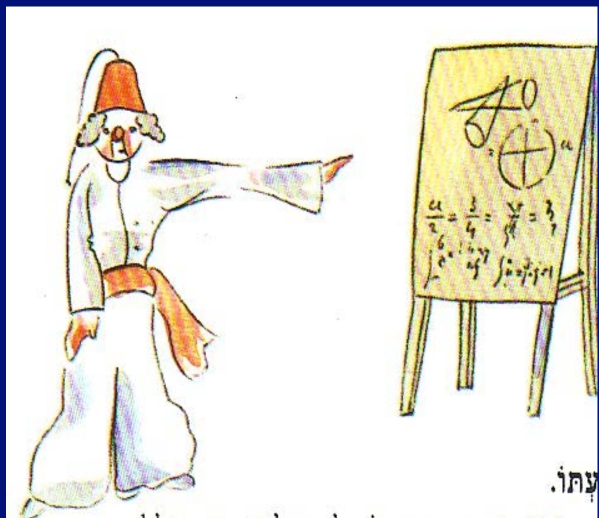
Dark-matter halos in CDM naturally form cusps due to merging compact satellites

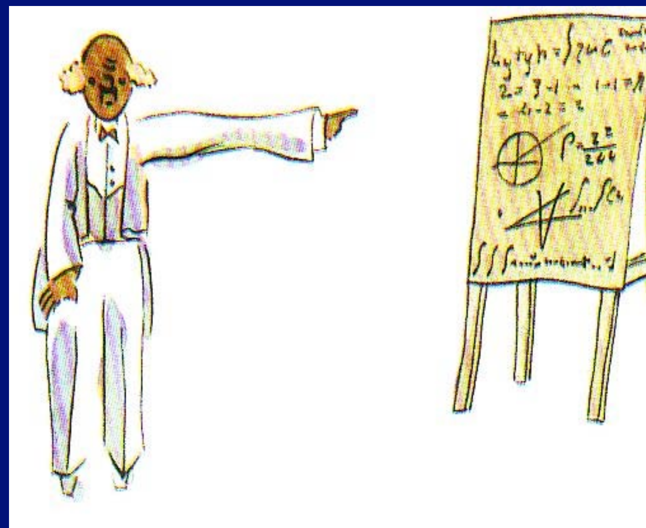
Observed Core

‰‡ÒË¯ÂÂÌ ‰˙Â¯ÎÈ (‰ÒÈÍ ‰˜ËÔ)

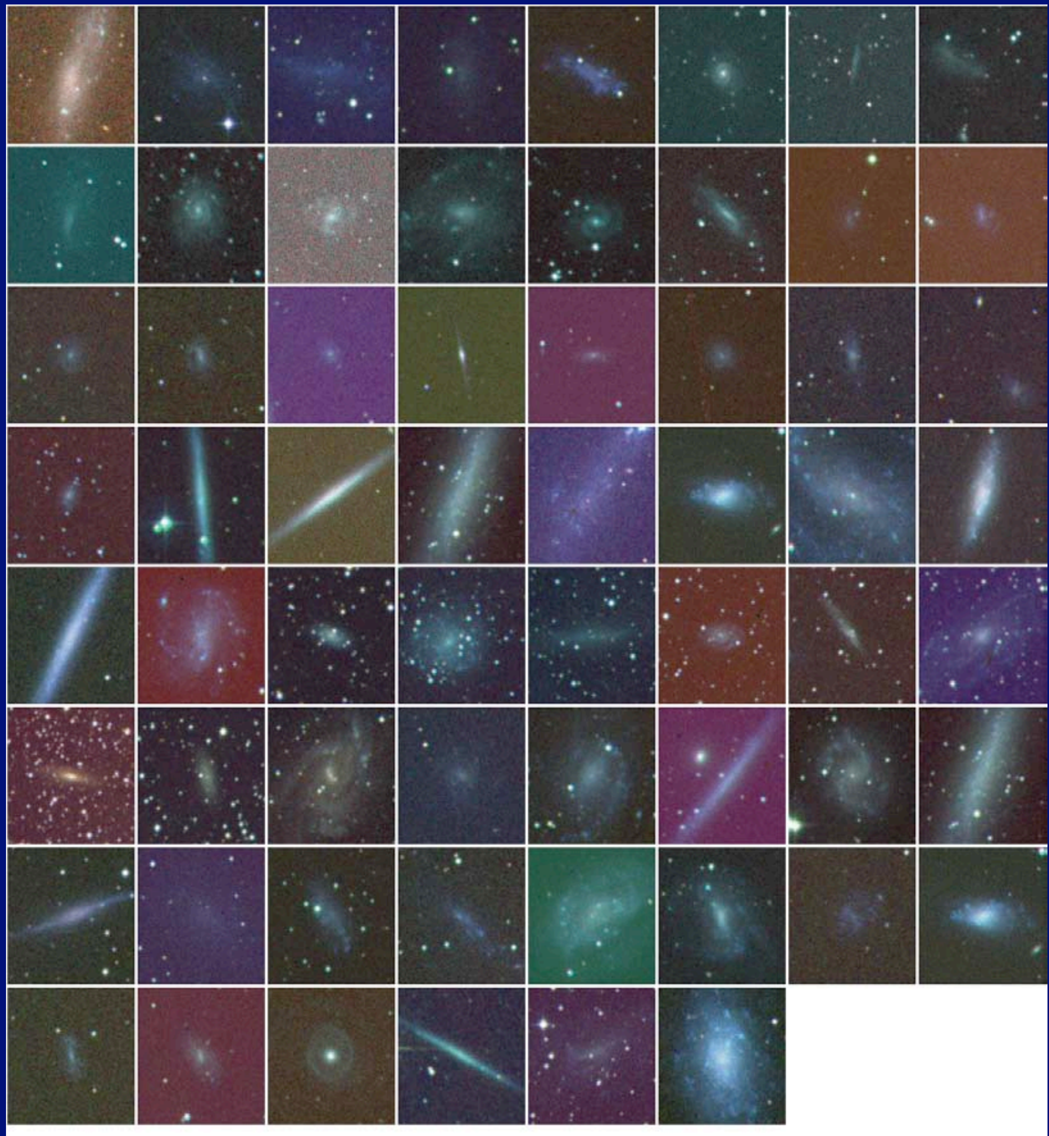


לְמָשָׁל: "אַסְטֵרוֹאִיד 3251".
יֵשׁ לִי יְסוֹד מוּצָק לְשַׁעֵר,
שֶׁהַכּוֹכָב שֶׁהַנָּסִיךְ הַקָּטָן בָּא מִמֶּנּוּ
הוּא אַסְטֵרוֹאִיד 612ב'.
אֶת הָאַסְטֵרוֹאִיד הַזֶּה רָאוּ בְּטֶלֶסְקוֹפּ
רַק פַּעַם אַחַת, בִּשְׁנַת 1909.
אַסְטְרוֹנוֹם תּוּרְכִּי רָאָה אוֹתוֹ, וּבְכֶנֶס
בֵּינְלְאֻמִּי לְאַסְטְרוֹנוֹמְיָה הוּא הִצִּיג אֶת
תַּגְלִיתוֹ בְּעֵסֶק גָּדוֹל, אֲבָל בִּגְלַל לְבוּשׁוֹ הַמְּיֻחָד אִישׁ לֹא
15





Low Surface Brightness Galaxies



Compare simulated $V_c(r)$ with rotation curves of dark-matter dominated LSB galaxies

Observations:

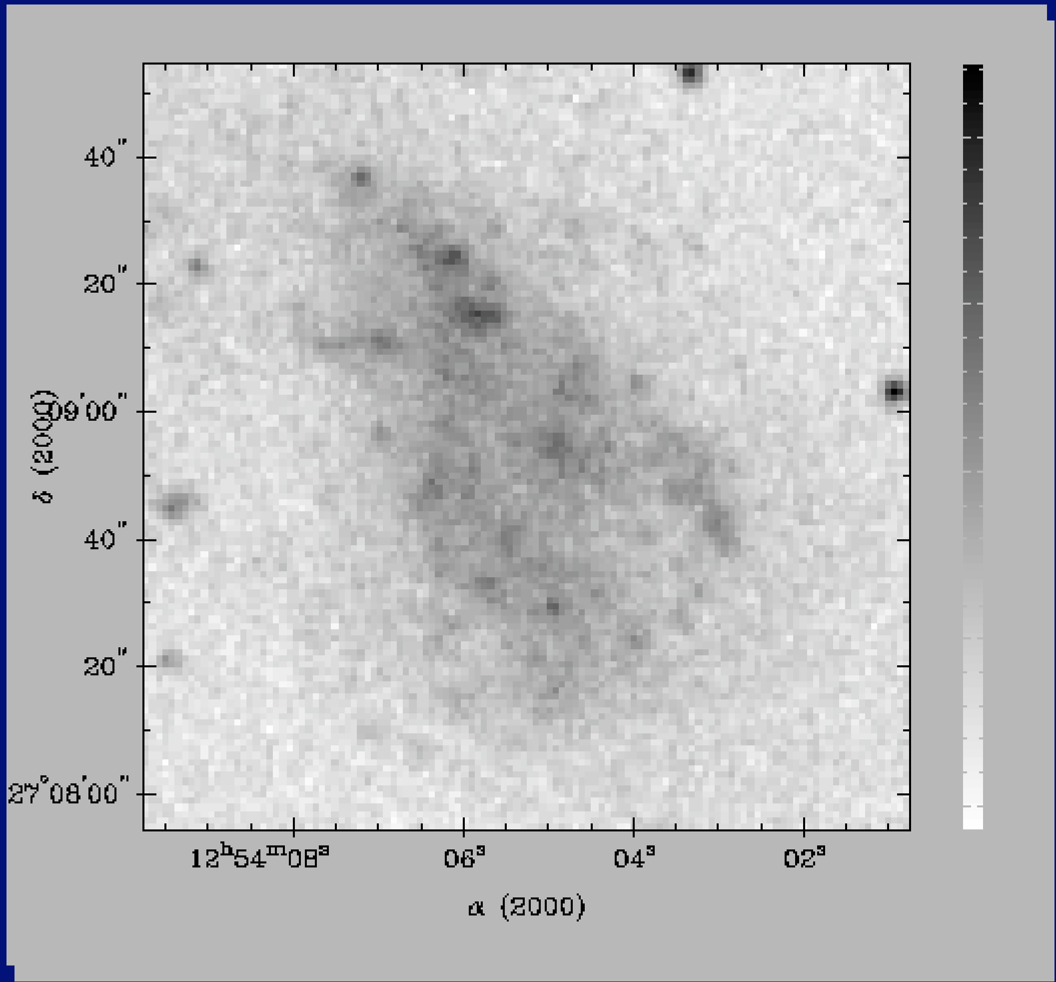
de Blok et al (2001) (B01),

de Blok & Bosma (2002) (B02),

and Swaters et al (2003) (S03)

Peak velocities range from 25 km/s to 270 km/s

These measurements are hard!



