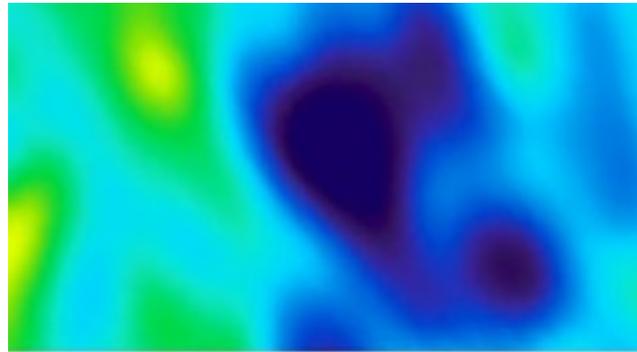


Structure Formation with Cold Dark Matter

- Nearly scale-invariant initial fluctuations from Inflation
- How structure grows with CDM
- How galaxies and clusters form in the CDM cosmology
- My favorite slides from the Dark Matter 2014 conference*

*At the Dark Matter 2014 conference, there were several talks each about new data indicating (1) 7 keV sterile neutrino and (2) about 35 GeV WIMP dark matter. They could even both be right -- i.e., cold dark matter of (at least) two different types. An embarrassment of riches! The evidence is not yet convincing, and alternative explanations need to be considered and ruled out. If either is right, there should be additional detections in the next year or two, and also possible production at the LHC when it goes into operation at higher energy next year.

Late Cosmological Epochs



380 kyr $z \sim 1000$

recombination
last scattering



dark ages



~ 100 Myr $z \sim 30$

first stars

~ 480 Myr $z \sim 10$

"reionization"



galaxy formation



13.7 Gyr $z=0$

today

BBN is a Prototype for Hydrogen Recombination and DM Annihilation

All three are examples of the universe dropping out of equilibrium!

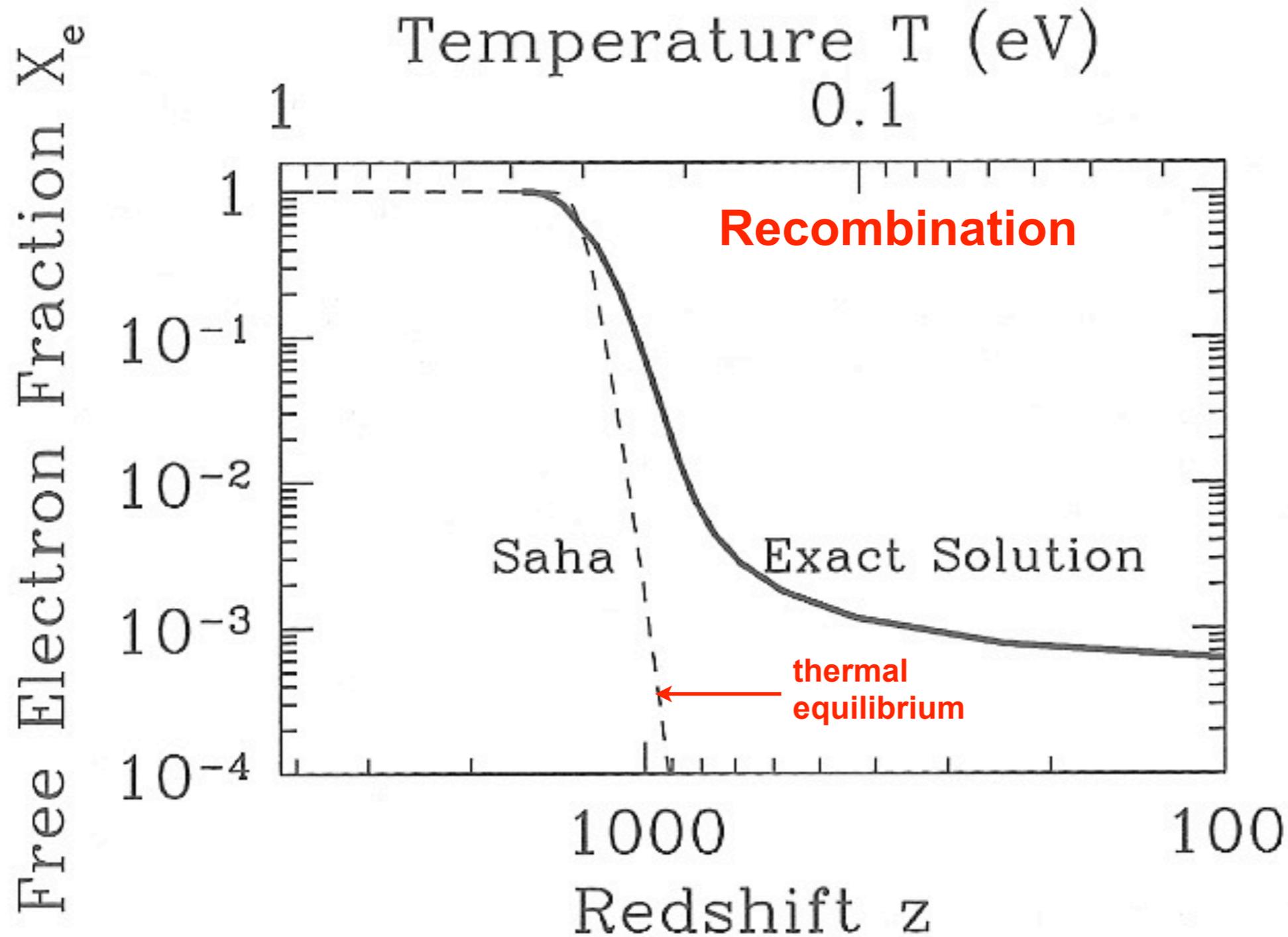


Figure 3.4. Free electron fraction as a function of redshift. Recombination takes place suddenly at $z \sim 1000$ corresponding to $T \sim 1/4$ eV. The Saha approximation, Eq. (3.37), holds in equilibrium and correctly identifies the redshift of recombination, but not the detailed evolution of X_e . Here $\Omega_b = 0.06$, $\Omega_m = 1$, $h = 0.5$.

Dodelson, Modern Cosmology, p. 72

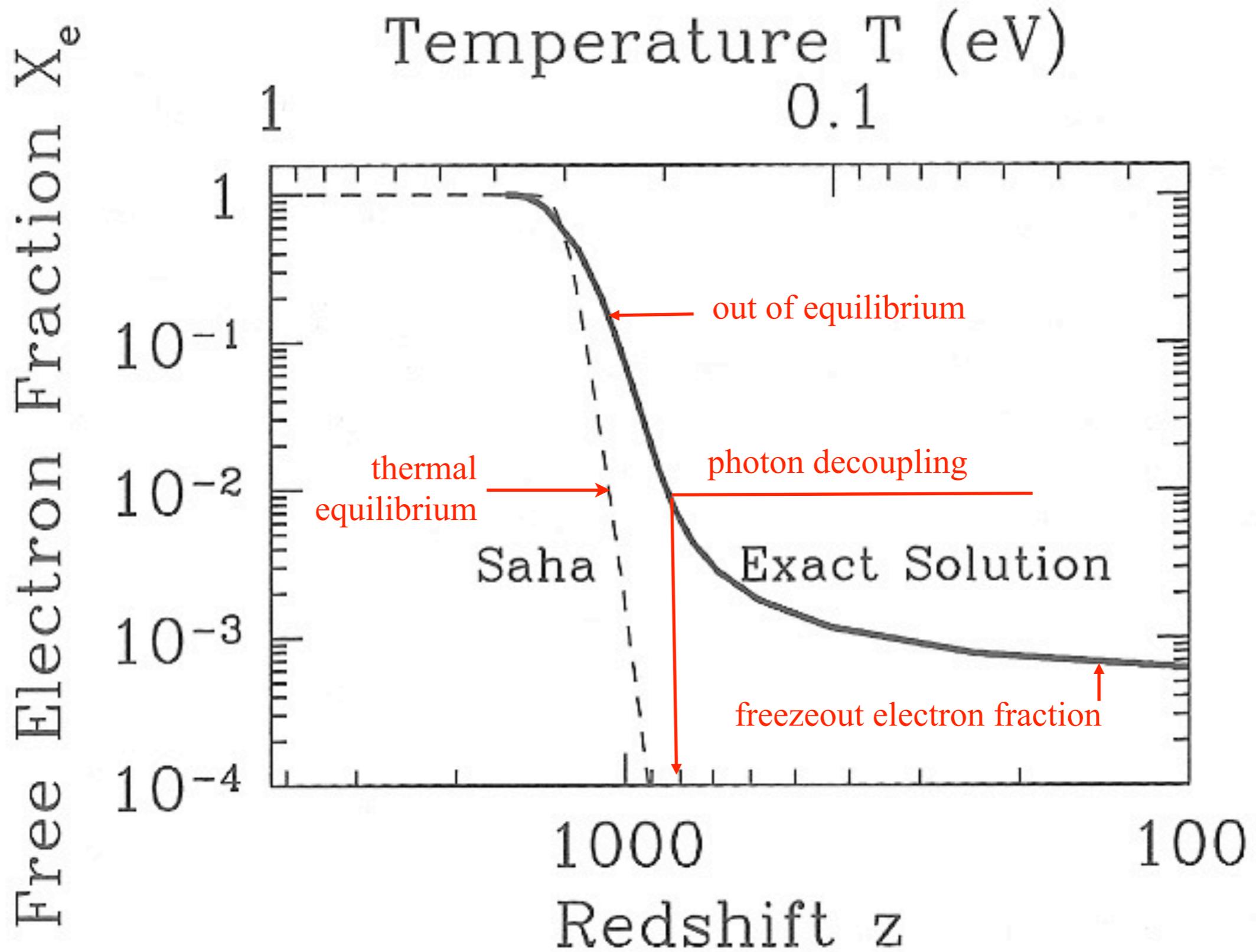
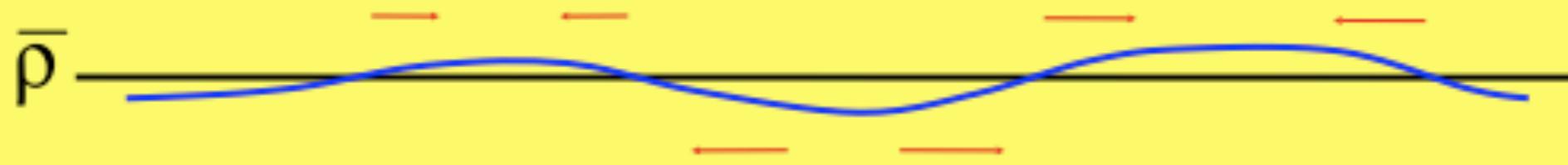


Figure 3.4. Free electron fraction as a function of redshift. Recombination takes place suddenly at $z \sim 1000$ corresponding to $T \sim 1/4$ eV. The Saha approximation, Eq. (3.37), holds in equilibrium and correctly identifies the redshift of recombination, but not the detailed evolution of X_e . Here $\Omega_b = 0.06$, $\Omega_m = 1$, $h = 0.5$.

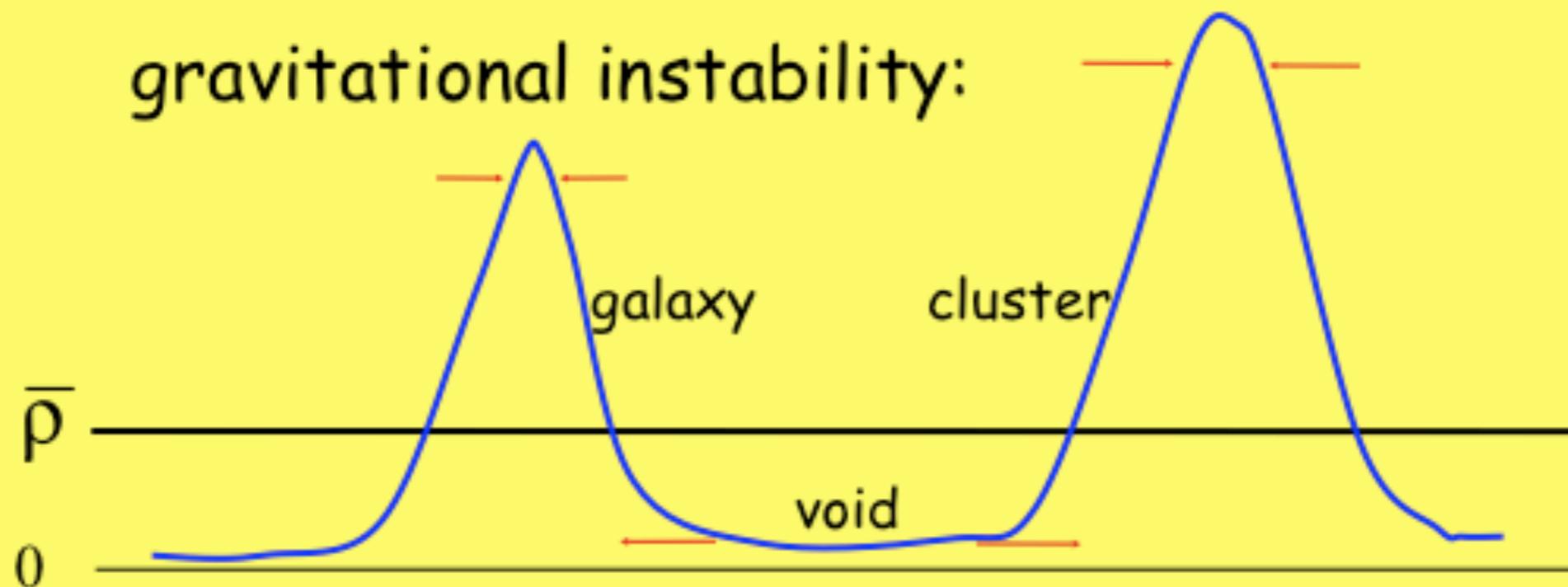
Dodelson, Modern Cosmology, p. 72

Gravitational instability

small-amplitude fluctuations:



gravitational instability:



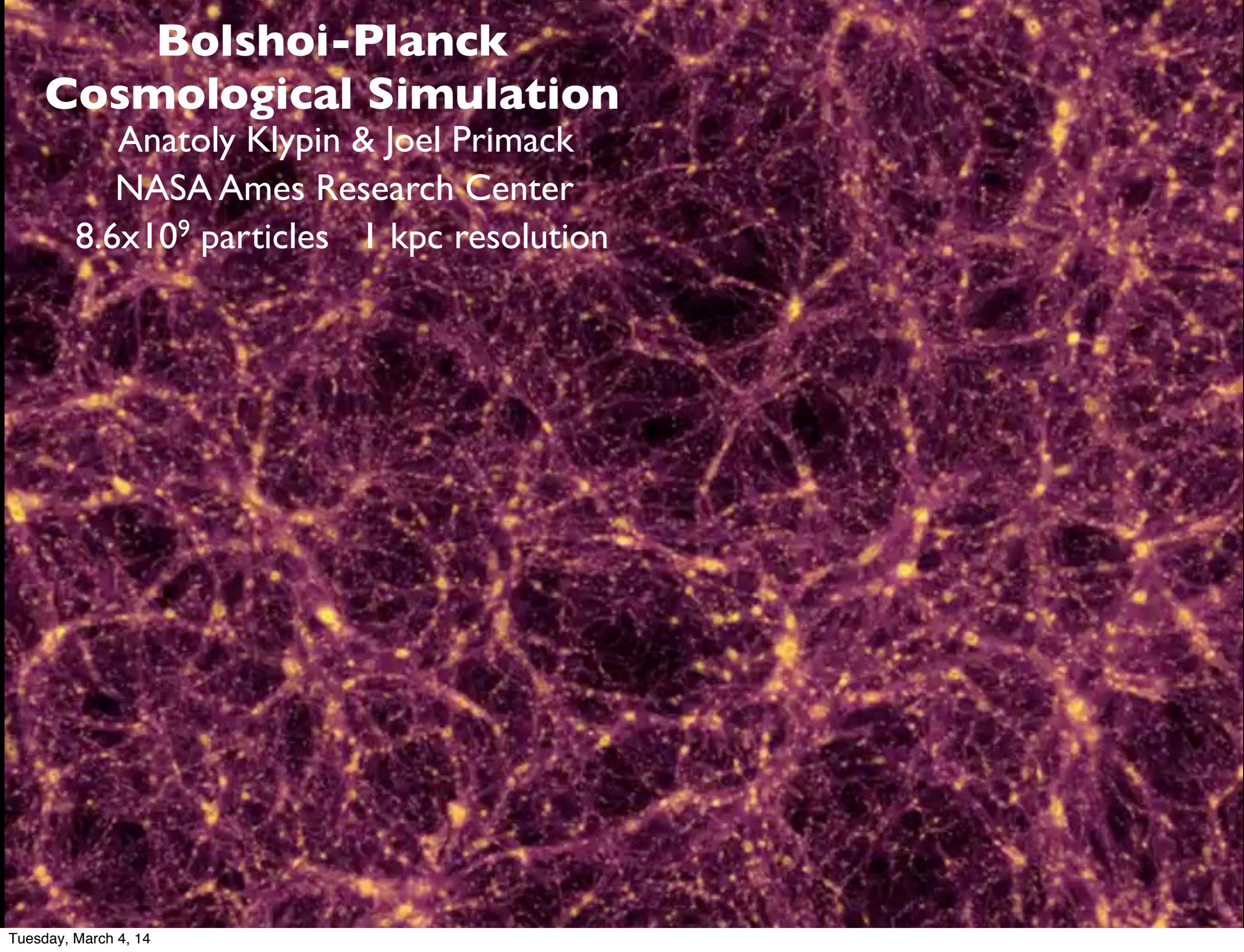
Constrained Local Universe Simulations CLUES

Bolshoi-Planck Cosmological Simulation

Anatoly Klypin & Joel Primack

NASA Ames Research Center

8.6×10^9 particles 1 kpc resolution



Bolshoi-Planck

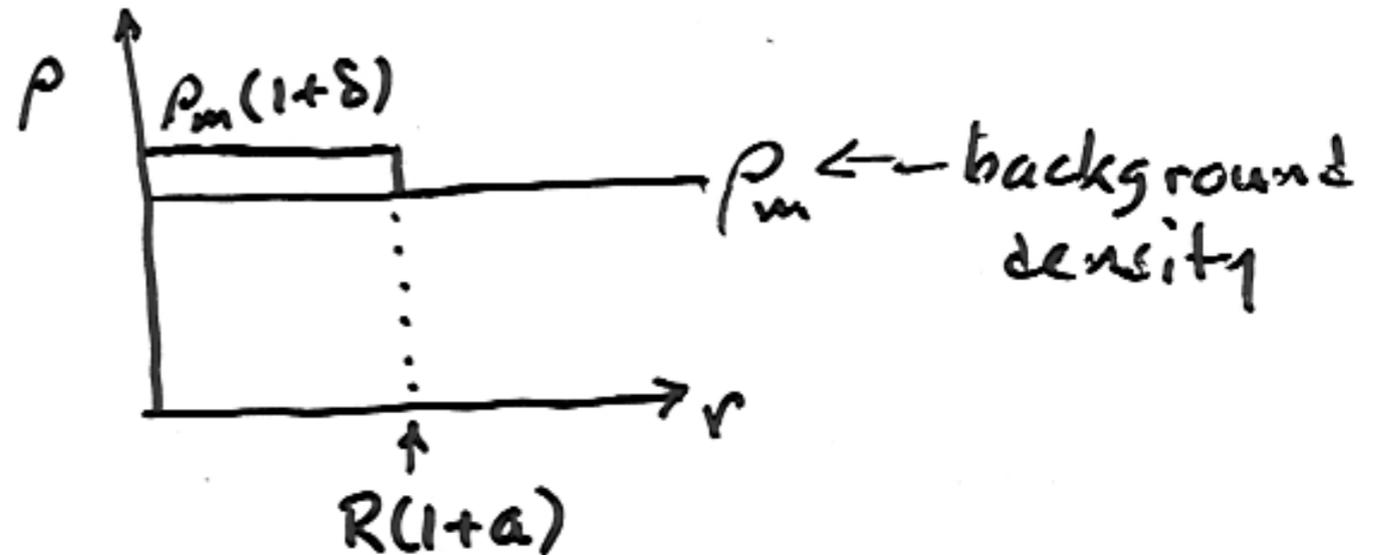
Cosmological Simulation

Merger Tree of a Large Halo

FLUCTUATIONS: LINEAR THEORY

Recall: $E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi Gp + \frac{2}{3}\Lambda$ (here $a = R, \Lambda=0$)

