Physics 129 LECTURE 4 January 16, 2014

- Special Relativity Doppler shift
- SR + QM ⇒ Spin & Statistics & PCT

- Introduction to General Relativity:

  In accelerating reference frames there are "pseudoforces" F_{pseudo} = ma, where a is the acceleration of the frame. General Relativity: gravity is a pseudoforce that is eliminated in inertial (freely falling) reference frames.

- Classic GR tests: precession, light deflection, redshift
- GR passes much more precise tests
- GR is essential for the GPS system - so, for our everyday lives!
- Black Holes in the Universe

Lorentz transformations and the Doppler effect

The Lorentz transformation between inertial reference frame (x,t) and inertial frame (x',t') moving at speed v in the x-direction is (setting $c = 1$ and writing $v/c = \beta$)

$$t' = \gamma(t - \beta x), \quad x' = \gamma(x - \beta t), \quad y' = y, \quad z' = z$$

Since all 4-vectors transform the same way, the Lorentz transformation between the same frames for the energy-momentum 4-vector $P^\mu = (E,\boldsymbol{p})$ is

$$E' = \gamma(E - \beta p_x), \quad p_x' = \gamma(p_x - \beta E), \quad p_y' = p_y, \quad p_z' = p_z$$

Since $|\boldsymbol{p}| = p = E$ for photons, this says that for a photon moving in the x-direction $E' = \gamma E\,(1 - \beta) = E\,[(1 - \beta)/(1 + \beta)]^{1/2}$. Since $E = hf$ where h = Planck's constant and f is the frequency, the frequencies are related by

$$f' = f\,[(1 - \beta)/(1 + \beta)]^{1/2} \approx f\,(1 - \beta),$$

which is 1st order in velocity β. Frequency decreases, so wavelength $\lambda = c/f$ increases, or "red-shifts" $\lambda' \approx \lambda\,(1 + \beta)$ so $\Delta\lambda \equiv \lambda' - \lambda \approx \lambda\,\beta$ and $\Delta\lambda/\lambda \approx \beta$. (We will derive a different formula in the expanding universe.) For a photon moving in the transverse (y or z) direction, the only effect is time dilation, and the redshift is second order in velocity β.

Schrodinger Equation: $i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$ where $\hat{H} = p^2/2m$ or $E = p^2/2m$

for free particle

Newtonian

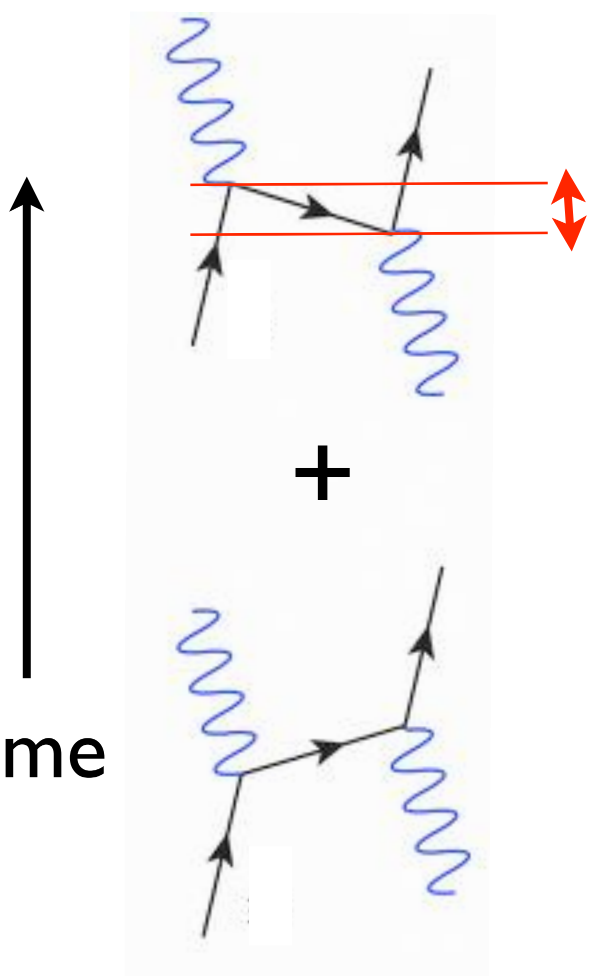
matrices

Dirac Equation: $i\hbar\frac{\partial}{\partial t}\Psi = (\beta mc^2 + \boldsymbol{\alpha}\cdot\boldsymbol{p}c)\Psi$, with $E^2 = \boldsymbol{p}^2c^2 + m^2c^4$

Relativistic

Special Relativity + Quantum Mechanics
⇒ Antiparticles (& CPT) & Spin-Statistics

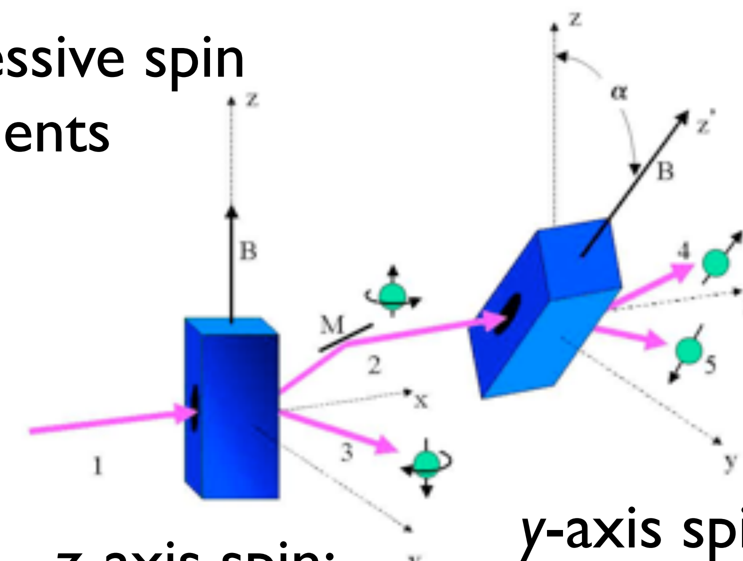
Feynman diagrams for electron-photon scattering (Compton scattering)



time

Electron goes backward in time = anti-electron (positron)

Two successive spin measurements



z-axis spin: "white-black"

2 possibilities (spin-1/2)

y-axis spin: "sheep-goats" (perpendicular: α=90°)

Two successive w-b or s-g measurements give the same answer, but if all the w's from the w-b measurement are subjected to a s-g measurement, half the s's will be w and half will be b, and the same for the g's. That is, measuring s-g interferes with measuring w-b!

Special Relativity is based on two postulates

- All the laws of physics are the same in all inertial reference frames.

- The speed of light is the same for all inertial observers, regardless of their velocity or that of the source of the light.

Einstein realized that Newton's theory of gravity, with instantaneous action at a distance, could not be compatible with special relativity -- which undermined the concept of simultaneous events at a distance. It took 10 years for Einstein to get the right idea for the right theory, but then in only two months in late 1915 he worked out the theory and its main initial predictions: the precession of the orbit of Mercury, bending of light by the sun, and the slowing of clocks by gravity.

General Relativity is also based on two postulates

- Equivalence Principle: All the effects of gravity on small scales are the same as those of acceleration. (Thus gravity is eliminated in local inertial = free fall frames.)

- Einstein's Field Equations: $G_{\mu\nu} = -(8\pi G/c^4)\,T_{\mu\nu}$ where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu}$ describes the curvature of space-time at each point and $T_{\mu\nu}$ describes the mass-energy, momentum, and stress density at the same point.