DDO154 (a dwarf LSB)

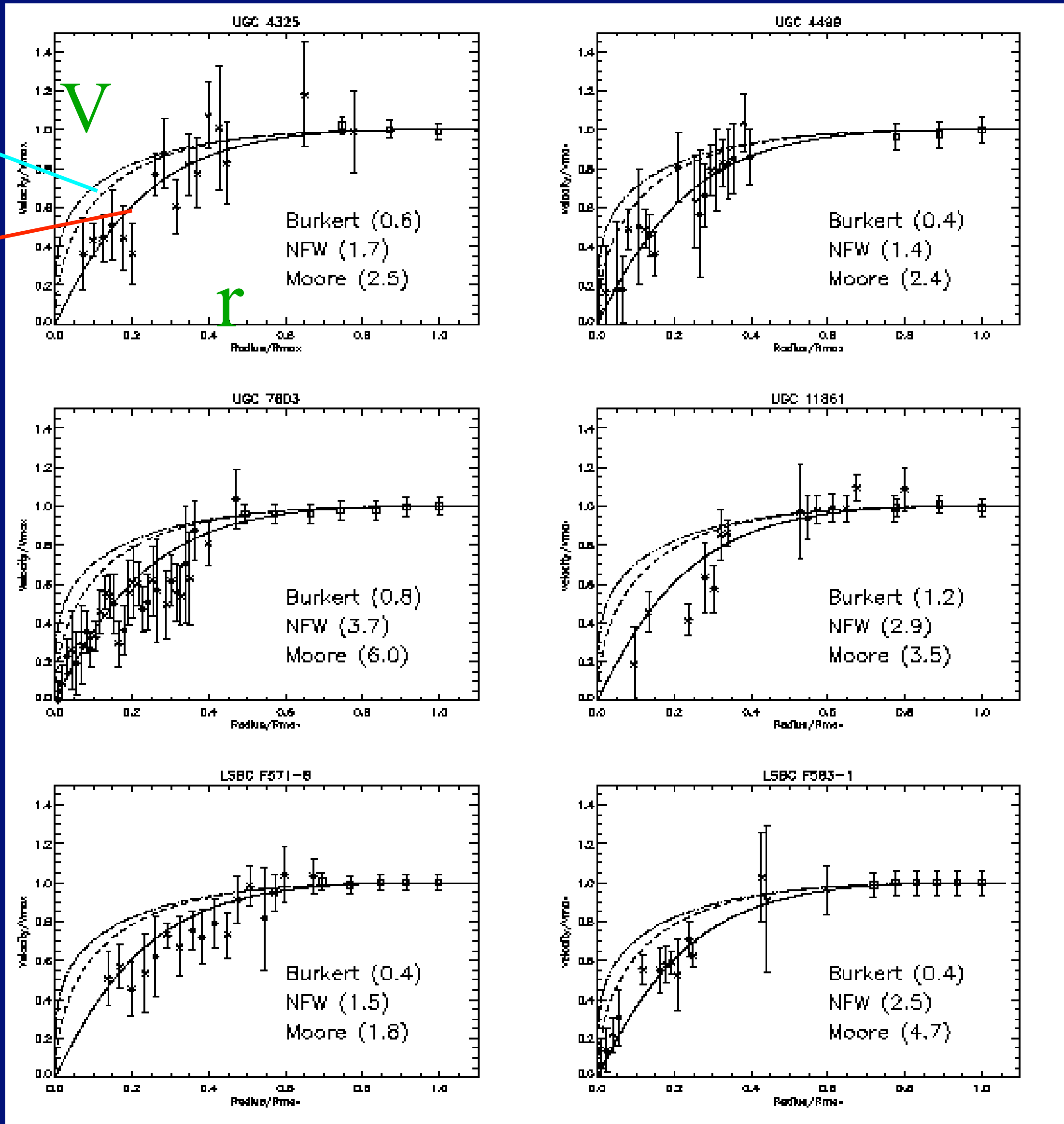
Observed cores vs. simulated cusps

cusp $\alpha \geq 1$

core $\alpha = 0$

$$V^2 = \frac{GM(r)}{r}$$

$$\rightarrow V \propto r^{1-\alpha/2}$$



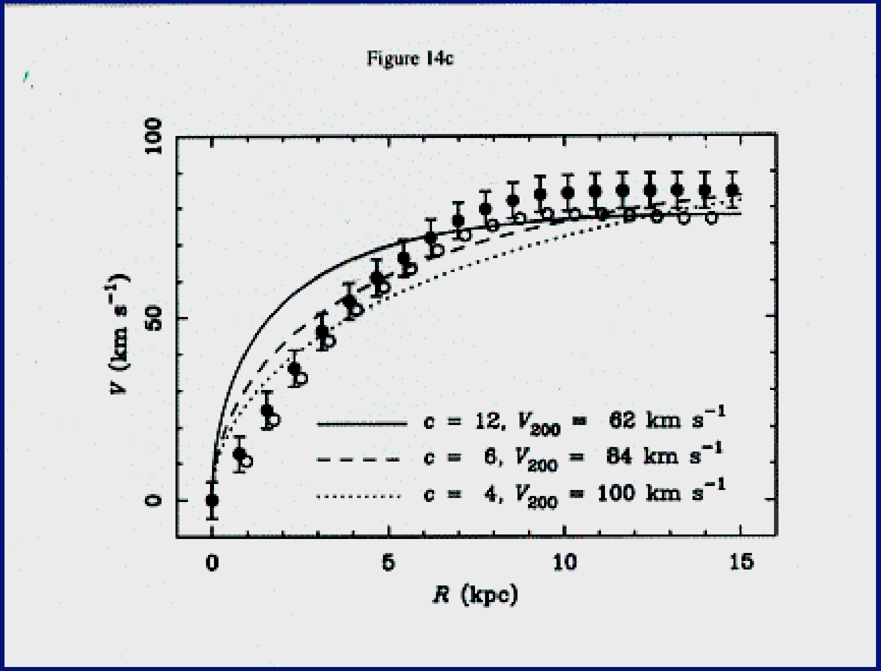
Marchesini, D'Onghia, et al.

LSB rotation curves and CDM halos

Two problems:

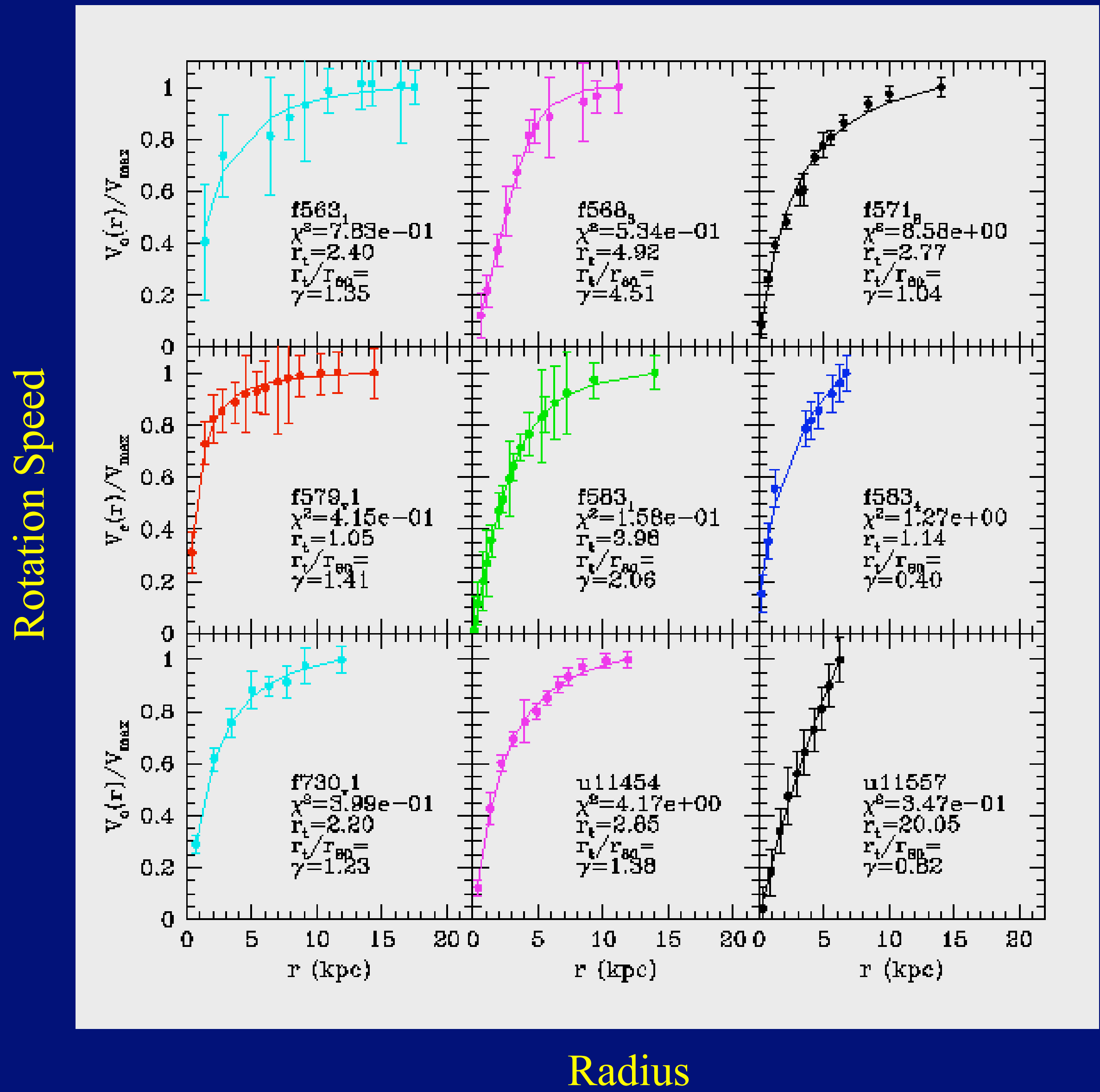
The shape of LSB galaxy rotation curves is inconsistent with the circular velocity curves of CDM halos.