"TOP HAT MODEL"



MASS CONS. \Rightarrow

$$\rho_m(1+\delta)R^3(1+a)^3 = \text{const.} \Rightarrow$$

$$\delta = -3a$$

GRAVITY: $\ddot{R} = -\frac{4\pi G}{3}(\rho + 3p)R$

$$\ddot{\delta} + 2\frac{\dot{R}}{R}\dot{\delta} = 4\pi G\rho_m\delta$$

RAD ERA $\dot{R}/R = \frac{1}{2}t^{-1}$,
 MATTER ERA $= \frac{2}{3}t^{-1}$,

APPLIED BOTH TO FLUCT. + BCG. \Rightarrow

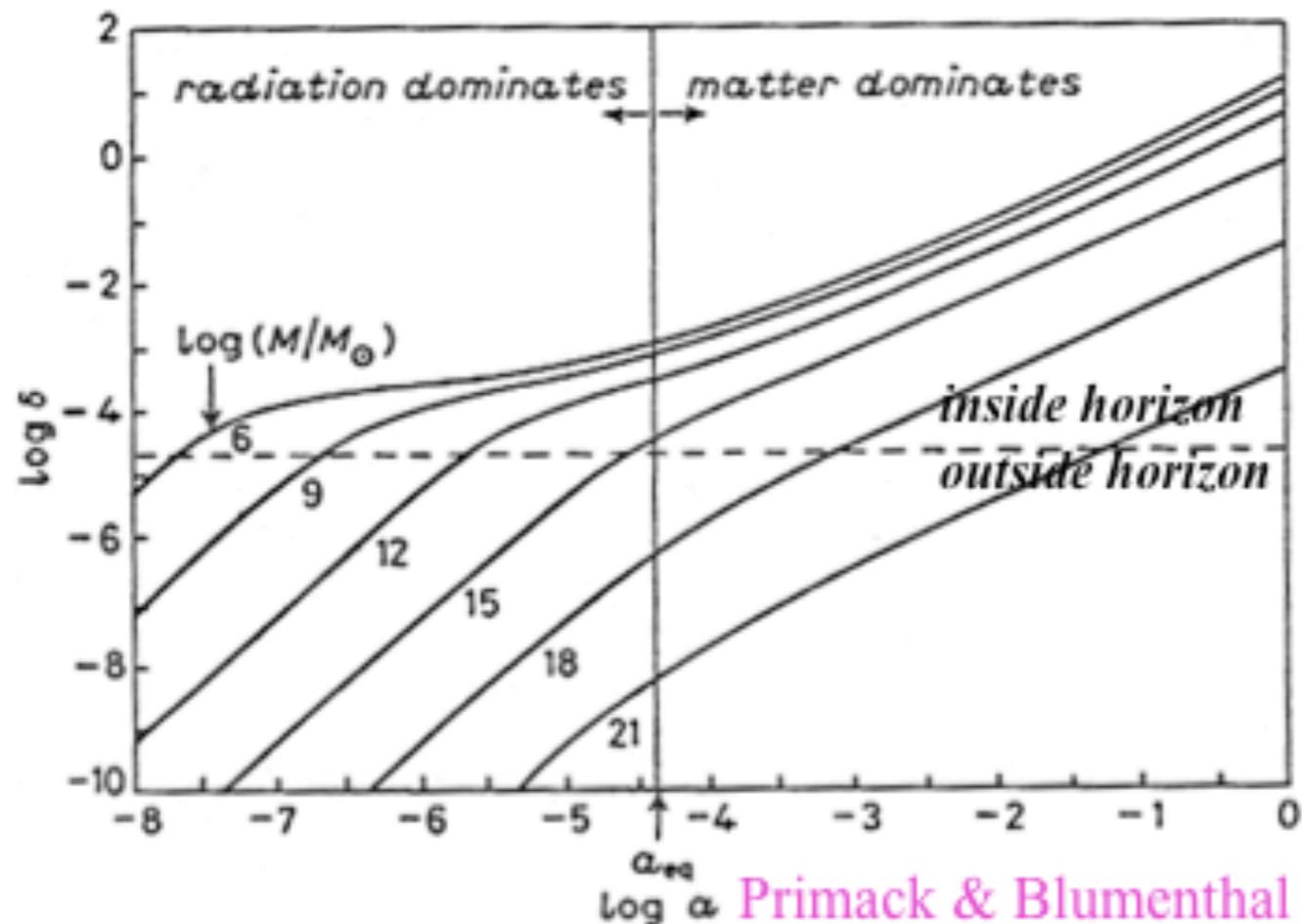
Try $\delta = t^\alpha$

$$\delta = At + Bt^{-1} = \underline{AR^2} + BR^{-2}$$

$$\delta = At^{2/3} + Bt^{-1} = \underline{AR} + BR^{-3/2}$$

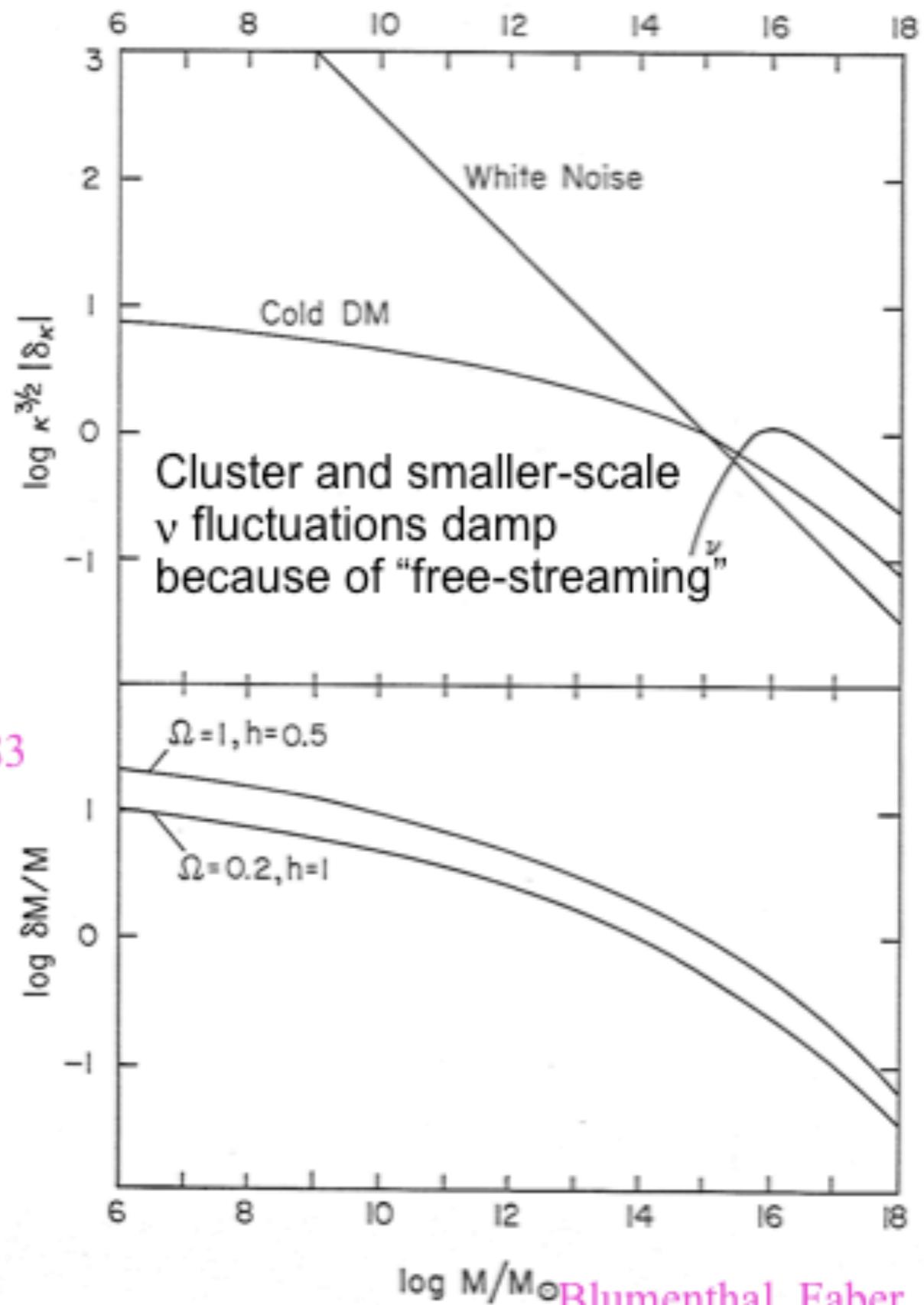
GROWING MODE

CDM Structure Formation: Linear Theory



Primack & Blumenthal 1983

Matter fluctuations that enter the horizon during the radiation dominated era, with masses less than about $10^{15} M_{\odot}$, grow only $\propto \log a$, because they are not in the gravitationally dominant component. But matter fluctuations that enter the horizon in the matter-dominated era grow $\propto a$. This explains the characteristic shape of the CDM fluctuation spectrum, with $\delta(k) \propto k^{-n/2-2} \log k$ for $k \gg k_{eq}$.



Blumenthal, Faber, Primack, & Rees 1984

The Initial Fluctuations

At Inflation: Gaussian, adiabatic

Fourier transform:

$$\delta(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}$$

Power Spectrum:

$$P(k) \equiv \langle |\tilde{\delta}(\vec{k})|^2 \rangle \propto k^n$$

rms perturbation:

$$\delta_{rms} = \langle \delta^2 \rangle^{1/2} \propto \int_0^{k_{max}} P_k d^3k \propto M^{-(n+3)/6}$$

Correlation function:

$$\xi(r) \equiv \langle \delta(\vec{x})\delta(\vec{x} + \vec{r}) \rangle \propto \int |\tilde{\delta}(\vec{k})|^2 e^{-i\vec{k}\cdot\vec{r}} d^3k \propto r^{-(n+3)}$$

$$dP = [1 + \xi(r)] n dV$$

**Λ CDM
PREDICTS
EVOLUTION
IN THE GALAXY
CORRELATION
FUNCTION**

$\xi_{gg}(r)$

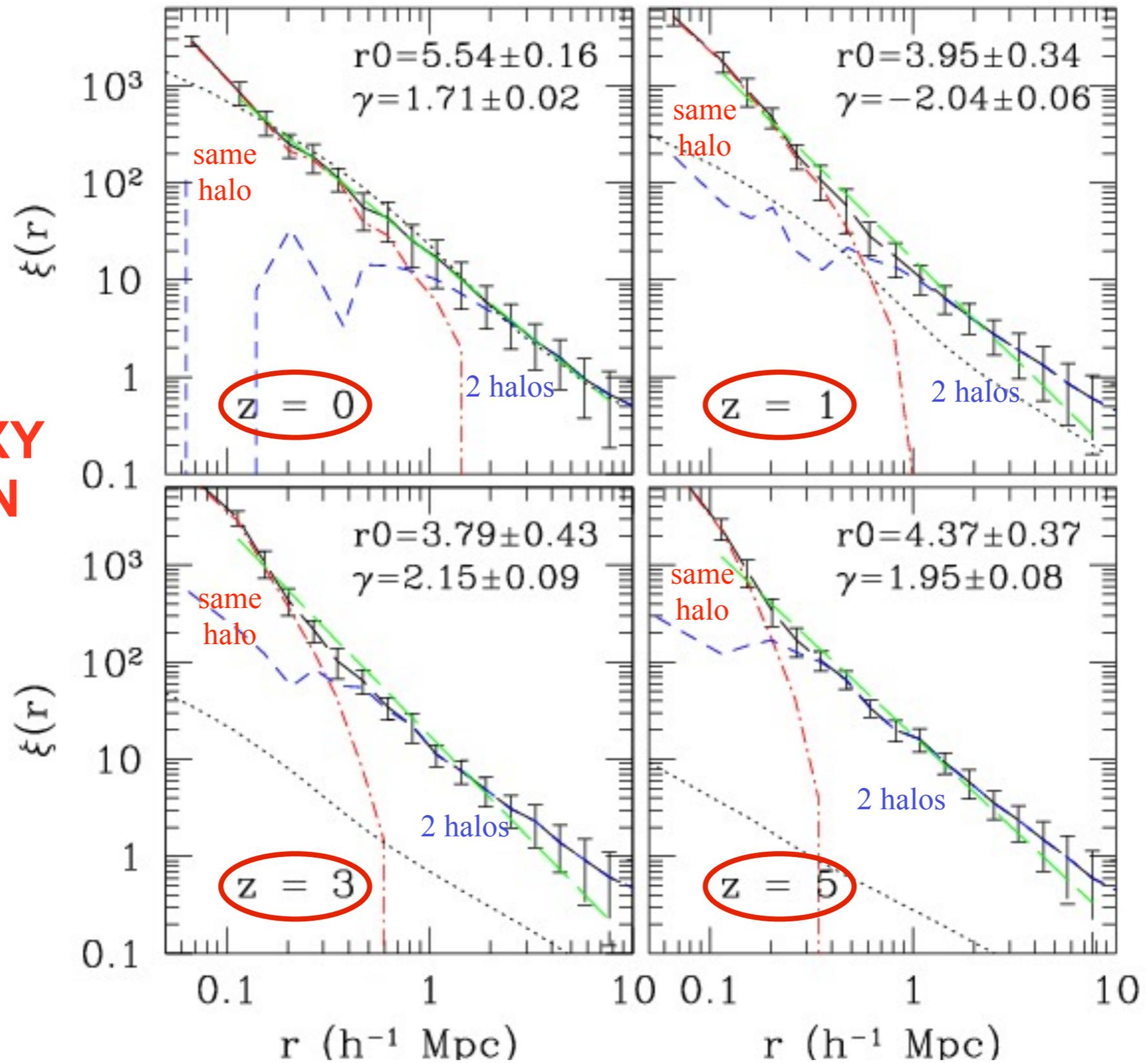
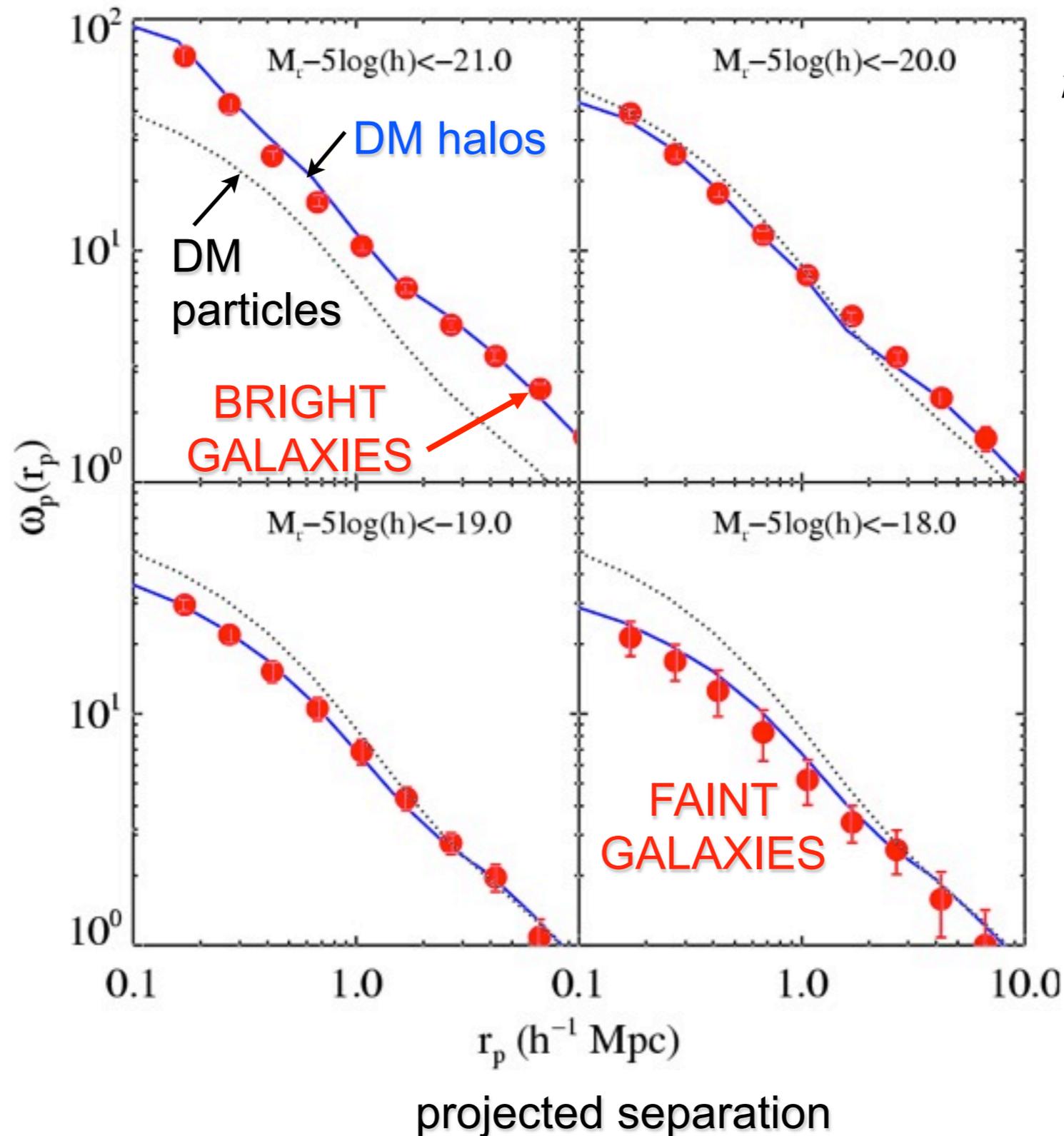


FIG. 8.— Evolution of the two-point correlation function in the $80h^{-1}$ Mpc simulation. The solid line with error bars shows the clustering of halos of the fixed number density $n = 5.89 \times 10^{-3} h^3 \text{ Mpc}^{-3}$ at each epoch. The error-bars indicate the “jack-knife” one sigma errors and are larger than the Poisson error at all scales. The dot-dashed and dashed lines show the corresponding one- and two-halo term contributions. The long-dashed lines show the power-law fit to the correlation functions in the range of $r = [0.1 - 8h^{-1} \text{ Mpc}]$. Although the correlation functions can be well fit by the power law at $r \gtrsim 0.3h^{-1} \text{ Mpc}$ in each epoch, at $z > 0$ the correlation function steepens significantly at smaller scales due to the one-halo term.