General Relativity

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

MATTER TELLS SPACE HOW TO CURVE

Einstein Field Equations

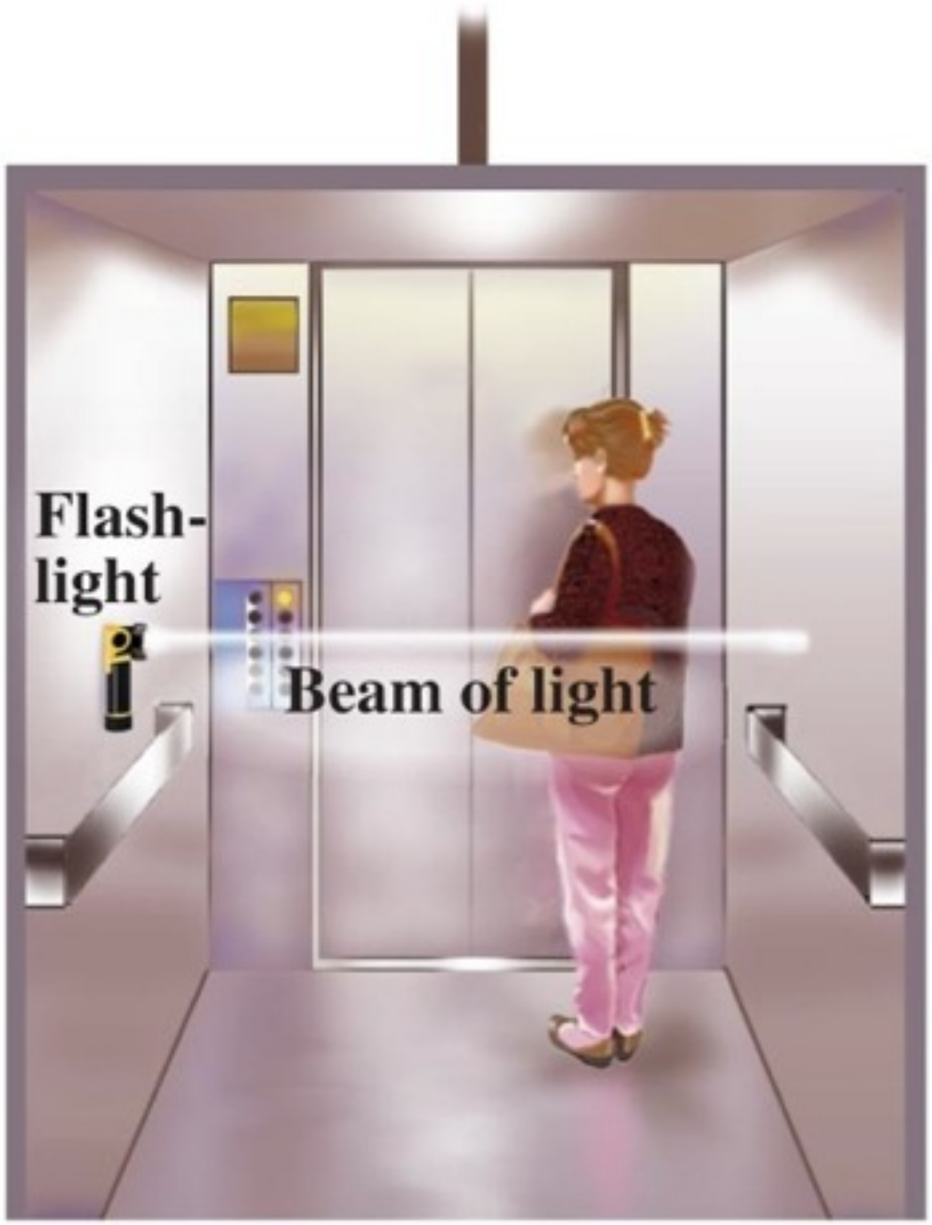
$$G^{\mu\nu} \equiv R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} = -\,8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

Here u^α is the velocity 4-vector of a particle. The Ricci curvature tensor $R_{\mu\nu} \equiv R_{\lambda\mu\sigma\nu} g^{\lambda\sigma}$, the Riemann curvature tensor $R^\lambda{}_{\mu\sigma\nu}$, and the affine connection $\Gamma^\mu{}_{\alpha\beta}$ can be calculated from the metric tensor $g_{\lambda\sigma}$. If the metric is just that of flat space, then $\Gamma^\mu{}_{\alpha\beta} = 0$ and the first equation above just says that the particle is unaccelerated -- i.e., it satisfies the law of inertia (Newton's 1st law).

Equivalence Principle:

All the effects of gravity on small scales are the same as those of acceleration.

This predicts the path of a beam of light in a gravitational field -- it will be a parabola.

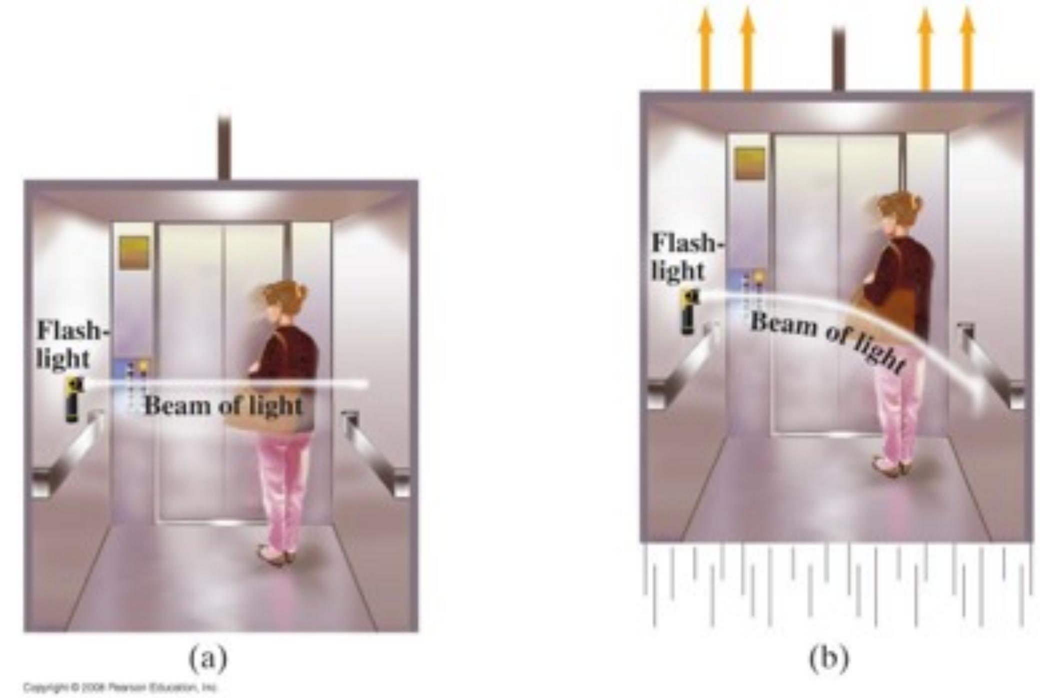


(a)

(b)

Equivalence Principle: All the effects of gravity on small scales are the same as those of acceleration.

This predicts the path of a beam of light in a gravitational field -- it will be a parabola.



In accelerating reference frames there are "pseudoforces"
F_{pseudo} = ma, where a is the acceleration of the frame.

General Relativity says: gravity is a pseudoforce which is eliminated in inertial (freely falling) reference frames.

In GR it's often helpful to analyze a system from a conveniently chosen inertial (freely falling) frame.

