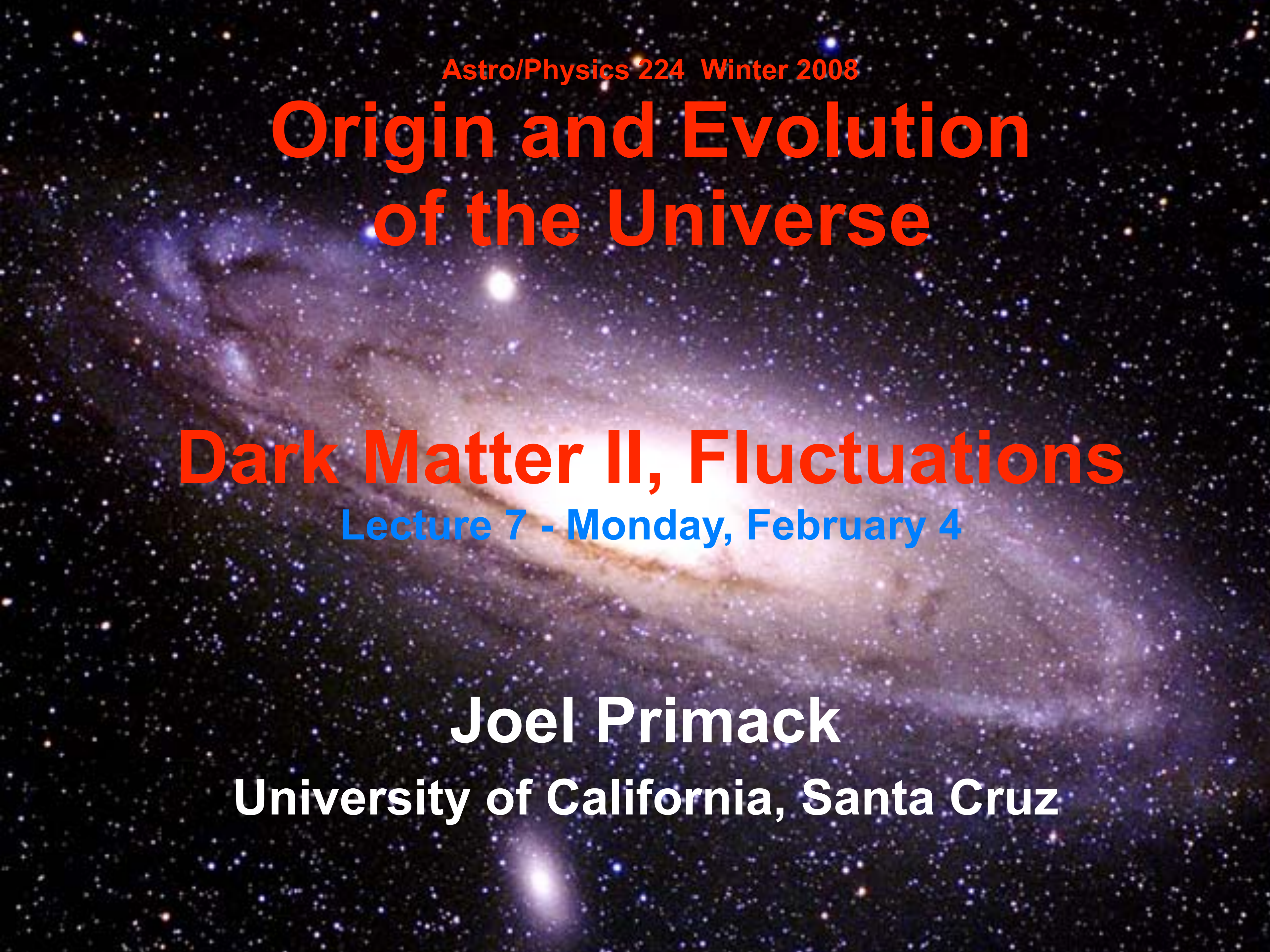

Astro/Physics 224 Winter 2008

Origin and Evolution of the Universe

Dark Matter II, Fluctuations

Lecture 7 - Monday, February 4

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Physical Constants for Cosmology

parsec	pc	$= 3.09 \times 10^{18}$ cm $= 3.26$ light years (lyr)
Newton's const.	G	$= 6.67 \times 10^{-8}$ dyne cm^2 g^{-2}
Hubble parameter	H_0	$= 100\,h$ km s^{-1} Mpc^{-1} , $h \approx 0.7$
Hubble time	H_0^{-1}	$= h^{-1}$ 9.78 Gyr
Hubble radius	R_H	$= cH^{-1} = 3.00\,h^{-1}$ Gpc
critical density	ρ_c	$= 3H^2/8\pi G = 1.88 \times 10^{-29} h^2$ g cm^{-3}
		$= 10.5\,h^2$ keV cm^{-3} $= 2.78 \times 10^{11} h^2\, M_\odot$ Mpc^{-3}
speed of light	c	$= 3.00 \times 10^{10}$ cm s^{-1} $= 306$ Mpc Gyr^{-1}
solar mass	$M_\odot$	$= 1.99 \times 10^{33}$ g
solar luminosity	$L_\odot$	$= 3.85 \times 10^{33}$ erg s^{-1}
Planck's const.	$\hbar$	$= 1.05 \times 10^{-27}$ erg s $= 6.58 \times 10^{-16}$ eV s
Planck mass	$m_{P\ell}$	$= (\hbar c/G)^{1/2} = 2.18 \times 10^{-5}$ g $= 1.22 \times 10^{19}$ GeV

Some Dark Matter Candidates

Candidates		
axion SUSY LSP neutralino technibaryon pseudo Higgs ⋮ shadow matter topological relics non-top. solitons	Weakly Interacting Massive Particles	COLD DARK MATTER
Primordial BH jupiters brown dwarfs white dwarfs neutron stars stellar BH massive BH	Massive Astrophysical Compact Halo Objects	COLD DARK MATTER
gravitino right-handed ν	WARM DM	
decaying dark matter ⋮	VOLATILE DM	
neutrinos ν_e ν_μ ν_τ (ν_s?) majorons?	HOT DARK MATTER	

Types of Dark Matter

Ω_i represents the fraction of the critical density $\rho_c = 10.54\ h^2$ keV/cm^3 needed to close the Universe, where h is the Hubble constant H_0 divided by 100 km/s/*Mpc*.