Kravtsov, Berlind, Wechsler, Klypin, Gottloeber, Allgood, & Primack 2004

Galaxy clustering in SDSS at $z \sim 0$ agrees with Λ CDM simulations

projected
2-point
correlation
function

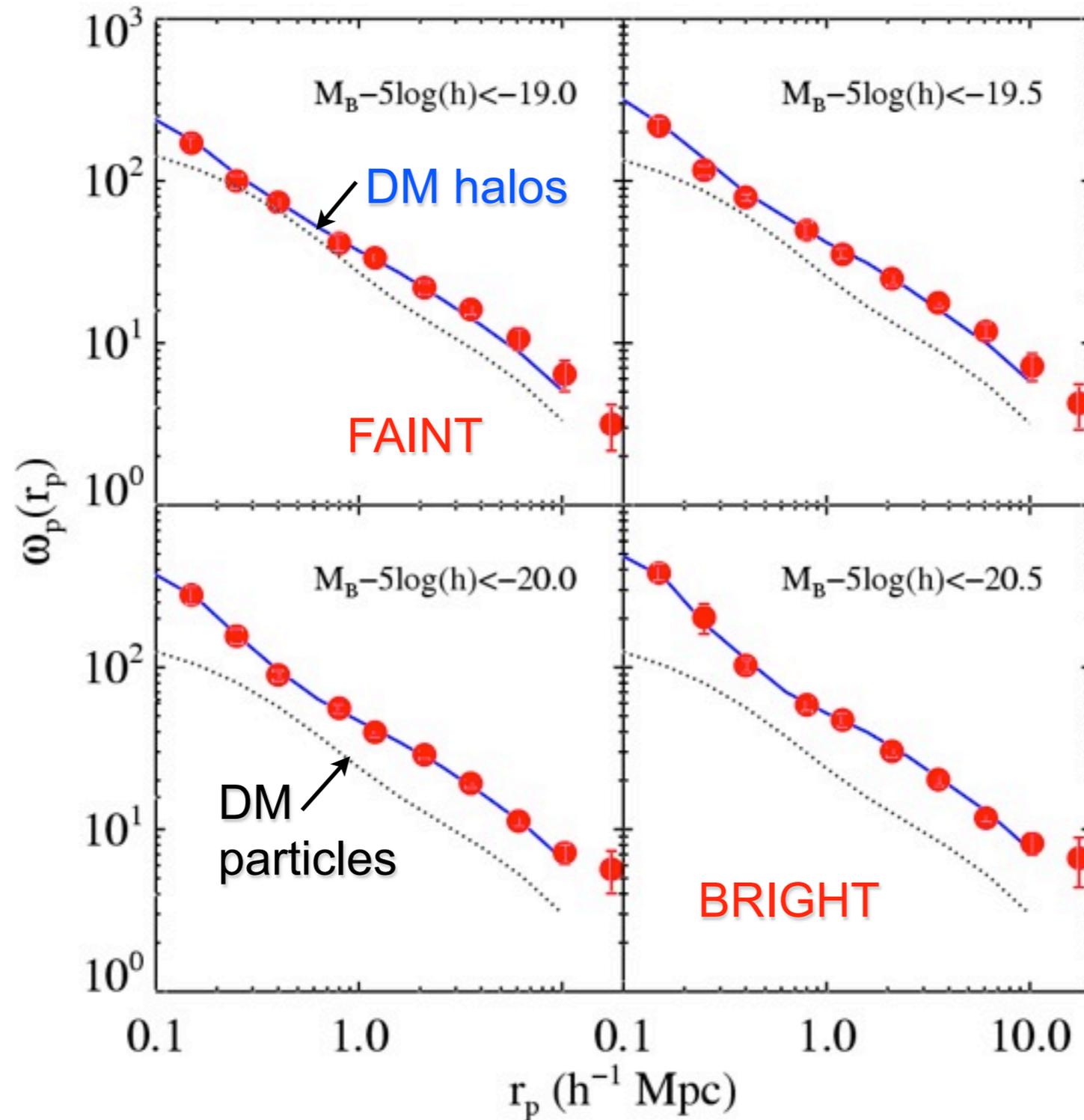


$$n(>V_{\text{max,acc}}) = n(>L)$$

Conroy,
Wechsler &
Kravtsov
2006, ApJ 647, 201

and at redshift $z \sim 1$ (DEEP2)

projected
2-point
correlation
function

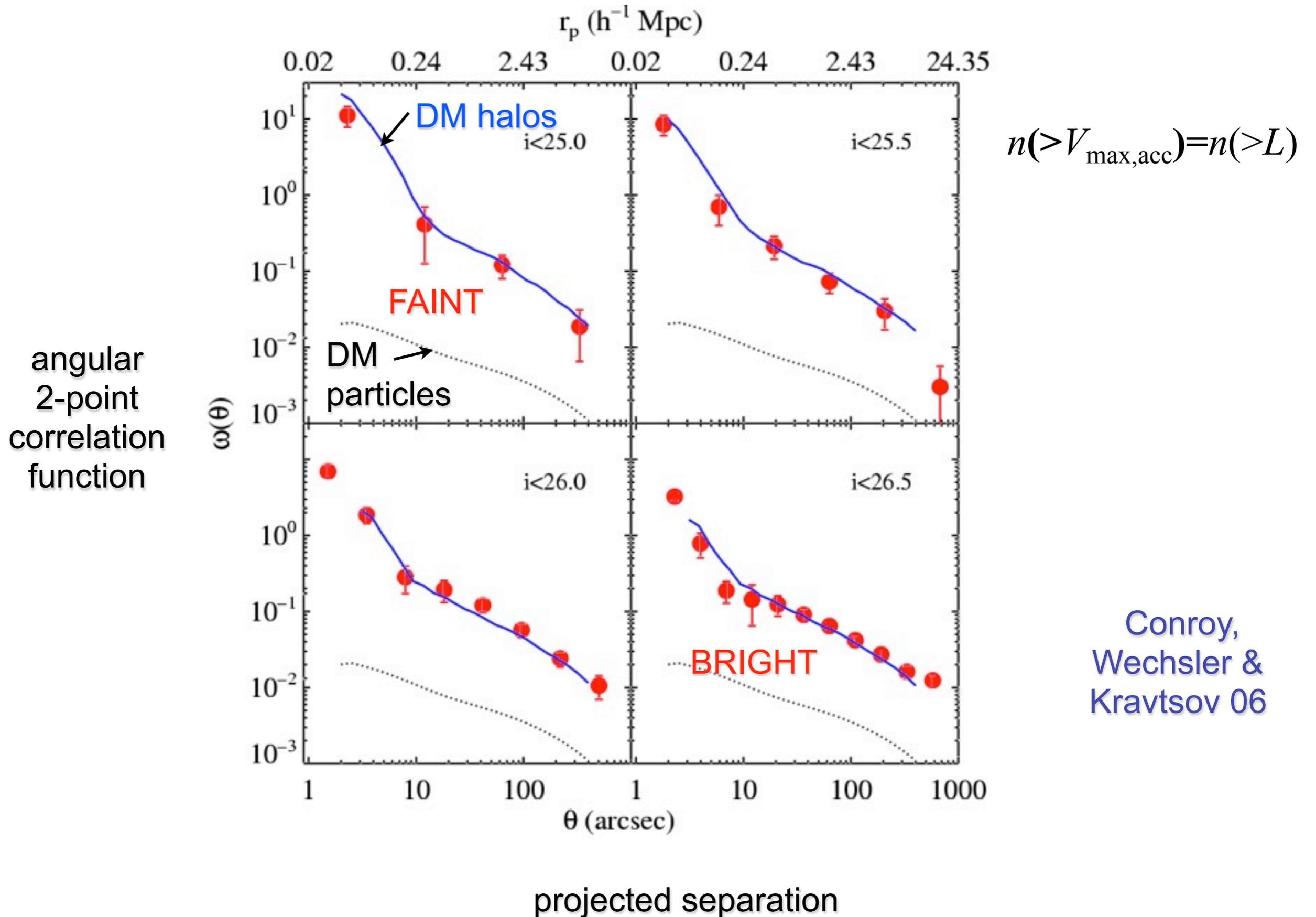


$$n(>V_{\text{max,acc}}) = n(>L)$$

projected separation

Conroy,
Wechsler &
Kravtsov 06

and at $z \sim 4-5$ (LBGs, Subaru)!



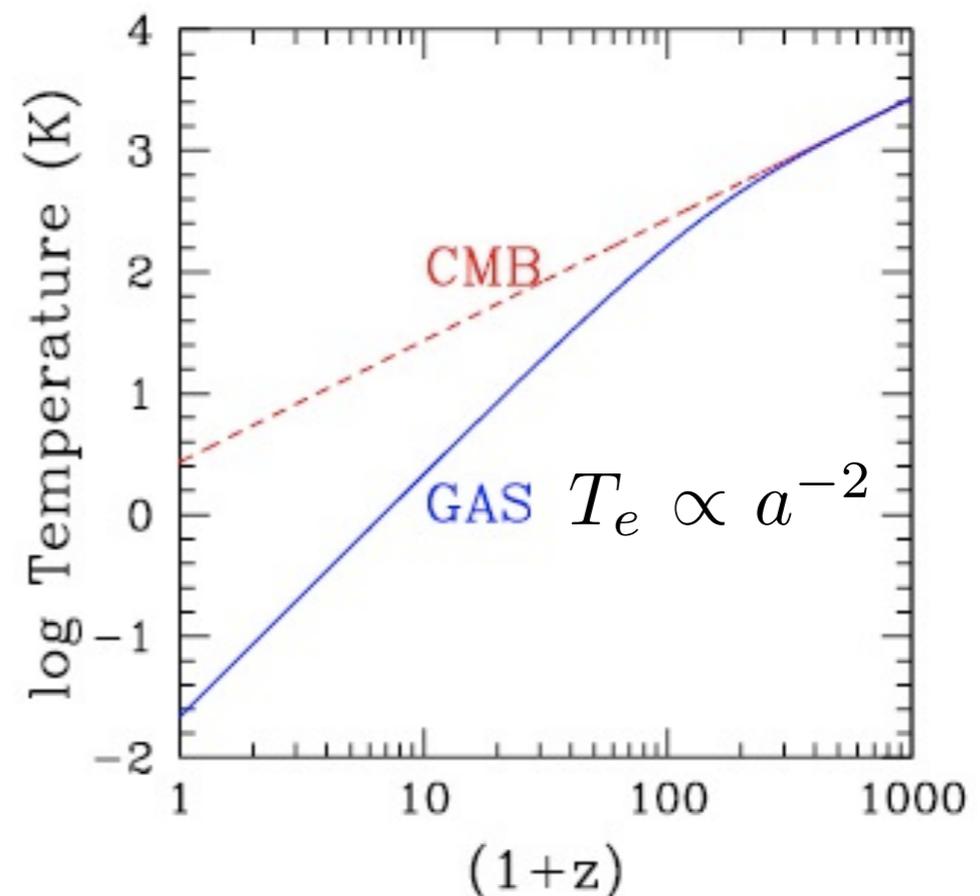
Thus far, we have considered only the evolution of fluctuations in the dark matter. But of course we have to consider also the ordinary matter, known in cosmology as “baryons” (implicitly including the electrons). See Madau’s lectures “The Astrophysics of Early Galaxy Formation” (<http://arxiv.org/abs/0706.0123v1>) for a summary. The baryons are primarily in the form of atoms after $z \sim 1000$, with a residual ionization fraction of order 10^{-4} . They become fully reionized by $z \sim 6$, but they were not reionized at $z \sim 20$ since the COBE satellite found that “Compton parameter” $y \leq 1.5 \times 10^{-5}$, where

$$y = \int_0^z \frac{k_B T_e}{m_e c^2} \frac{d\tau_e}{dz} dz \quad \text{with} \quad (n_e \sigma_T c dt) = d\tau_e, \quad \sigma_T = (8\pi/3)(e^2/mc^2)^2$$

This implies that $\langle x_e T_e \rangle [(1+z)^{3/2} - 1] < 4 \times 10^7 \text{ K}$. Thus, for example, a universe that was reionized and reheated at $z = 20$ to $(x_e, T_e) = (1, > 4 \times 10^5 \text{ K})$ would violate the COBE y -limit.

The figure at right shows the evolution of the radiation (dashed line, labeled **CMB**) and matter (solid line, labeled **GAS**) temperatures after recombination, in the absence of any reheating mechanism.

(From Madau’s lectures, at physics.ucsc.edu/~joel/Phys129.)



The linear evolution of sub-horizon density perturbations in the dark matter-baryon fluid is governed in the matter-dominated era by two second-order differential equations:

$$\ddot{\delta}_{\text{dm}} + 2H\dot{\delta}_{\text{dm}} = \frac{3}{2}H^2\Omega_m^z (f_{\text{dm}}\delta_{\text{dm}} + f_b\delta_b) \quad (1)$$

for the dark matter, and

$$\ddot{\delta}_b + 2H\dot{\delta}_b = \frac{3}{2}H^2\Omega_m^z (f_{\text{dm}}\delta_{\text{dm}} + f_b\delta_b) - \frac{c_s^2}{a^2}k^2\delta_b$$

“Hubble friction”

for the baryons, where $\delta_{\text{dm}}(\mathbf{k})$ and $\delta_b(\mathbf{k})$ are the Fourier components of the density fluctuations in the dark matter and baryons,† f_{dm} and f_b are the corresponding mass fractions, c_s is the gas sound speed, k is the (comoving) wavenumber, and the derivatives are taken with respect to cosmic time. Here

$$\Omega_m^z \equiv 8\pi G\rho(t)/3H^2 = \Omega_m(1+z)^3/[\Omega_m(1+z)^3 + \Omega_\Lambda] \quad (2)$$

is the time-dependent matter density parameter, and $\rho(t)$ is the total background matter density. Because there is ~ 6 times more dark matter than baryons, it is the former that defines the pattern of gravitational wells in which structure formation occurs. In the case where $f_b = 0$ and the universe is static ($H = 0$), equation (1) above becomes

† For each fluid component ($i = b, \text{dm}$) the real space fluctuation in the density field,

$\delta_i(\mathbf{x}) \equiv \delta\rho_i(\mathbf{x})/\rho_i$, can be written as a sum over Fourier modes,

$\delta_i(\mathbf{x}) = \int d^3\mathbf{k} (2\pi)^{-3} \delta_i(\mathbf{k}) \exp i\mathbf{k}\cdot\mathbf{x}$.

$$\ddot{\delta}_{\text{dm}} = 4\pi G\rho\delta_{\text{dm}} \equiv \frac{\delta_{\text{dm}}}{t_{\text{dyn}}^2},$$

where t_{dyn} denotes the dynamical timescale. This equation has the solution

$$\delta_{\text{dm}} = A_1 \exp(t/t_{\text{dyn}}) + A_2 \exp(-t/t_{\text{dyn}}).$$

After a few dynamical times, only the exponentially growing term is significant: gravity tends to make small density fluctuations in a static pressureless medium grow exponentially with time. Sir James Jeans (1902) was the first to discuss this.

The additional term $\propto H\dot{\delta}_{\text{dm}}$ present in an expanding universe can be thought as a “**Hubble friction**” term that acts to slow down the growth of density perturbations. Equation (1) has the general solution for the growing mode:

$$\delta_{\text{dm}}(a) = \frac{5\Omega_m}{2} H_0^2 H \int_0^a \frac{da'}{(\dot{a}')^3}, \quad (3)$$

so that an Einstein-de Sitter universe gives the familiar scaling $\delta_{\text{dm}}(a) = a$ with coefficient unity. The right-hand side of equation (3) is called the linear growth factor $D(a) = D_+(a)$. Different values of Ω_m, Ω_Λ lead to different linear growth factors.

Growing modes actually decrease in density, but not as fast as the average universe. Note how, in contrast to the exponential growth found in the static case, the growth of perturbations even in the case of an Einstein-de Sitter ($\Omega_m = 1$) universe is just algebraic rather than exponential. This was discovered by the Russian physicist Lifshitz (1946).

Since cosmological curvature is at most marginally important at the present epoch, it was negligible during the radiation-dominated era and at least the beginning of the matter-dominated era. But for $k = -1$, i.e. $\Omega < 1$, the growth of δ slows for $(R/R_o) \gtrsim \Omega_o$, as gravity becomes less important and the universe begins to expand freely. To discuss this case, it is convenient to introduce the variable

$$x \equiv \Omega^{-1}(t) - 1 = (\Omega_o^{-1} - 1)R(t)/R_o.$$

(Note that $\Omega(t) \rightarrow 1$ at early times.) The general solution in the matter-dominated era is then (Peebles, 1980, §11)

$$\delta = \tilde{A}D_1(t) + \tilde{B}D_2(t),$$

where the growing solution is

$$D_1 = 1 + \frac{3}{x} + \frac{3(1+x)^{1/2}}{x^{3/2}} \ln \left[(1+x)^{1/2} - x^{1/2} \right]$$

and the decaying solution is

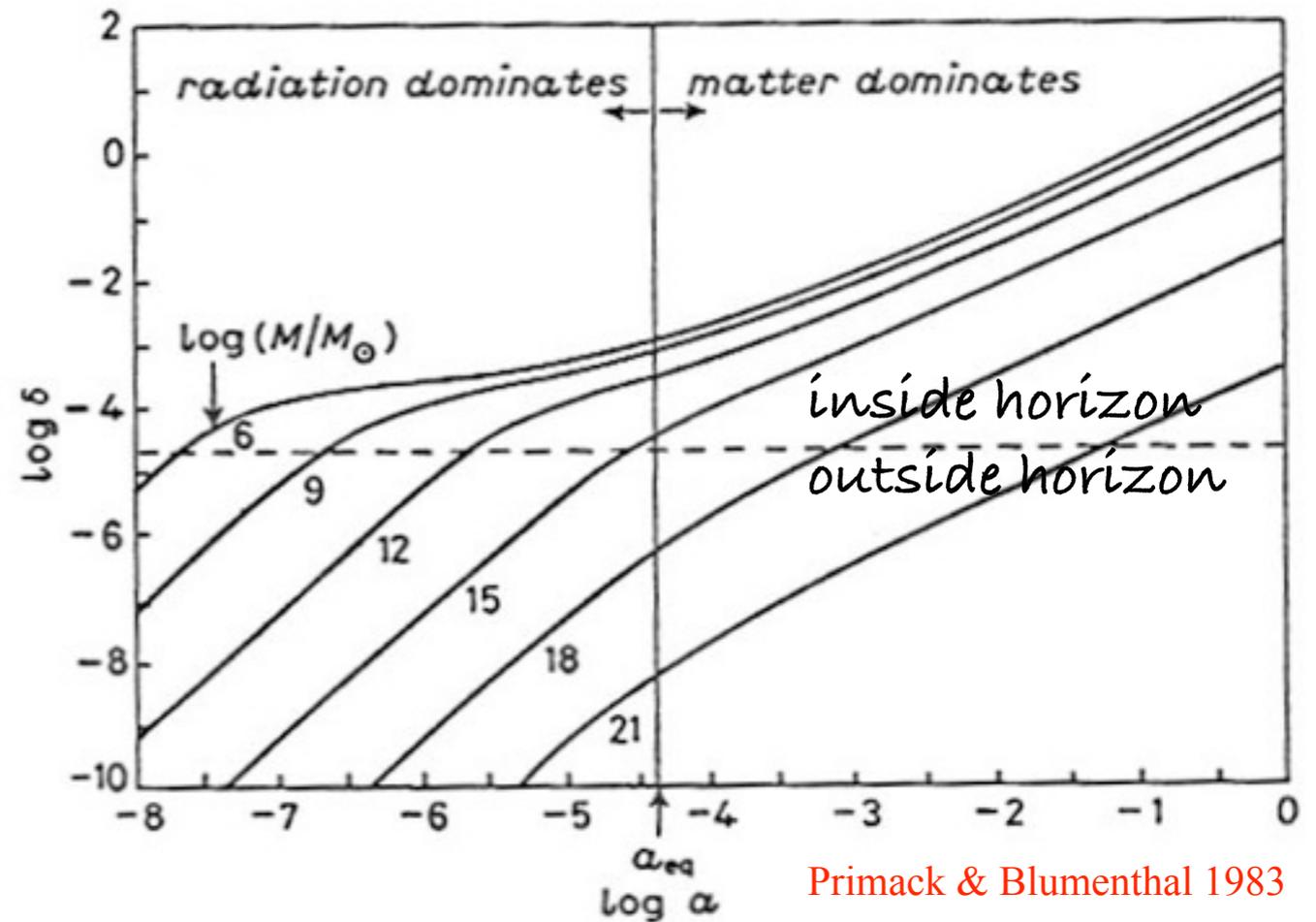
$$D_2 = (1+x)^{1/2}/x^{3/2}.$$

These agree with the Einstein-de Sitter results (2.53) at early times ($t \ll t_o, x \ll 1$). For late times ($t \gg t_o, x \gg 1$) the solutions approach

$$D_1 = 1, D_2 = x^{-1};$$

in this limit the universe is expanding freely and the amplitude of fluctuations stops growing.

The consequence is that dark matter fluctuations grow proportionally to the scale factor $a(t)$ when matter is the dominant component of the universe, but only logarithmically when radiation is dominant. Thus there is not much difference in the amplitudes of fluctuations of mass $M < 10^{15} M_{\text{sun}}$, which enter the horizon before $z_{\text{mr}} \sim 4 \times 10^3$, while there is a stronger dependence on M for fluctuations with $M > 10^{15} M_{\text{sun}}$.