General Relativity: Observational Implications & Tests

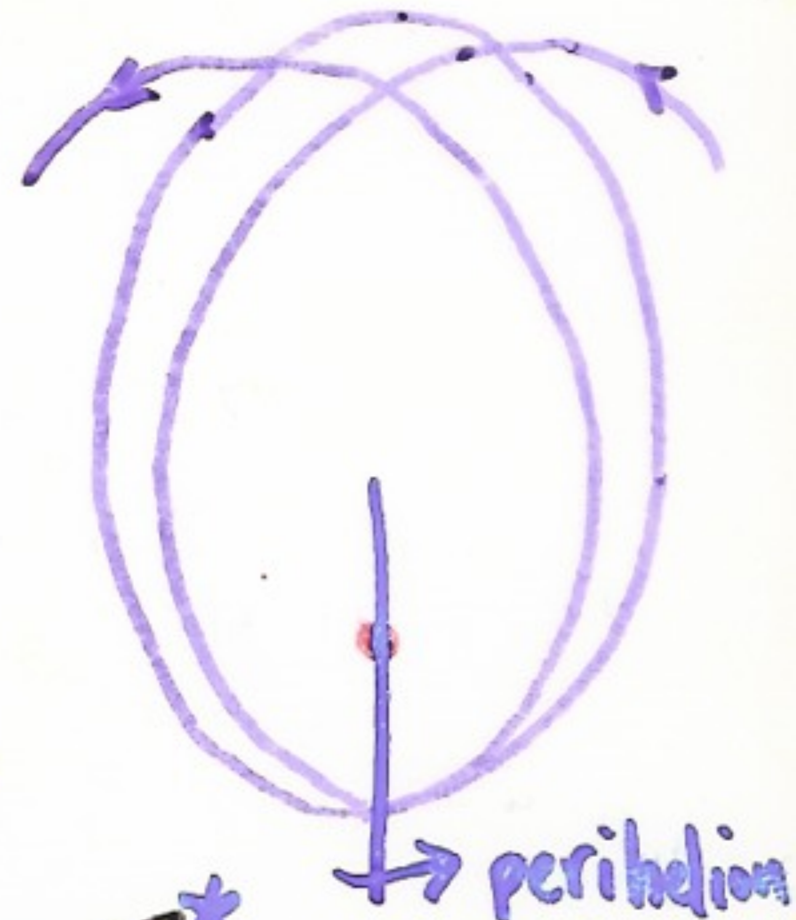
PERIHELION SHIFT OF ORBITS

finite speed of gravity, curvature

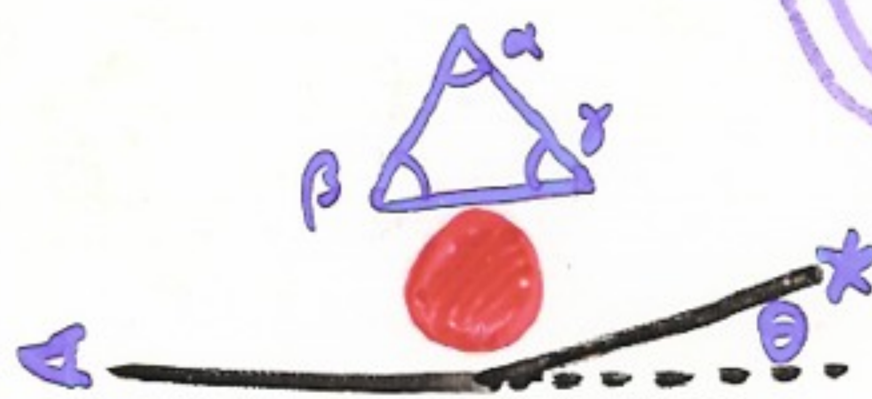
⇒ axis of elliptical orbit rotates

For Mercury, GR predicts 43"/century

– just what Simon Newcomb saw!



DEFLECTION OF LIGHT

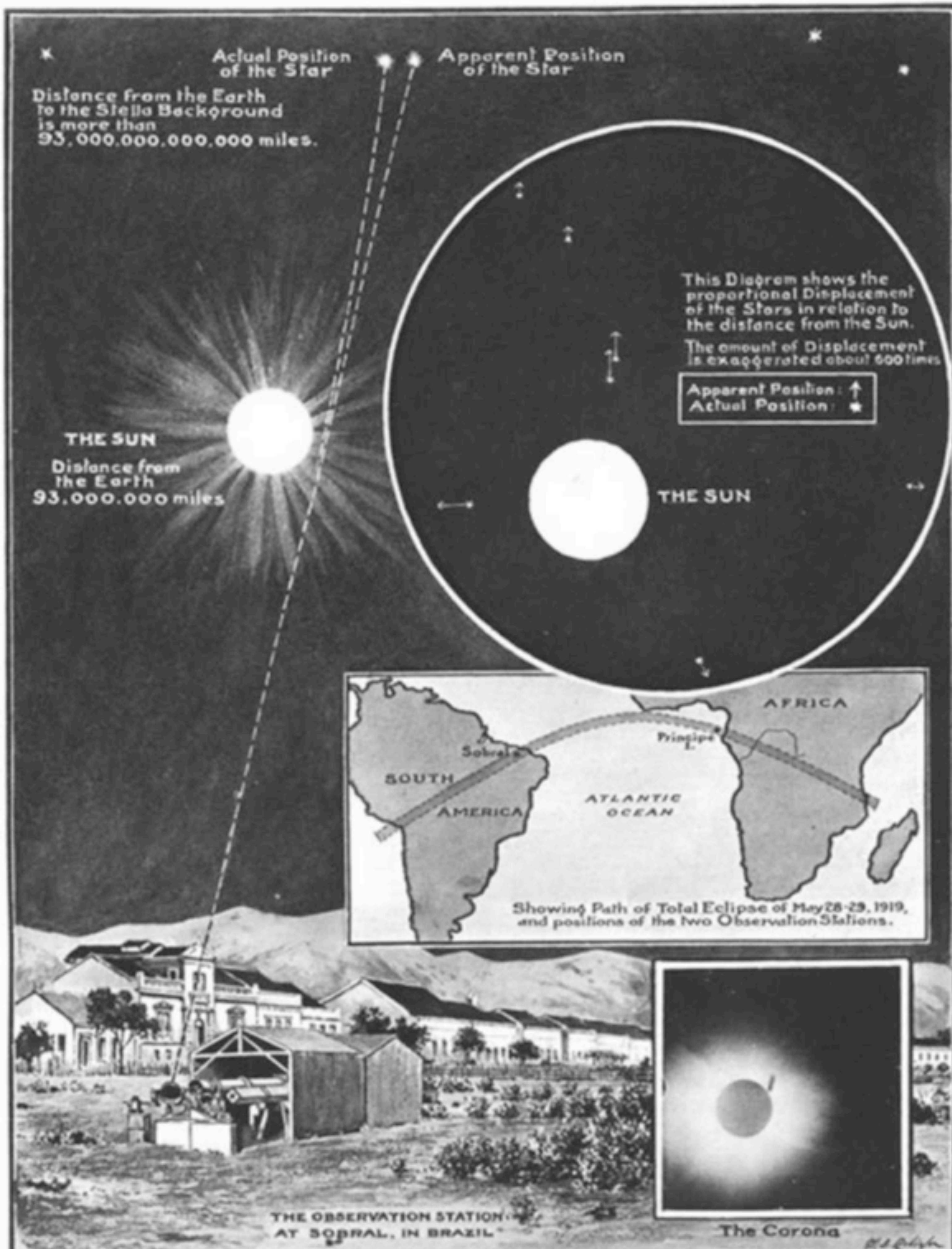


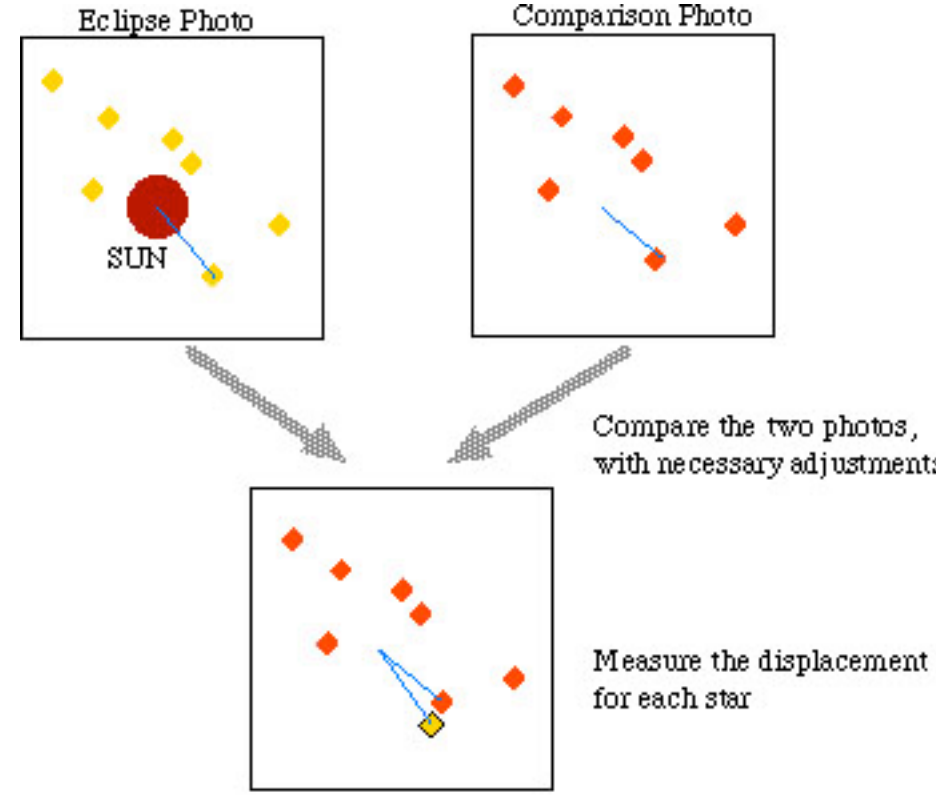
Newtonian & simple

equivalence principle prediction θ = 0.875"

GR, including curvature ($\alpha+\beta+\gamma = 180° - 0.875"$) ⇒ θ = 1.75"

