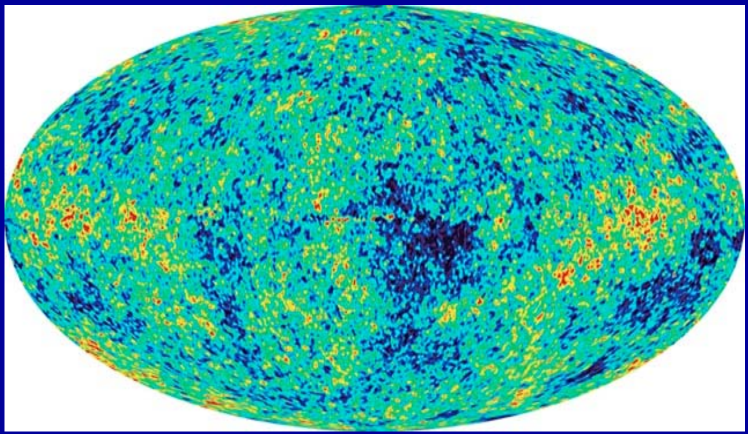
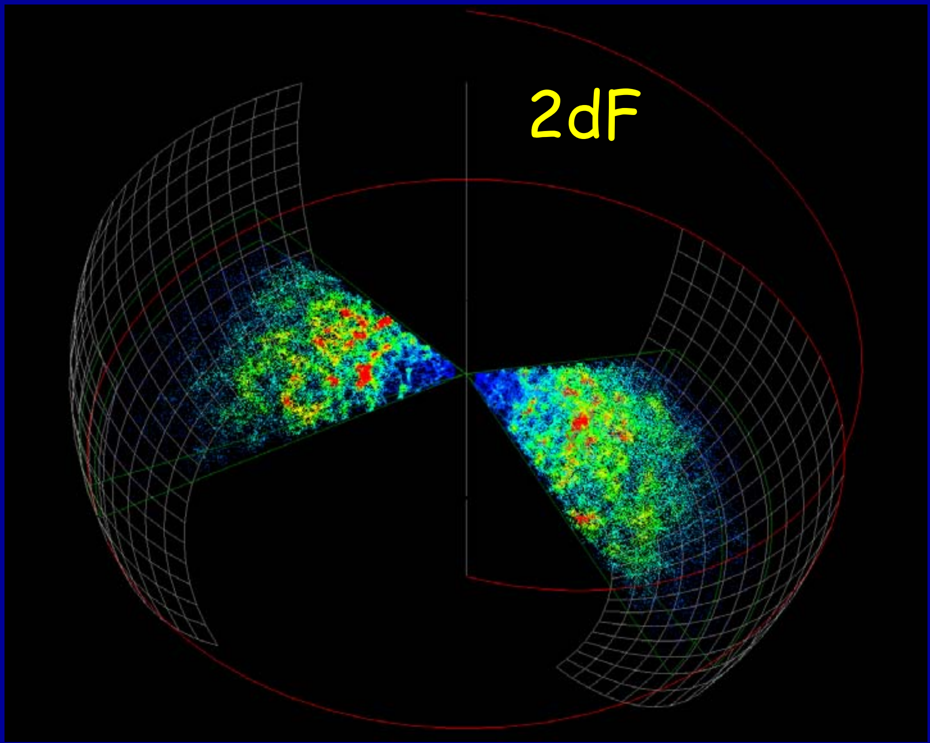
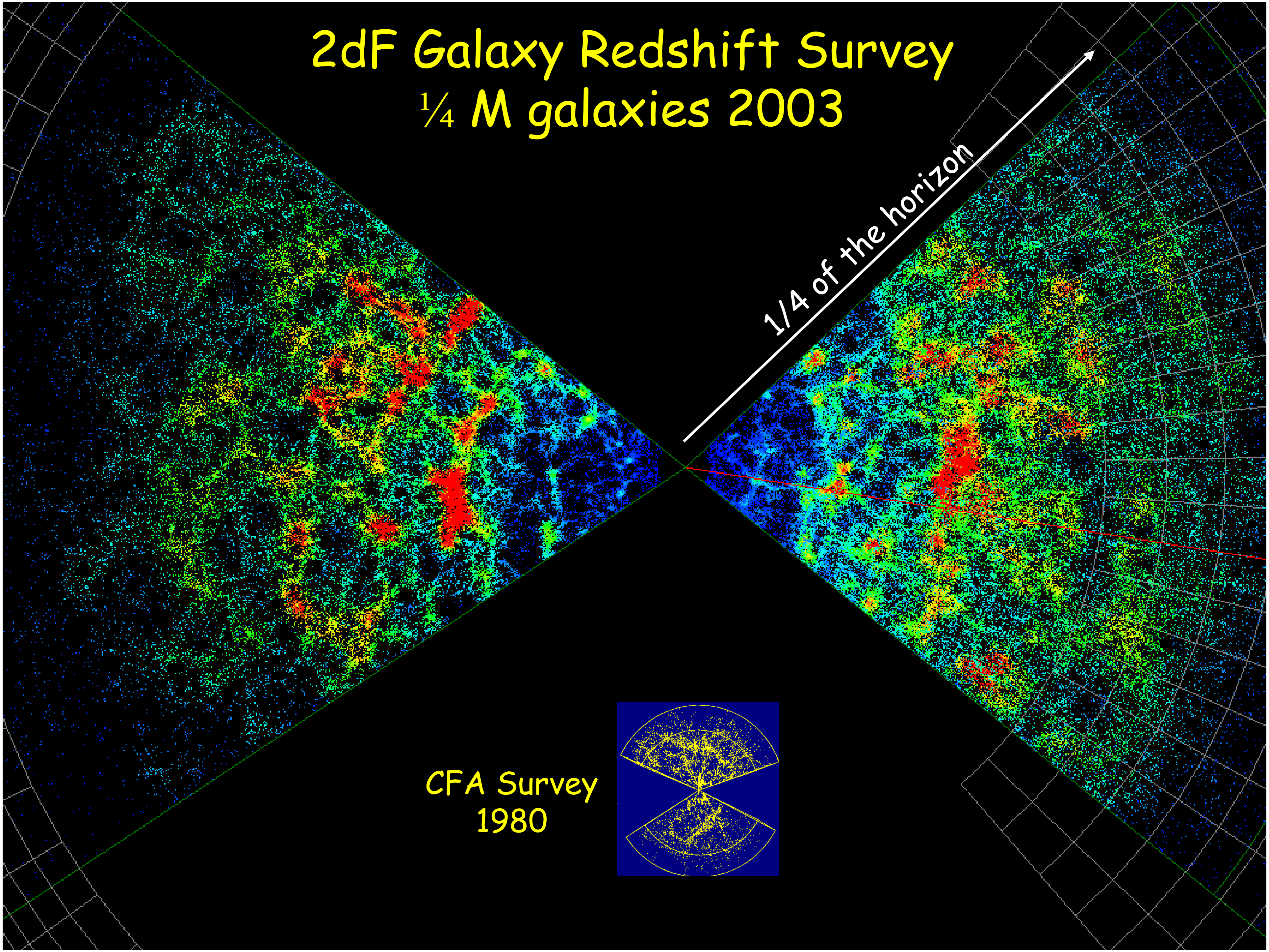
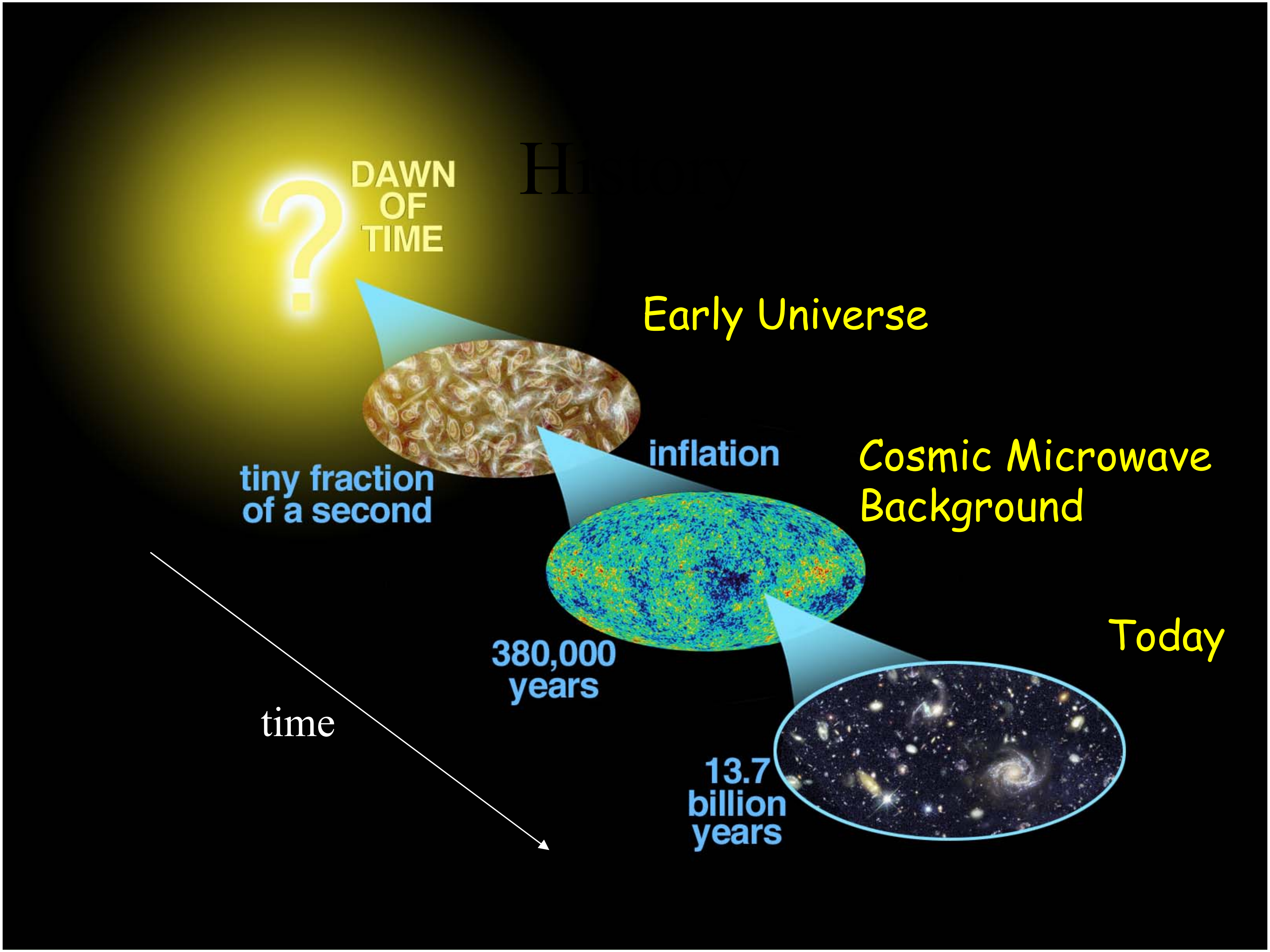
Linear Growth of Fluctuations by Gravitational Instavility