The concentration of dark matter halos is inconsistent with rotation curve data: there is too much dark matter in the inner regions of LSB galaxies.



McGaugh & de Block 1998
see also Moore 1994
Flores & Primack 1994

LSB rotation curves (McGaugh et al sample)



The shape of V(r) varies from galaxy to galaxy

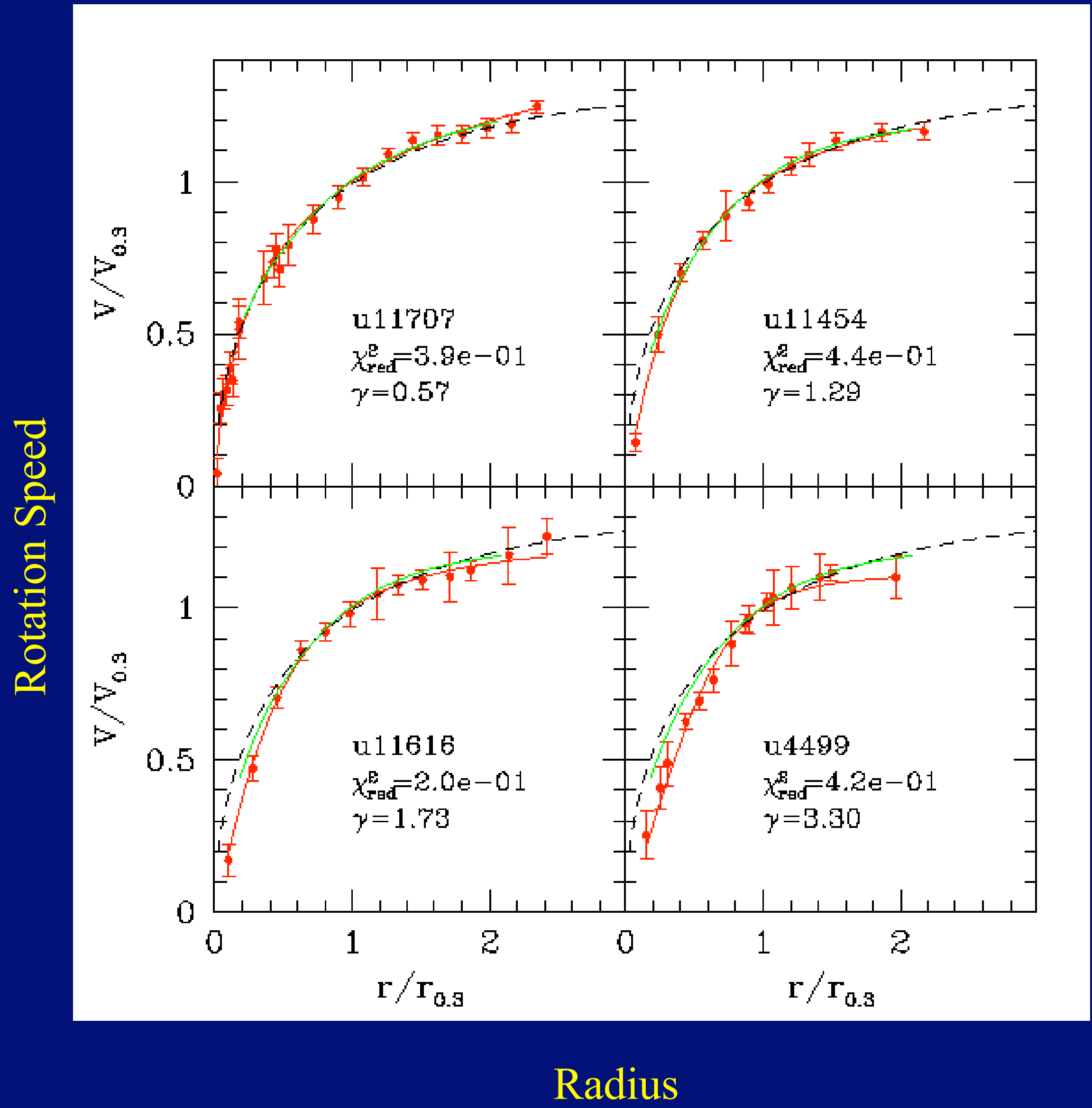
A fitting function:

$V_{\gamma}(r)=V_0\,(1+(r/r_t)^{-\gamma})^{-1/\gamma}$

The parameter γ is a good indicator of the shape of the rotation curve, the rate of change from rising to flat.

Hayashi et al 2003

Scaled LSB rotation curves: a representative sample

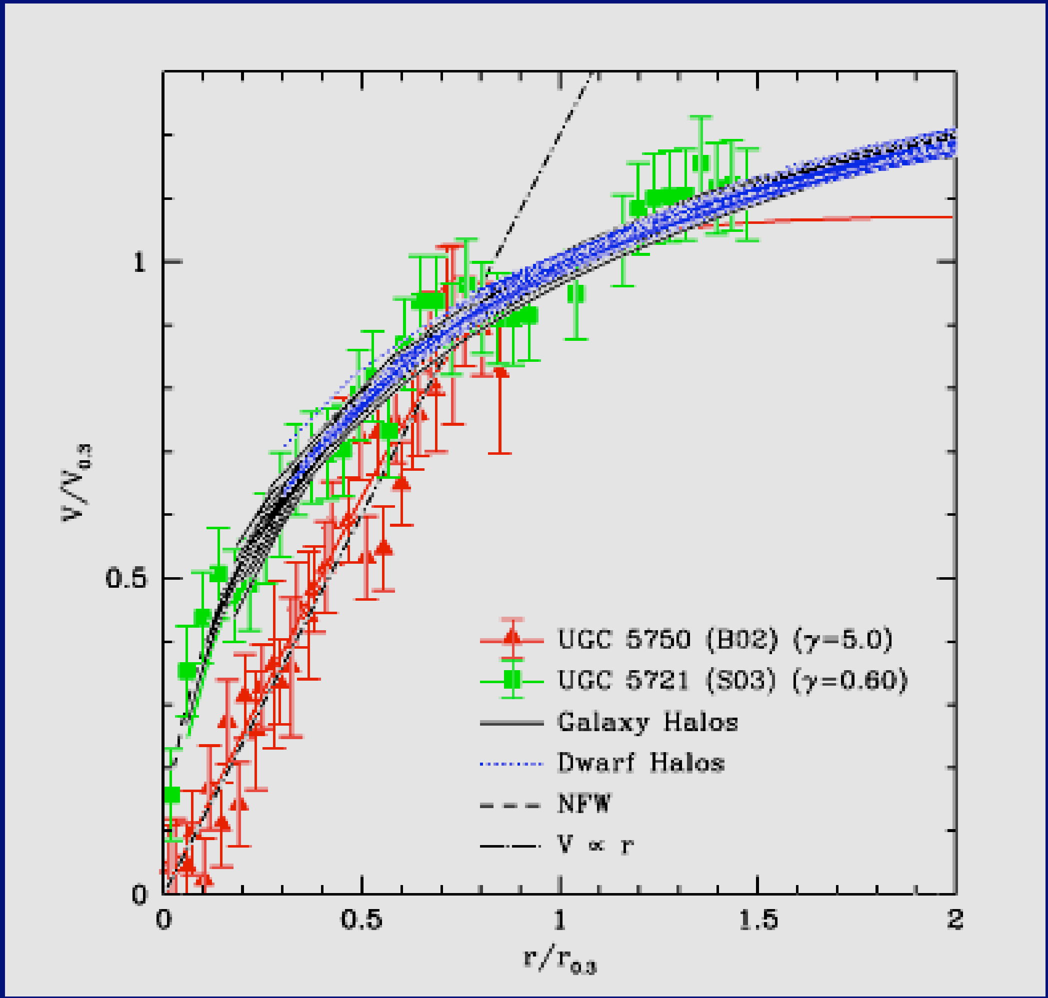


75% of LSB have $0.5<\gamma<2$
(~CDM halos)

25% have $\gamma>>2$
(in conflict with CDM halos)

