Homework Set 2
DUE: Thursday May 5

1. One version of the "tired light" hypothesis states that the universe is not expanding, but that photons lose energy per unit distance

$$\frac{dE}{dr} = -KE\,.$$

Show that this hypothesis gives a distance-redshift relation that is linear in the limit $z \ll 1$. What value of K gives a Hubble constant $h = 0.7$? Give some arguments against the "tired light" hypothesis?

2. Consider a "power-law" cosmology, where the scale factor $a(t) = (t/t_0)^n$ where n is some number. For example, for the Einstein-de Sitter cosmology, $n = 2/3$.

(a) Verify that the Hubble radius is $d_H \equiv c/H_0 = ct_0/n$. Consider a galaxy that radiated at emission redshift z_e, corresponding to scale factor $a_e = 1/(1+z_e)$, light that we see today. Using

$$d_p(t_0) = r_e = \int_{t_e}^{t_0} \frac{dt}{a(t)} \quad ,$$

show that the proper distance today to this galaxy is

$$d_p(t_0) = \frac{n}{1-n} d_H (1 - {a_e}^{(1-n)/n})$$

and that the proper distance at the time of emission was

$$d_p(t_e) = a_e r_e \quad .$$

Note that the emission distance $d_p(t_e)$ vanishes for $t_e = 0$ (the Big Bang) and for $t_e = t_0$. Check that these formulas agree with the usual results for the E-dS case.

(b) Show using Hubble's law that the velocity of this galaxy away from us today (i.e., at $t = t_0$) is

$$v(t_0) = \frac{nc}{1-n}(1 - {a_e}^{(1-n)/n})$$

and that the galaxy's velocity at the time of emission t_e was

$$v(t_e) = \frac{nc}{1-n}({a_e}^{(n-1)/n} - 1) \quad .$$

Note that when z_e is small (i.e., a_e is near 1), both recession velocities reduce to $v \approx cz_e$, as expected. (A graph of these velocities vs. redshift for the E-dS case was shown in class.)

(c) Show that the distance to the particle horizon is $d_H n/(1-n)$. Show that when $n > 1/2$ the radius of the observable universe is larger than the Hubble radius, and that in the limit $n?1$ there is no particle horizon. (The case $n = 1$ corresponds to an empty universe, also called the Milne cosmology.)

(d) Show that Hubble's law implies that the velocity at the particle horizon is $cn/(1-n)$. Show that the velocity of the particle horizon itself is $c/(1-n)$, and that this means that the particle horizon sweeps past the galaxies at the particle horizon at the speed of light.

3. Short calculations:

(a) If a neutrino has mass m_ν and decouples at $T_{\nu d} \sim 1$ MeV, show that the contribution of this neutrino and its antiparticle to the cosmic density today is (Dodelson Eq. 2.80)

$$\Omega_\nu = \frac{m_\nu}{94 h^2 \mathrm{eV}} \quad .$$

(b) Verify that $\eta_b \equiv n_b/n_\gamma$ is given by (Dodelson Eq. 3.11; Weinberg, *Cosmology*, pp. 168-169)

$$\eta_b = 5.5 \times 10^{-10} \left(\frac{\Omega_b h^2}{0.020} \right) \quad .$$

(c) Verify the time-temperature relation (Dodelson Eq. 3.30)

$$t = 132 \,\mathrm{sec}\, (0.1 \mathrm{MeV}/T)^2 \quad .$$

(d) Calculate the redshift of matter-radiation equality z_{eq} in terms of Ω_m and h. Assume that the photon temperature today is $T_\gamma = 2.73$K and use the fact, derived in class (and Weinberg, *Cosmology*, Eq. 3.1.21), that the total energy density in radiation (i.e., photons and three species of neutrinos of negligible mass) after e^+e^- annihilation is $\rho_r = 1.68\rho_\gamma$, where ρ_γ is the photon energy density.

4. It is possible that the universe contains a quantum field called "quintessence" which in the simplest version has an equation of state parameter $w_Q = p_Q/\rho_Q$ with energy density ρ_Q positive (of course) but pressure p_Q negative. Suppose that the universe contains nothing but pressureless matter, i.e. with $w_m = 0$, and quintessence, with $w_Q = -3/4$. The current density parameter of matter is $\Omega_m \approx 0.3$ and that of quintessence is $\Omega_Q = 1 - \Omega_m$. At what scale factor a_{mQ} will the energy density of quintessence and matter be equal? Solve the Friedmann equation to find $a(t)$ for this universe. What is $a(t)$ in the limit $a \gg a_{mQ}$? What is the current age of the universe, expressed in terms of H_0 and $\Omega_{m,0}$?

5. Suppose that the neutron decay time were $\tau_n = 1890$ s instead of $\tau_n = 890$ s, with all other physical parameters unchanged. Estimate Y_p, the primordial mass fraction of nucleons in ^{4}He, assuming for simplicity that all available neutrons are incorporated into ^{4}He.