WMAP

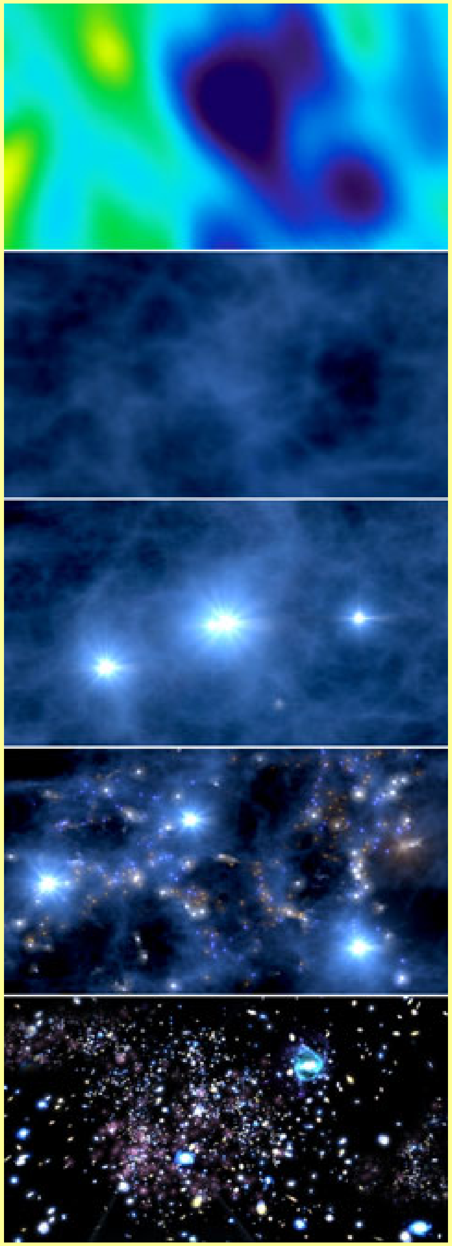





2dF Galaxy Redshift Survey
¼ M galaxies 2003
1/4 of the horizon
CFA Survey
1980


DAWN OF TIME
Early Universe
tiny fraction of a second
inflation
Cosmic Microwave Background
380,000 years
Today
time
13.7 billion years