Dark Matter Type	Fraction of Critical Density	Comment
Baryonic	$\Omega_b \sim 0.04$	about 10 times the visible matter
Hot	$\Omega_\nu \sim 0.001–0.1$	light neutrinos
Cold	$\Omega_c \sim 0.3$	most of the dark matter in galaxy halos

Dark Matter and Associated Cosmological Models

Ω_m represents the fraction of the critical density in all types of matter.
Ω_Λ is the fraction contributed by some form of "dark energy."

Acronym	Cosmological Model	Flourished
HDM	hot dark matter with $\Omega_m = 1$	1978–1984
SCDM	standard cold dark matter with $\Omega_m = 1$	1982–1992
CHDM	cold + hot dark matter with $\Omega_c \sim 0.7$ and $\Omega_\nu = 0.2–0.3$	1994–1998
ΛCDM	cold dark matter $\Omega_c \sim 1/3$ and $\Omega_\Lambda \sim 2/3$	1996–today

Joel Primack, *Beam Line*, Fall 2001

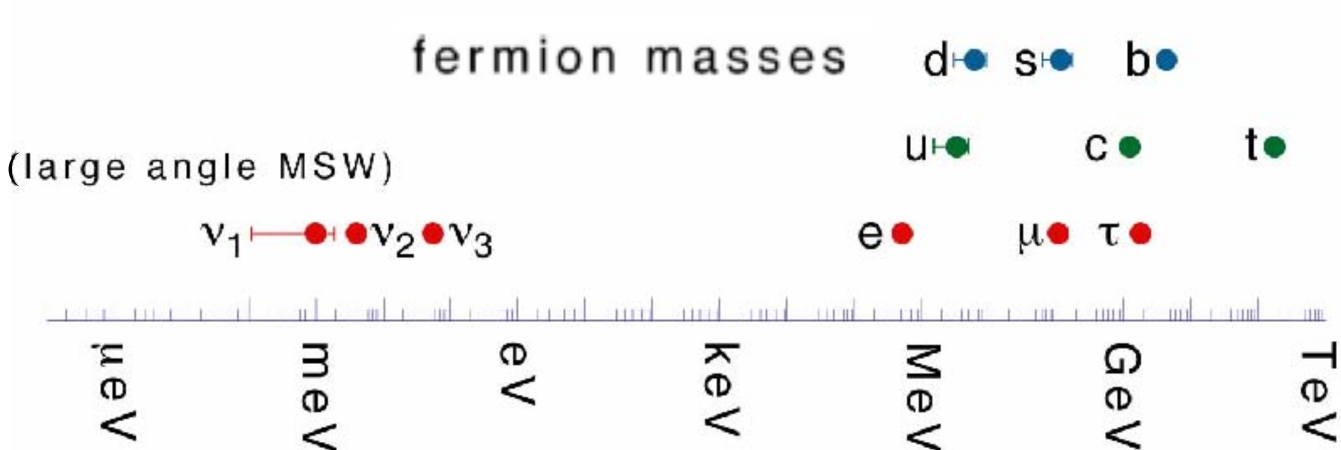
THE ATMOSPHERIC-NEUTRINO DATA from the Super-Kamiokande underground neutrino detector in Japan provide strong evidence of muon to tau neutrino oscillations, and therefore that these neutrinos have nonzero mass (see the article by John Learned in the Winter 1999 *Beam Line*, Vol. 29, No. 3). This result is now being confirmed by results from the K2K experiment, in which a muon neutrino beam from the KEK accelerator is directed toward Super-Kamiokande and the number of muon neutrinos detected is about as expected from the atmospheric-neutrino data (see article by Jeffrey Wilkes and Koichiro Nishikawa, this issue).

But oscillation experiments cannot measure neutrino masses directly, only the squared mass difference $\Delta m^2_{ij} = |m_i^2 - m_j^2|$ between the oscillating species. The Super-Kamiokande atmospheric neutrino data imply that $1.7\times10^{-4} < \Delta m^2_{\tau\mu} < 4\times10^{-3}$ eV^2 (90 percent confidence), with a central value $\Delta m^2_{\tau\mu} = 2.5\times10^{-3}$ eV^2. If the neutrinos have a hierarchical mass pattern $m_{\nu_e} << m_{\nu_\mu} << m_{\nu_\tau}$ like the quarks and charged leptons, then this implies that $\Delta m^2_{\tau\mu} \cong m^2_{\nu_\tau}$ so $m_{\nu_\tau} \sim 0.05$ eV.

These data then imply a lower limit on the HDM (or light neutrino) contribution to the cosmological matter density of $\Omega_\nu > 0.001$—almost as much as that of all the stars in the disks of galaxies. There is a connection between neutrino mass and the corresponding contribution to the cosmological density, because the thermodynamics of the early Universe specifies the abundance of neutrinos to be about 112 per cubic centimeter for each of the three species (including both neutrinos and antineutrinos). It follows that the density Ω_ν contributed by neutrinos is $\Omega_\nu = m(\nu)/(93\, h^2$ eV), where $m(\nu)$ is the sum of the masses of all three neutrinos. Since $h^2 \sim 0.5$, $m_{\nu_\tau} \sim 0.05$ eV corresponds to $\Omega_\nu \sim 10^{-3}$.

This is however a lower limit, since in the alternative case where the oscillating neutrino species have nearly equal masses, the values of the individual masses could be much larger. The only other laboratory approaches to measuring neutrino masses are attempts to detect neutrinoless double beta decay, which are sensitive to a possible Majorana component of the electron neutrino mass, and measurements of the endpoint of the tritium beta-decay spectrum. The latter gives an upper limit on the electron neutrino mass, currently taken to be 3 eV. Because of the small values of both squared-mass differences, this tritium limit becomes an upper limit on all three neutrino masses, corresponding to $m(\nu) < 9$ eV. A bit surprisingly, cosmology already provides a stronger constraint on neutrino mass than laboratory measurements, based on the effects of neutrinos on large-scale structure formation.

Joel Primack, *Beam Line*, Fall 2001



Hitoshi Murayama

Neutrino Properties

See the note on "Neutrino properties listings" in the Particle Listings.

Mass $m < 2$ eV (tritium decay)
Mean life/mass, $\tau/m > 300$ s/eV, CL = 90% (reactor)
Mean life/mass, $\tau/m > 7 \times 10^9$ s/eV (solar)
Mean life/mass, $\tau/m > 15.4$ s/eV, CL = 90% (accelerator)
Magnetic moment $\mu < 0.9 \times 10^{-10}\ \mu_B$, CL = 90% (reactor)

Number of Neutrino Types

Number $N = 2.994 \pm 0.012$ (Standard Model fits to LEP data)
Number $N = 2.93 \pm 0.05$ (S = 1.2) (Direct measurement of invisible Z width)

Neutrino Mixing

The following values are obtained through data analyses based on the 3-neutrino mixing scheme described in the review "Neutrino mass, mixing, and flavor change" by B. Kayser in this *Review*.