Hayashi et al 2003

Scaled LSB rotation curves



Rotation Speed

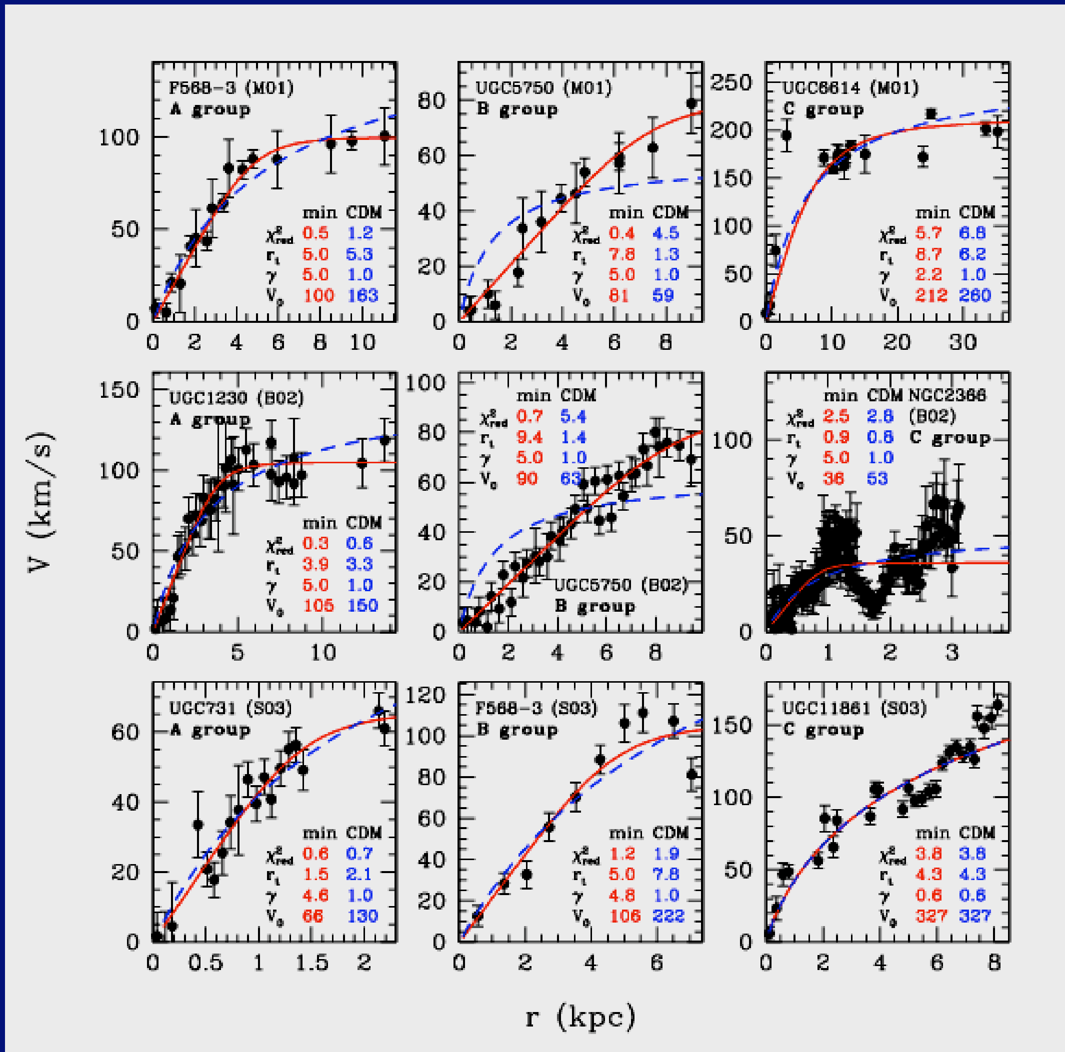
Radius

75% of LSB have 0.5<γ<2 (~CDM halos)

25% have γ>>2 (in conflict with CDM halos)

Hayashi et al 2003

Rotation Curves Inconsistent with CDM Halos



Three categories of rotation curves:

A) Well fit by V_g with LCDM compatible parameters (70%)

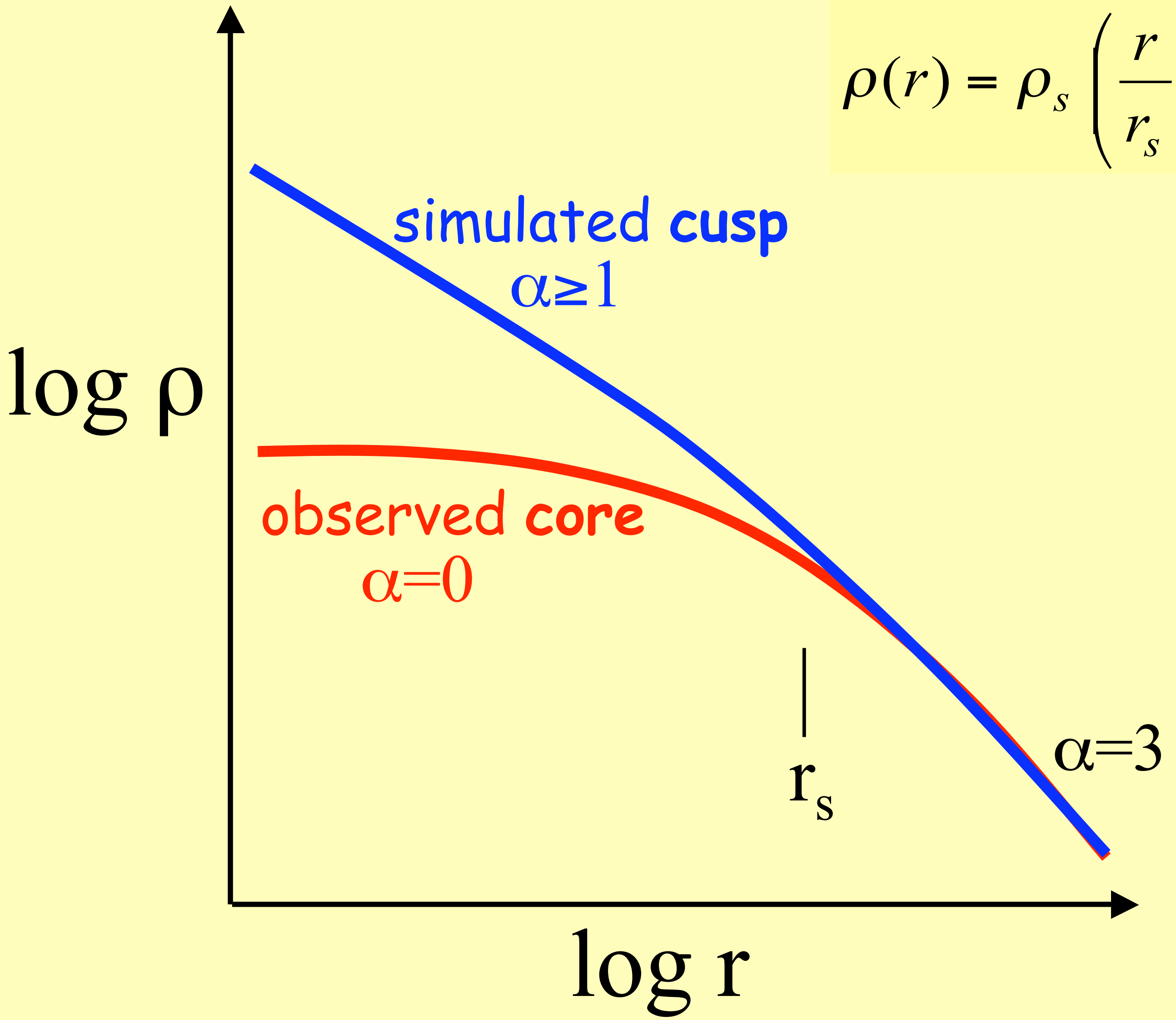
B) Poorly fit by V_g with LCDM-compatible parameters (10%)

C) Poorly fit by V_g with any parameters (20%)

Only 10% of LSB rotation curves are robustly inconsistent with LCDM halo structure

The dark-halo cusp/core problem

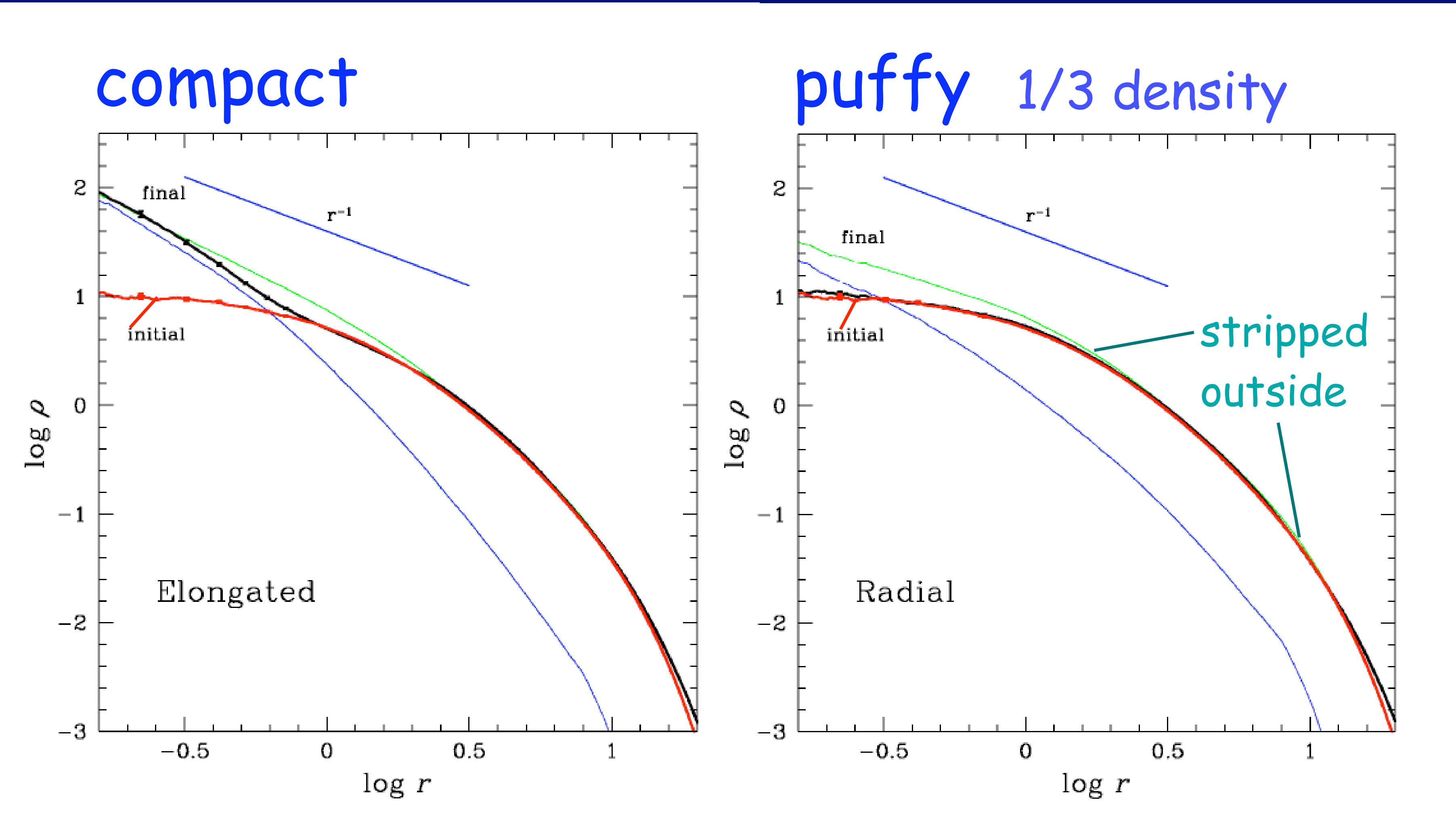
$$\rho(r) = \rho_s \left(\frac{r}{r_s}\right)^{-\alpha_0} \left(1 + \frac{r}{r_s}\right)^{-3+\alpha_0}$$