Late Cosmological Epochs



380 kyr   z~1000 — recombination / last scattering

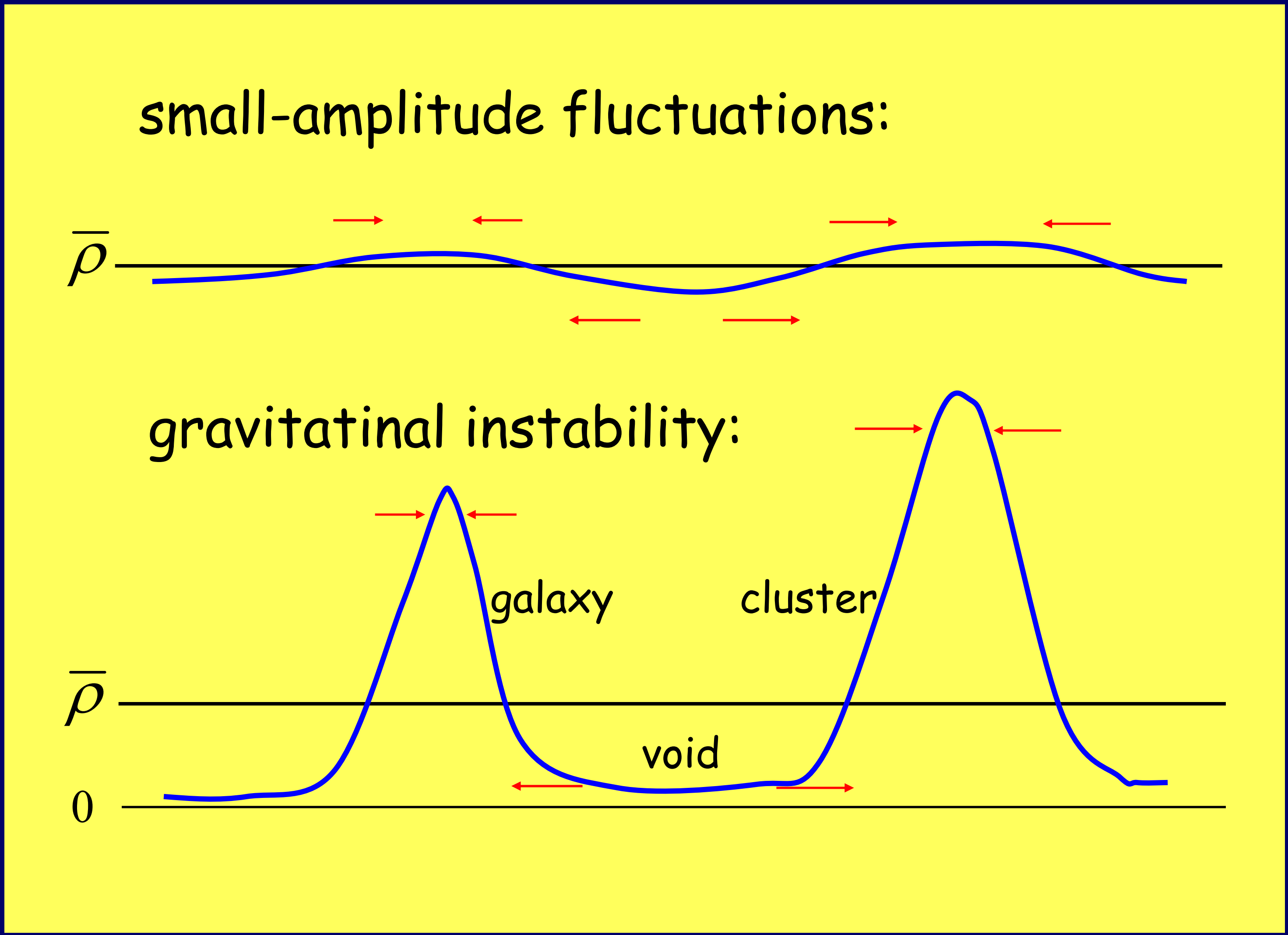
dark ages

180 Myr   z~20 — first stars / reionization

galaxy formation

13.7 Gyr   z=0 — today

Gravitational instability



Gravitational Instability: linear, matter-era

Fluid equations:

(1) $\dot{\rho} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$ continuity

(2) $\dot{\vec{V}} + (\vec{V}\cdot\vec{\nabla})\vec{V} = -\vec{\nabla}\Phi - \vec{\nabla}P/\rho$ Euler

(3) $\nabla^2\Phi = 4\pi G\rho$ Poisson

Uniform background: $\rho(\vec{r}) = const.$

$$\vec{r} \equiv a\vec{x} \qquad \vec{v} = \frac{\dot{a}}{a}\vec{r} \qquad \rho^{(1)} = \frac{\rho_0}{a^3} \qquad \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho$$

$$H \equiv \dot{a}/a \qquad \dot{\rho}^{(1)} = -3\rho H$$

Perturbations: $\rho(\vec{r},t) = \rho_u(t)[1+\delta(\vec{r},t)] \quad \vec{V} = H(t)\vec{r} + \vec{v} \quad \Phi = \Phi_u + \varphi \quad P = p$

1^{st} order +: $\delta << 1$ etc.

(1) $\dot{\delta} + H\vec{r}\cdot\vec{\nabla}\delta + \vec{\nabla}\cdot\vec{v} + \vec{\nabla}\cdot(\delta\vec{v}) = 0$

(2) $\dot{\vec{v}} + H(\vec{r}\cdot\vec{\nabla})\vec{v} + H\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla}\varphi - c_s^{\,2}\vec{\nabla}\delta$

(3) $\nabla^2\varphi = 4\pi G\rho_u\delta$

$$\vec{\nabla}P = \frac{\partial P}{\partial\rho}\vec{\nabla}\rho \qquad c_s^{\,2} \equiv \frac{\partial P}{\partial\rho} \overset{\text{(ideal gas)}}{=} \frac{P}{\rho} = \frac{kT}{m_p}$$

Comoving coordinates : $\vec{x} \equiv \dfrac{\vec{r}}{a}$ $\qquad \left.\dfrac{\partial}{\partial t}\right|_x = \left.\dfrac{\partial}{\partial t}\right|_r + \dfrac{\dot{a}}{a}\vec{r}\cdot\left.\vec{\nabla}_r\right|_t$ $\qquad \nabla_x = a\nabla_r$

$$\vec{w} \equiv \vec{v}/a \quad \psi \equiv \varphi / a$$

$$(1) \quad \dot{\delta} + \vec{\nabla}\cdot\vec{w} + \vec{\nabla}\cdot(\delta\vec{w}) = 0$$

$$(2) \quad \dot{\vec{w}} + 2H\vec{w} + (\vec{w}\cdot\vec{\nabla})\vec{w} = -\vec{\nabla}\psi - a^{-1}c_s^{\,2}\vec{\nabla}\delta$$

$$(3) \quad \nabla^2\psi = 4\pi G\rho_u\delta \qquad [=(3/2)H^2\Omega\delta]$$

$a^{-1}\vec{\nabla}\cdot(\text{eq. }2) \quad \partial/\partial t(\text{eq. }1) \quad \rightarrow$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_u\delta + a^{-2}c_s^{\,2}\nabla^2\delta$$

gravity pressure

$$\delta(\vec{x},t) \qquad a(t) \quad H(t) = \frac{\dot{a}}{a} \quad \rho_u(t) \propto a^{-3}$$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_u \delta + a^{-2} c_s^{\ 2} \nabla^2 \delta$$

$$a(t) \quad H(t) = \frac{\dot{a}}{a} \quad \rho_u(t) \propto a^{-3}$$

Static background : $\dot{a} = 0 \quad a = const. \equiv 1 \quad \rho_u = const.$

$$\delta \propto \exp[i(\vec{k}\cdot\vec{x} + \omega t)] \qquad \omega^2 = k^2 c_s^{\ 2} - 4\pi G \rho_u$$

pressure gravity

Jeans scale : $k_J = \left(\frac{4\pi G\rho_u}{c_s^{\ 2}}\right)^{1/2} \quad \lambda_J \equiv \frac{2\pi}{k_J} \quad M_J \equiv \frac{4\pi}{3}\rho_m \left(\frac{\pi c_s^{\ 2}}{G\rho}\right)^{3/2} \propto \frac{T^{3/2}}{\rho^{1/2}}$