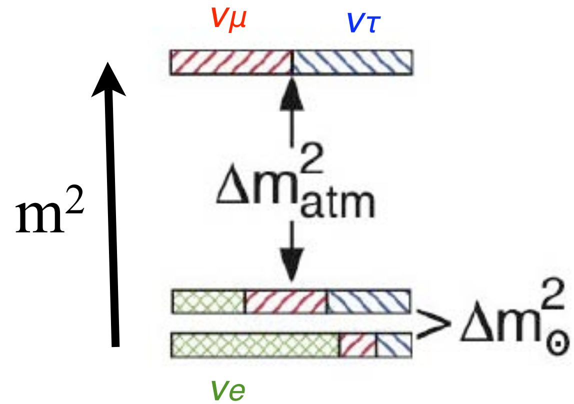
$\sin^2(2\theta_{12}) = 0.86^{+0.03}_{-0.04}$
$\Delta m^2_{21} = (8.0^{+0.4}_{-0.3}) \times 10^{-5}$ eV^2

The ranges below for $\sin^2(2\theta_{23})$ and Δm^2_{32} correspond to the projections onto the appropriate axes of the 90% CL contours in the $\sin^2(2\theta_{23})$-Δm^2_{32} plane.

$\sin^2(2\theta_{23}) > 0.92$
$\Delta m^2_{32} = 1.9$ to 3.0×10^{-3} eV^2 [i]

$\sin^2(2\theta_{13}) < 0.19$, CL = 90%

Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) (URL: http://pdg.lbl.gov)

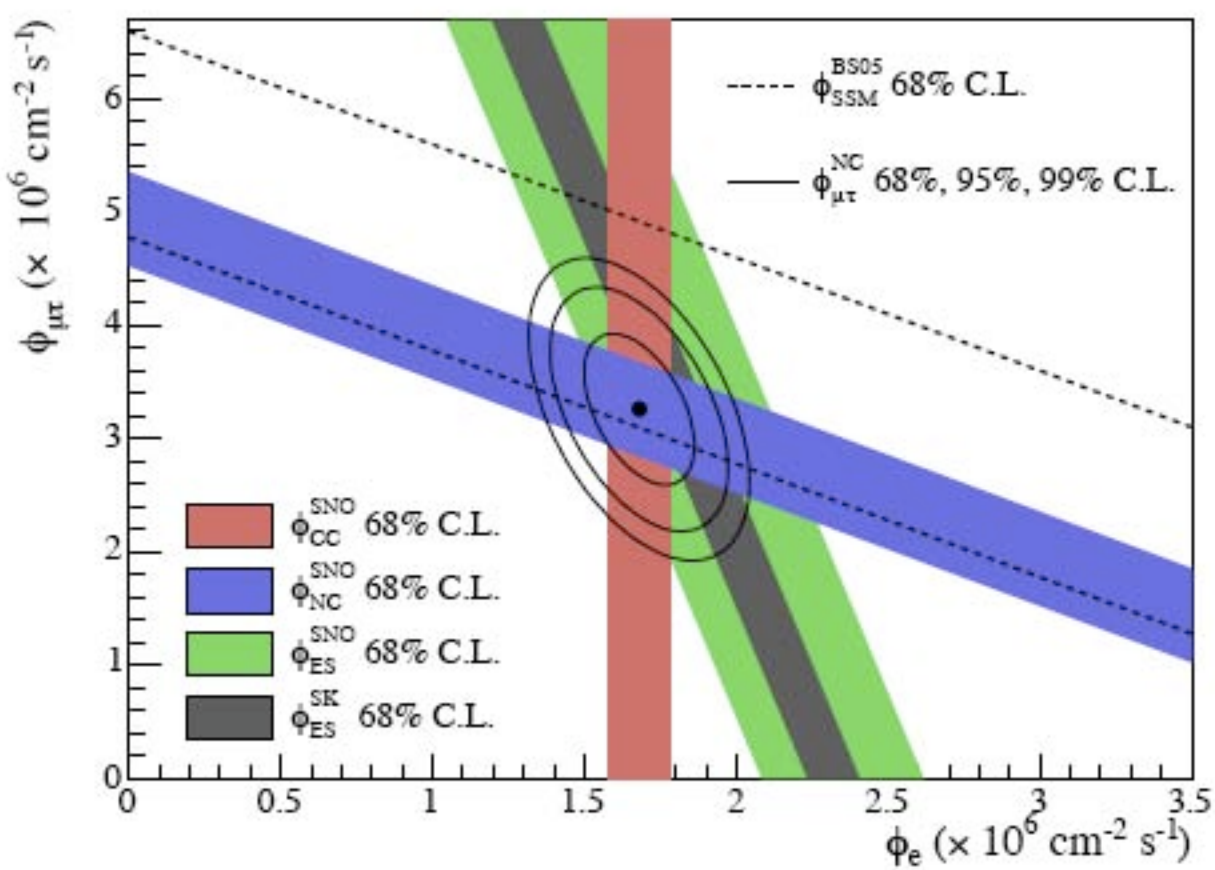


A three-neutrino squared-mass spectrum that accounts for the observed flavor changes of solar, reactor, atmospheric, and long-baseline accelerator neutrinos. The ν_e fraction of each mass eigenstate is crosshatched, the ν_μ fraction is indicated by right-leaning hatching, and the ν_τ fraction by left-leaning hatching. From B. Kaiser, http://pdg.lbl.gov/2007/reviews/