How to make and maintain a core?

must suppress satellite mergers with the halo core!

Compact vs. puffy satellite



Dekel, Devor & Hetzroni 2003

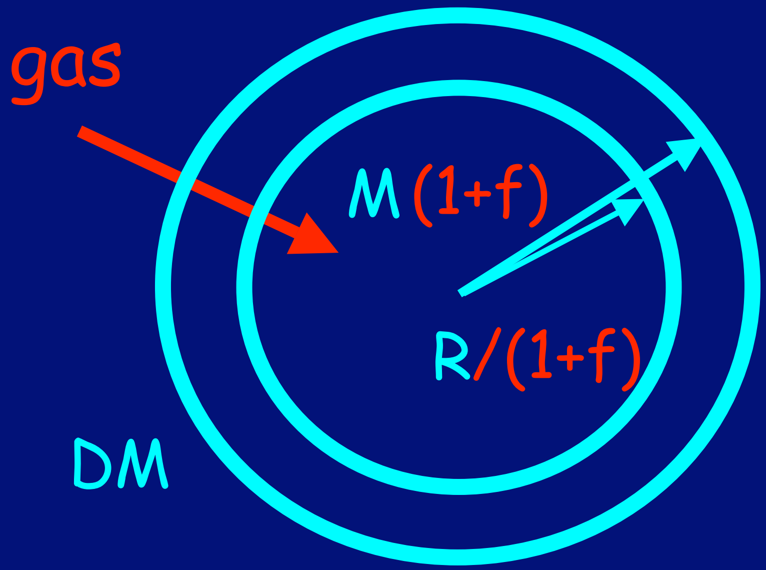
Adiabatic Contraction

Periodic motion under a slowly varying potential

Adiabatic invarinat:

$$I \approx \int_0^T v^2 dt \approx v^2 T$$

$$t_{dyn} \sim \frac{R}{V} \sim \frac{R}{(GM/R)^{1/2}} \sim (GM/R^3)^{-1/2} \sim (G\rho)^{-1/2}$$



$$I \approx \frac{GM}{R}\left(\frac{M}{R^3}\right)^{-1/2} \propto (M\,R)^{1/2}$$

$$R \propto M^{-1}$$

Instant Blowout

$$E_{before} = -\frac{GM^2}{R} + \frac{1}{2}MV^2$$

Lose M/2 while V^2 is unchanged:

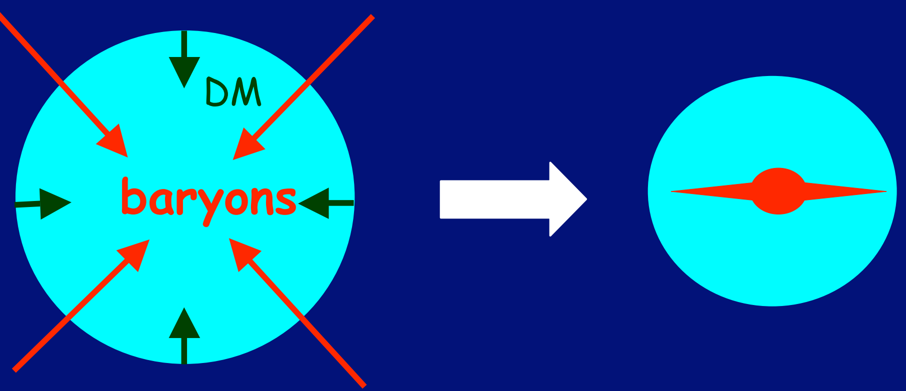
$$E_{after} = -\frac{G(M/2)^2}{R} + \frac{1}{2}(M/2)V^2 = 0$$

unbound!

Feedback

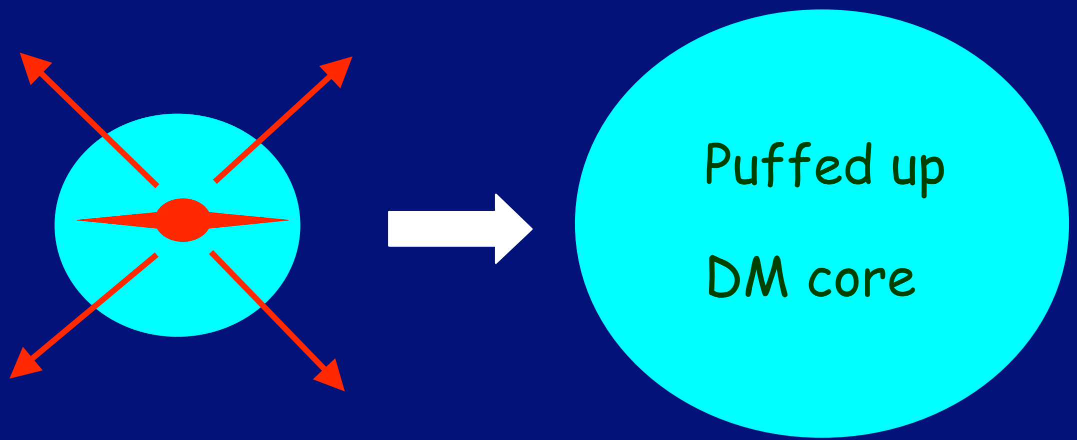
DM-halo reaction to blowout

Adiabatic contraction:



Instant blowout:

by supernova feedback

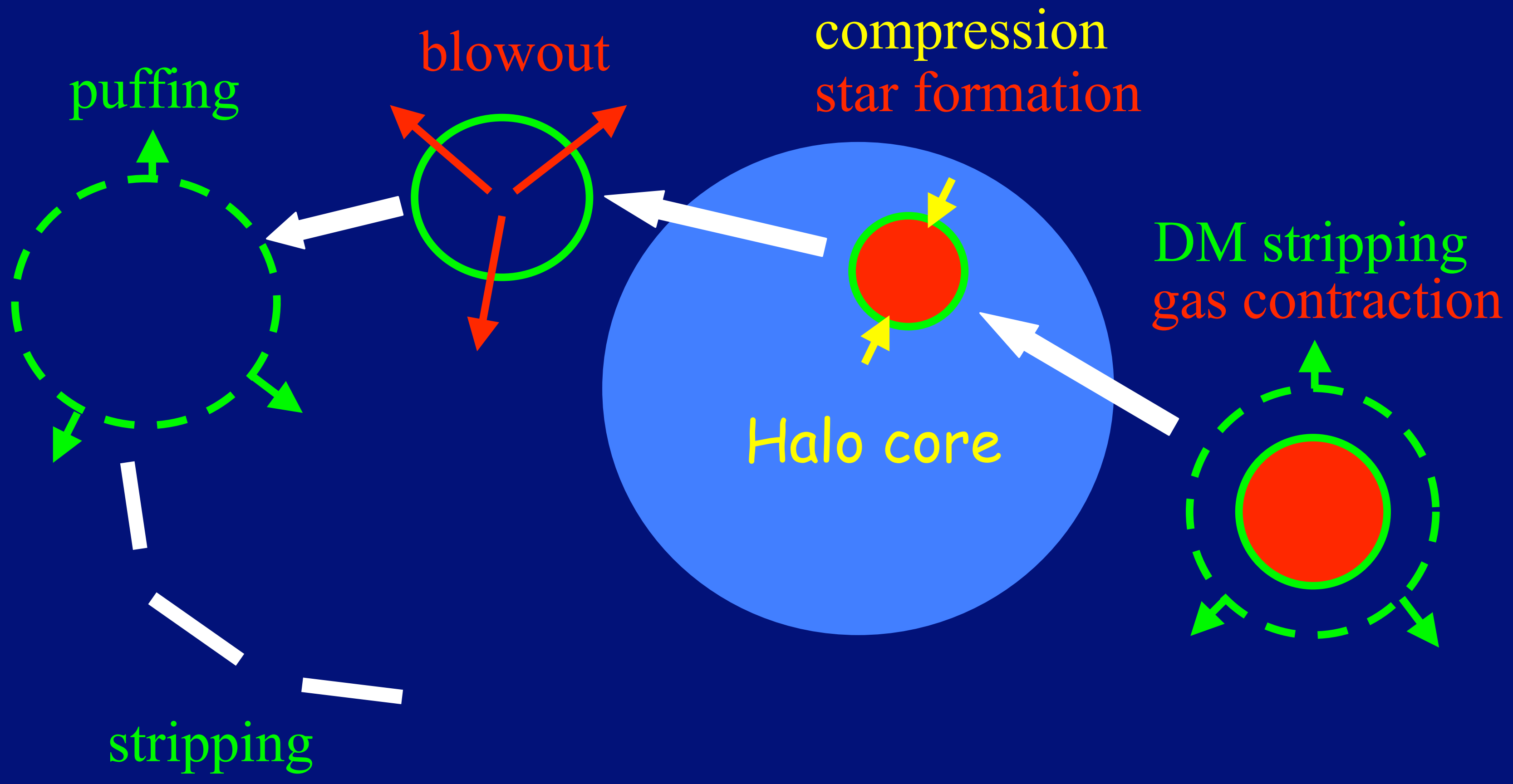


only 1/6 in density (Gnedin & Zhao 02)