$\lambda >> \lambda_J \quad (p = 0) \quad \to \delta = Ae^{\omega t} + Be^{-\omega t}$

$\lambda << \lambda_J$ $\to$ stable oscilations

Expanding background , $\lambda >> \lambda_J$:

$k = 0 \quad \to \quad a \propto t^{2/3} \quad \to \quad \ddot{\delta} + \frac{4}{3t}\dot{\delta} = \frac{2}{3t^2}\delta \quad \to \quad \delta = At^{2/3} + Bt^{-1}$

$k = -1 \to a \propto t \quad \to \ddot{\delta} + \frac{2}{t}\dot{\delta} = \frac{3\Omega_0 t_0}{2t^3}\delta \quad \to \delta = const.$ freezout

Properties of the linear growing mode:

linear $\ddot{\delta} + 2H\dot{\delta} = (3/2)H^2\Omega\delta \qquad H(t) \quad \Omega(t)$

growing mode: $\delta \propto D(t)$

$$f(\Omega) \equiv \frac{\dot{D}}{HD} \approx \Omega^{0.6} \quad \rightarrow \quad \frac{\ddot{D}}{D} = H^2(-2f + \frac{3}{2}\Omega)$$

continuity $\rightarrow \quad \delta = -\frac{1}{Hf}\vec{\nabla}\cdot\vec{v}$

Poisson $\rightarrow \quad \vec{v} = -\vec{\nabla}\varphi_v$ irrotational $\qquad \varphi = \frac{3H\Omega}{2f}\varphi_v$

The Jeans scale in an expanding universe:

In k-space: $\delta = \sum_{\vec{k}} \delta_{\vec{k}}(t)\, e^{i\vec{k}\cdot\vec{x}} \qquad r = ax$

for each $\vec{k}$: $\ddot{\delta}_k + 2H\dot{\delta}_k = (4\pi G\rho - k^2 c_s^{\,2})\,\delta_k \quad \rightarrow$ same Jeans scale

Lecture 3

Statistics of Fluctuations:

The Cold Dark Matter Scenario

The Initial Fluctuations

At Inflation: Gaussian, adiabatic

fluctuation field $\delta(x) = \frac{\rho(x) - <\rho>}{<\rho>}$ a realization of an ensemble

ensembe average ~ volume average

Fourier $\delta(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}}\, e^{i\vec{k}\cdot\vec{x}}$

Power Spectrum $P(k) \equiv <|\tilde{\delta}(\vec{k})|^2> \quad \propto k^n$

rms

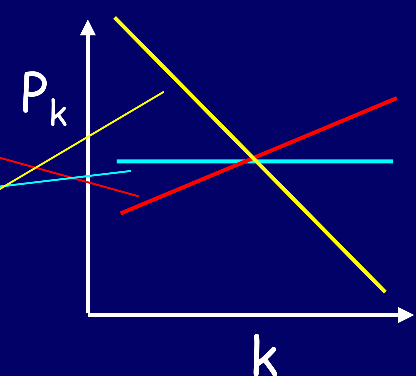
$$<\delta^2>_\lambda \sim < \int_{k=0}^{K} \int_{k'=0}^{K\sim 2\pi/\lambda} \exp[-i(k+k')\cdot x]\, d^3k'\, d^3k\, \delta_k \delta_{k'} > \quad \sim \int_{k=0}^{K} d^3k < \delta_k \delta_{-k} >$$

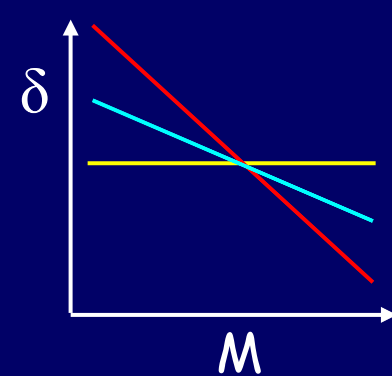
$\delta_{Dirac}(k+k')$

$$<\delta_k \delta_{-k}> = <|\delta_k|^2>$$

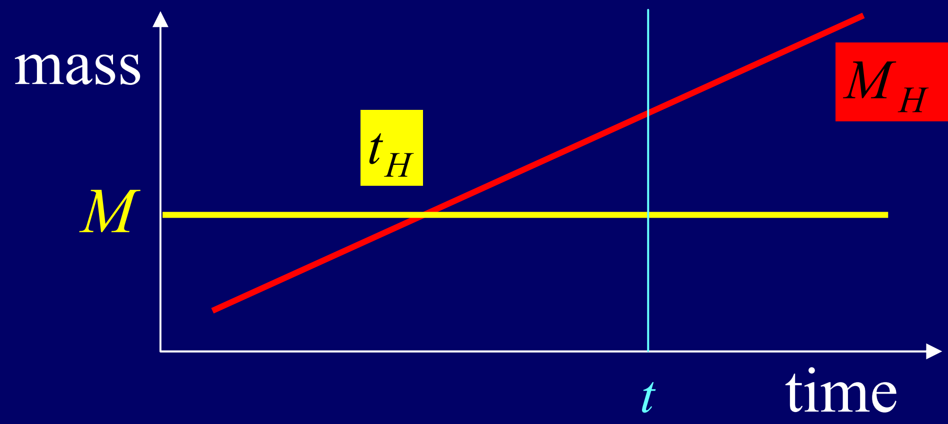
$$<\delta^2>_\lambda \propto \int_{k=0}^{2\pi/\lambda} P_k\, d^3k \quad \propto M^{-(n+3)/3}$$

$n = 1$	$\delta \propto M^{-2/3}$
$n = 0$	$\delta \propto M^{-1/2}$
$n = -3$	$\delta \propto const.$





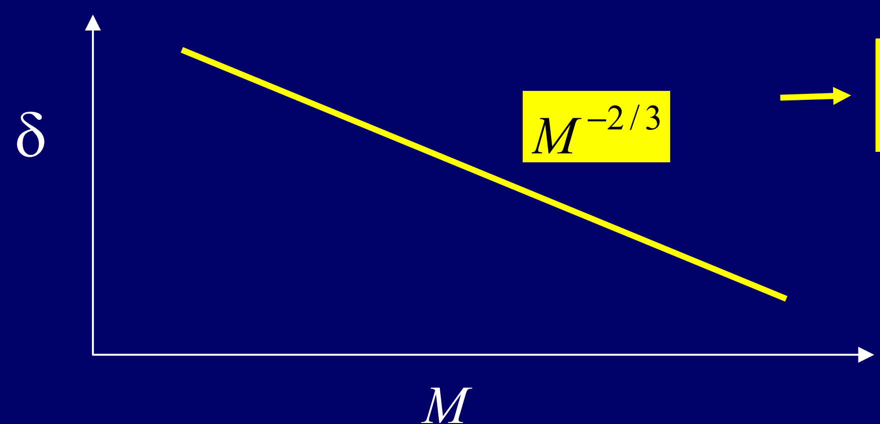
Scale-Invariant Spectrum (Harrison-Zel'dovich)



$$M_H \propto t \leftarrow \begin{array}{l} R_{Horizon} \propto t \\ \rho_m \propto a^{-3} \propto t^{-2} \end{array}$$