1919 Eddington eclipse expedition: 1.98" ± 0.12"





Arthur Stanley Eddington

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

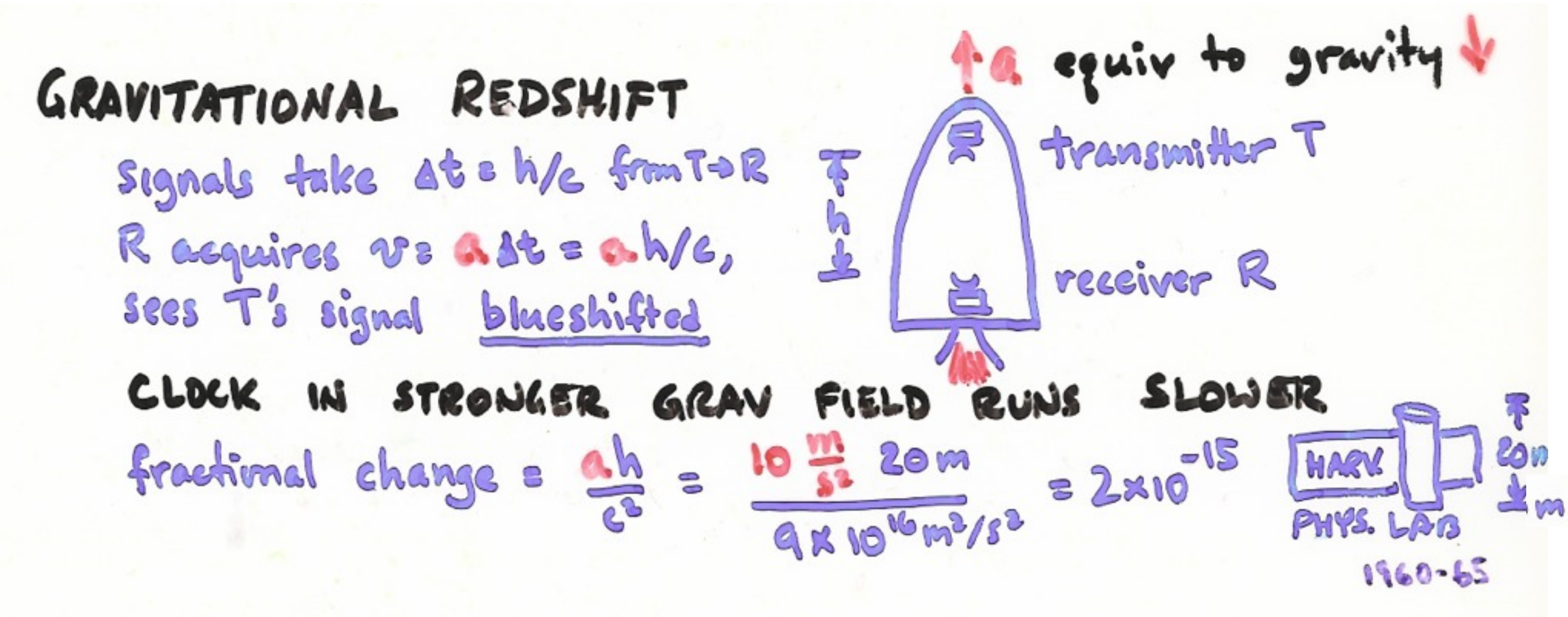
EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

General Relativity: Observational Implications & Tests



Note that we are using Einstein's **Principle of Equivalence:** locally, gravitation has exactly the same effect as acceleration ("strong" EEP version: this applies to all physical phenomena)

One elementary equivalence principle is the kind Newton had in mind when he stated that the property of a body called "mass" is proportional to the "weight", and is known as the weak equivalence principle (WEP). An alternative statement of WEP is that the trajectory of a freely falling "test" body (one not acted upon by such forces as electromagnetism and too small to be affected by tidal gravitational forces) is independent of its internal structure and composition. In the simplest case of dropping two different bodies in a gravitational field, WEP states that the bodies fall with the same acceleration (this is often termed the Universality of Free Fall, or UFF).

The Einstein equivalence principle (EEP) is a more powerful and far-reaching concept; it states that:

1. WEP is valid.
2. The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.
3. The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.

The second piece of EEP is called local Lorentz invariance (LLI), and the third piece is called local position invariance (LPI).

For example, a measurement of the electric force between two charged bodies is a local non-gravitational experiment; a measurement of the gravitational force between two bodies (Cavendish experiment) is not.

The Einstein equivalence principle is the heart and soul of gravitational theory, for it is possible to argue convincingly that if EEP is valid, then gravitation must be a "curved spacetime" phenomenon, in other words, the effects of gravity must be equivalent to the effects of living in a curved spacetime. As a consequence of this argument, the only theories of gravity that can fully embody EEP are those that satisfy the postulates of "metric theories of gravity", which are:

1. Spacetime is endowed with a symmetric metric.
2. The trajectories of freely falling test bodies are geodesics of that metric.
3. In local freely falling reference frames, the non-gravitational laws of physics are those written in the language of special relativity.

General Relativity

GR follows from the principle of equivalence and Einstein's equation $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G T_{\mu\nu}$.* Einstein had intuited the local equivalence of gravity and acceleration in 1907 (Pais, p. 179), but it was not until November 1915 that he developed the final form of the GR equation.

(*Gravitation & Cosmology*)

It can be derived from the following assumptions (Weinberg, p. 153):

1. The l.h.s. $G_{\mu\nu}$ is a tensor
2. $G_{\mu\nu}$ consists only of terms linear in second derivatives or quadratic in first derivatives of the metric tensor $g_{\mu\nu}$ ($\Leftrightarrow$ $G_{\mu\nu}$ has dimension L^{-2})
3. Since $T_{\mu\nu}$ is symmetric in $\mu\nu$, so is $G_{\mu\nu}$
4. Since $T_{\mu\nu}$ is conserved (covariant derivative $T^{\mu}{}_{\nu;\mu}=0$) so also $G^{\mu}{}_{\nu;\mu}=0$
5. In the weak field limit where $g_{00} \approx -(1+2\phi)$, satisfying the Poisson equation $\nabla^2\phi=4\pi G\rho$ (i.e., $\nabla^2 g_{00}= -8\pi G T_{00}$), we must have $G_{00}= \nabla^2 g_{00}$

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*Note: the r.h.s. sign depends other sign choices including the metric.

Einstein's equation can also be derived from an action principle, varying the total action $I = I_M + I_G$, where I_M is the action of matter and I_G is that of gravity:

$$I_G = -\frac{1}{16\pi G}\int R(x)\,\sqrt{g(x)}\,d^4x$$

(see, e.g., Weinberg, p. 364). The curvature scalar $R \equiv R_{\mu\nu}\, g^{\mu\nu}$ is the obvious term to insert in I_G since a scalar connected with the metric is needed and it is the only one, unless higher powers R^2, R^3 or higher derivatives $\Box R$ are used, which will lead to higher-order or higher-derivative terms in the gravity equation.

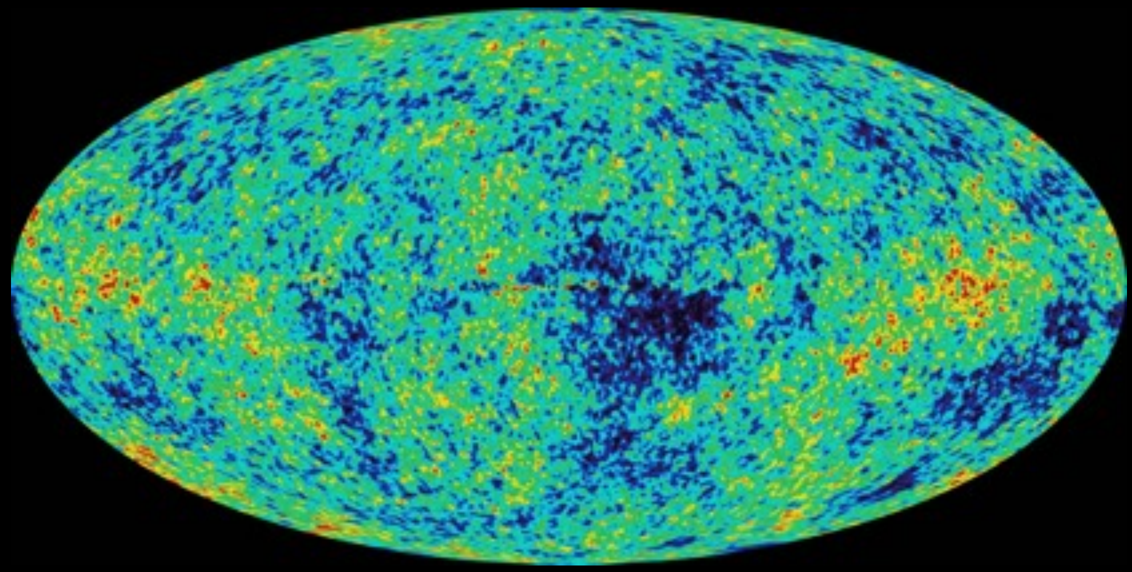
Einstein realized in 1916 that the 5th postulate above isn't strictly necessary – merely that the equation reduce to the Newtonian Poisson equation within observational errors, which allows the inclusion of a small cosmological constant term. In the action derivation, such a term arises if we just add a constant to R.