There is a similar suppression of the growth of matter fluctuations once the gravitationally dominant component of the universe is the dark energy, for example a cosmological constant. Lahav, Lilje, Primack, & Rees (1991) showed that the growth factor in this case is well approximated by

$$\delta_{\text{dm}}(a) = D(a) \simeq \frac{5\Omega_m^z}{2(1+z)} \left[(\Omega_m^z)^{4/7} - \frac{(\Omega_m^z)^2}{140} + \frac{209}{140}\Omega_m^z + \frac{1}{70} \right]^{-1}.$$

Here Ω_m^z is again given by $\Omega_m^z \equiv 8\pi G\rho(t)/3H^2 = \Omega_m(1+z)^3/[\Omega_m(1+z)^3 + \Omega_\Lambda]$

The Linear Transfer Function $T(k)$

The observed uniformity of the CMB guarantees that density fluctuations must have been quite small at decoupling, implying that the evolution of the density contrast can be studied at $z \lesssim z_{\text{dec}}$ using linear theory, and each mode $\delta(k)$ evolves independently. The inflationary model predicts a scale-invariant primordial power spectrum of density fluctuations $P(k) \equiv \langle |\delta(k)|^2 \rangle \propto k^n$, with $n = 1$ (the so-called Harrison-Zel'dovich spectrum). It is the index n that governs the balance between large and small-scale power. In the case of a Gaussian random field with zero mean, the power spectrum contains the complete statistical information about the density inhomogeneity. It is often more convenient to use the dimensionless quantity $\Delta_k^2 \equiv [k^3 P(k)/2\pi^2]$, which is the power per logarithmic interval in wavenumber k . In the matter-dominated epoch, this quantity retains its initial primordial shape ($\Delta_k^2 \propto k^{n+3}$) only on very large scales. Small wavelength modes enter the horizon earlier on and their growth is suppressed more severely during the radiation-dominated epoch: on small scales the amplitude of Δ_k^2 is essentially suppressed by four powers of k (from k^{n+3} to k^{n-1}). If $n = 1$, then small scales will have nearly the same power except for a weak, logarithmic dependence. Departures from the initially scale-free form are described by the transfer function $T(k)$, defined such that $T(0) = 1$:

$$P(k, z) = Ak^n \left[\frac{D(z)}{D(0)} \right]^2 T^2(k),$$

where A is the normalization.

An approximate fitting function for $T(k)$ in a Λ CDM universe is (Bardeen et al. 1986)

$$T_k = \frac{\ln(1 + 2.34q)}{2.34q} [1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4},$$

where (Sugayama 1995)

$$q \equiv \frac{k/\text{Mpc}^{-1}}{\Omega_m h^2 \exp(-\Omega_b - \Omega_b/\Omega_m)}.$$

For accurate work, for example for starting high-resolution N-body simulations, it is best to use instead of fitting functions the numerical output of highly accurate integration of the Boltzmann equations, for example from CMBFast, which is available at <http://lambda.gsfc.nasa.gov/toolbox/> which points to http://lambda.gsfc.nasa.gov/toolbox/tb_cmbfast_ov.cfm

W e l c o m e to the CMBFAST Website!

This is the most extensively used code for computing cosmic microwave background anisotropy, polarization and matter power spectra. The code has been tested over a wide range of cosmological parameters. We are continuously testing and updating the code based on suggestions from the cosmological community. Do not hesitate to contact us if you have any questions or suggestions.

U. Seljak & M. Zaldarriaga

CMB Toolbox Overview

We provide links to a number of useful tools for CMB and Astronomy in general.

CMB Tools

- [CMB Simulations](#) - High-resolution, full-sky microwave temperature simulations including secondary anisotropies.
- [Contributed Software](#) is an archive at a LAMBDA partner site for tools built by members of the community.
- [CMBFast](#) - A tool that computes spectra for the cosmic background for a given set of CMB parameters. LAMBDA provides a [web-based interface](#) for this tool. Seljak and Zaldarriaga
- [CAMB](#) - Code for Anisotropies in the Microwave Background that computes spectra for a set of CMB parameters. LAMBDA provides a [web-based interface](#) for this tool. Lewis and Challinor
- [CMBEASY](#) - A C++ package, initially based on CMBFAST, now featuring a parameter likelihood package as well. Doran
- [CMBview](#) - A Mac OS X program for viewing HEALPix-format CMB data on an OpenGL-rendered sphere. Portsmouth
- [COMBAT](#) - A set of computational tools for CMB analysis. Borrill et al.
- [CosmoMC](#) - A Markov-Chain Monte-Carlo engine for exploring cosmological parameter space. Lewis and Bridle
- [CosmoNet](#) - Accelerated cosmological parameter estimation using Neural Networks.
- [GLESP](#) - Gauss-Legendre sky pixelization package. Doroshkevich, et al.
- [GSM](#) - Predicted all-sky maps at any frequency from 10 MHz to 100 GHz. de Oliveira-Costa.
- [HEALPix](#) - A spherical sky pixelization scheme. The Wilkinson Microwave Anisotropy Probe (WMAP) data skymap products are supplied in this form. Górski et al.
- [IGLOO](#) - A sky pixelization package. Crittenden and Turok
- [MADCAP](#) - Microwave Anisotropy Data Computational Analysis Package. Borrill et al.
- [PICO](#) - Integrates with CAMB and/or CosmoMC for cosmological parameter estimation using machine learning. Wandelt and Fendt
- [RADPACK](#) - Radical Compression Analysis Package. Knox
- [RECFAST](#) - Software to calculate the recombination history of the Universe. Seager, Sasselov, and Scott
- [SkyViewer](#) - A LAMBDA-developed OpenGL-based program to display HEALPix-based skymaps stored in FITS format files. Phillips
- [SpiCE](#) - Spatially Inhomogenous Correlation Estimator. Szapudi et al.
- [WMAPViewer](#) - A LAMBDA-developed web-based CMB map viewing tool using a technology similar to that found on [maps.google.com](#). Phillips
- [WOMBAT](#) - Microwave foreground emission tools. Gawiser, Finkbeiner, Jaff et al.

Likelihood Software

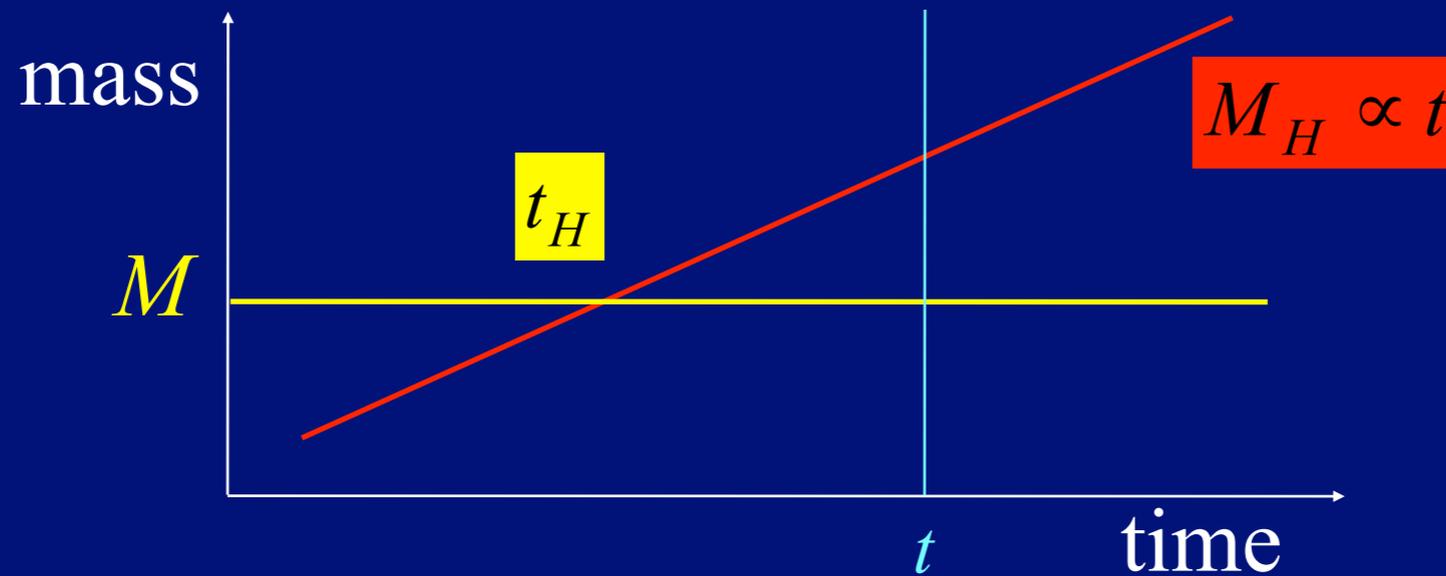
- [SDSS LRG DR7 Likelihood Software](#) - A software package that computes likelihoods for Luminous Red Galaxies (LRG) data from the seventh release of the Sloan Digital Sky Survey (SDSS).
- [WMAP Likelihood Software](#) - A software library used by the WMAP team to compute Fisher and Master matrices and to compute the likelihoods of various models. This is the same software found on the [WMAP products list](#); more information may be found [here](#).

Other Tools

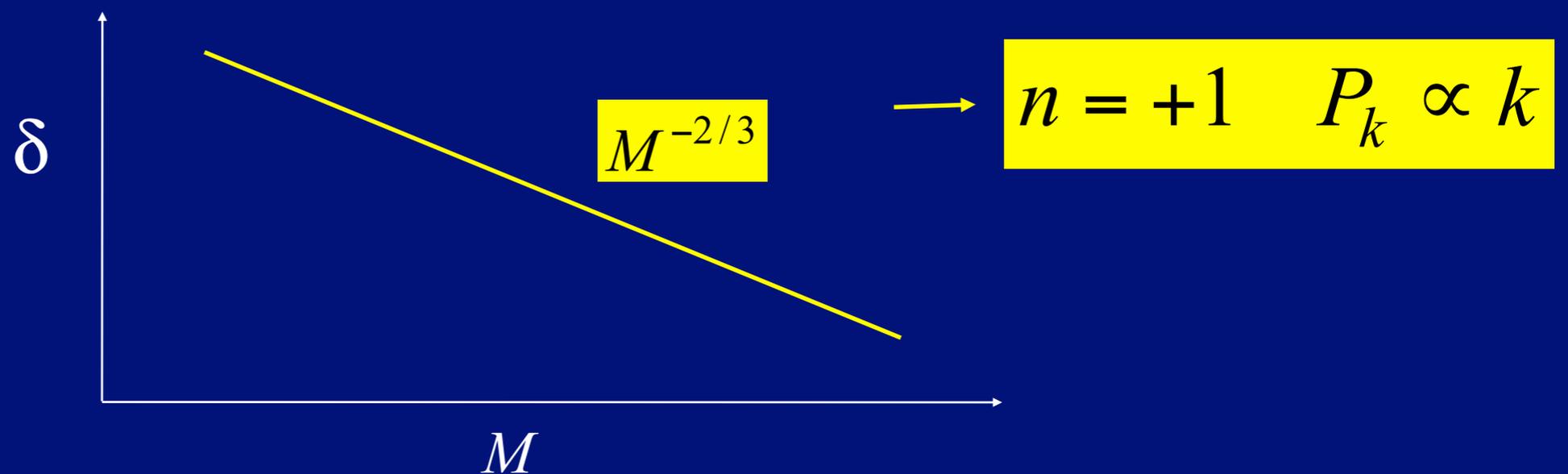
- [WMAP Effective Frequency Calculator](#) - A tool that calculates the effective frequencies of the five WMAP frequency bands.
- [CFITSIO](#) - A library of C and Fortran routines for reading and writing data in the FITS format.
- [IDL Astro](#) - The IDL Astronomy Users Library.
- [Conversion Utilities](#) - A small collection of astronomical conversion utilities.
- [Calculators](#) - A list of links to calculators.

This collection of tools can only be extended and improved with your input! Please feel free to send us [suggestions and comments](#).

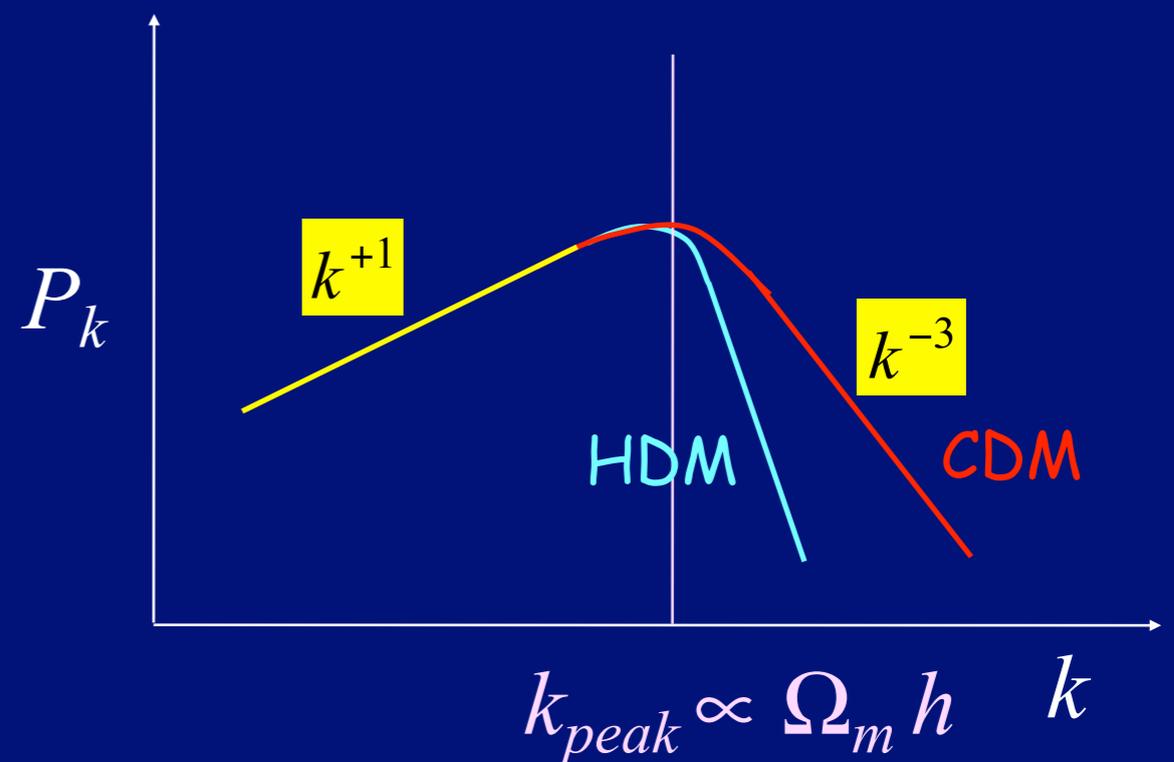
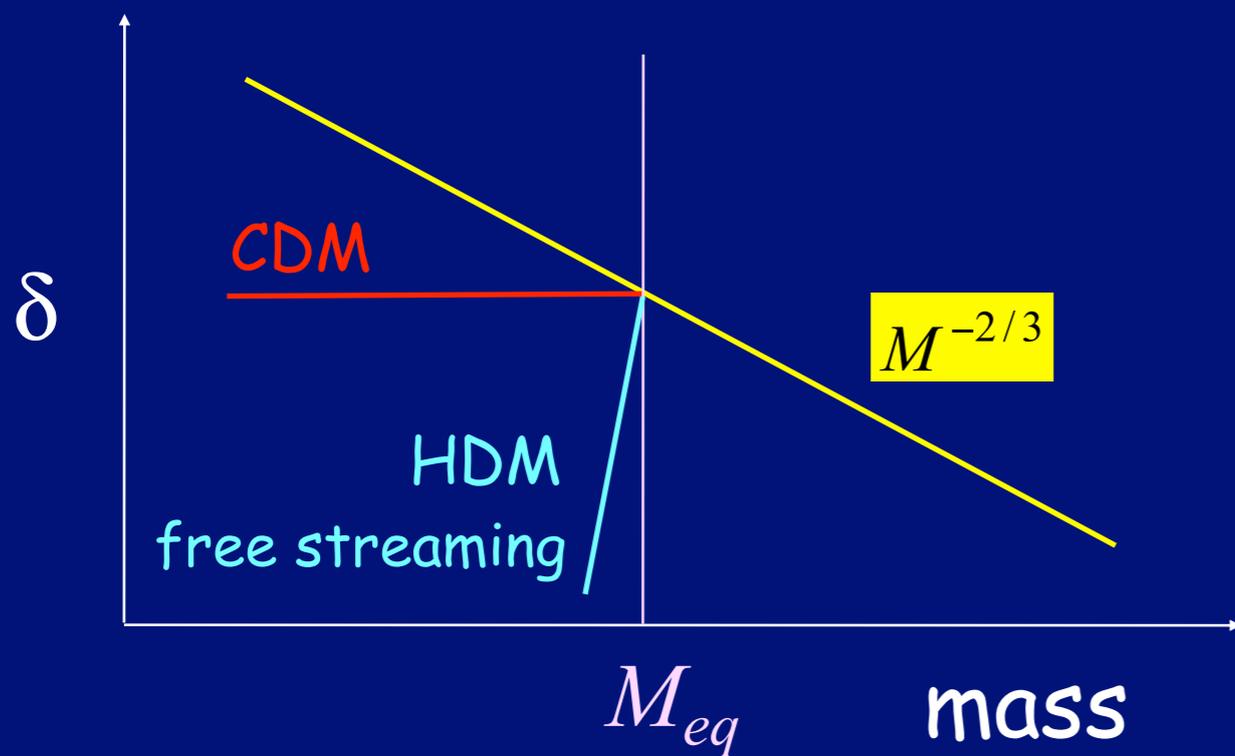
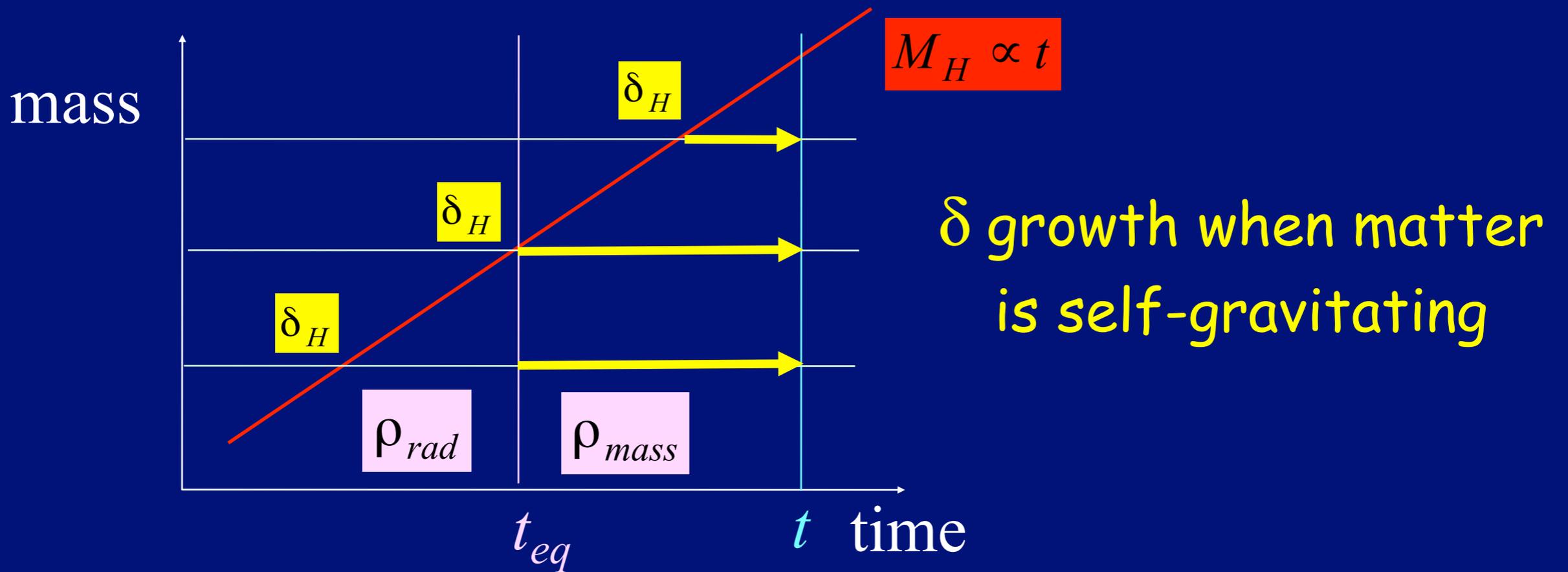
Scale-Invariant Spectrum (Harrison-Zel'dovich)



$$\delta(M, t) = \delta_H \left(\frac{t}{t_H(M)} \right)^{2/3} \propto M^{-2/3} t^{2/3}$$