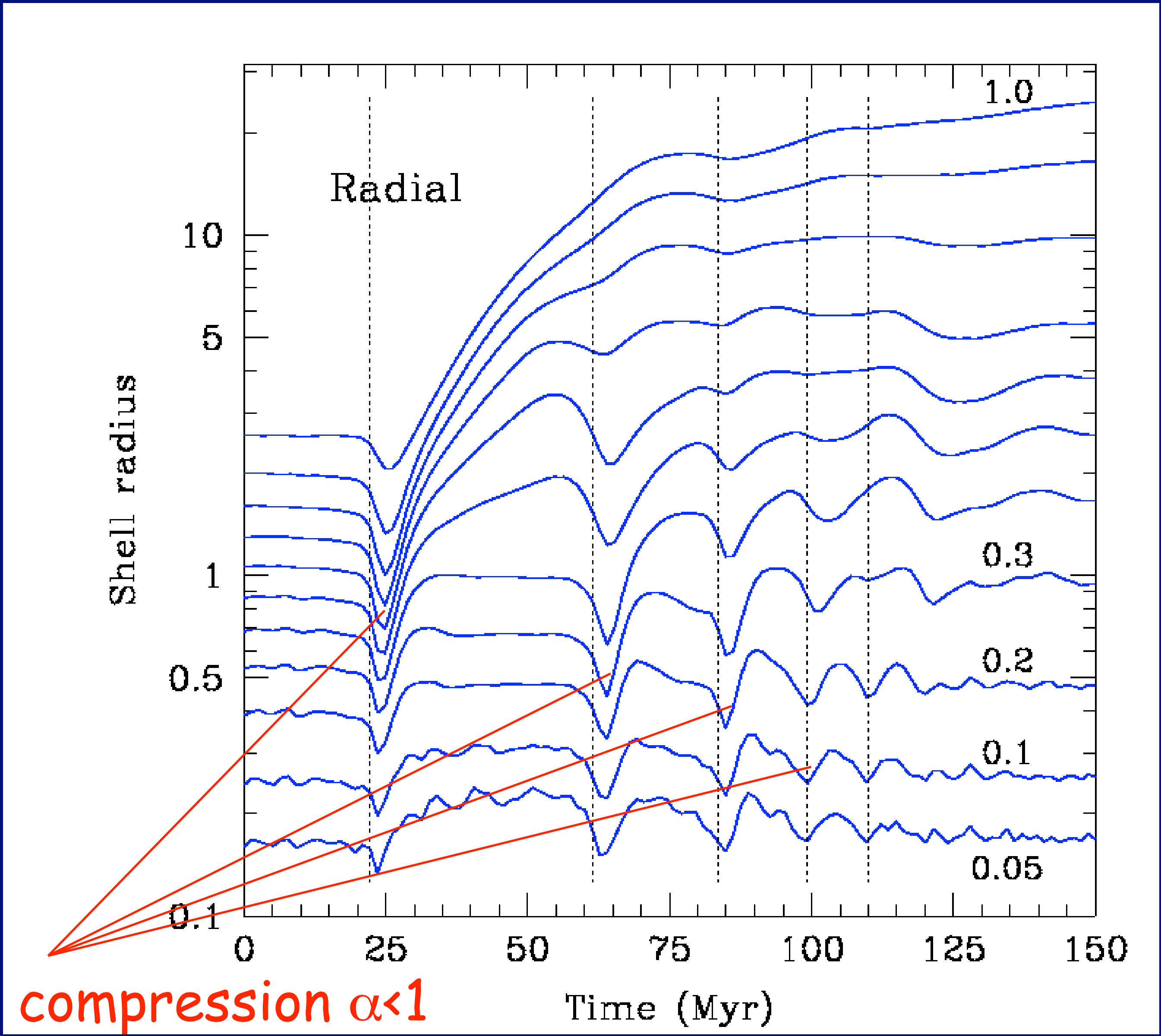
not enough in big galaxies?

Enough in satellites?

Satellite disruption by stimulated feedback



Compression in core



Summary: Core

Feedback may lead to a core
by puffing small satellites

Caveats

- Cusps (though flatter) form also in simulations where satellites are suppressed

- Cores detected in big galaxies and clusters (?)

→ Puffing-up of satellite halos is necessary for cores, but perhaps not sufficient

Other scenarios for core formation

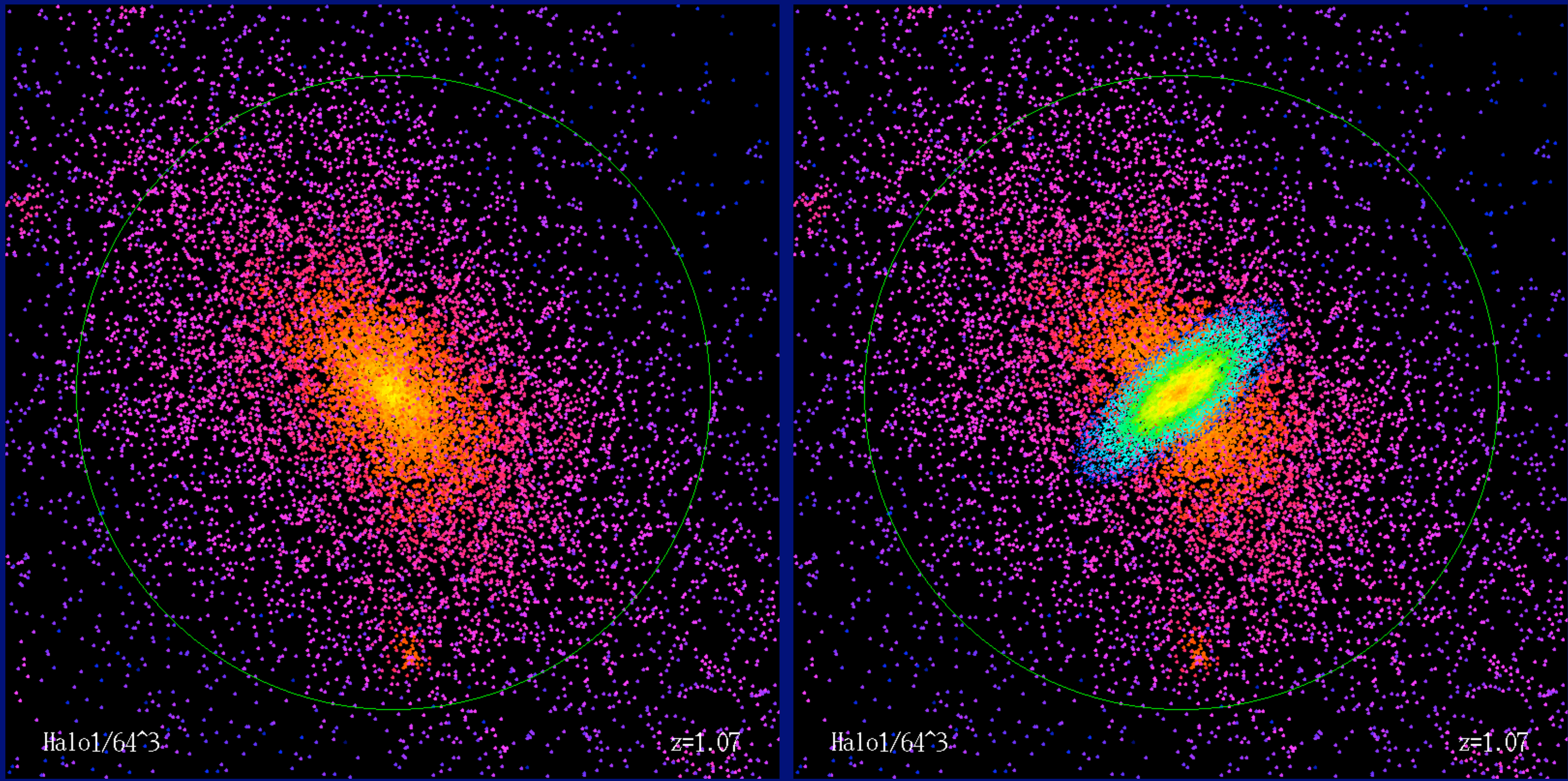
- **Warm** dark matter, **Interacting** dark matter
  → suppress satellites
- Disruption of satellites by a massive **black hole**
  (Merritt & Cruz 01)
- Angular-momentum transfer from a big **bar**
  to the halo core (Weinberg & Katz 02)
- Delicate **resonant** tidal reaction of halo-core orbits
  if the system is noise-less (Katz & Weinberg 02)
- Heating of the cusp by merging **clouds**
  (El-Zant, Shlosman & Hoffman 02)

Origin of Core:
Disk in Triaxial Halo

Disk Rotation curve is NOT $V^2=GM(r)/r$

Hayashi, Navarro et al.