Renaissance of General Relativity 1960-

1960 QUASARS 1967 PULSARS

1974 BINARY PULSAR

1965 COSMIC BACKGROUND RADIATION



WMAP 2003

1971 BLACK HOLE CANDIDATES Cygnus X1…

1980 GRAVITATIONAL LENSES

1995 GPS SYSTEM OPERATIONAL

Experimental Tests of General Relativity

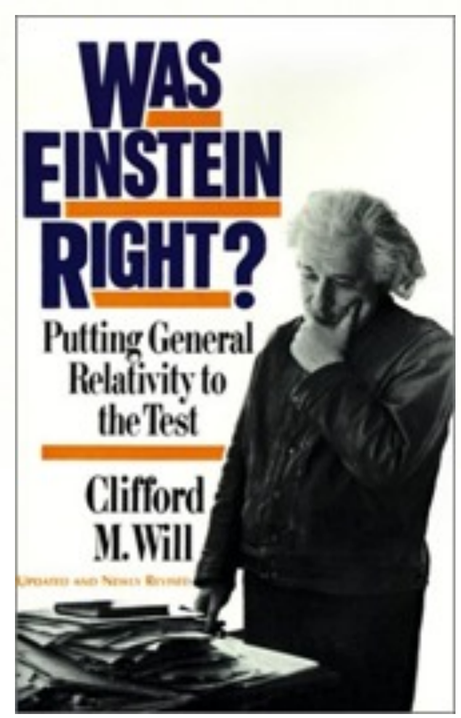
	Early	1960-70	1970-	Frontier
$m_{inertial} = m_{gravitational}$	Newton 10^{-3}; Eötvös 10^{-8}	Dicke 10^{-11}	Braginsky 10^{-12}	
Gravitational redshift		Pound 10^{-2}	airliner 10^{-1}; Scout 10^{-4}	
Perihelion precession	Newcomb (43"/century) 10^{-2}	(Solar oblateness ?)	binary pulsar (4°/yr)	Star-probe satellite
Light deflection	Eddington 10^{-1}		QSOs 10^{-2}	Grav. Lenses
Time delay		Venus, Mercury 10^{-1}	Mars lander 10^{-3}	

Experimental Tests of General Relativity

	Early	1960-70	1970-	frontier
Constancy of G			Mars lander ($\lvert\dot{G}/G\rvert < 10^{-11}$/yr)	
Gravity waves			binary pulsar $\sim 10^{-2}$	Antennas, Lasers (LIGO,LISA)
Draging of inertial frames				Gyro Satellite
Antimatter falls up?				Anti-proton experiment
F vs. r ("5th force")				$1/r^2$ r>mm

see Clifford Will, ***Was Einstein Right***

2nd Edition (Basic Books, 1993)



Clifford M. Will
"The Confrontation Between General Relativity and Experiment"
Living Reviews in Relativity (2001)

www.livingreviews.org

(his latest update was in 2006)

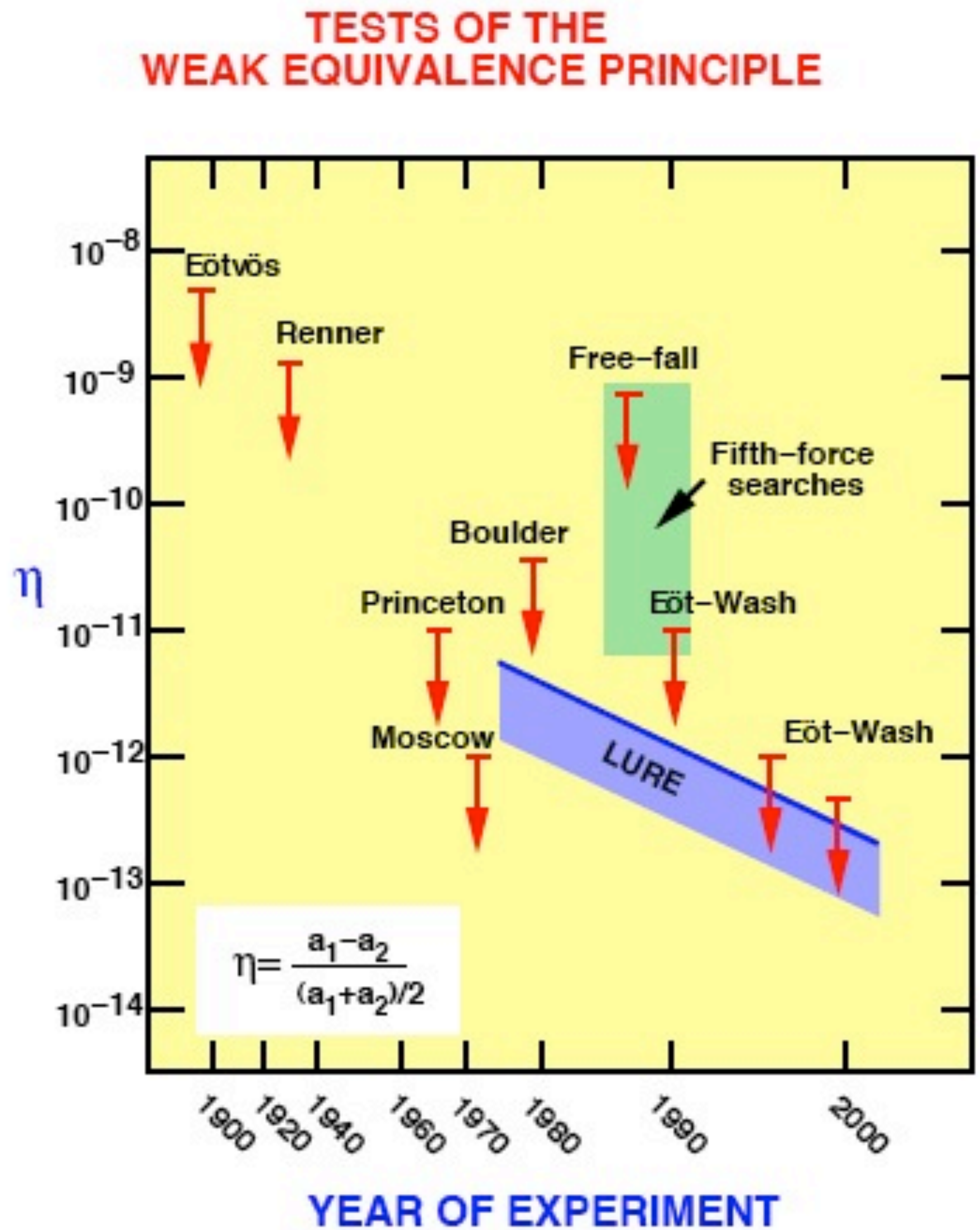


Figure 1: *Selected tests of the weak equivalence principle, showing bounds on η, which measures fractional difference in acceleration of different materials or bodies. The free-fall and Eöt-Wash experiments were originally performed to search for a fifth force. The blue band shows current bounds on η for gravitating bodies from lunar laser ranging (LURE).*