numixrpp.pdf

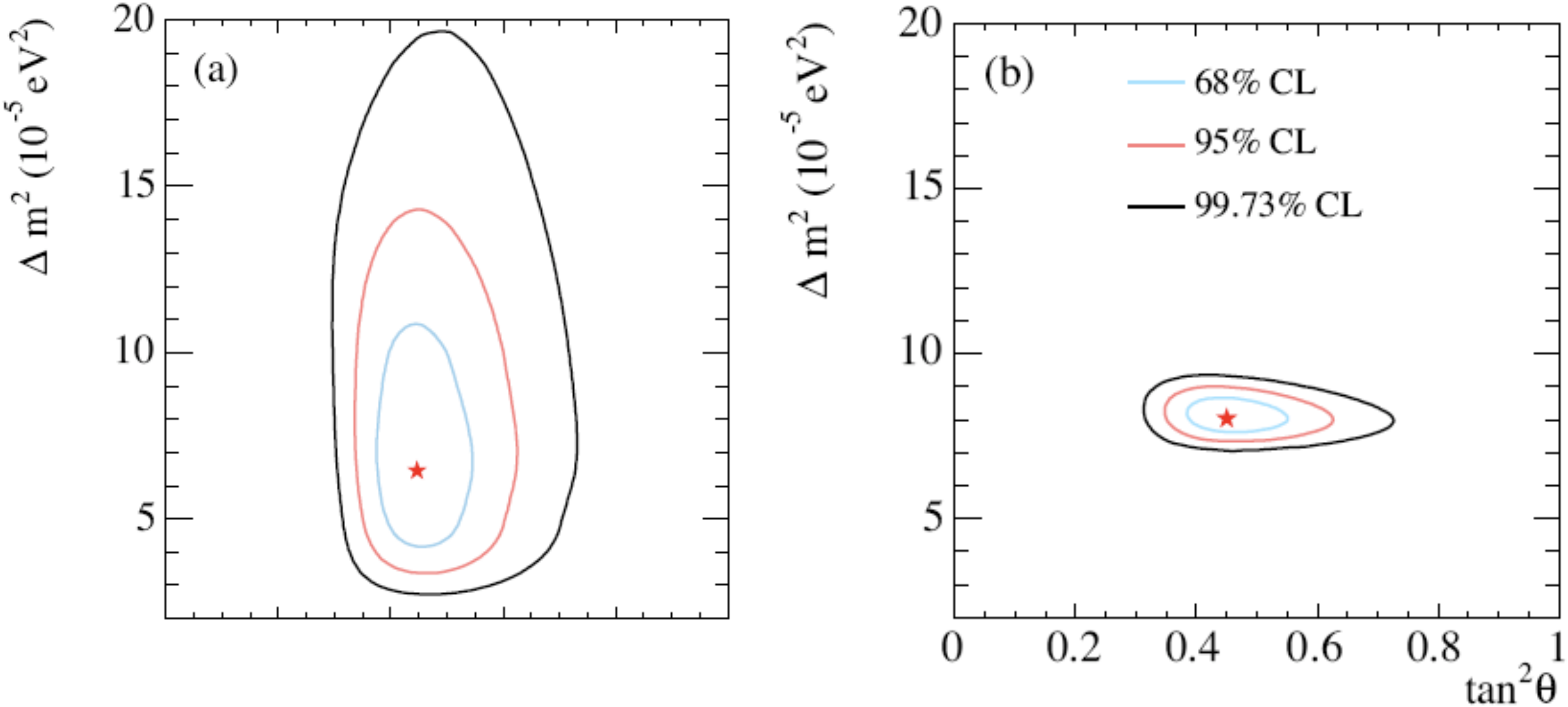
Sudbury Neutrino Observatory Confirms Solar Neutrinos Oscillate

$n \rightarrow p\, e^- \bar{\nu}_e$ must happen twice per ^{4}He, and then ~1/3 of the electron antineutrinos oscillate to mu or tau neutrinos



Fluxes of ^{8}B solar neutrinos, $\phi(\nu_e)$,and $\phi(\nu_{\mu \text{ or } \tau})$, deduced from the SNO's charged current (CC), ν_e elastic scattering (ES), and neutral-current (NC) results for the salt phase measurement. The Super-Kamiokande ES flux and the BS05(OP) standard solar model prediction are also shown. The bands represent the 1σ error. The contours show the 68%, 95%, and 99% joint probability for $\phi(\nu_e)$ and $\phi(\nu_{\mu \text{ or } \tau})$.

[From PDG 2005 review by K. Nakamura.]

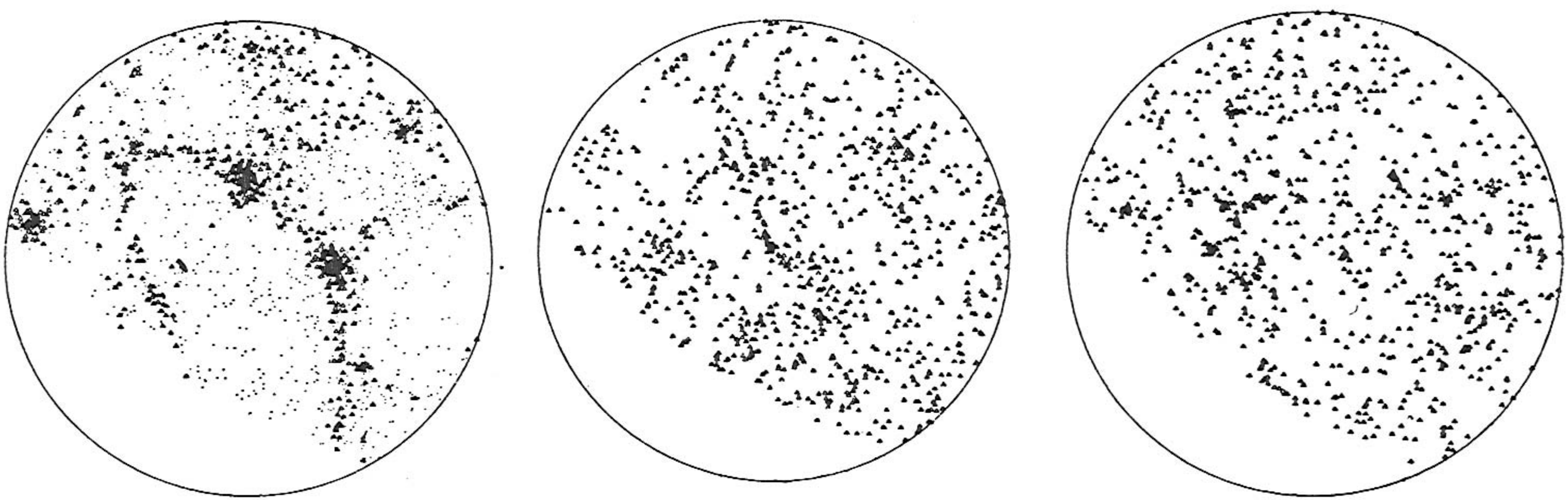


Update of the global neutrino oscillation contours given by the SNO Collaboration assuming that the ^8B neutrino flux is free and the *hep* neutrino flux is fixed. (a) Solar global analysis. (b) Solar global + KamLAND. [From PDG 2005 review by K. Nakamura.]

$$\Delta m_{12}{}^2 = 8\text{x}10^{-5}\ \text{eV}^2 \Rightarrow m_2 \geq 9\text{x}10^{-3}\ \text{eV}$$

Whatever Happened to Hot Dark Matter?