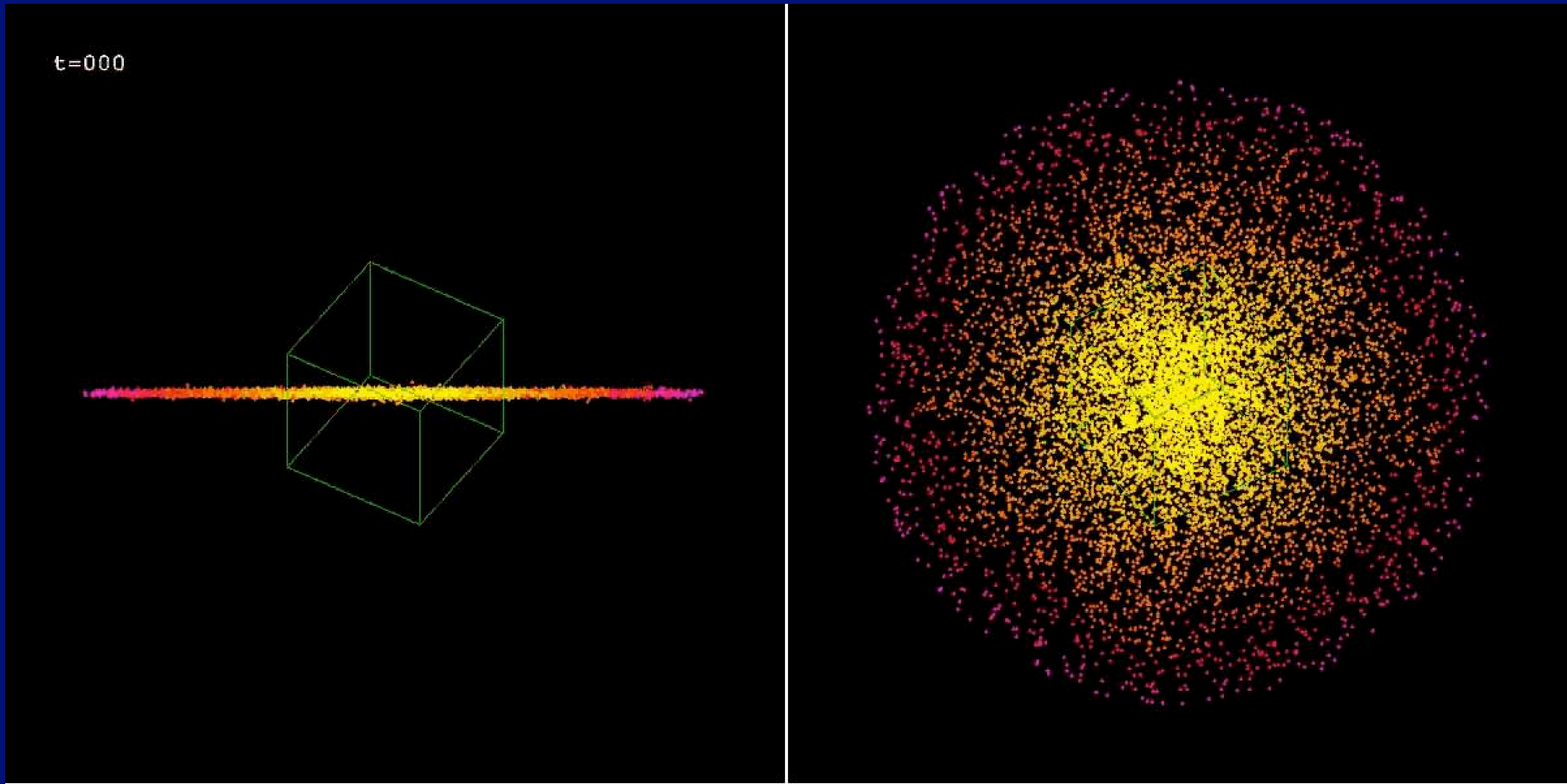
Disks in realistic dark matter halos



Massless isothermal gaseous disk in the non-spherical DM halo potential tracks the closed orbits within this potential