GRAVITATIONAL REDSHIFT TESTS

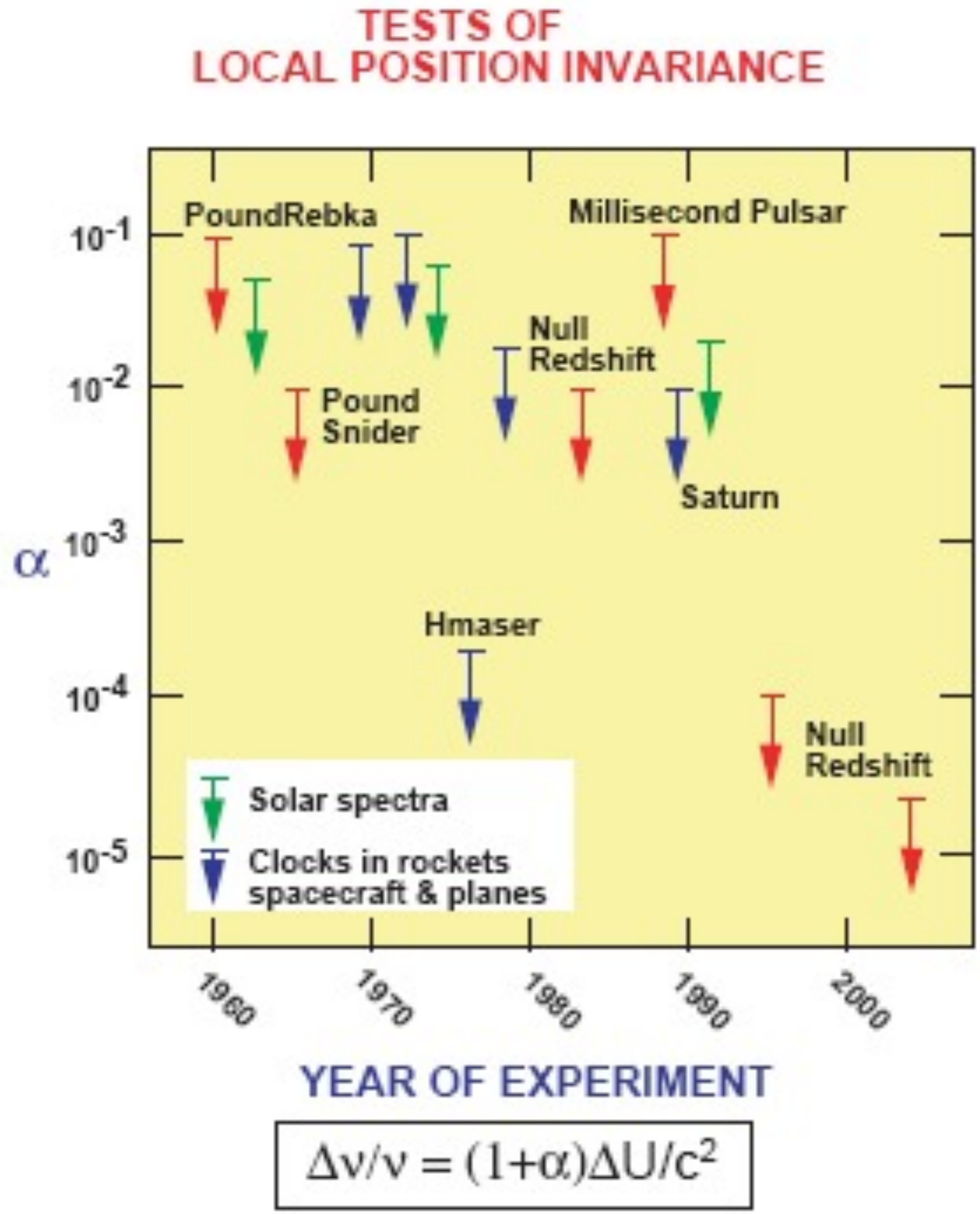


Figure 3: *Selected tests of local position invariance via gravitational redshift experiments, showing bounds on α, which measures degree of deviation of redshift from the formula $\Delta\nu/\nu = \Delta U/c^2$.*

Light Deflection; Shapiro Time Delay

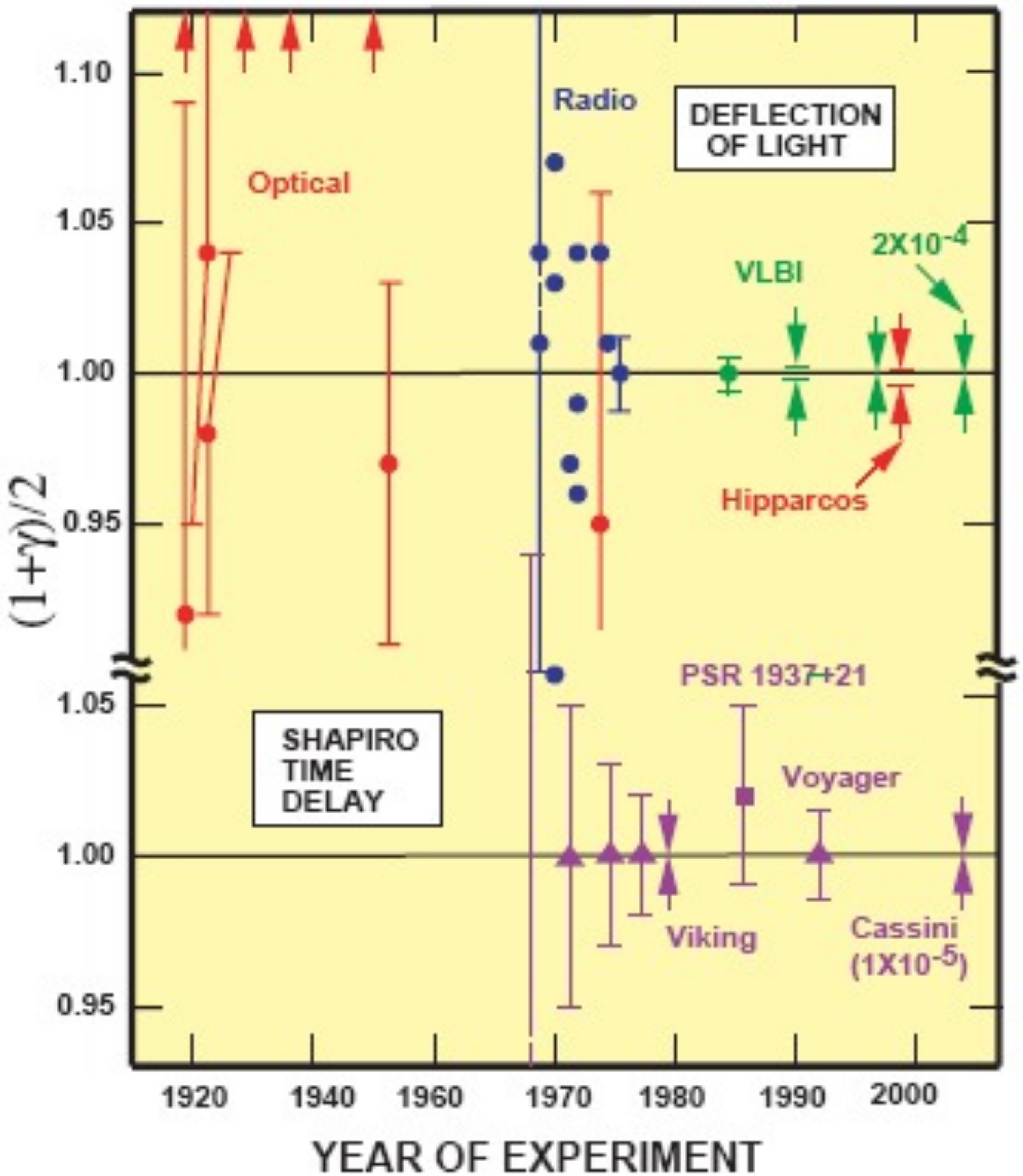
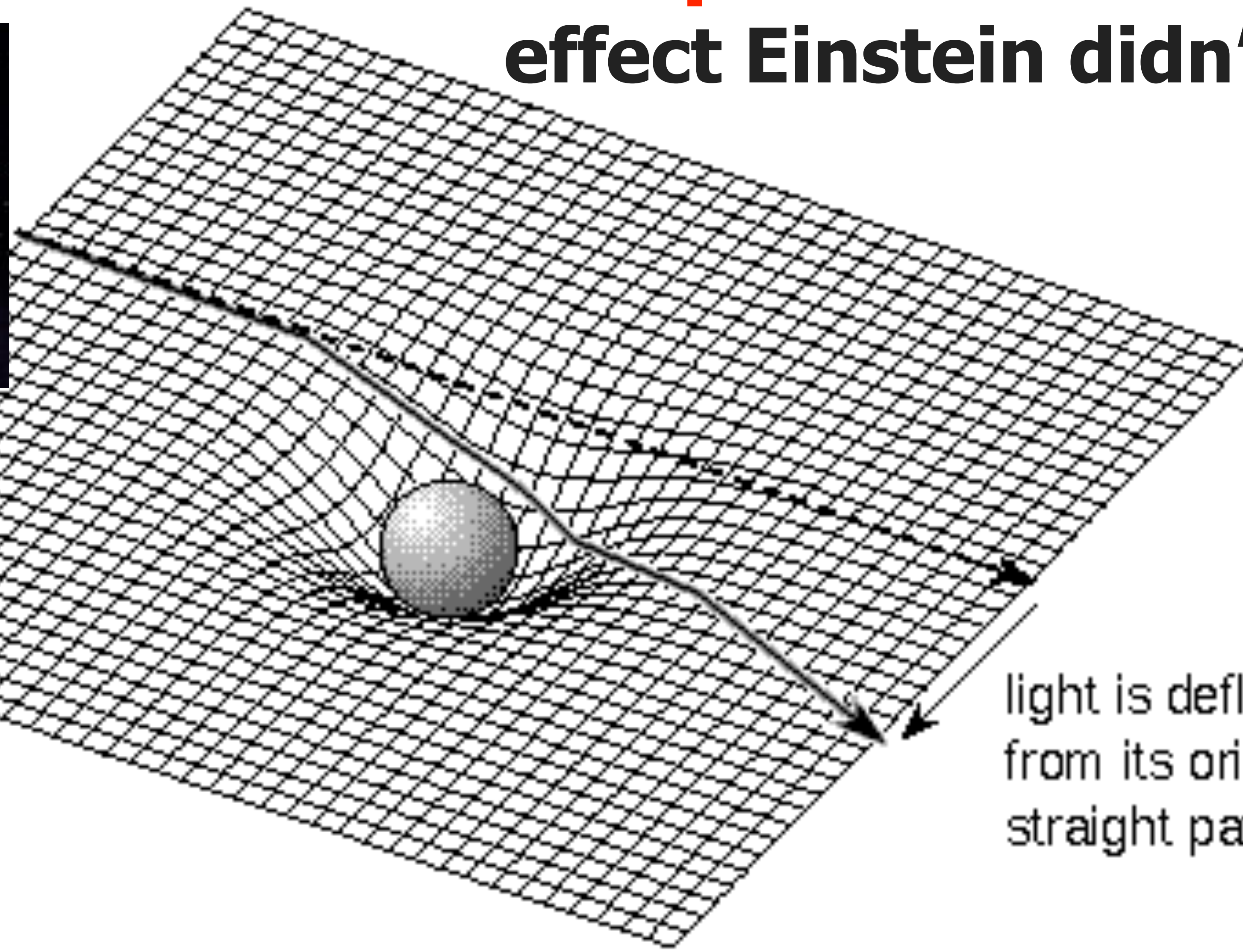