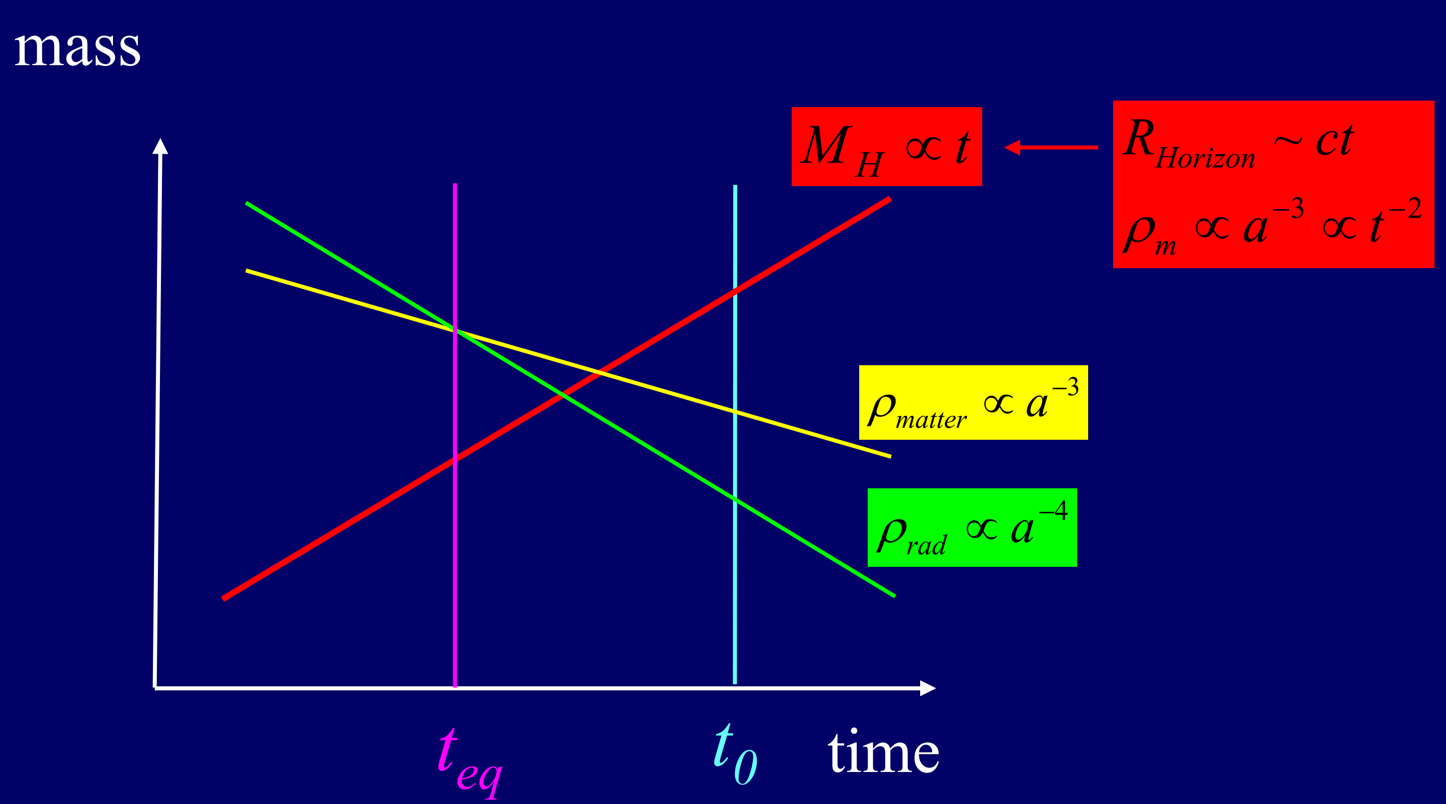
$$\delta(M,t) = \delta_H \left(\frac{t}{t_H(M)} \right)^{2/3} \propto M^{-2/3} t^{2/3}$$

$$\delta_H = const.$$



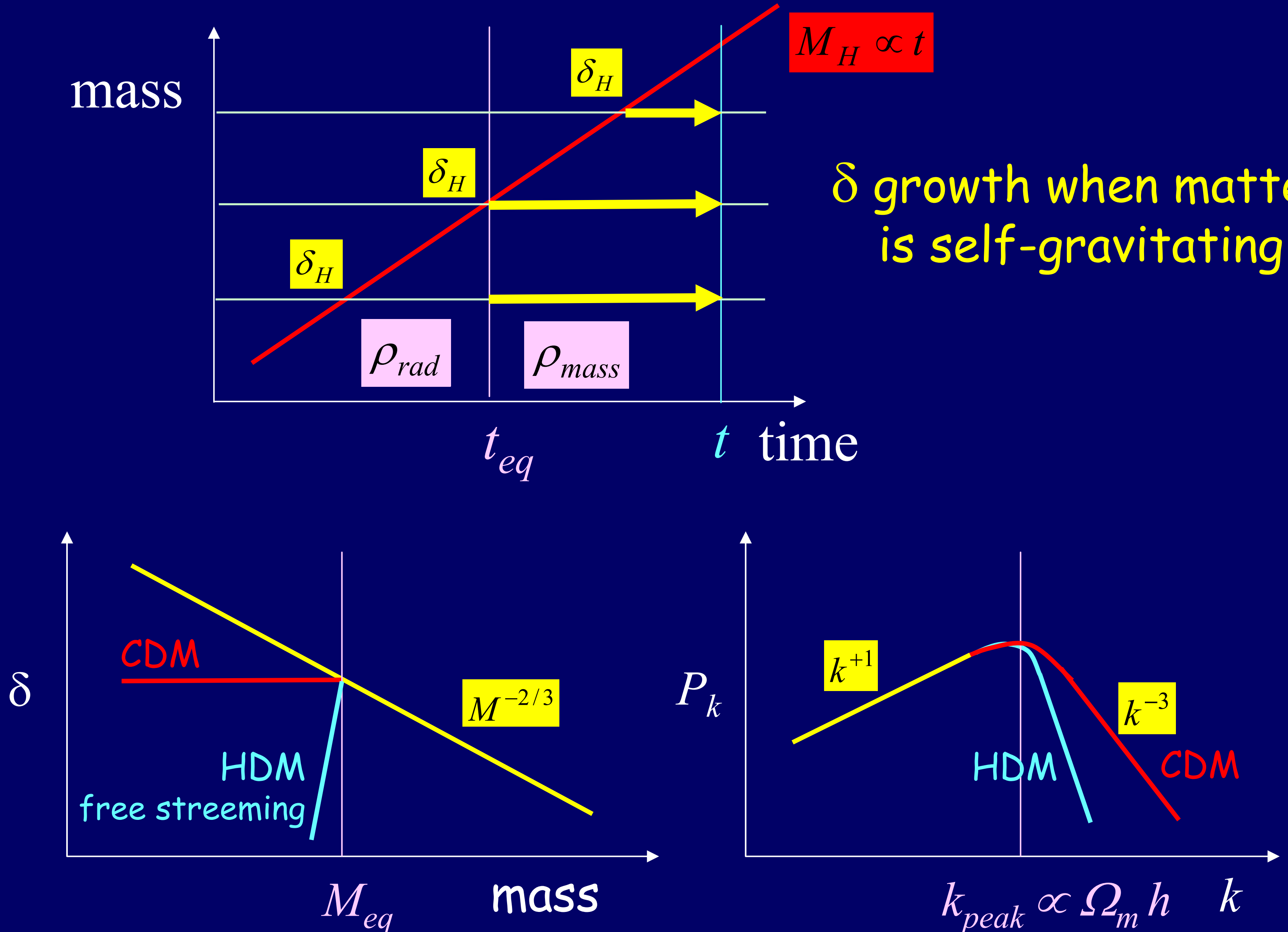
$$\rightarrow \quad n = +1 \quad P_k \propto k$$