In ~1980, when purely baryonic adiabatic fluctuations were ruled out by the improving upper limits on CMB anisotropies, theorists led by Zel'dovich turned to what we now call the HDM scenario, with light neutrinos making up most of the dark matter. However, in this scheme the fluctuations on small scales are damped by relativistic motion ("free streaming") of the neutrinos until T becomes less than m_ν, which occurs when the mass entering the horizon is about 10^{15} solar masses, the supercluster mass scale. Thus superclusters would form first, and galaxies later by fragmentation. This predicted a galaxy distribution much more inhomogeneous than observed.



HDM Observed Galaxy Distribution CDM

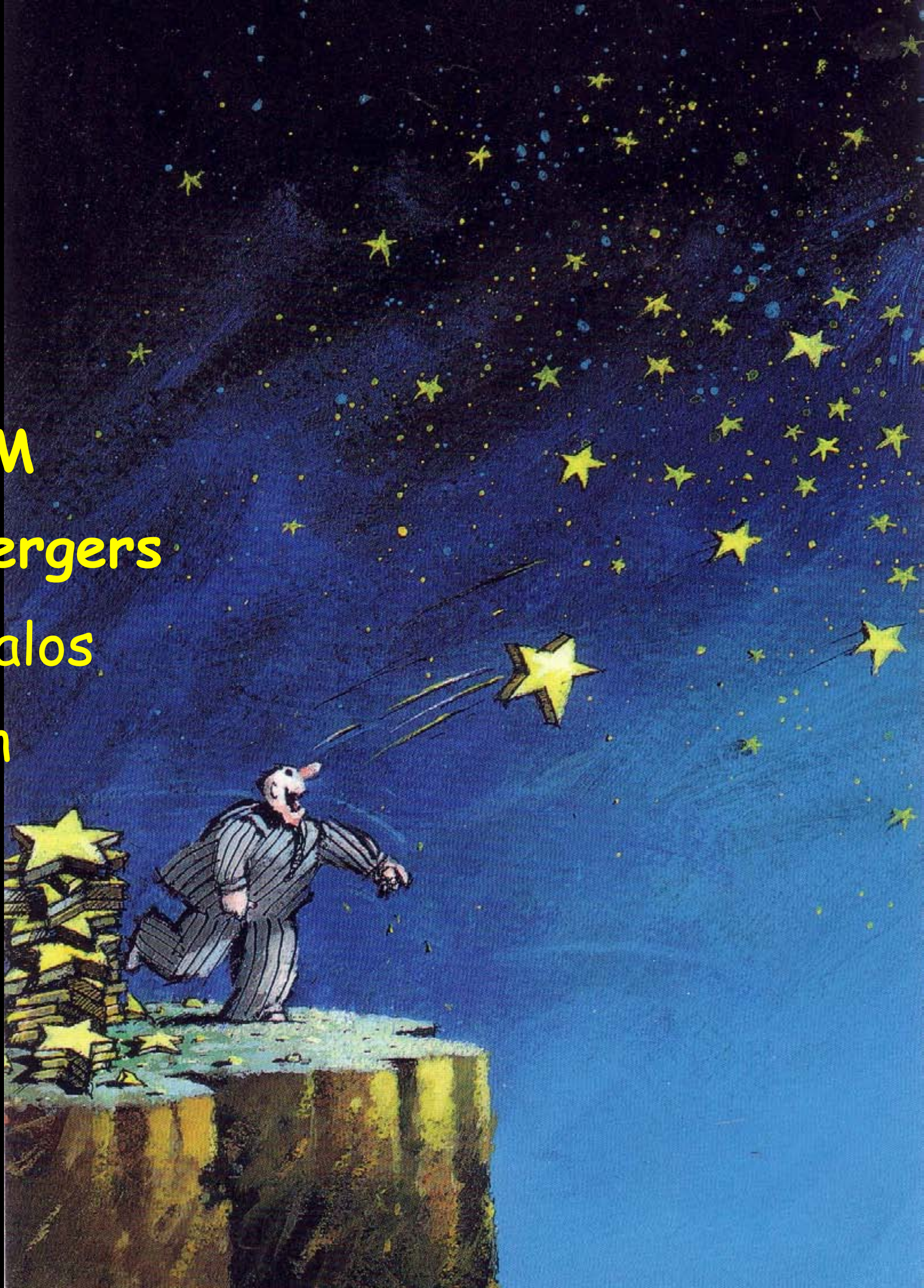
White 1986

Since 1984, the most successful structure formation scenarios have been those in which most of the matter is CDM. With the COBE CMB data in 1992, two CDM variants appeared to be viable: ΛCDM with $\Omega_m \approx 0.3$, and $\Omega_m = 1$ Cold+Hot DM with $\Omega_\nu \approx 0.2$. A potential problem with CHDM was that, like all $\Omega_m = 1$ theories, it predicted rather late structure formation. A potential problem with ΛCDM was that the correlation function of the dark matter was higher around 1 Mpc than the power-law $\xi_{gg}(r) = (r/r_0)^{-1.8}$ observed for galaxies, so "scale-dependent anti-biasing" was required (Klypin, Primack, & Holtzman 1996, Jenkins et al. 1998). When better ΛCDM simulations could resolve halos that could host galaxies, they were found to have the same correlations as observed for galaxies.