CDM Power Spectrum



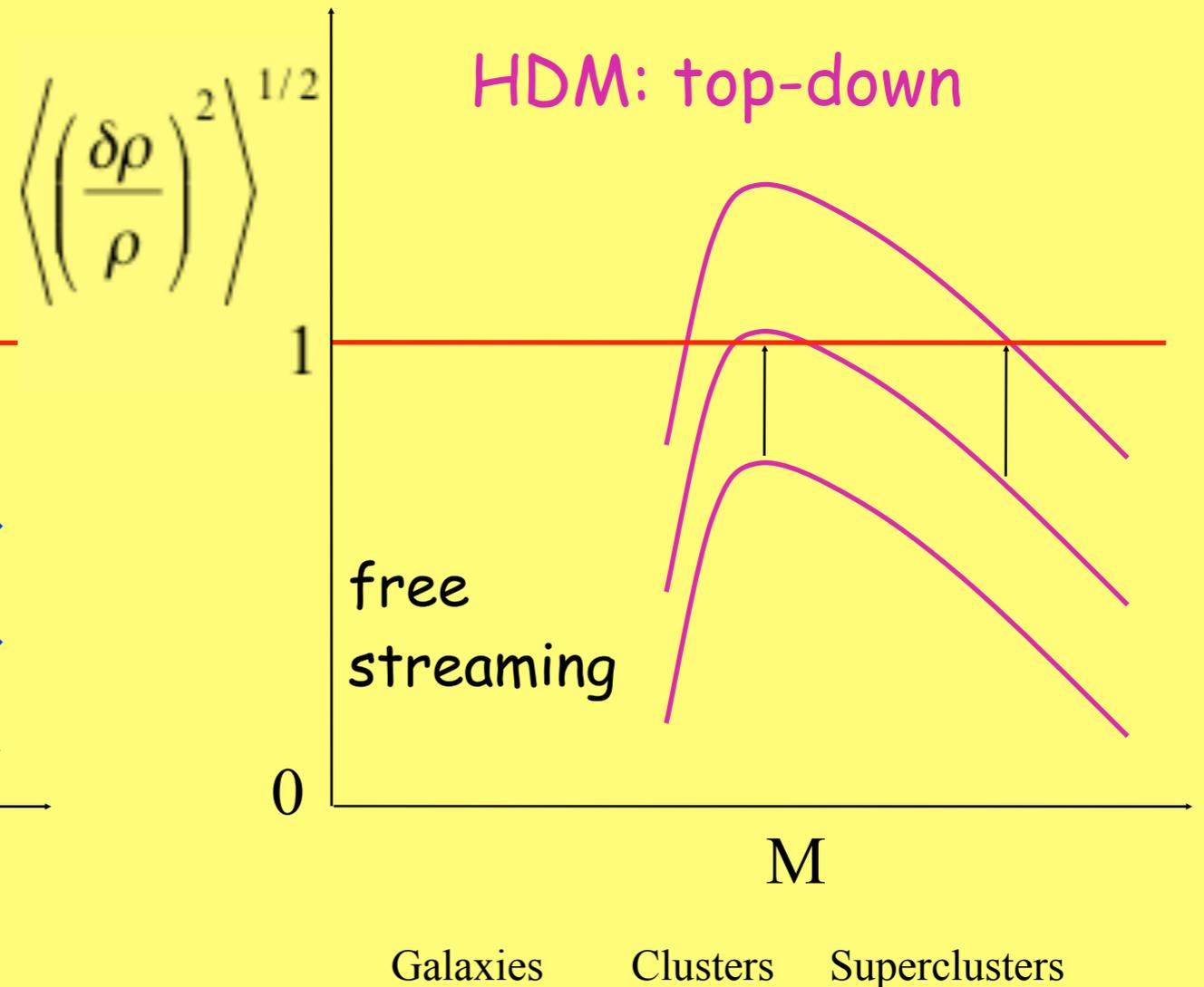
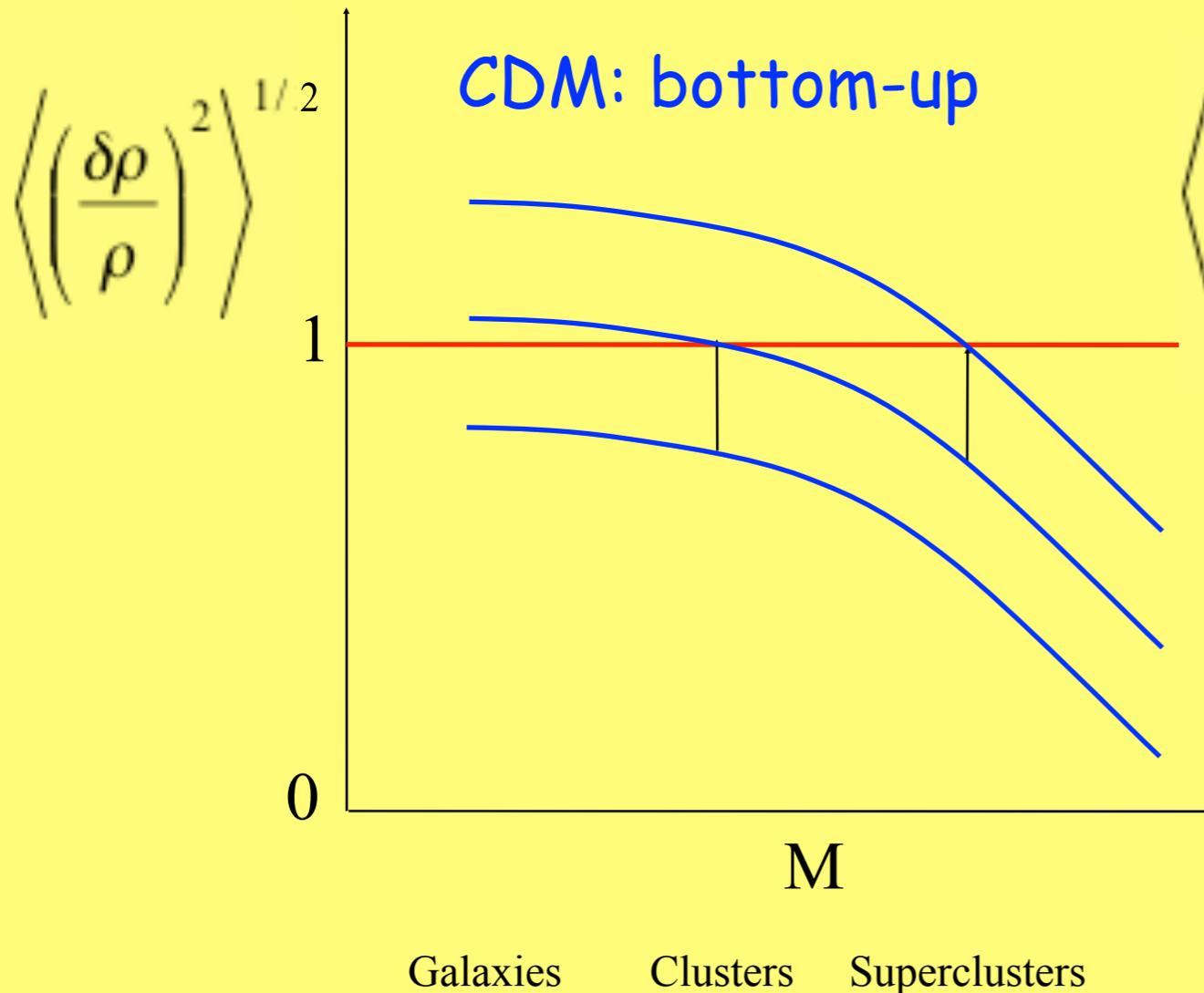
Formation of Large-Scale Structure

Fluctuation growth in the linear regime:

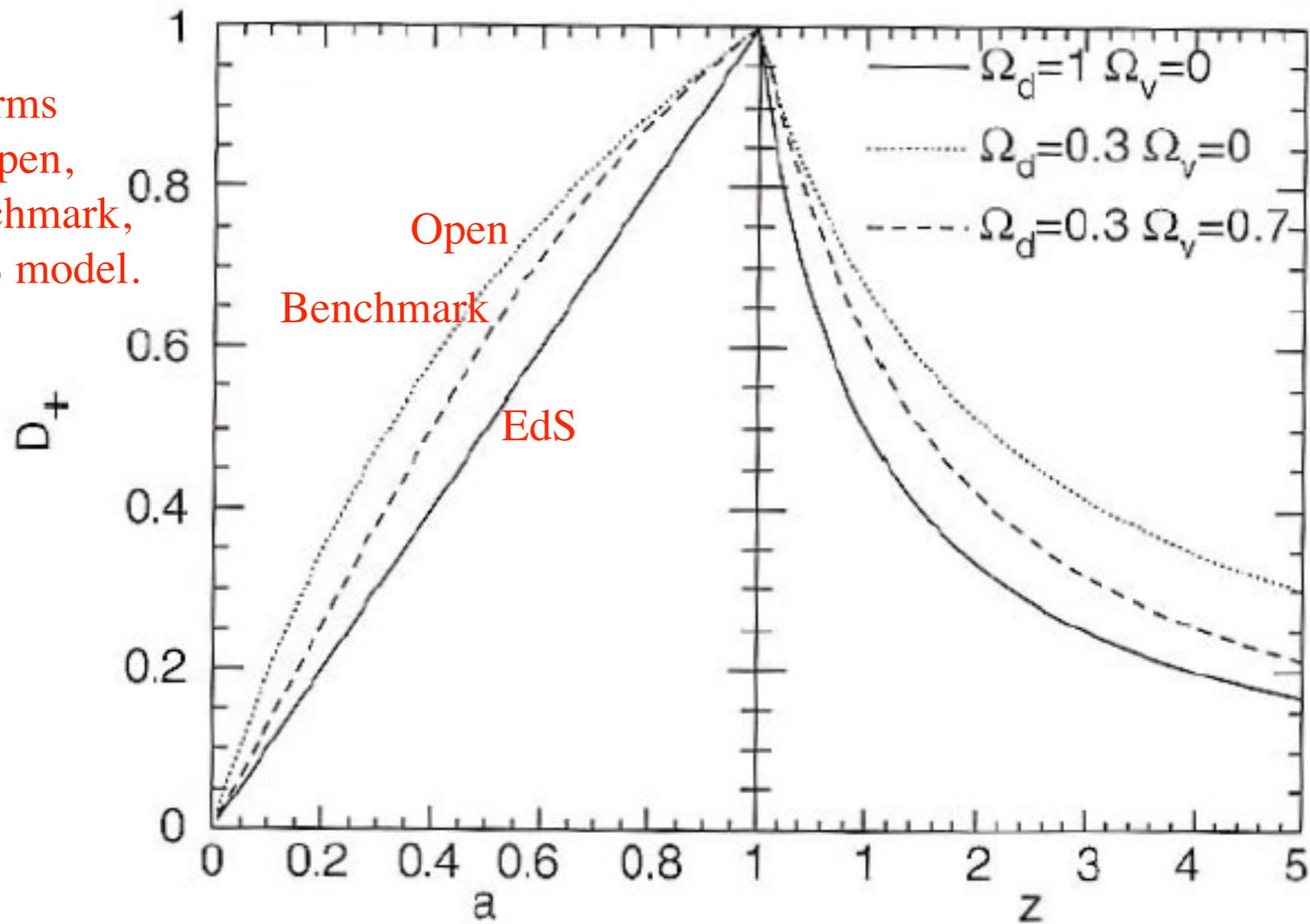
$$\delta \ll 1 \longrightarrow \delta \propto t^{2/3}$$

rms fluctuation at mass scale M :

$$\delta \propto M^{-\alpha} \quad 0 < \alpha \leq 2/3$$



Structure forms earliest in Open, next in Benchmark, latest in EdS model.



Einstein-de Sitter
Open universe
Benchmark model

Fig. 7.3. Growth factor D_+ for three different cosmological models, as a function of the scale factor a (left panel) and of redshift (right panel). It is clearly visible how quickly D_+ decreases with increasing redshift in the EdS model, in comparison to the models of lower density

From Peter Schneider, *Extragalactic Astronomy and Cosmology* (Springer, 2006)

Linear Growth Rate Function $D(a)$

For completeness, here we present some approximations used in the text. For the family of flat cosmologies ($\Omega_m + \Omega_\Lambda = 1$) an accurate approximation for the value of the virial overdensity Δ_{vir} is given by the analytic formula (Bryan & Norman 1998):

$$\Delta_{\text{vir}} = (18\pi^2 + 82x - 39x^2)/\Omega(z), \quad (\text{A1})$$

where $\Omega(z) \equiv \rho_m(z)/\rho_{\text{crit}}$ and $x \equiv \Omega(z) - 1$.

The linear growth-rate function $\delta(a)$, used in eqs. (14-15) and also in $\sigma_8(a)$ is defined as

$$\delta(a) = D(a)/D(1), \quad (\text{A2})$$

where $a = 1/(1+z)$ is the expansion parameter and $D(a)$ is:

$$D(a) = \frac{5}{2} \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}} \right)^{1/3} \frac{\sqrt{1+x^3}}{x^{3/2}} \int_0^x \frac{x^{3/2} dx}{[1+x^3]^{3/2}}, \quad (\text{A3})$$

$$x \equiv \left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} \right)^{1/3} a, \quad (\text{A4})$$

where $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ are density contributions of matter and cosmological constant at $z = 0$. For $\Omega_m > 0.1$ the growth rate factor $D(a)$ can be accurately approximated by the following expressions (Lahav et al. 1991; Carroll et al. 1992):

$$D(a) = \frac{(5/2)a\Omega_m}{\Omega_m^{4/7} - \Omega_\Lambda + (1 + \Omega_m/2)(1 + \Omega_\Lambda/70)}, \quad (\text{A5})$$

$$\Omega_m(a) = \Omega_{m,0}/(1+x^3), \quad (\text{A6})$$

$$\Omega_\Lambda(a) = 1 - \Omega_m(a) \quad (\text{A7})$$

For $\Omega_{m,0} = 0.27$ the error of these approximation is less than 7×10^{-4} .

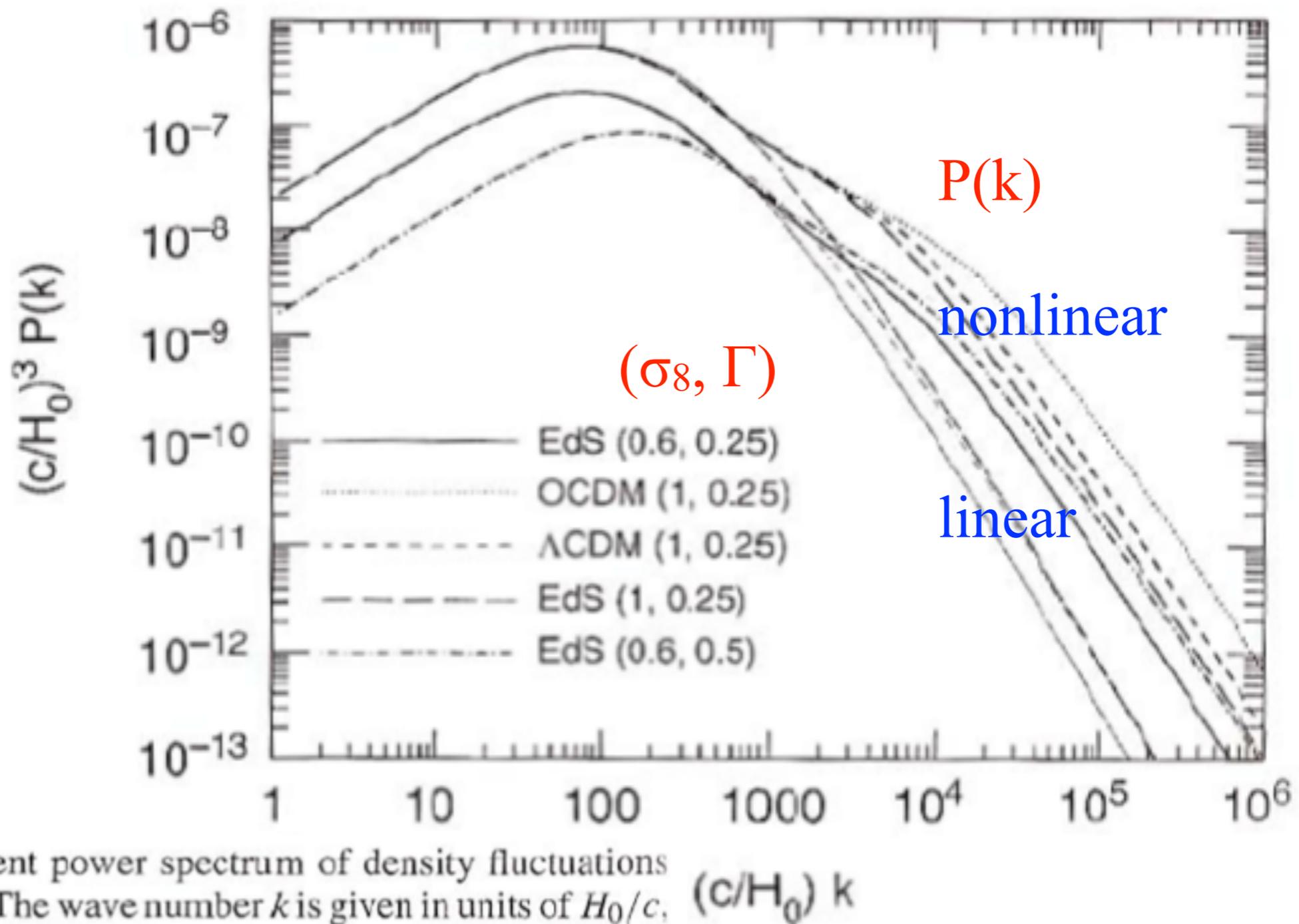
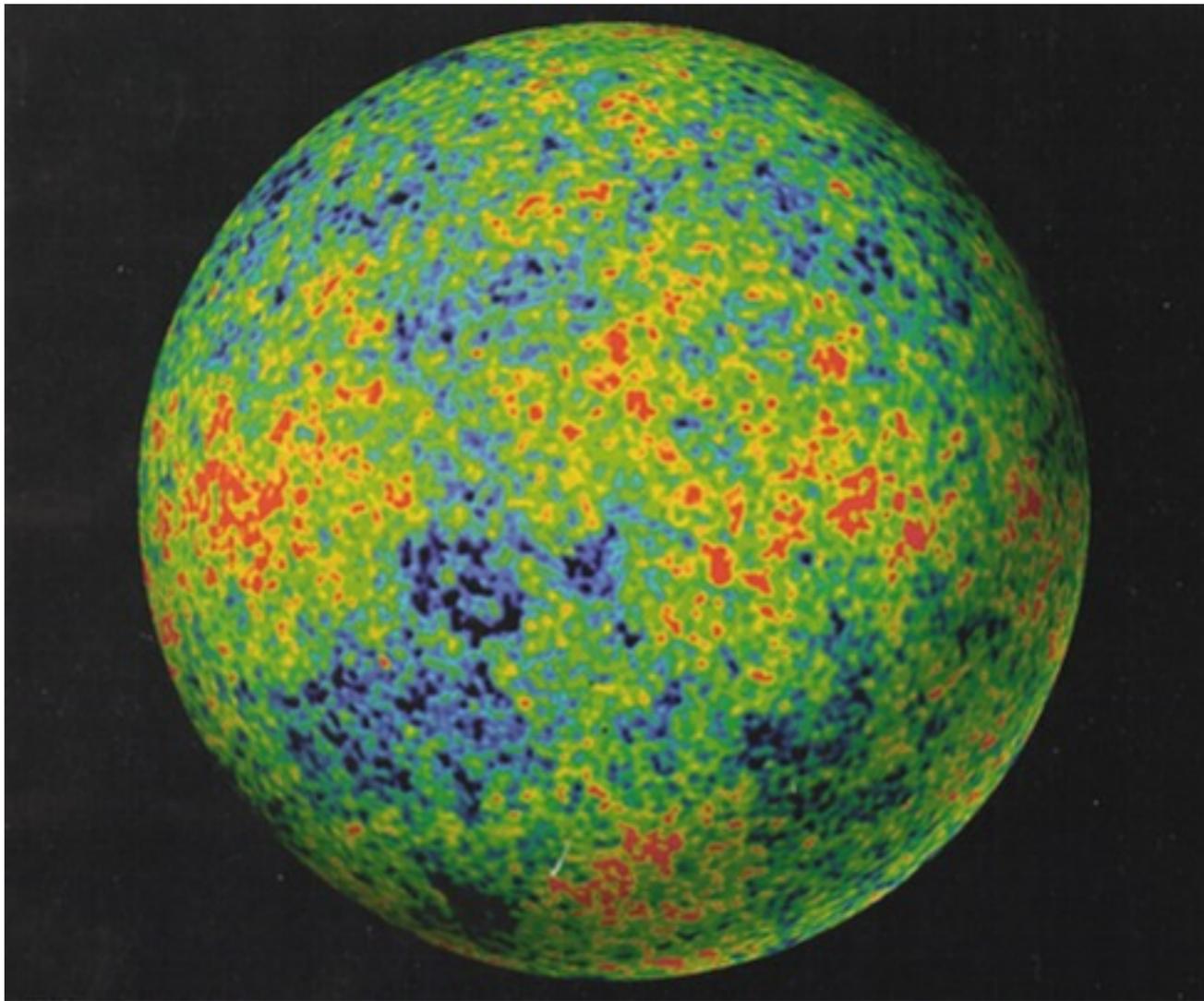


Fig. 7.6. The current power spectrum of density fluctuations for CDM models. The wave number k is given in units of H_0/c , and $(H_0/c)^3 P(k)$ is dimensionless. The various curves have different cosmological parameters: EdS: $\Omega_m = 1$, $\Omega_\Lambda = 0$; OCDM: $\Omega_m = 0.3$, $\Omega_\Lambda = 0$; Λ CDM: $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$. The values in parentheses specify (σ_8, Γ) , where σ_8 is the normalization of the power spectrum (which will be discussed below), and where Γ is the shape parameter. The thin curves correspond to the power spectrum $P_0(k)$ linearly extrapolated to the present day, and the bold curves take the non-linear evolution into account

From Peter Schneider,
*Extragalactic Astronomy and
 Cosmology* (Springer, 2006)

GRAVITY – The Ultimate Capitalist Principle

Astronomers say that a region of the universe with more matter is “richer.” Gravity magnifies differences—if one region is slightly denser than average, it will expand slightly more slowly and grow relatively denser than its surroundings, while regions with less than average density will become increasingly less dense. The rich always get richer, and the poor poorer.