Hayashi et al 2003

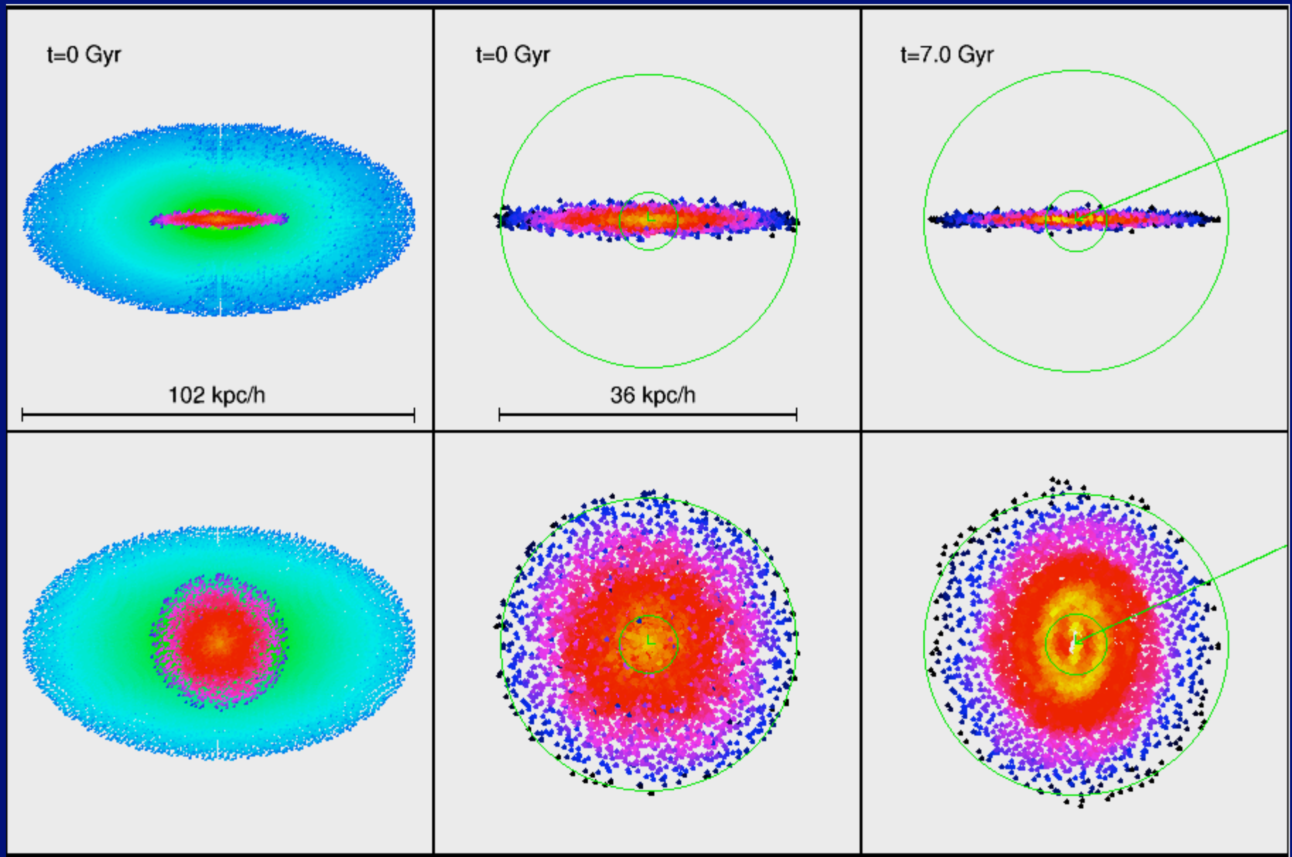
Disks in realistic dark matter halos



Massless isothermal gaseous disk in the DM halo potential

Hayashi et al 2003

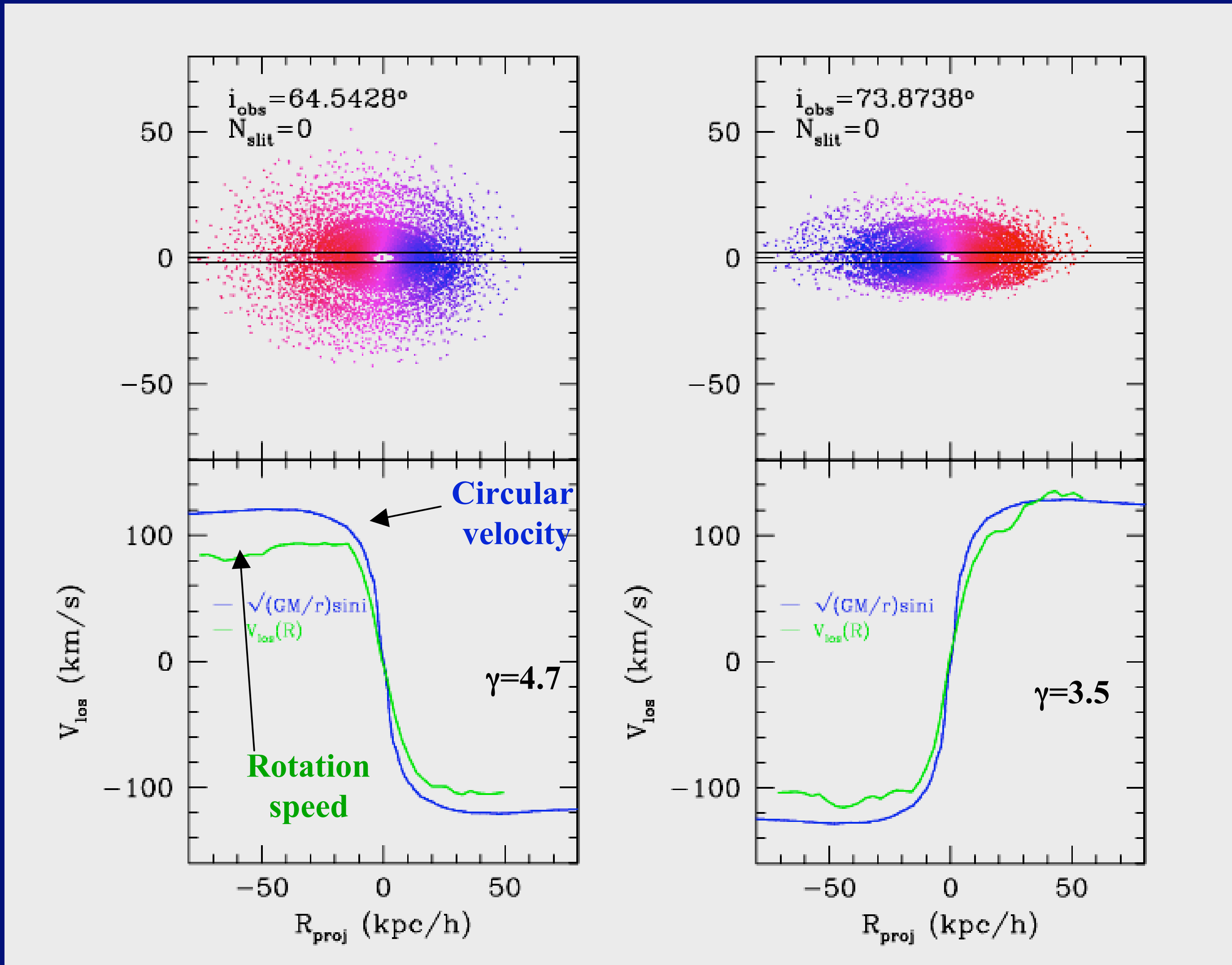
Dynamics of a Gaseous Disk



Closed orbits in triaxial potentials are not circular, and not limited to a plane.

High _?

Disks in triaxial dark matter halos

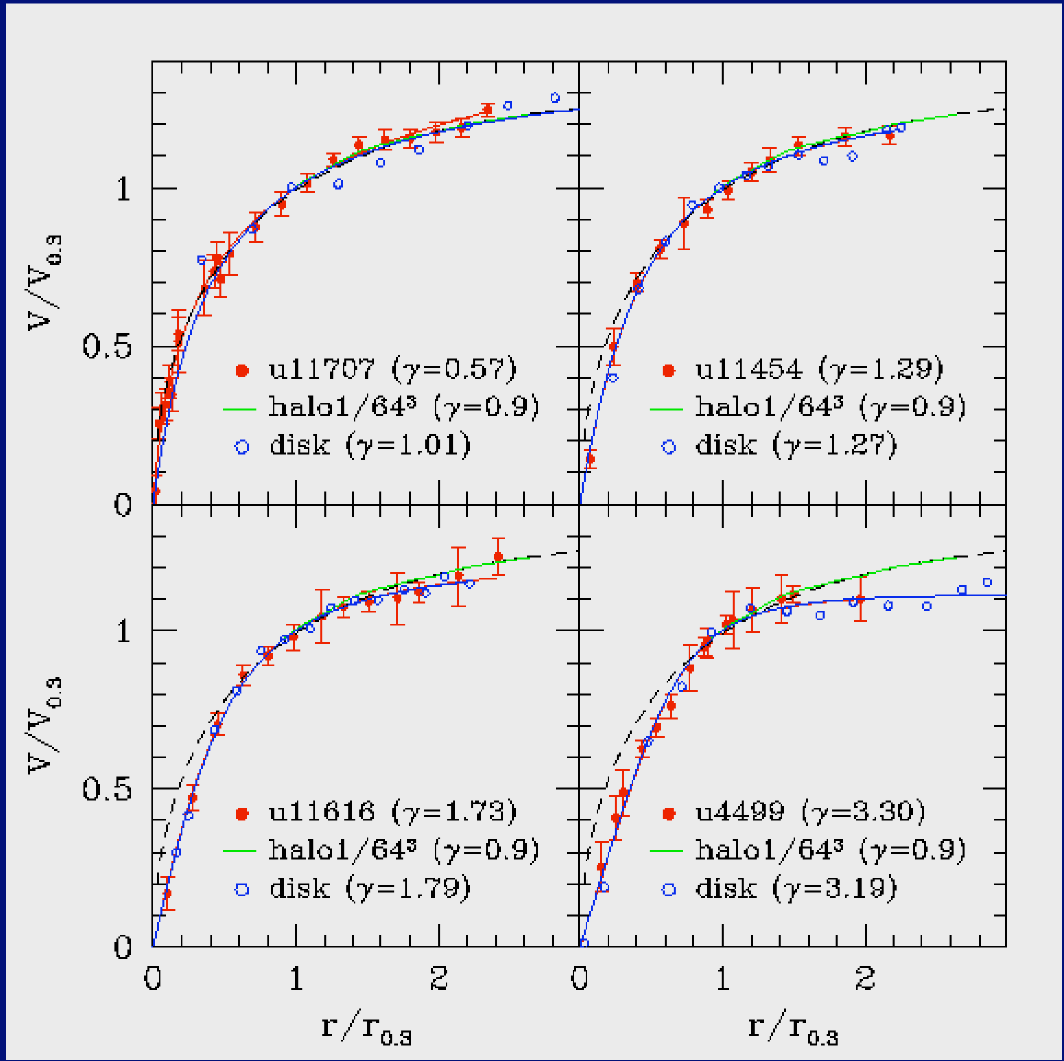


Inferred rotation speeds may differ significantly from actual circular velocity.

Inclination: 50 degrees 67 degrees

Hayashi et al 2003

Scaled Rotation Curves: disk in CDM halo vs LSBs



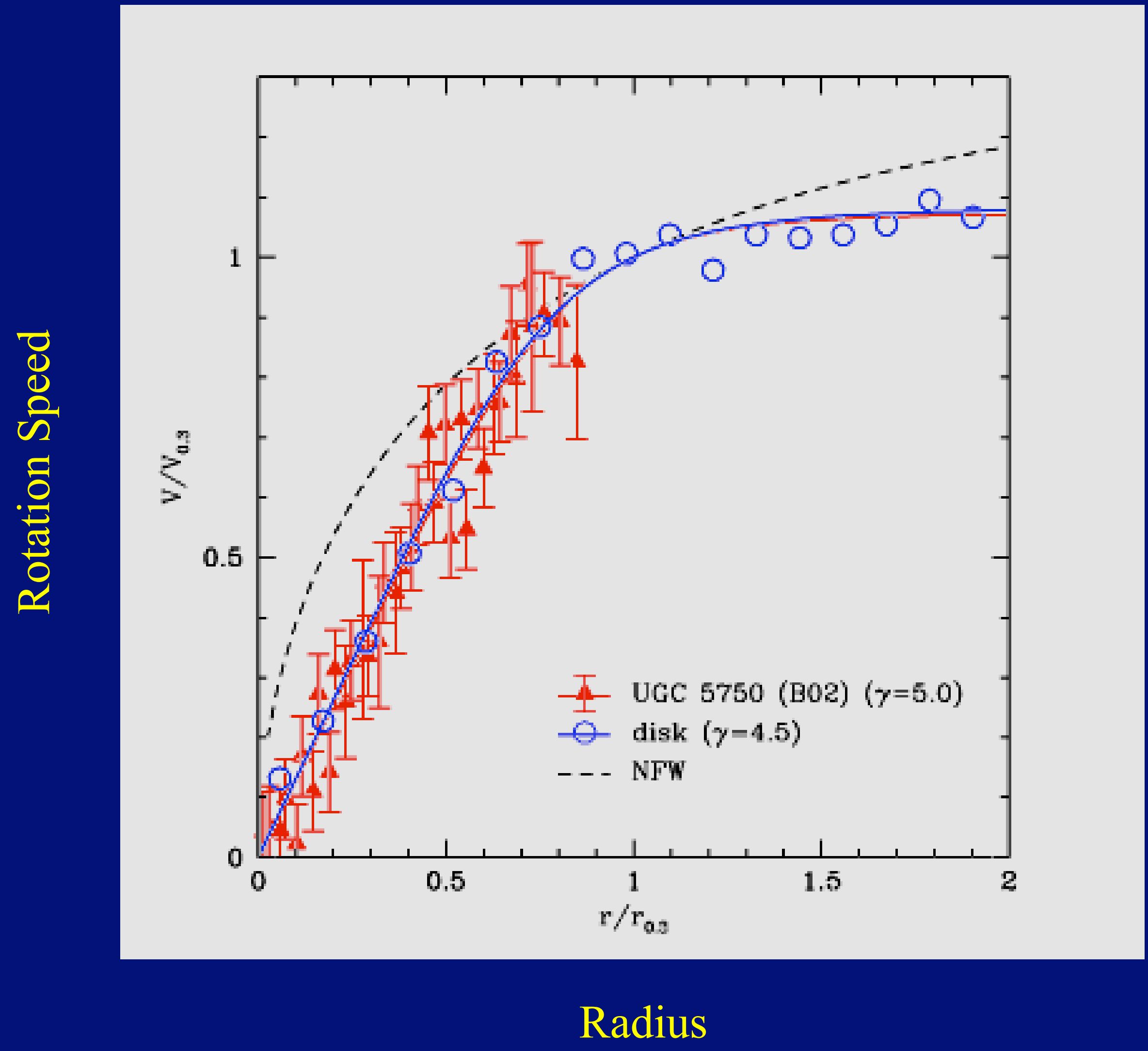
Scaled Rotation Speed

Scaled radius

All LSB rotation curve shapes may be accounted for by various projections of a disk in a single CDM halo

Hayashi et al 2003

Scaled LSB rotation curves: a representative sample



LSB rotation curve shapes may be accounted for by various projections of a disk in a single CDM halo

Triaxiality in the halo potential may be enough to explain the "cusp-core" discrepancy.

Hayashi et al 2003

Halo Shape