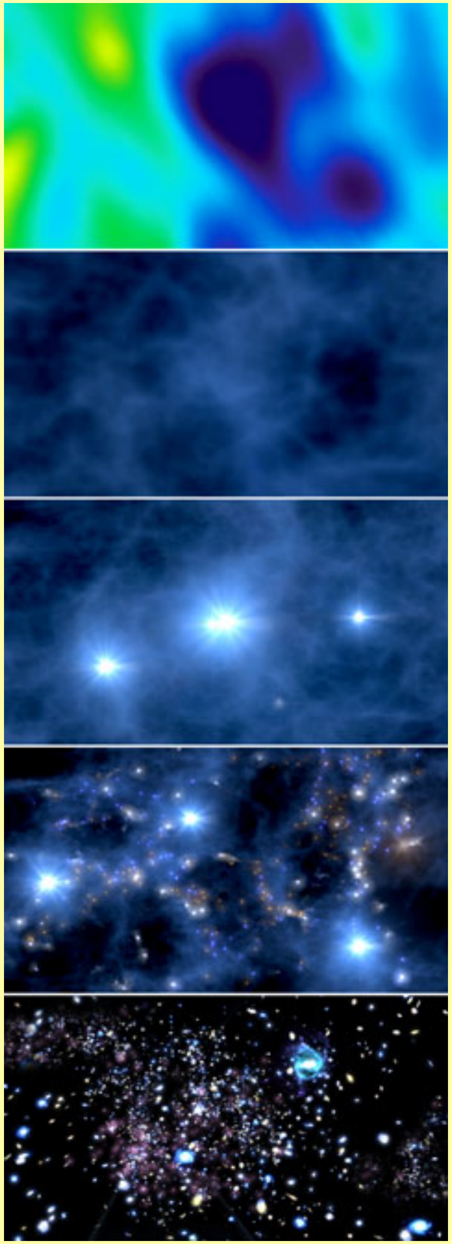
By 1998, the evidence of early galaxy and cluster formation and the increasing evidence that $\Omega_m \approx 0.3$ had doomed CHDM. But now we also know from neutrino oscillations that neutrinos have mass. The upper limit from cosmology is $\Omega_\nu h^2 < 0.002$, corresponding to $m_\nu < 0.17$ eV (95% CL) for the most massive neutrino (Seljak et al. 2006).

Outline of Next Few Lectures

1. **growth of structure: CDM**
2. **dark halos: collapse & mergers**
3. luminous galaxies in dark halos
4. puzzles in galaxy formation