The early universe expands *almost* perfectly uniformly. But there are small differences in density from place to place (about 30 parts per million). Because of gravity, denser regions expand more slowly, less dense regions more rapidly. Thus gravity amplifies the contrast between them, until...

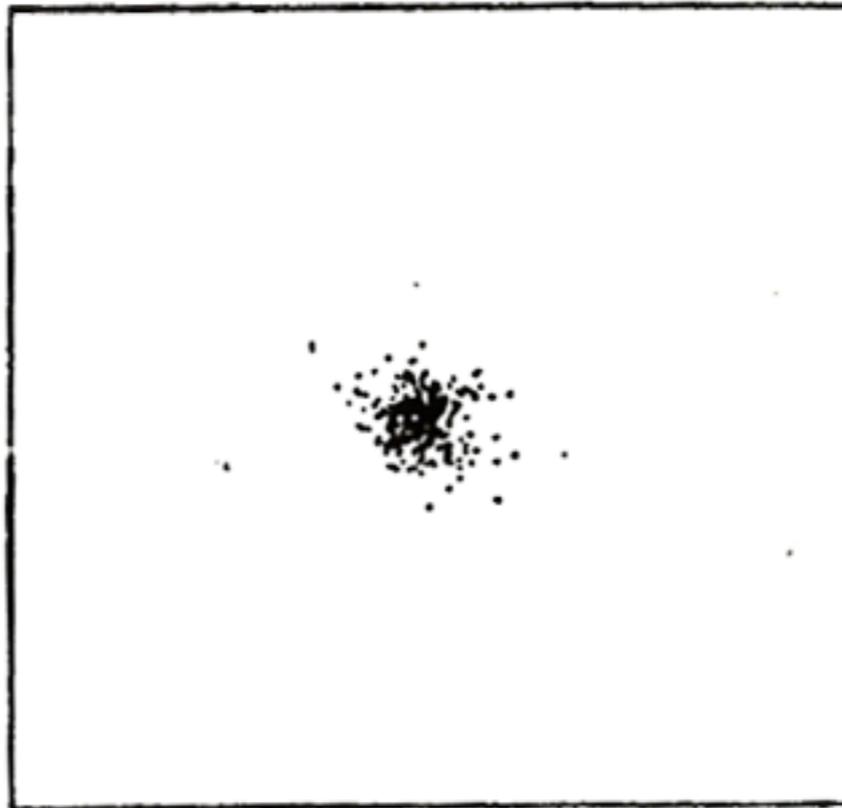
Temperature map at 380,000 years after the Big Bang. **Blue** (cooler) regions are slightly denser. From NASA's **WMAP** satellite, 2003.

Structure Formation by Gravitational Collapse



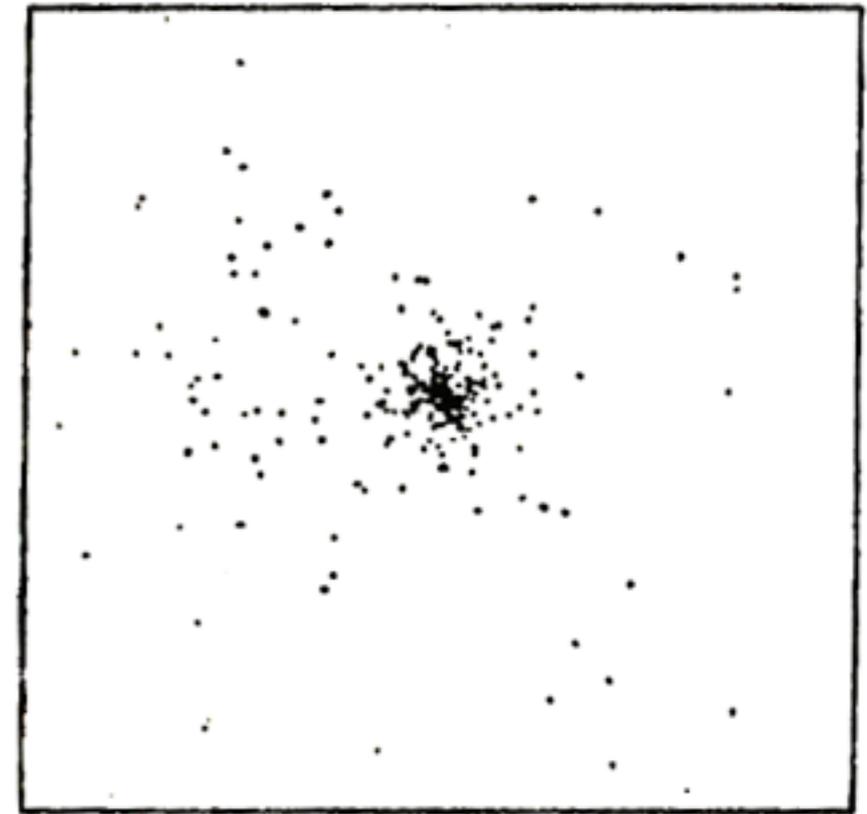
When any region becomes about twice as dense as typical regions its size, it reaches a maximum radius, *stops expanding,*

Simulation of top-hat collapse:
P.J.E. Peebles 1970, ApJ, 75, 13.

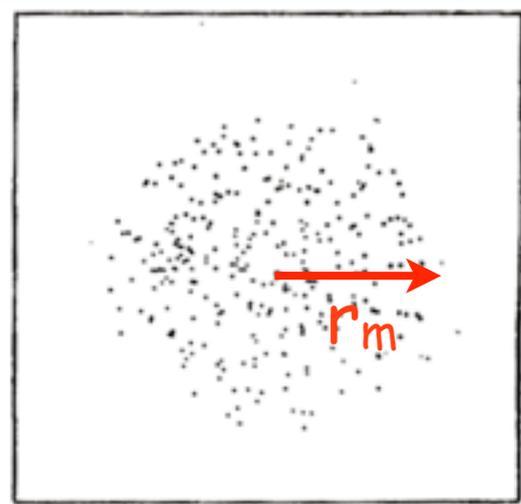


and starts falling together. The forces between the subregions generate velocities which *prevent* the material from *all falling toward the center.*

Used in my 1984 summer school lectures “Dark matter, Galaxies, and Large Scale Structure,” <http://tinyurl.com/3bjkn3>

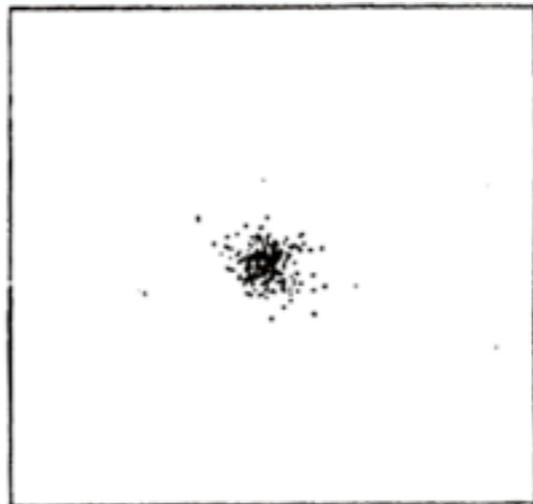


Through Violent Relaxation the dark matter quickly reaches a *stable configuration* that’s about half the maximum radius but denser in the center.

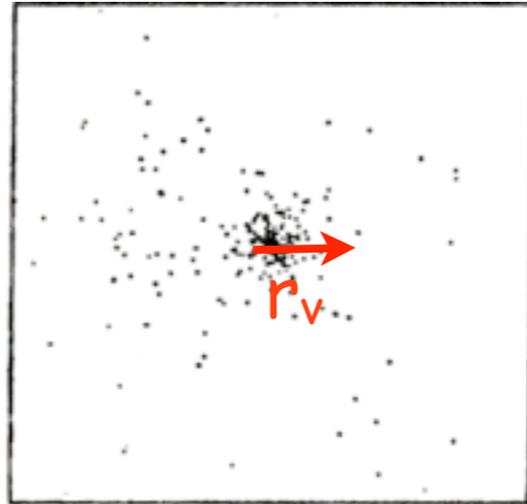


TOP HAT

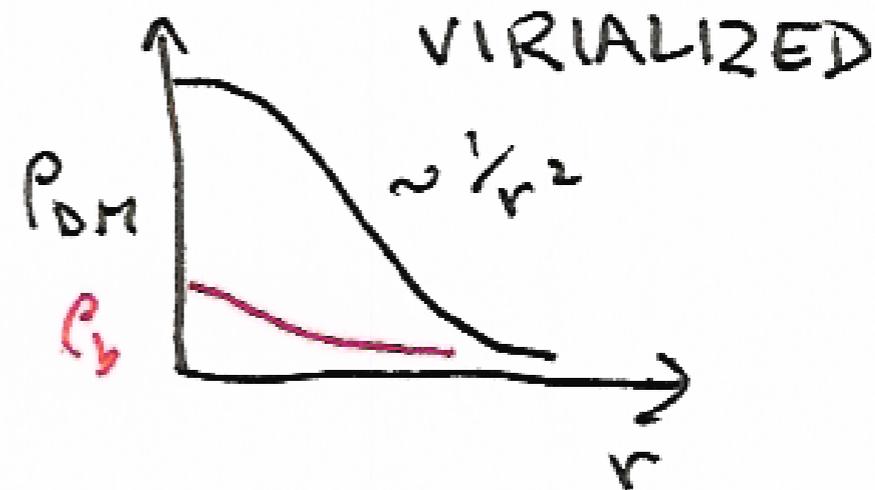
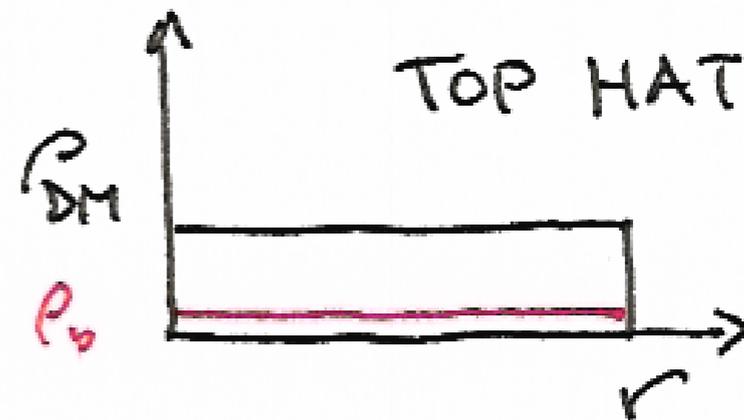
Max Expansion



VIOLENT
RELAXATION



VIRIALIZED



Virial Theorem: $\langle K \rangle = -\frac{1}{2} \langle W \rangle$

$W_m = \frac{C}{r_m}$, so after virialization

$\frac{C}{r_m} = E = W + K = \frac{1}{2} \langle W \rangle = \frac{C}{2r_v}$

$\Rightarrow r_v = \frac{1}{2} r_m, \rho_v = 8\rho_m \approx 50 \bar{\rho}(t_m)$

$\langle v^2 \rangle \approx \frac{GM}{r_v}$

Structure of Dark Matter Halos

Navarro, Frenk, White

1996

1997

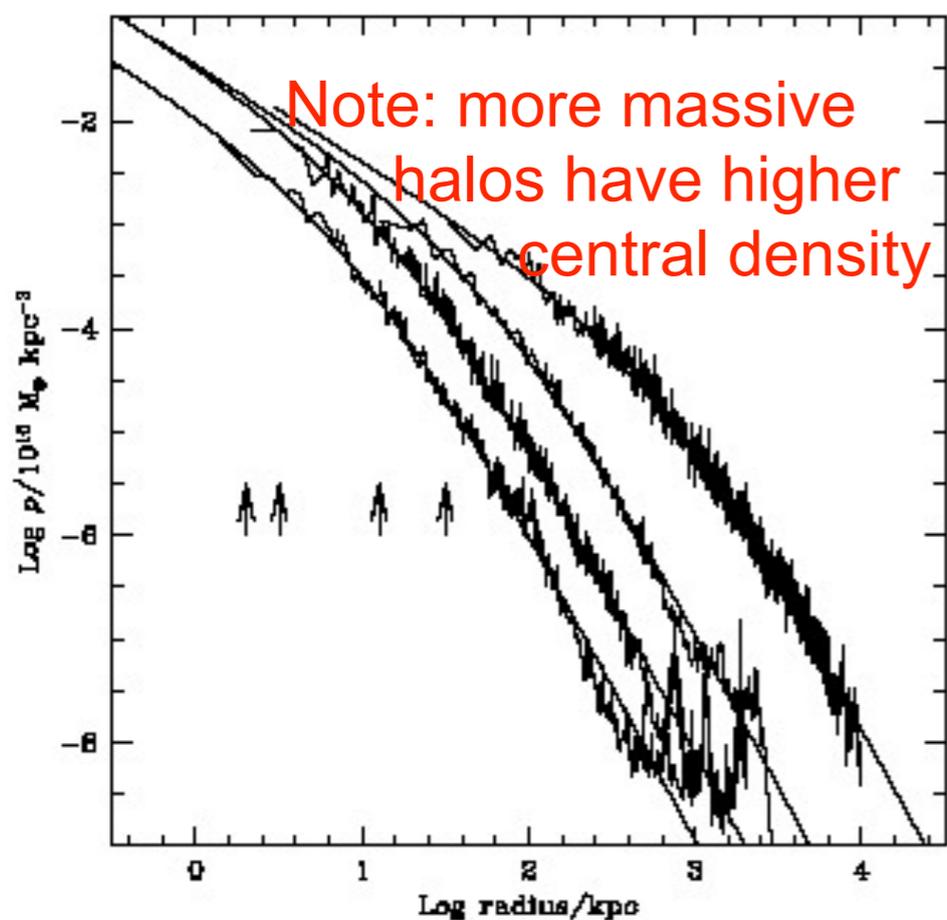
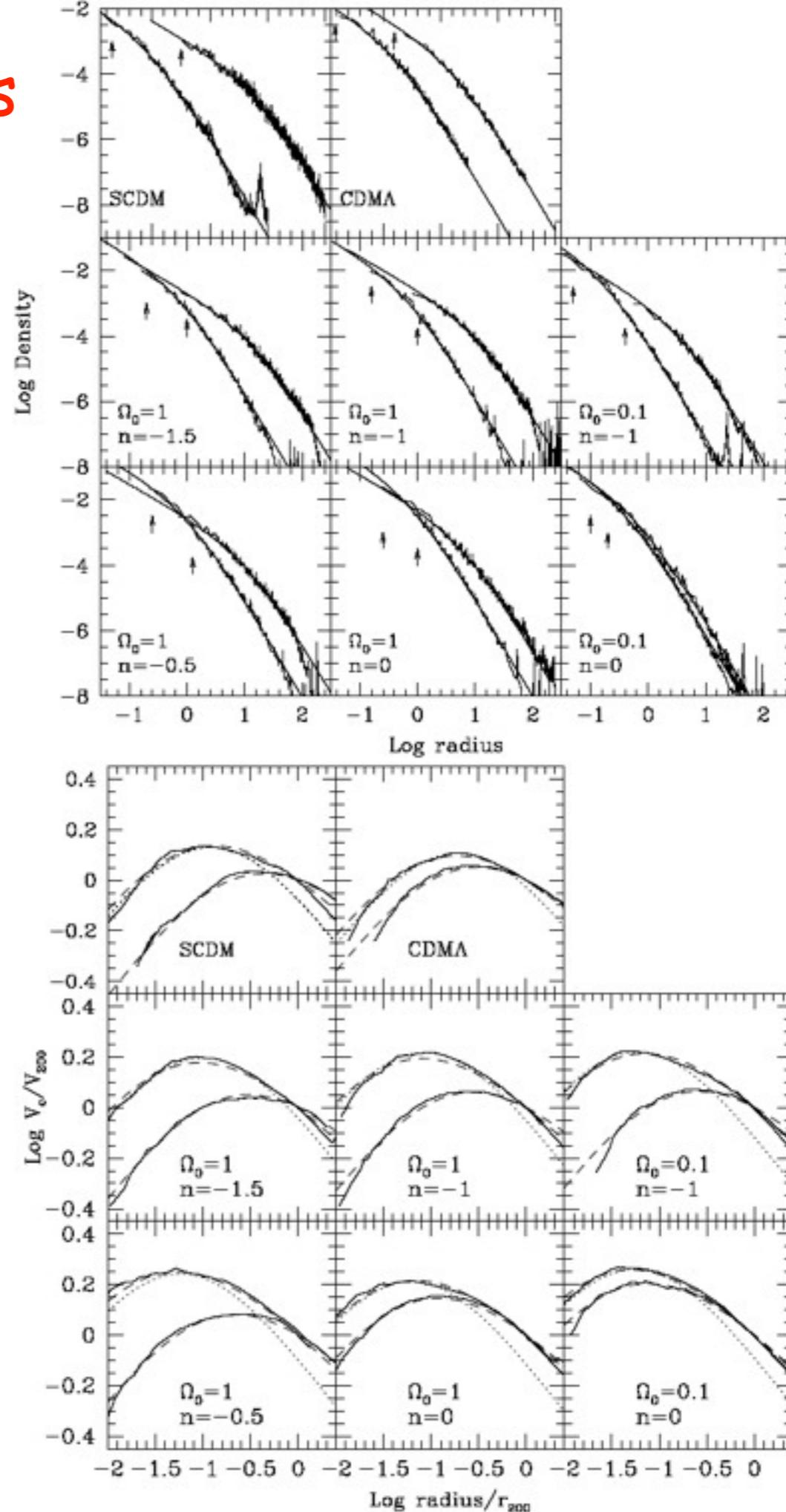


Fig. 3.— Density profiles of four halos spanning four orders of magnitude in mass. The arrows indicate the gravitational softening, h_g , of each simulation. Also shown are fits from eq.3. The fits are good over two decades in radius, approximately from h_g out to the virial radius of each system.