Cosmological Scales



$z_{eq} \sim 10^4$

CDM Power Spectrum



δ growth when matter is self-gravitating

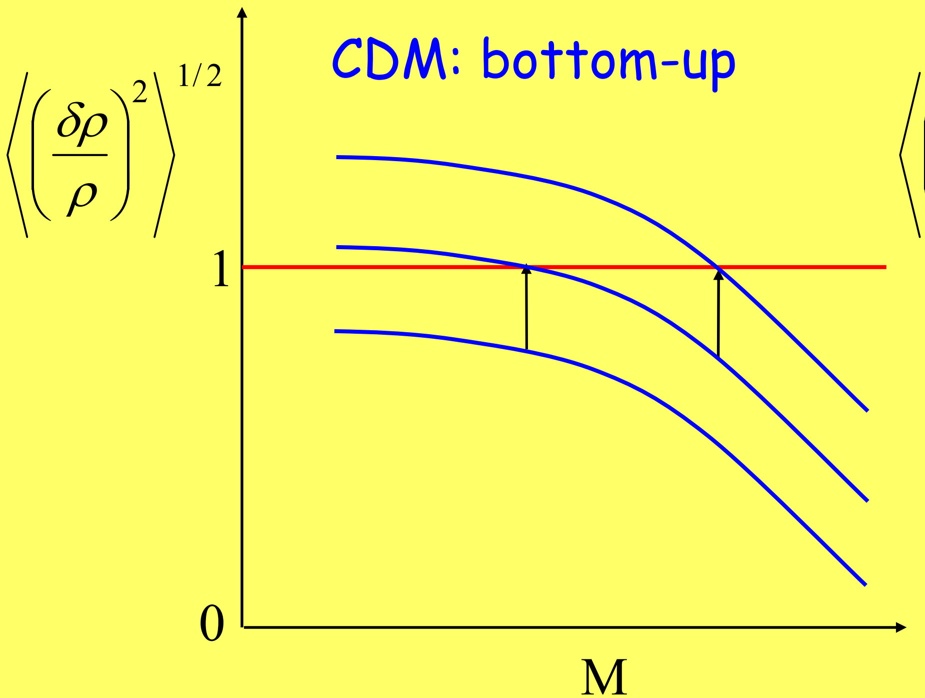
Formation of Large-Scale Structure

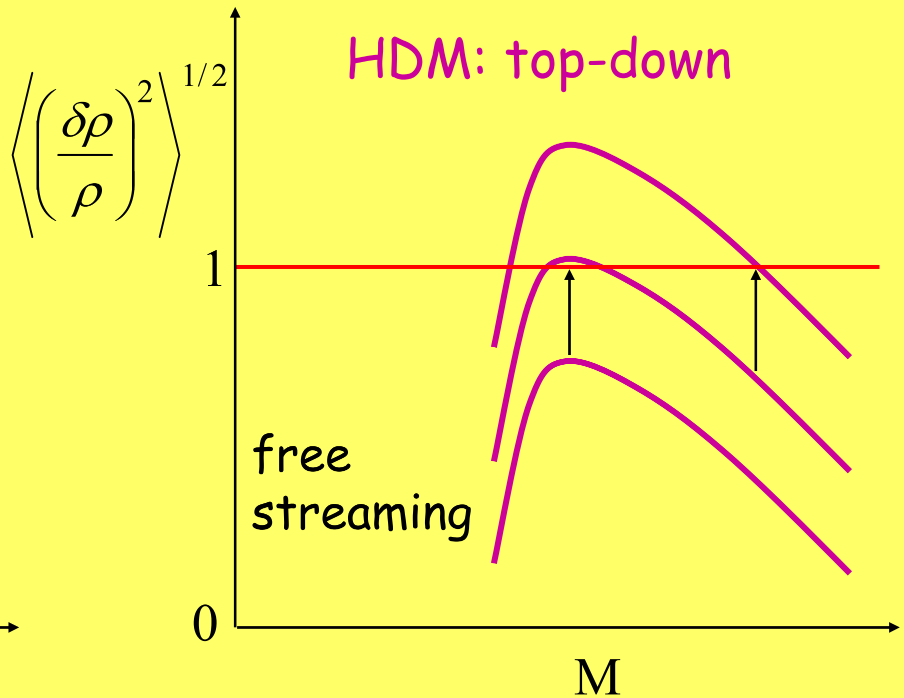
Fluctuation growth in the linear regime: $\delta << 1 \quad \to \quad \delta \propto a \propto t^{2/3}$

rms fluctuation at mass scale M: $\delta \propto M^{-\alpha} \quad 0 < \alpha = (n+3)/6 \leq 2/3$

Typical objects forming at t: $1 \sim \delta \propto M^{-\alpha} a \quad \to M_* \propto a^{1/\alpha}$