Late Cosmological Epochs



380 kyr z~1000 — recombination last scattering

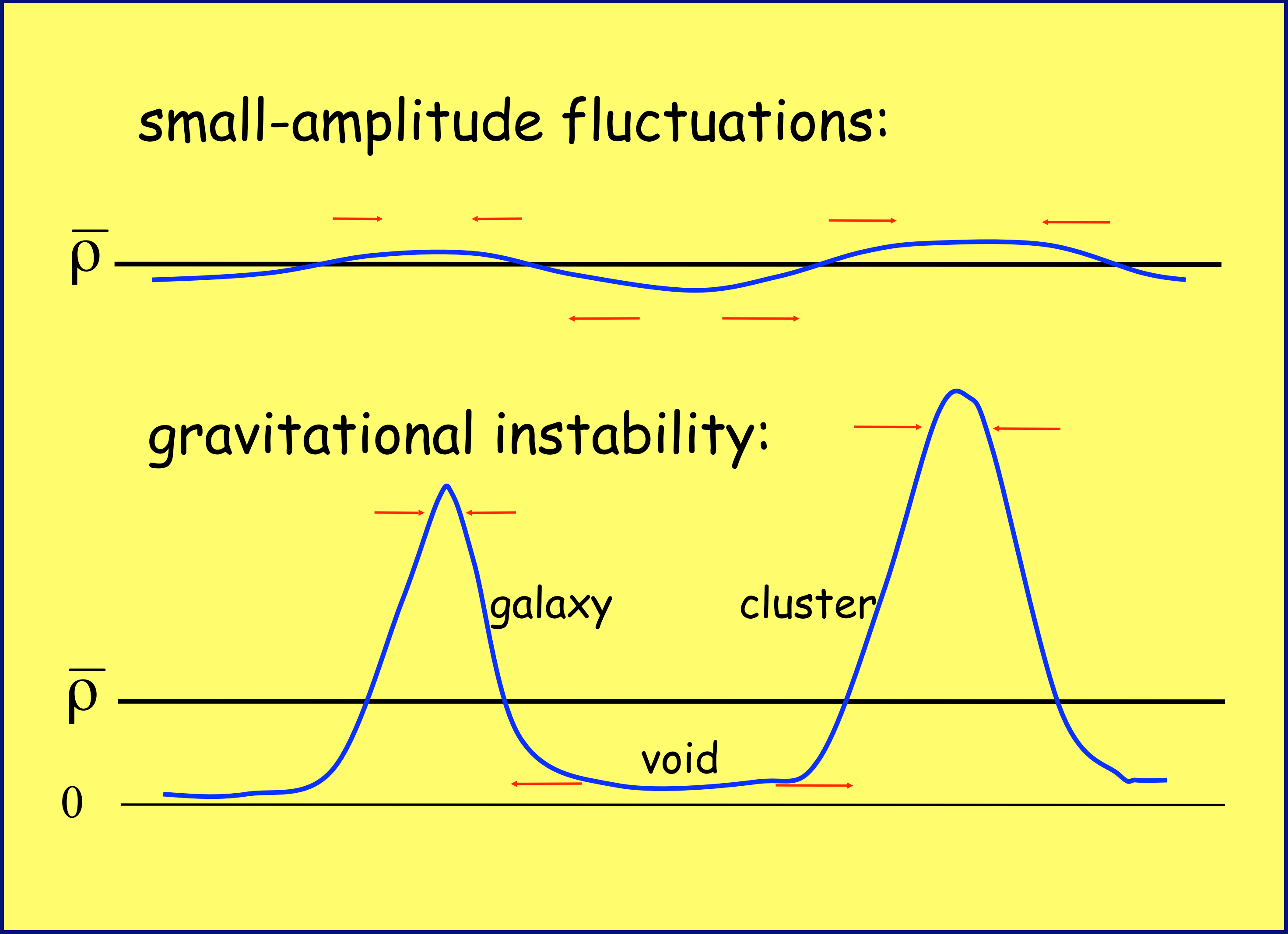
dark ages

180 Myr z~20 — first stars reionization

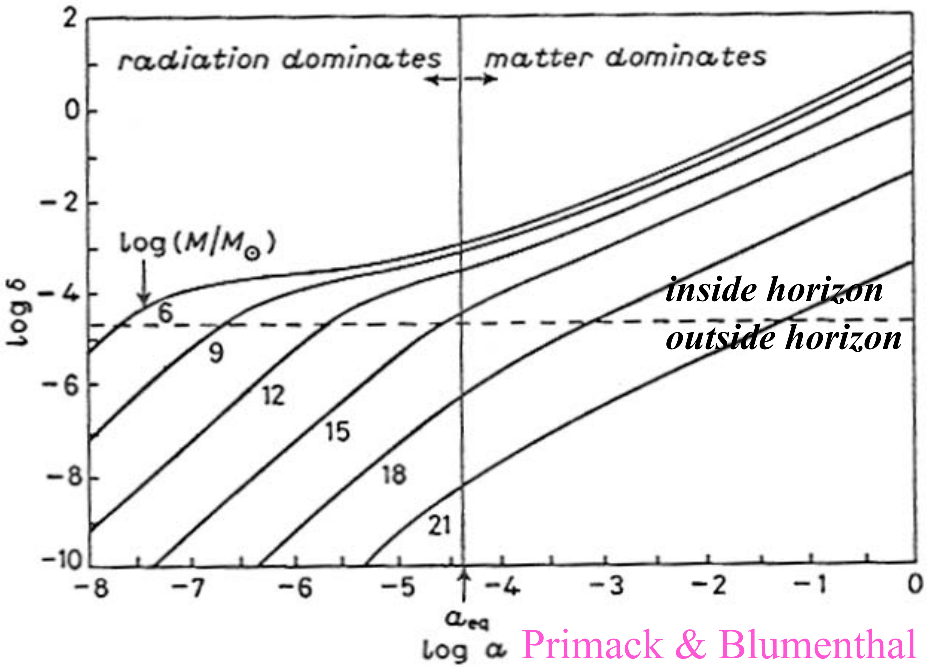
galaxy formation

13.7 Gyr z=0 — today

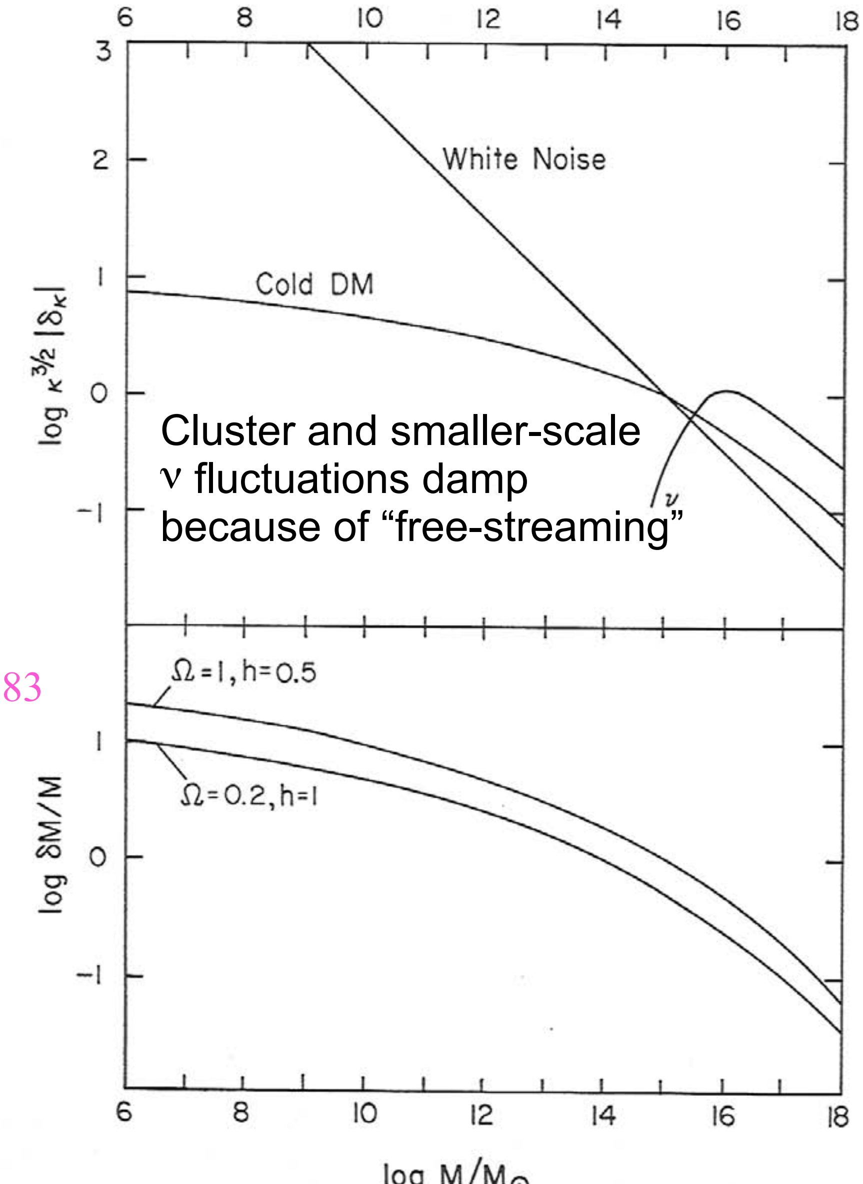
Gravitational instability



CDM Structure Formation: Linear Theory



Primack & Blumenthal 1983



Matter fluctuations that enter the horizon during the radiation dominated era, with masses less than about $10^{15} M_{\odot}$, grow only $\propto \log a$, because they are not in the gravitationally dominant component. But matter fluctuations that enter the horizon in the matter-dominated era grow $\propto a$. This explains the characteristic shape of the CDM fluctuation spectrum, with $\delta(k) \propto k^{-n/2-2} \log k$ for $k \gg k_{eq}$. Blumenthal, Faber, Primack, & Rees 1984

GROWTH OF THE SCALE FACTOR a(t) = R(t)

CONTINUITY EQ $\frac{\partial}{\partial R}(\rho R^3) = -3pR^2$

PLUS EQ OF STATE $p = w\rho \Rightarrow \rho \propto R^{-3(1+w)}$

+ FRIEDMANN EQ $\frac{\dot{R}^2}{R^2} = \frac{8}{3}\pi G\rho - \frac{k}{R^2} + \frac{\Lambda}{3}$ NEGLIGIBLE AT $t \ll t_0$

$\Rightarrow R \propto t^{2/3(1+w)}$

STANDARD CASES

RADIATION ERA $w = \frac{1}{3}$, $\rho \propto R^{-4}$, $R \propto t^{1/2}$

MATTER ERA $w = 0$, $\rho \propto R^{-3}$, $R \propto t^{2/3}$

CROSSOVER (R_{eq}: RADIATION-MATTER EQUALITY) AT

$$R_{eq} = 4.05 \times 10^{-5} (\Omega h^2)^{-1} \theta^4 \quad (\theta \equiv T/2.7K)$$