$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2}, \quad (3)$$

NFW is a good approximation for all models



Aquarius Simulation: Formation of a Milky-Way-size Dark Matter Halo

**Diameter of Milky Way Dark Matter Halo
1.6 million light years**

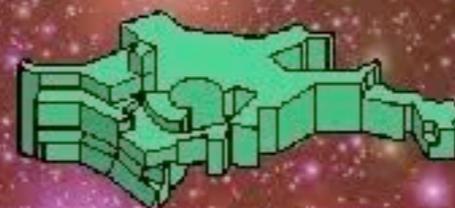
Diameter of visible Milky Way
30 kpc = 100,000 light years



Diameter of Milky Way Dark Matter Halo
1.6 million light years



500 kpc



Volker Springel
Max-Planck-Institute
for Astrophysics



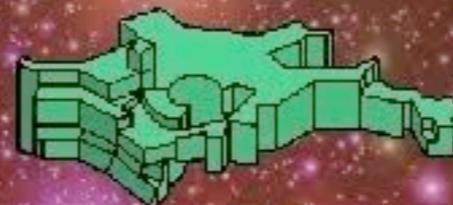
Diameter of visible Milky Way
30 kpc = 100,000 light years



Diameter of Milky Way Dark Matter Halo
1.6 million light years



500 kpc

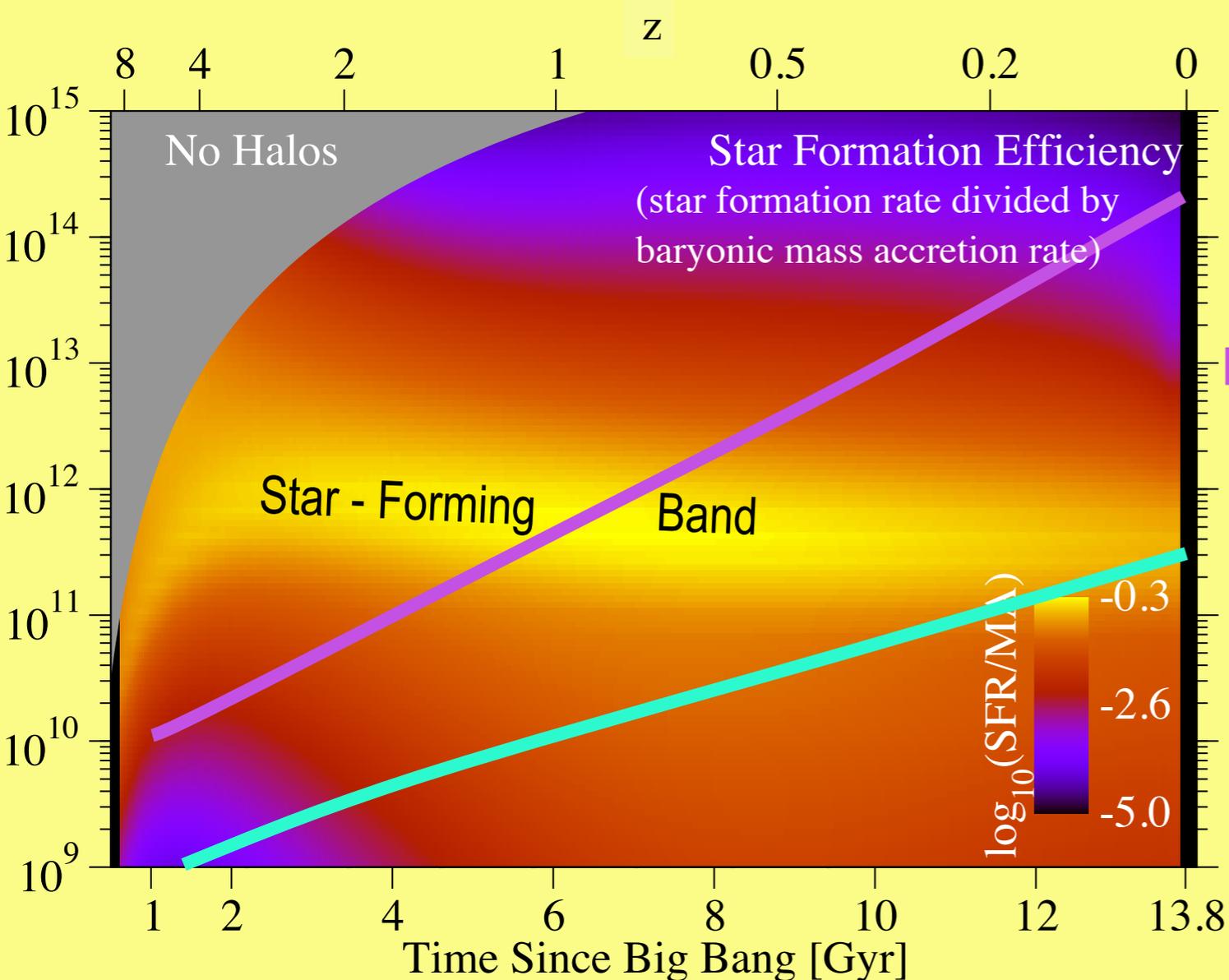


Volker Springel
Max-Planck-Institute
for Astrophysics



Implications of the Star-Forming Band Model

(from Lecture 13 - Dark Matter)



Massive galaxies:

- Started forming stars early.
- Shut down early.
- Are red today.
- Populate dark halos that are much more massive than their stellar mass.

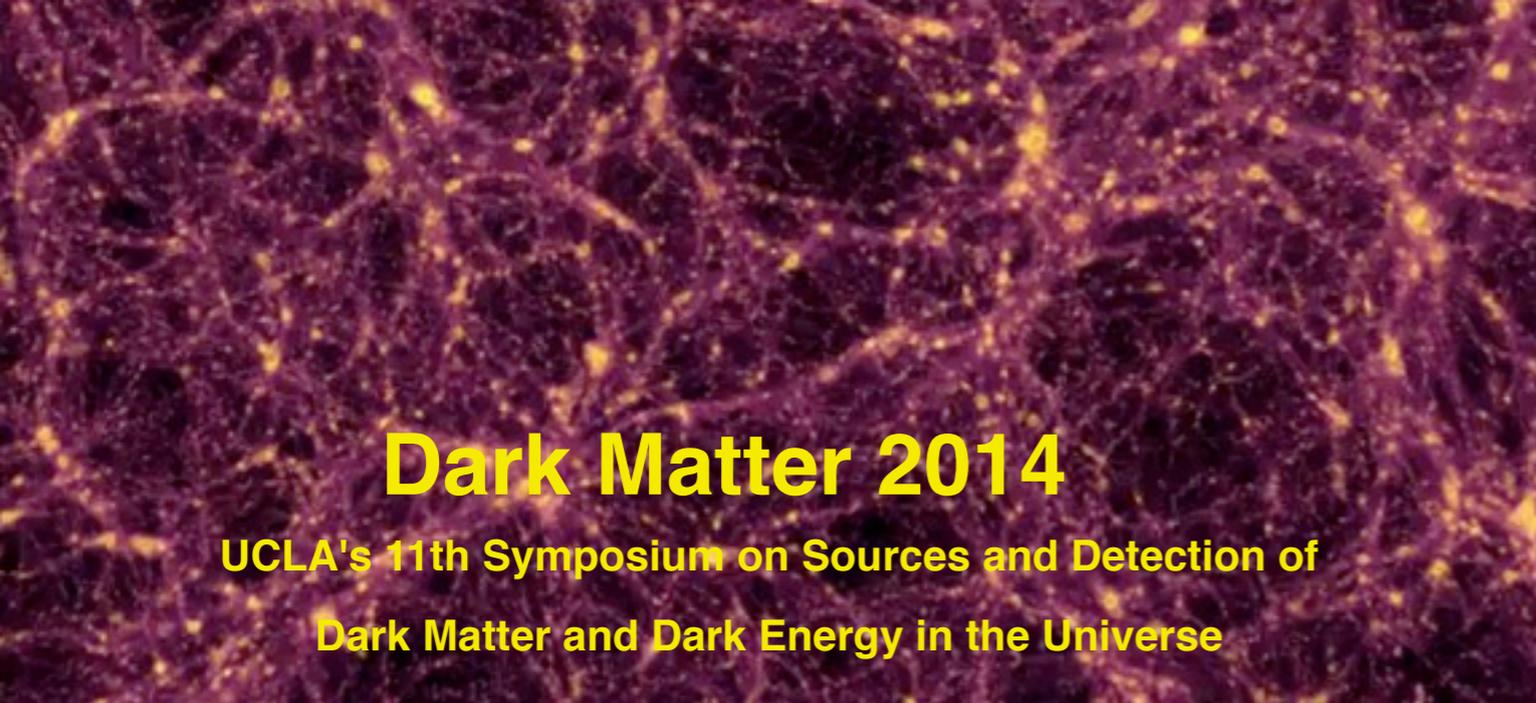
Small galaxies:

- Started forming stars late.
- Are still making stars today.
- Are blue today.
- Populate dark halos that match their stellar mass.

"Downsizing"

Star formation is a wave that started in the largest galaxies and swept down to smaller masses later (Cowie et al. 1996).

From Figure 1 of Behroozi, Wechsler, Conroy ApJL, 762, L31 (2013)



Dark Matter 2014

UCLA's 11th Symposium on Sources and Detection of
Dark Matter and Dark Energy in the Universe



Λ CDM cosmology: successes, challenges, and opportunities for progress

Joel Primack, UC Santa Cruz

- **Successes:** CMB, Expansion History, Large Scale Structure
But: WMAP9 vs. Planck, Planck Clusters vs. CMB?
- **Challenges:** Cusp-Core, Too Big to Fail, Satellite Galaxies
- **Opportunities for Progress Now:** Deeper Surveys, Halo Substructure, Early Galaxies, Galactic Archeology

Slides at <https://hepconf.physics.ucla.edu/dm14/agenda.html>

The Dark Matter Questionnaire

Tim Tait, UC Irvine



“Cold Dark Matter: An Exploded View” by Cornelia Parker

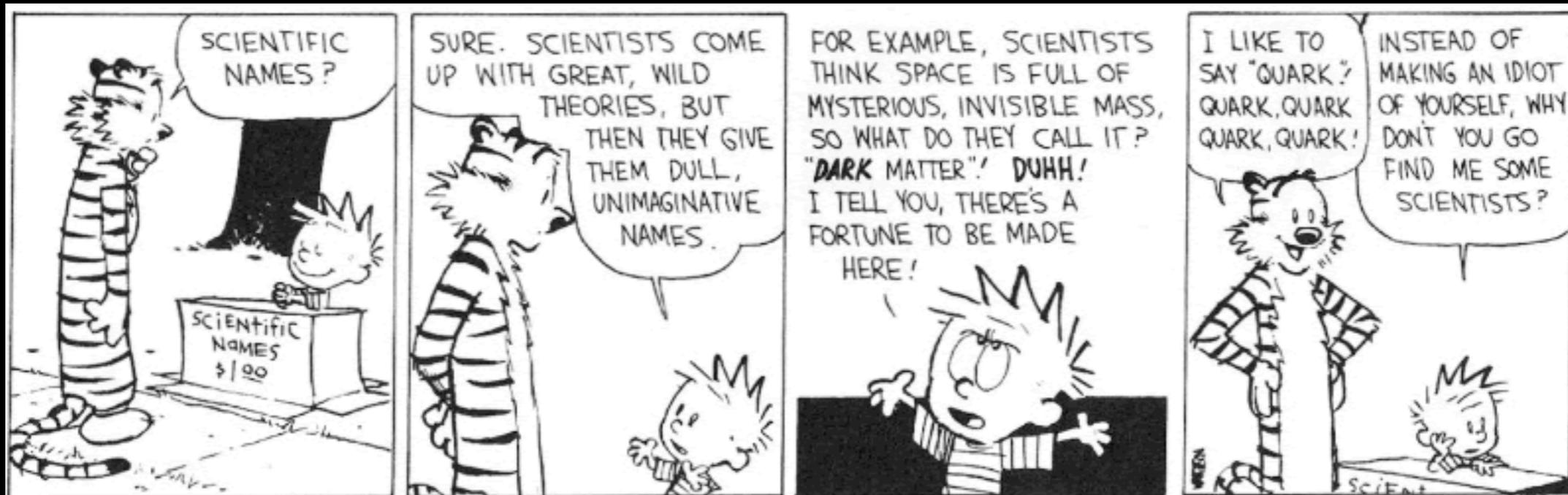
- Mass
- Spin
- Stable?
 - Yes
 - No

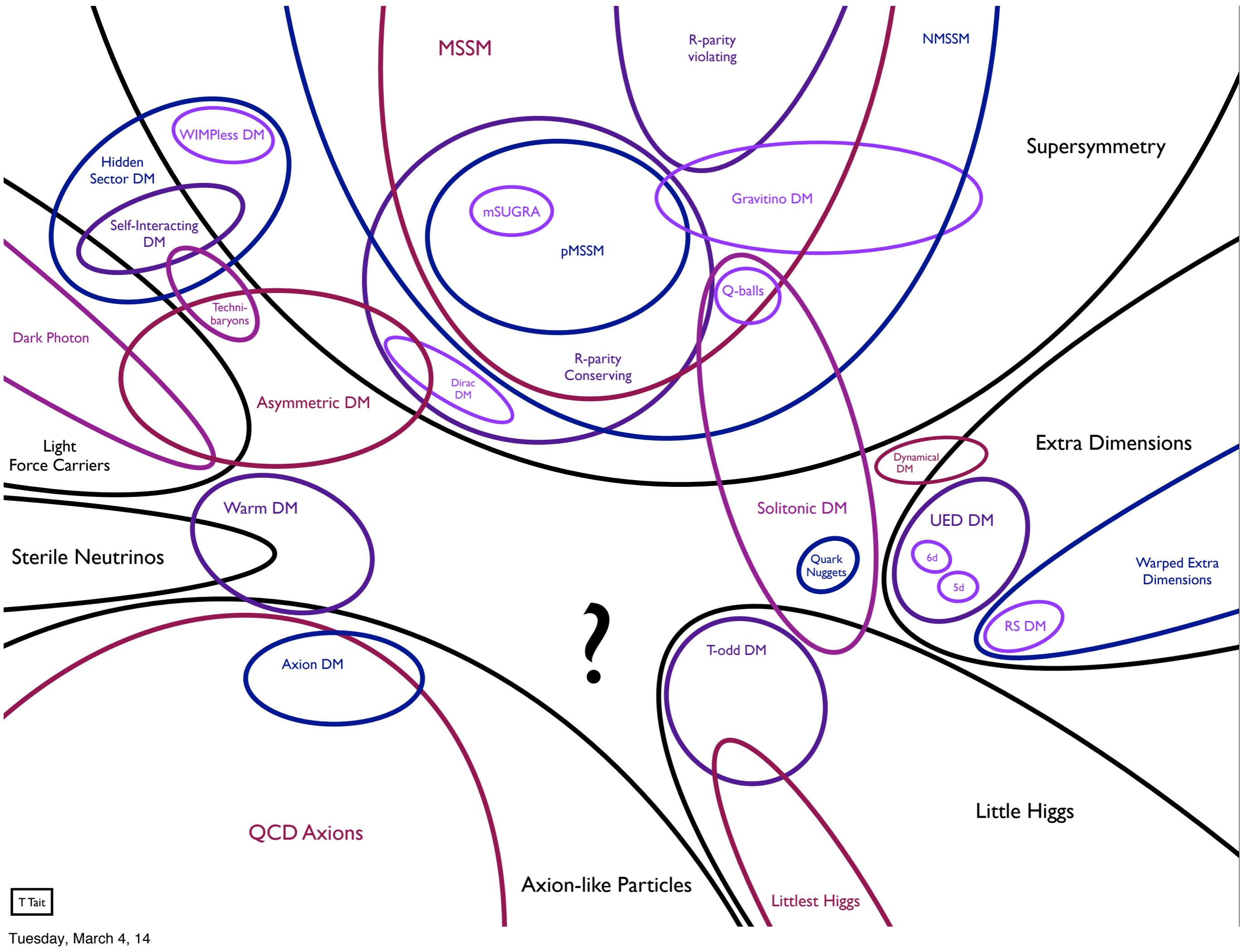
Couplings:

- Gravity
- Weak Interaction?
- Higgs?
- Quarks / Gluons?
- Leptons?

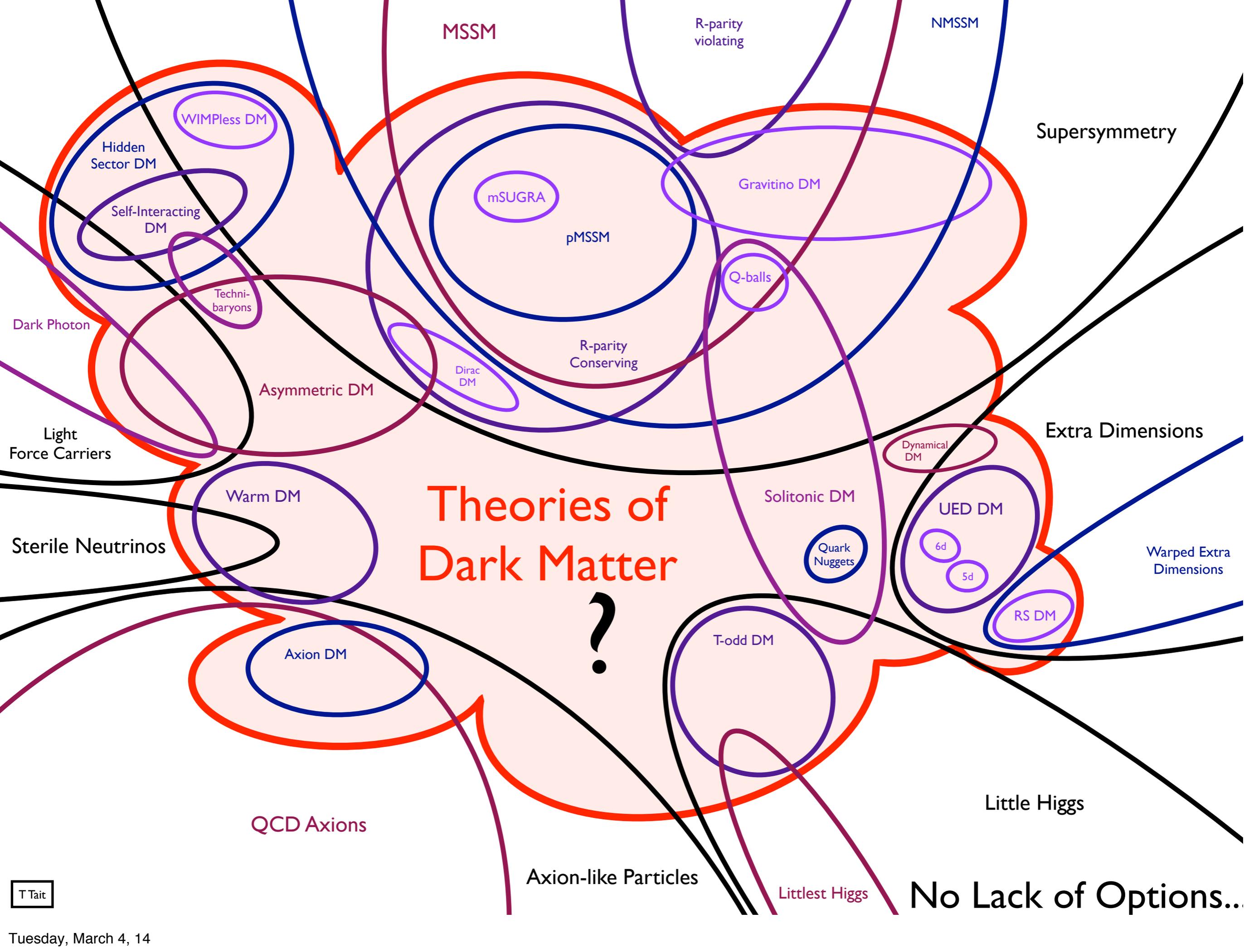
Thermal Relic?

- Yes
- No





Theories of Dark Matter



No Lack of Options...