Figure 5: *Measurements of the coefficient $(1+\gamma)/2$ from light deflection and time delay measurements. Its GR value is unity. The arrows at the top denote anomalously large values from early eclipse expeditions. The Shapiro time-delay measurements using the Cassini spacecraft yielded an agreement with GR to 10^{-3} percent, and VLBI light deflection measurements have reached 0.02 percent. Hipparcos denotes the optical astrometry satellite, which reached 0.1 percent.*

Constancy of G

Method	$\dot{G}/G(10^{-13}\ \mathrm{yr}^{-1})$
Lunar Laser Ranging	4 ± 9
Binary Pulsar 1913 + 16	40 ± 50
Helioseismology	0 ± 16
Big Bang nucleosynthesis	0 ± 4

Shapiro Time Delay – an effect Einstein didn't notice.



General Relativity: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent. The Shapiro time delay is the combined effect of the light slowing down when it passes near the sun and the extra length of the bent path. It was dramatically confirmed by the transponders on the Viking Lander spacraft (1976) measuring the time of radio waves from Earth to the Landers and back.

Deflection and Delay of Light

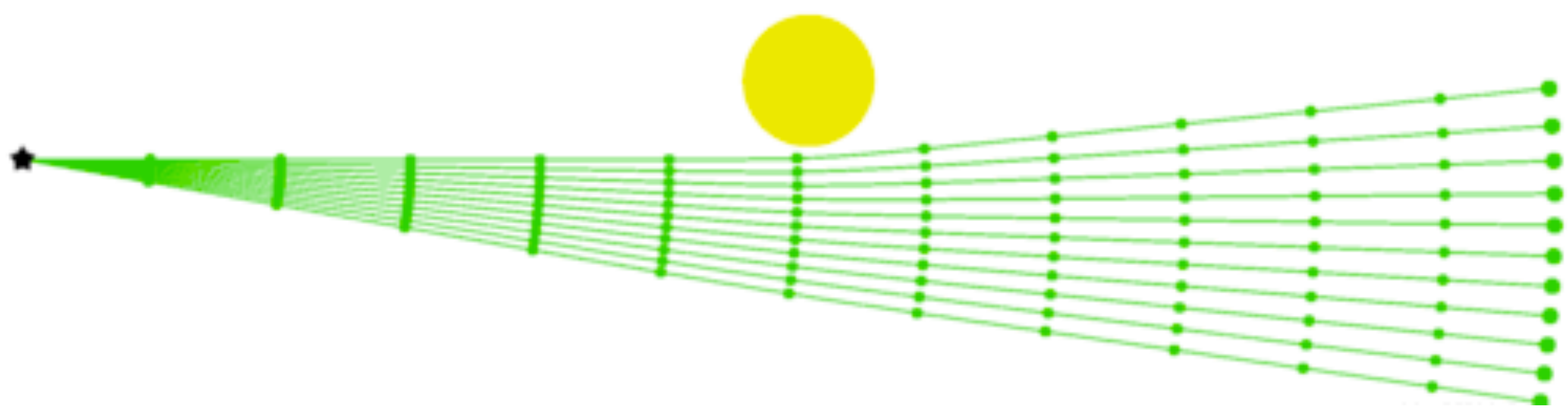
Everybody knows that light travels in straight lines, but while that is its natural tendency light can be deflected by lenses, mirrors, and by gravitational fields. Newtonian mechanics predicts that a particle traveling at the speed of light which just grazes the edge of the Sun will be deflected by 0.875 seconds of arc. That means that the image we see of a star will be displaced away from the Sun by this angle. The figure below shows this with the black showing the situation when the Sun is not close to the star. When the Sun is nearly blocking the star its image is deflected outward giving the red image. This Newtonian model also predicts that the gravitational attraction of the Sun will make light travel faster close to the Sun, so according to Newton the deflected light arrives before the undeflected light. The figure shows the red light pulse arriving before the black light pulse. Of course the travel time for starlight is very hard to measure, and the deflection of starlight can only be measured during a total eclipse of the Sun. The deflection angle is actually very small, and in the figure it has been increased by a factor of nearly 10,000 for clarity.

Before Einstein developed the full theory of General Relativity he also predicted a deflection of 0.875 arcseconds in 1913, and asked astronomers to look for it. But World War I intervened, and during the war Einstein changed his prediction to 1.75 arcseconds, which is twice the Newtonian deflection. The final Einstein prediction is shown in green in the figure above. An expedition to a solar eclipse in 1919 measured this larger value for the deflection. Currently the deflection of "light" is best measured using radio astronomy, since radio waves can be measured during the day without waiting for an eclipse of the Sun. Lebach et al. (1995, PRL, 75, 1439) find a deflection of 0.9998 +/- 0.0008 times Einstein's prediction. This agrees with Einstein within 0.3 standard deviations, but differs from the Newtonian deflection by 600 standard deviations. Einstein predicts that light will be delayed instead of accelerated when passing close to the Sun. In the figure above, the green light pulse arrives after the black light pulse.

Deflection and Delay of Light

This time delay effect is closely related to the deflection of starlight. Since times can be measured to much greater accuracy than arcsecond angles, the greatest accuracy on this effect is now given by measuring the time delay instead of the angle. In order to measure the time delay one needs a a spacecraft behind the Sun instead of a star. This was first done by Irwin Shapiro (Shapiro et al. 1977, JGR, 82, 4329), and more recently done by Bertotti, Iess & Tortora (2003, Nature, 425, 374-376). The current result is 1.00001 +/- 0.000012 times the general relativity prediction, or -2.000021 +/- 0.000023 times the Newtonian prediction. So the delay observations agree with Einstein within 0.9 standard deviations, but are 130,000 standard deviations away from the Newtonian prediction.