It is also possible to obtain a simple expression for $t(R)$ that is valid in both radiation- and matter-dominated eras, for the case of a flat universe (i.e., $k = 0$). Simply integrate the Einstein equation (2.8) with

$$\rho = \rho_{rel} + \rho_{nonrel} \approx \rho_{c,o}\Omega_o(R_{eq}R^{-4} + R^{-3}), \tag{2.45}$$

The result is

$$t = \frac{2}{3}H_o^{-1}\Omega_o^{-1/2}\left[(R - 2R_{eq})(R + R_{eq})^{1/2} + 2R_{eq}^{3/2}\right], \tag{2.46}$$

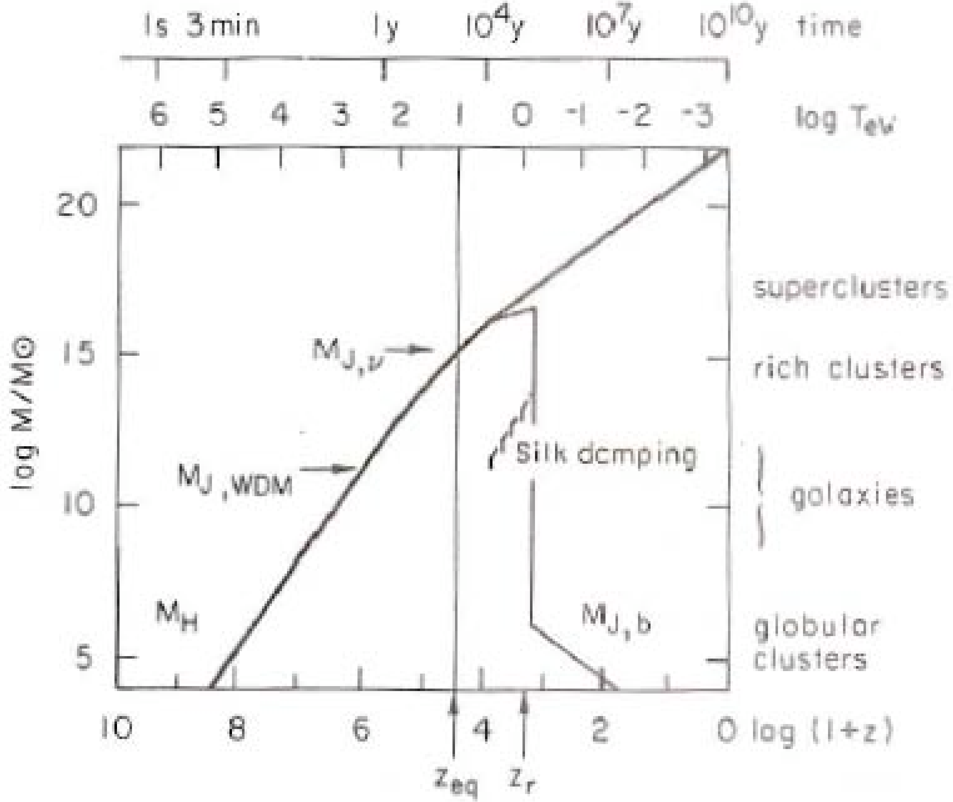
with the following limiting behaviors:

$$\begin{aligned} R \ll R_{eq}: &\qquad t \approx \frac{1}{2}H_o^{-1}\Omega_o^{-1/2}R_{eq}^{-1/2}R^2 \\ R = R_{eq}: &\qquad t_{eq} = 0.3905 H_o^{-1}\Omega_o^{-1/2}R_{eq}^{3/2} \\ R \gg R_{eq}: &\qquad t \approx \frac{2}{3}H_o^{-1}\Omega_o^{-1/2}R^{3/2}. \end{aligned} \tag{2.47}$$

It is now easy to calculate the mass M_H of nonrelativistic matter encompassed by the horizon $ct(R)$ as a function of scale factor R:

$$M_H = \frac{4}{3}\pi c^3 t^3 \frac{\rho_{c,o}\Omega_o}{R^3} = \frac{2.41 \times 10^{15} M_\odot}{\Omega_o^2 h^4}\left[\frac{(y-2)(y+1)^{1/2}+2}{y}\right]^3, \tag{2.48}$$

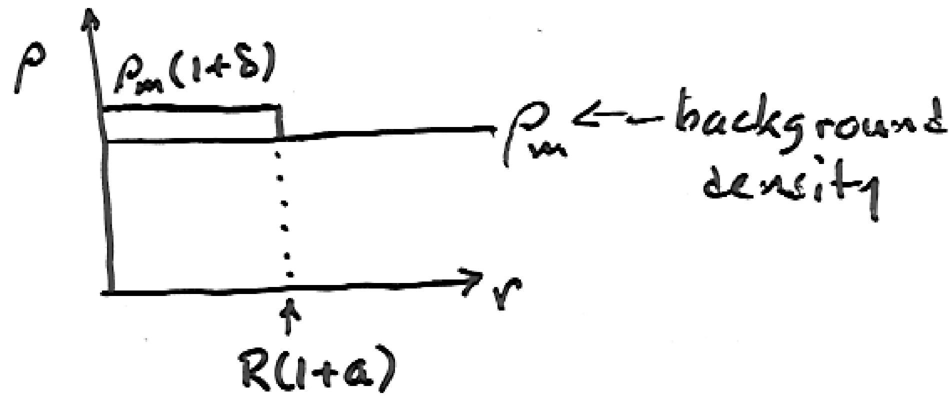
where $y \equiv R/R_{eq}$. The behavior of M_H is sketched in Fig. 2.12 (heavy solid curve).



Primack, 1984 Varenna Summer School Lectures

FLUCTUATIONS: LINEAR THEORY

"TOP HAT" MODEL



MASS CONS. ⇒

$$\rho_m (1+\delta) R^3 (1+a)^3 = \text{const.} \Rightarrow$$

$$\delta = -3a$$

GRAVITY: $\ddot{R} = -\frac{4\pi G}{3}(\rho + 3p) R$ APPLIED BOTH TO FLUCT. + BCG. ⇒

$$\boxed{\ddot{\delta} + 2\frac{\dot{R}}{R}\dot{\delta} = 4\pi G \rho_m \delta}$$

Try $\delta = t^\alpha$

RAD ERA $\dot{R}/R = \frac{1}{2} t^{-1}$, $\delta = At + Bt^{-1} = AR^2 + BR^{-2}$

MATTER ERA $= \frac{2}{3} t^{-1}$, $\delta = At^{2/3} + Bt^{-1} = AR + BR^{-3/2}$

GROWING MODE