Basic Features of the GeV Excess

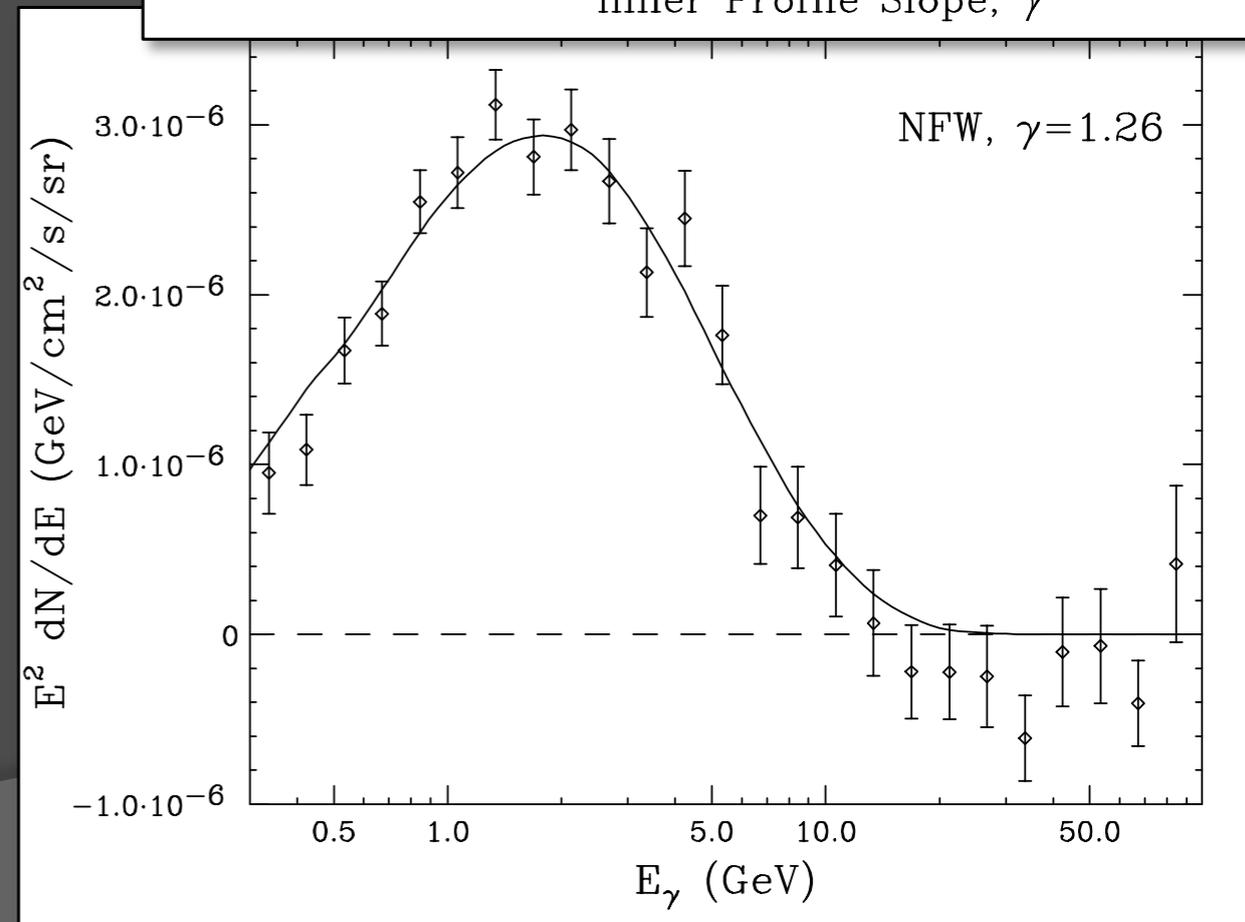
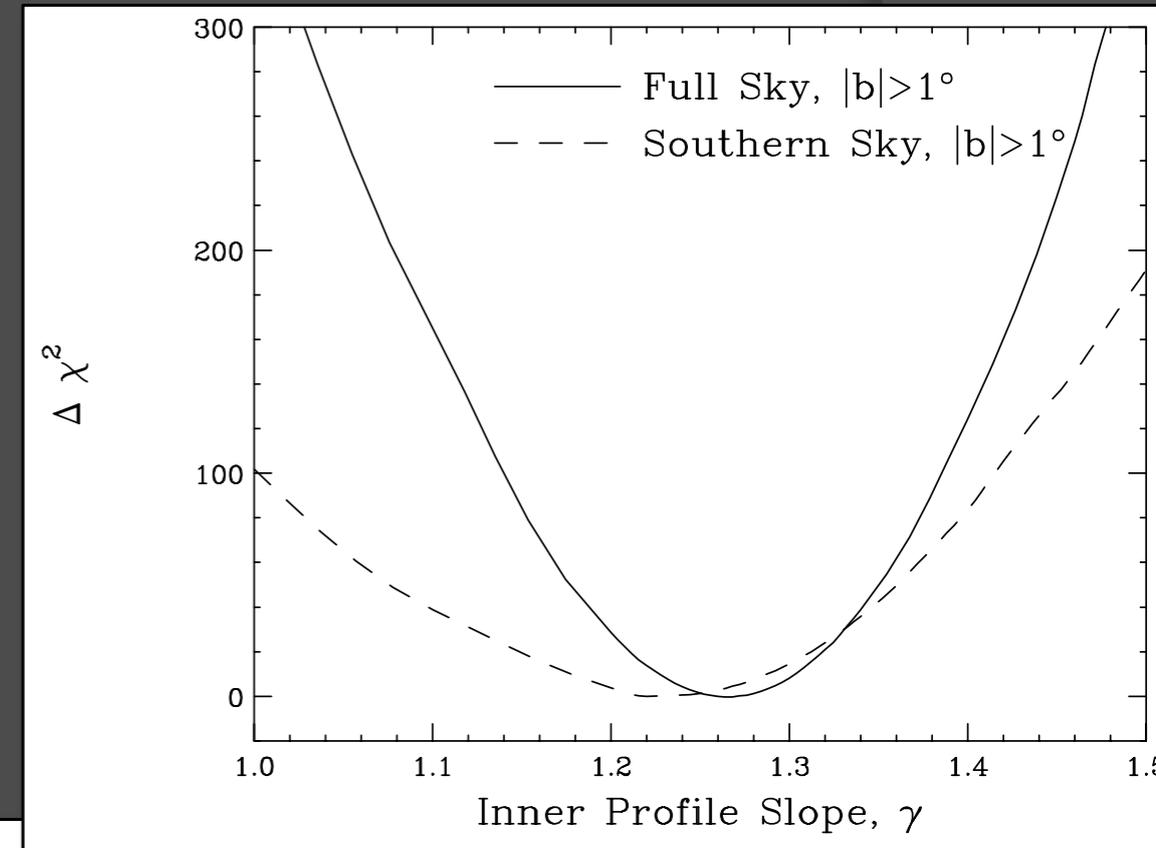
Dan Hooper's talk

The excess is distributed around the Galactic Center with a flux that falls off approximately as $r^{-2.4}$ (if interpreted as dark matter annihilation products, this implies $\rho_{\text{DM}} \sim r^{-1.2}$)

The spectrum of this excess peaks at $\sim 1\text{-}3$ GeV, and is in very good agreement with that predicted from a 30-40 GeV WIMP (annihilating to b quarks)

To normalize the observed signal with annihilating dark matter, a cross section of $\sigma v \sim (1\text{-}2) \times 10^{-26} \text{ cm}^3/\text{s}$ is required (for $\rho_{\text{local}} = 0.3 \text{ GeV}/\text{cm}^3$)

(See Tim Linden's talk)



The Characterization of the Gamma-Ray Signal from the Central Milky Way: A Compelling Case for Annihilating Dark Matter

Tansu Daylan,¹ Douglas P. Finkbeiner,^{1,2} Dan Hooper,^{3,4} Tim Linden,⁵
Stephen K. N. Portillo,² Nicholas L. Rodd,⁶ and Tracy R. Slatyer^{6,7}

¹*Department of Physics, Harvard University, Cambridge, MA*

²*Harvard-Smithsonian Center for Astrophysics, Cambridge, MA*

³*Fermi National Accelerator Laboratory, Theoretical Astrophysics Group, Batavia, IL*

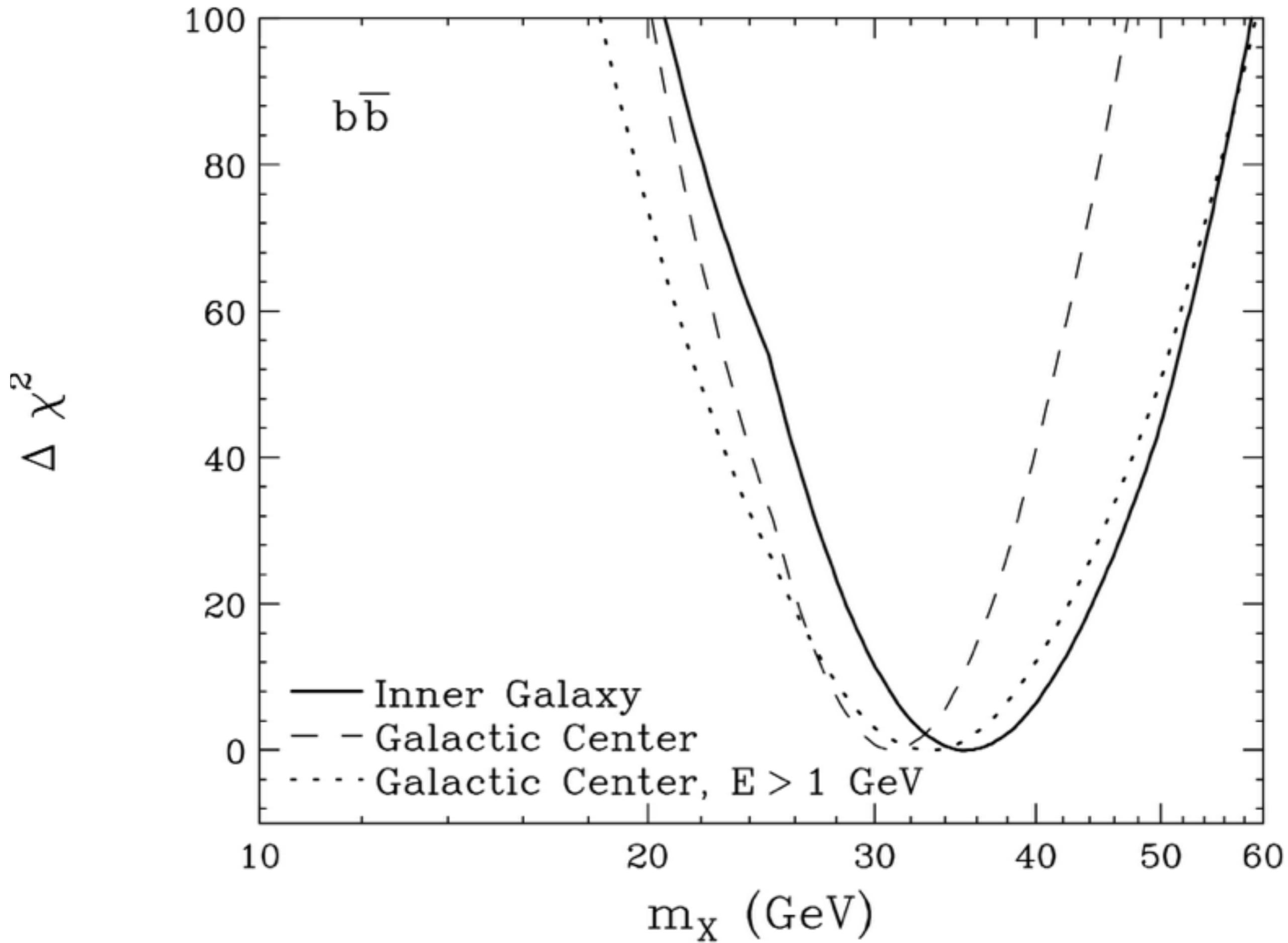
⁴*University of Chicago, Department of Astronomy and Astrophysics, Chicago, IL*

⁵*University of Chicago, Kavli Institute for Cosmological Physics, Chicago, IL*

⁶*Center for Theoretical Physics, Massachusetts Institute of Technology, Boston, MA*

⁷*School of Natural Sciences, Institute for Advanced Study, Princeton, NJ*

Past studies have identified a spatially extended excess of $\sim 1\text{-}3$ GeV gamma rays from the region surrounding the Galactic Center, consistent with the emission expected from annihilating dark matter. We revisit and scrutinize this signal with the intention of further constraining its characteristics and origin. By applying cuts to the *Fermi* event parameter CTBCORE, we suppress the tails of the point spread function and generate high resolution gamma-ray maps, enabling us to more easily separate the various gamma-ray components. Within these maps, we find the GeV excess to be robust and highly statistically significant, with a spectrum, angular distribution, and overall normalization that is in good agreement with that predicted by simple annihilating dark matter models. For example, the signal is very well fit by a 31-40 GeV dark matter particle annihilating to $b\bar{b}$ with an annihilation cross section of $\sigma v = (1.4 - 2.0) \times 10^{-26}$ cm³/s (normalized to a local dark matter density of 0.3 GeV/cm³). Furthermore, we confirm that the angular distribution of the excess is approximately spherically symmetric and centered around the dynamical center of the Milky Way (within $\sim 0.05^\circ$ of Sgr A*), showing no sign of elongation along or perpendicular to the Galactic Plane. The signal is observed to extend to at least $\simeq 10^\circ$ from the Galactic Center, disfavoring the possibility that this emission originates from millisecond pulsars.



What's new?

Why should we take this seriously?

Reason 1: Overwhelming Statistical Significance and Detailed Information

- This excess consists of $\sim 10^4$ photons per square meter, per year (>1 GeV, within 10° of the Galactic Center)
- In our Inner Galaxy analysis, the quality of the best-fit found with a dark matter component improves over the best-fit without a dark matter component by over 40σ (the Galactic Center analysis “only” prefers a dark matter component at the level of 17σ)
- This huge data set allows us to really scrutinize the signal, extracting its characteristics in some detail

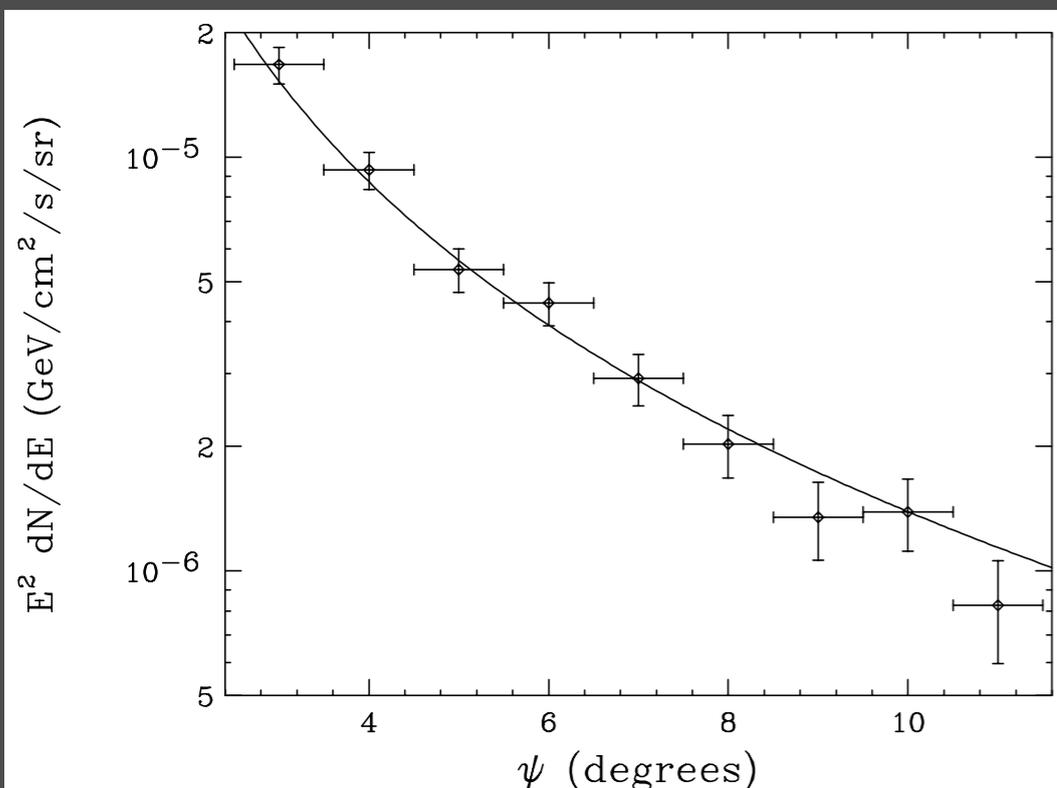
Reason 2: The Signal is Well-Fit by Simple, Predictive Dark Matter Models

On the other hand, the Milky Way's gamma-ray excess is fit by very simple and predictive dark matter models. We tune only 1) the halo profile's slope, 2) the dark matter's mass, and 3) the dark matter's annihilation cross section and final state

Reason 3: The Lack of a Plausible Alternative Interpretation

This signal does not correlate with the distribution of gas, dust, magnetic fields, cosmic rays, star formation, or radiation (It does, however, trace quite well the square of the dark matter density, for a profile slightly steeper than NFW)

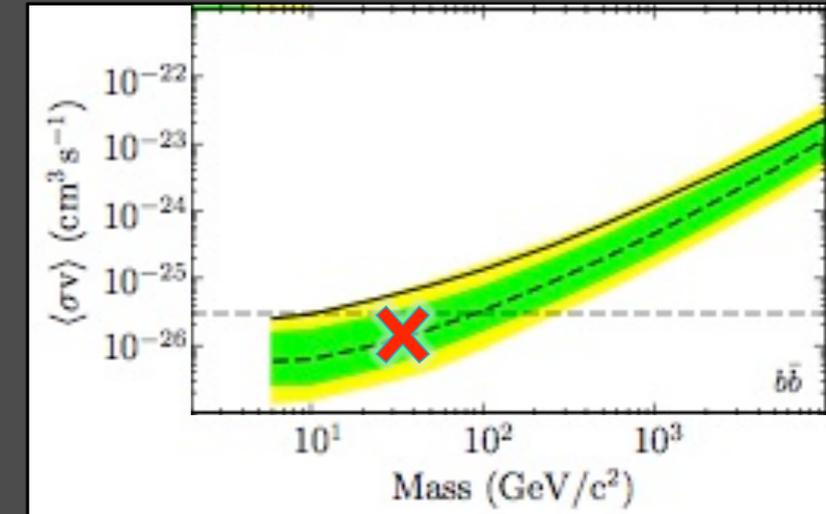
No known diffuse emission mechanisms can account for this excess



Dwarf Spheroidal Galaxies

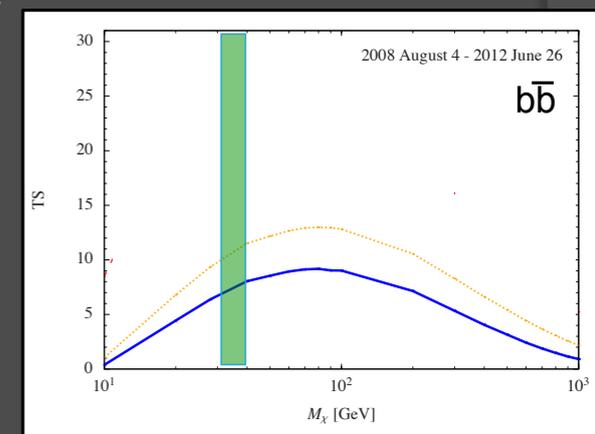
Other ways to test this

- The Fermi Collaboration has recently presented their analysis of 25 dwarf spheroidal galaxies, making use of 4 years of data
- They find a modest excess, $\sim 2-3\sigma$ (local)
- If interpreted as a signal of dark matter, this would imply a mass and cross section that is very similar to that required to account for the Galactic Center/ Inner Galaxy excess
- With more data from Fermi, this hint could potentially become statistically significant



Galaxy Clusters

- Galaxy clusters are also promising targets for indirect dark matter searches, competitive with dwarf galaxies
- Two groups have reported a gamma-ray excess from the Virgo cluster, at the level of $\sim 2-3\sigma$
- The results of these analyses depend critically on the treatment of point sources and diffuse cosmic ray induced emission, making it difficult to know how seriously one should take this result
- If the excess from Virgo arises from dark matter annihilation, it also suggests a similar mass and cross section that that implied by the Galactic Center excess (upto uncertainties in the boost factor; see talk by Miguel Sanchez-Conde)
- Again, more data should help to clarify



Summary

- We have revisited and scrutinized the gamma-ray emission from the Central Milky Way, as observed by Fermi
- The previously reported GeV excess persists, and is highly statistically significant and robust
- The spectrum and angular distribution of this signal is very well fit by a 31-40 GeV WIMP (annihilating to b quarks), distributed as $\rho \sim r^{-1.2}$
- The normalization of this signal requires a dark matter annihilation cross section of $\sigma v \sim (1.4-2.0) \times 10^{-26} \text{ cm}^3/\text{s}$ (for $\rho_{\text{local}} = 0.3 \text{ GeV}/\text{cm}^3$); this is in remarkable agreement with the value predicted for a simple thermal relic
- The excess is distributed with approximate spherical symmetry and extends out to at least 10° from the Galactic Center
- Although a population of several thousand millisecond pulsars might have been able to account for much of the excess observed within $1-2^\circ$ of the Galactic Center, the very extended nature of this signal strongly disfavors pulsars as the primary sources of this emission