example $n = -2 \quad \to \quad M_* \propto a^6$



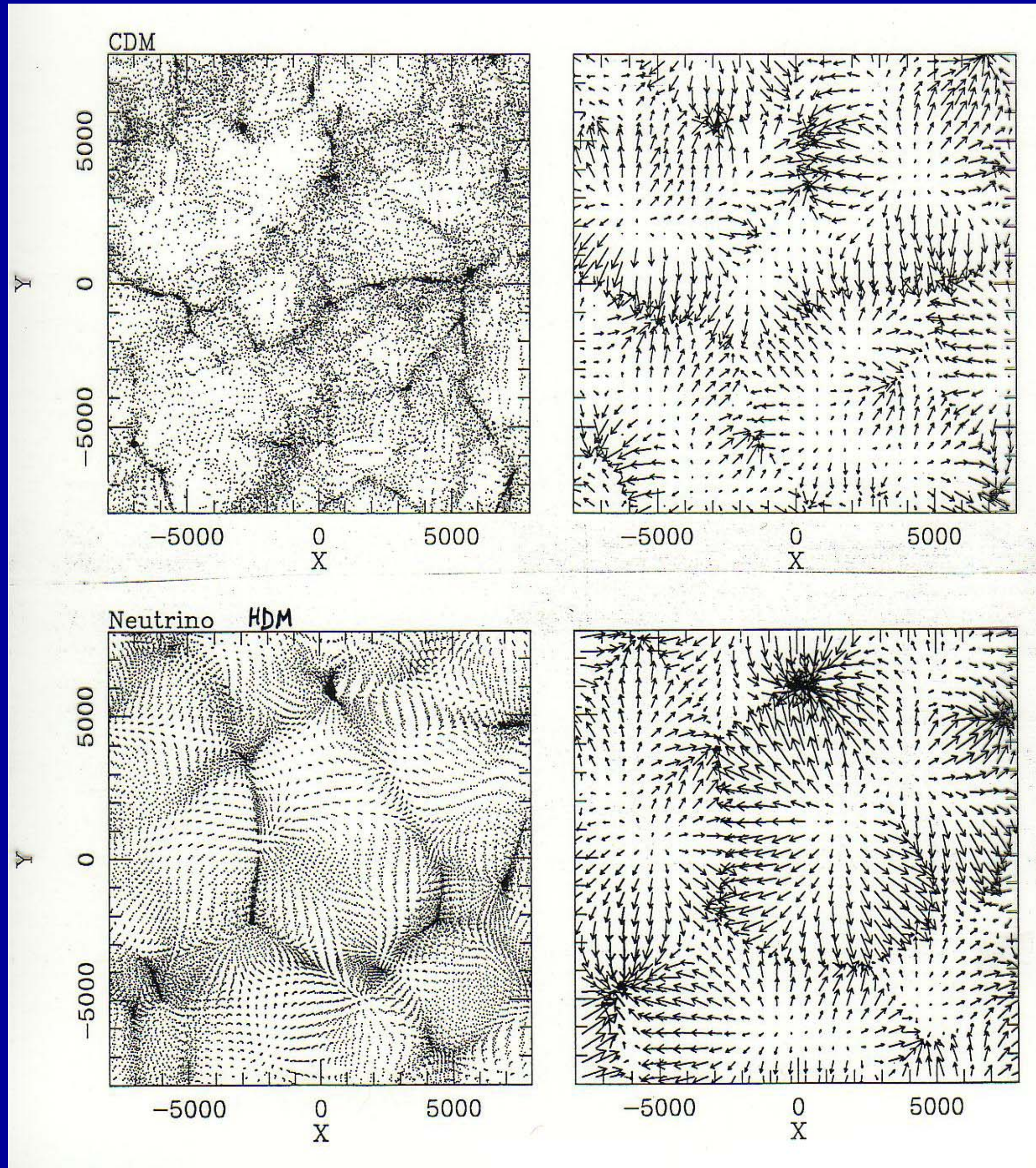


Micro-Macro Connection

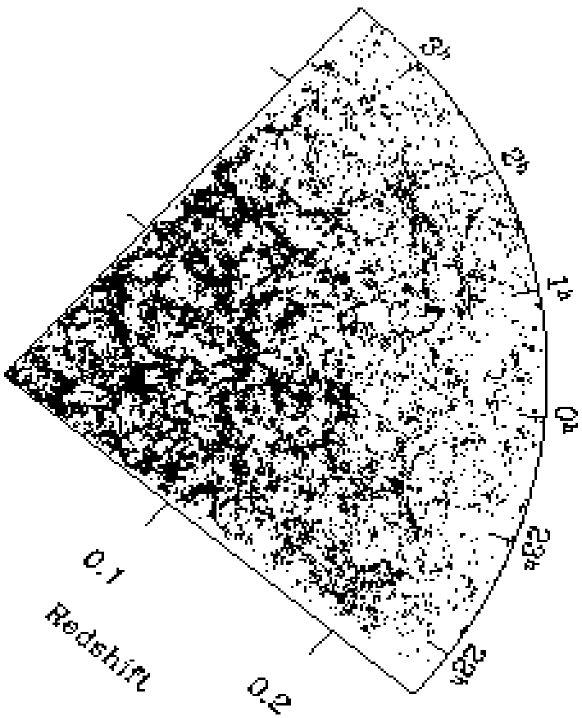
Cold Dark Matter

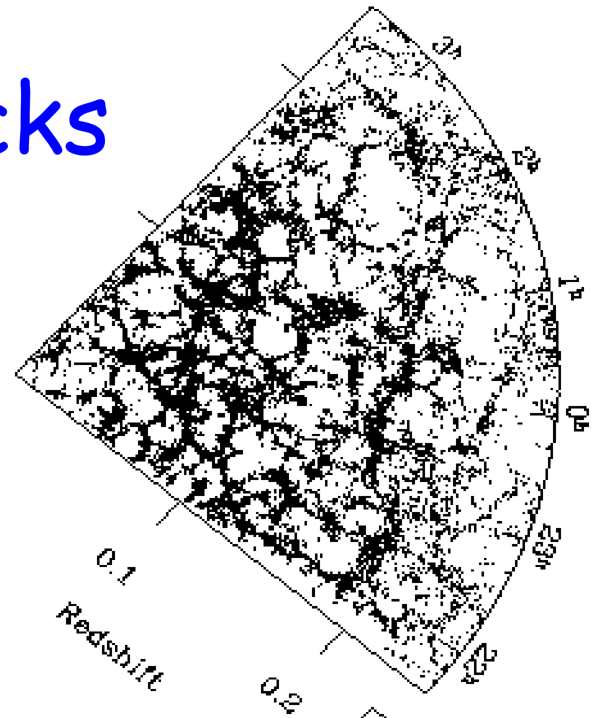
Hot Dark Matter

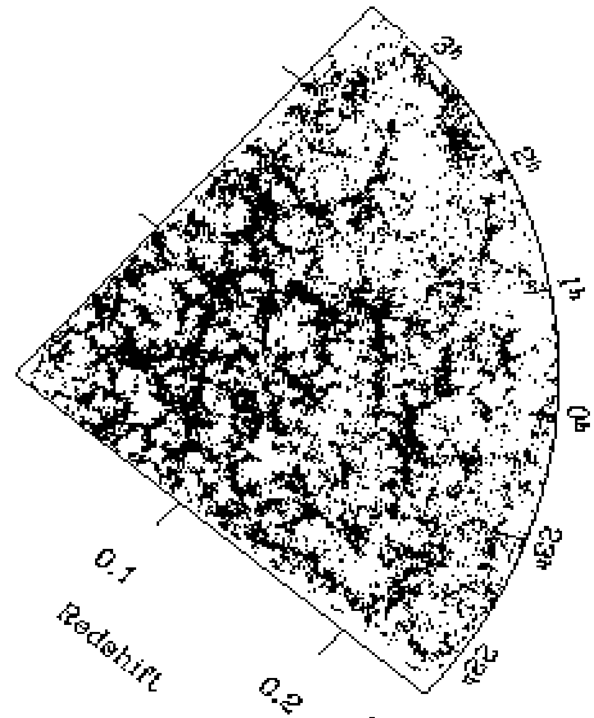
ν

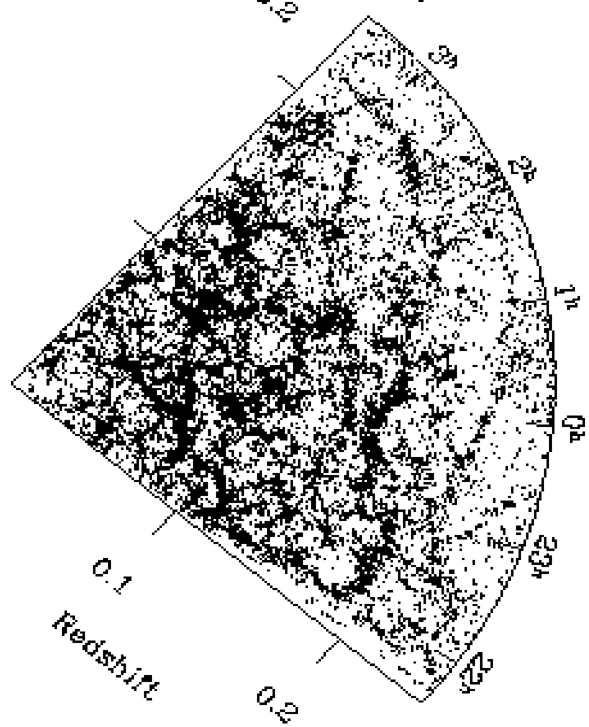


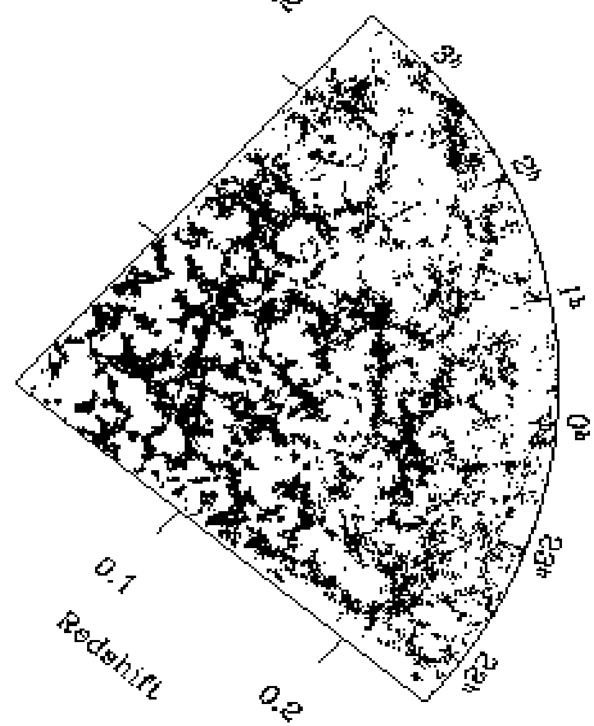
2dF and Mocks



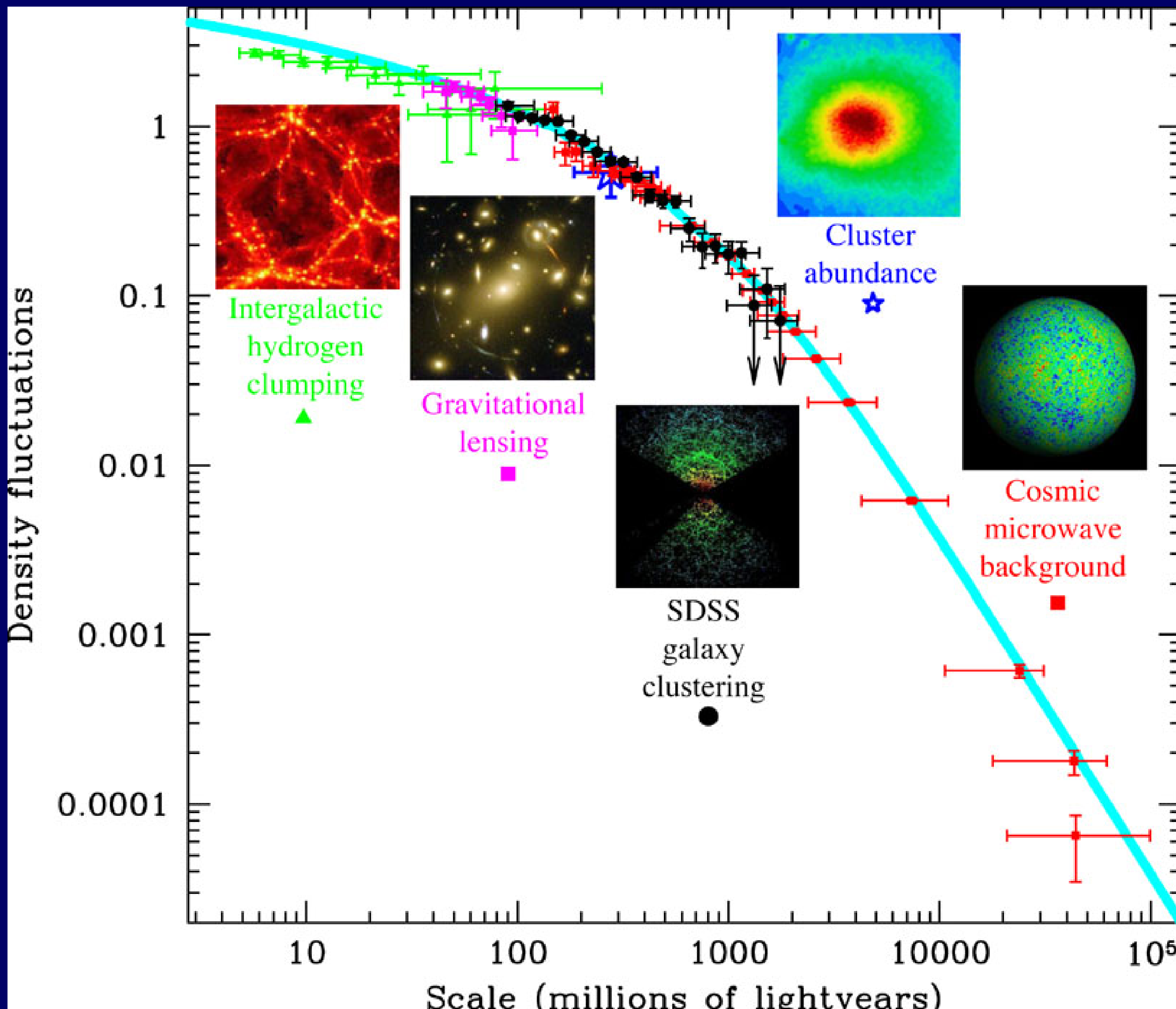








Power Spectrum



ΛCDM Power Spectrum

$$P(k) \propto k\,T^2(k)$$

$$T(k) = \frac{\ln(1+2.34q)}{2.34q}\left(1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right)^{-1/4} \qquad q = \frac{k}{\Omega_m h^2 Mpc^{-1}}$$

normalization: $\sigma_8 \equiv \sigma_{tophat}(R = 8h^{-1}Mpc)$