In a very real sense, the delay experienced by light passing a massive object is responsible for the deflection of the light. The figure below shows a bundle of rays passing the Sun at various distances. The rays are always perpendicular to the wavefronts which mark the set of points with constant travel time from the star. In order to bend the light toward the star one needs to delay the wavefront near the star.



Another way to delay the wavefronts of light is to send the light through glass, as in a lens or a prism. The deflection and delay of light caused by massive objects is called gravitational lensing.

http://www.astro.ucla.edu/~wright/deflection-delay.html

Schwarzschild Metric

Einstein derived the precession of Mercury and the deflection of light near the sun by perturbing around flat space. But a few months after Einstein invented GR, Karl Schwarzschild discovered the exact solution of Einstein's equations around a massive object:

$$ds^2 = \left[1 - \frac{2GM}{(rc^2)}\right] c^2 dt^2 - \left[1 - \frac{2GM}{(rc^2)}\right]^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) \tag{2.21}$$

If we set d*r* = dθ = dφ = 0, the proper time interval dτ is given by

$$d\tau = \sqrt{\left[1 - \frac{2GM}{(rc^2)}\right]} \cdot dt$$

The quantity $r_S = 2GM/c^2 = 2.95$ km $(M/M_\odot)$ is called the Schwarzschild radius. Perkins uses the Schwarzschild metric to discuss precession, light deflection, and the Shapiro time delay.

BINARY PULSAR

In 1993, the Nobel Prize in Physics was awarded to Russell Hulse and Joseph Taylor of Princeton University for their 1974 discovery of a pulsar, designated PSR1913+16, in orbit with another star around a common center of mass.

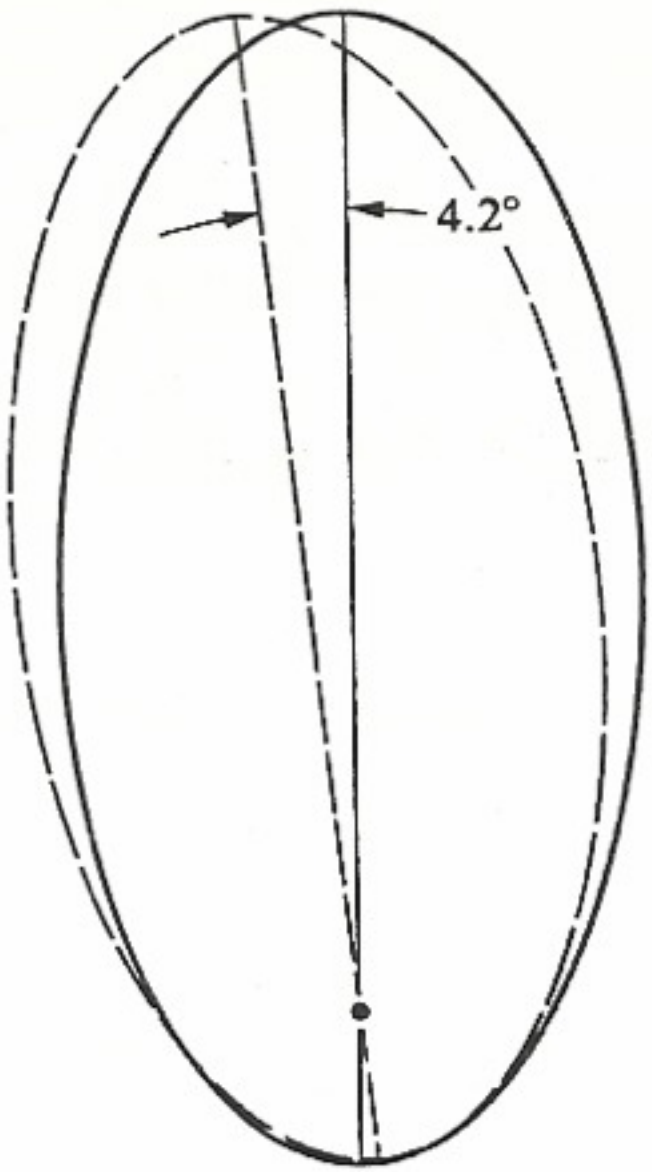


Figure 10.28. The rotation of the line of apsides in the binary pulsar amounts to 4.2 degrees per year. This effect has been interpreted to arise because the gravitational attraction of the companion deviates from a $1/r^2$ force law because of general-relativistic corrections.

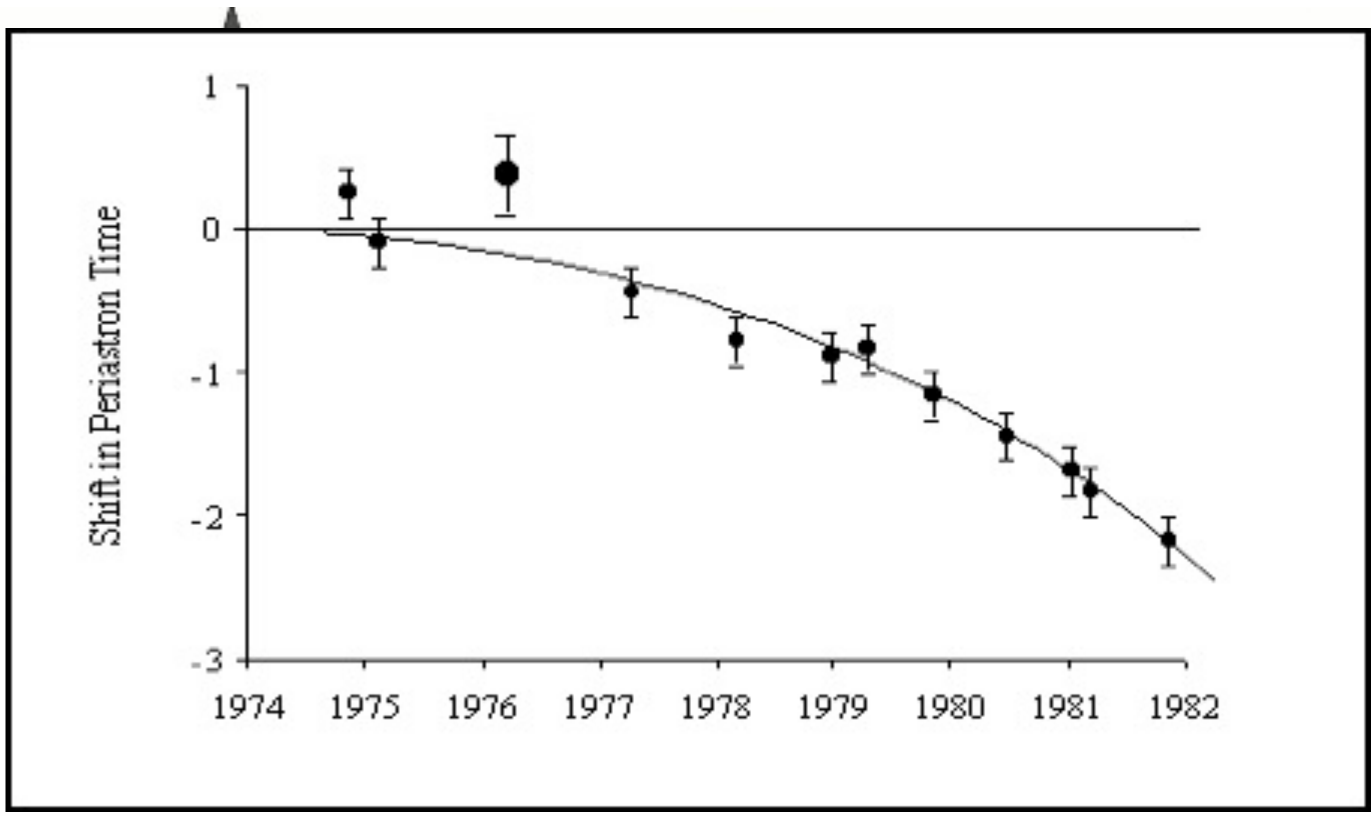


Figure 10.29. The decrease of the orbital period of the binary pulsar. This decrease is consistent with gravitational radiation causing a slow decay of the orbital separation as the two stars spiral slowly toward one another.

The pulsar is a rapidly rotating, highly magnetized neutron star which rotates on its axis 17 times per second. The pulsar is in a binary orbit with another star with a period of 7.75 hours.

BINARY PULSAR test of gravitational radiation

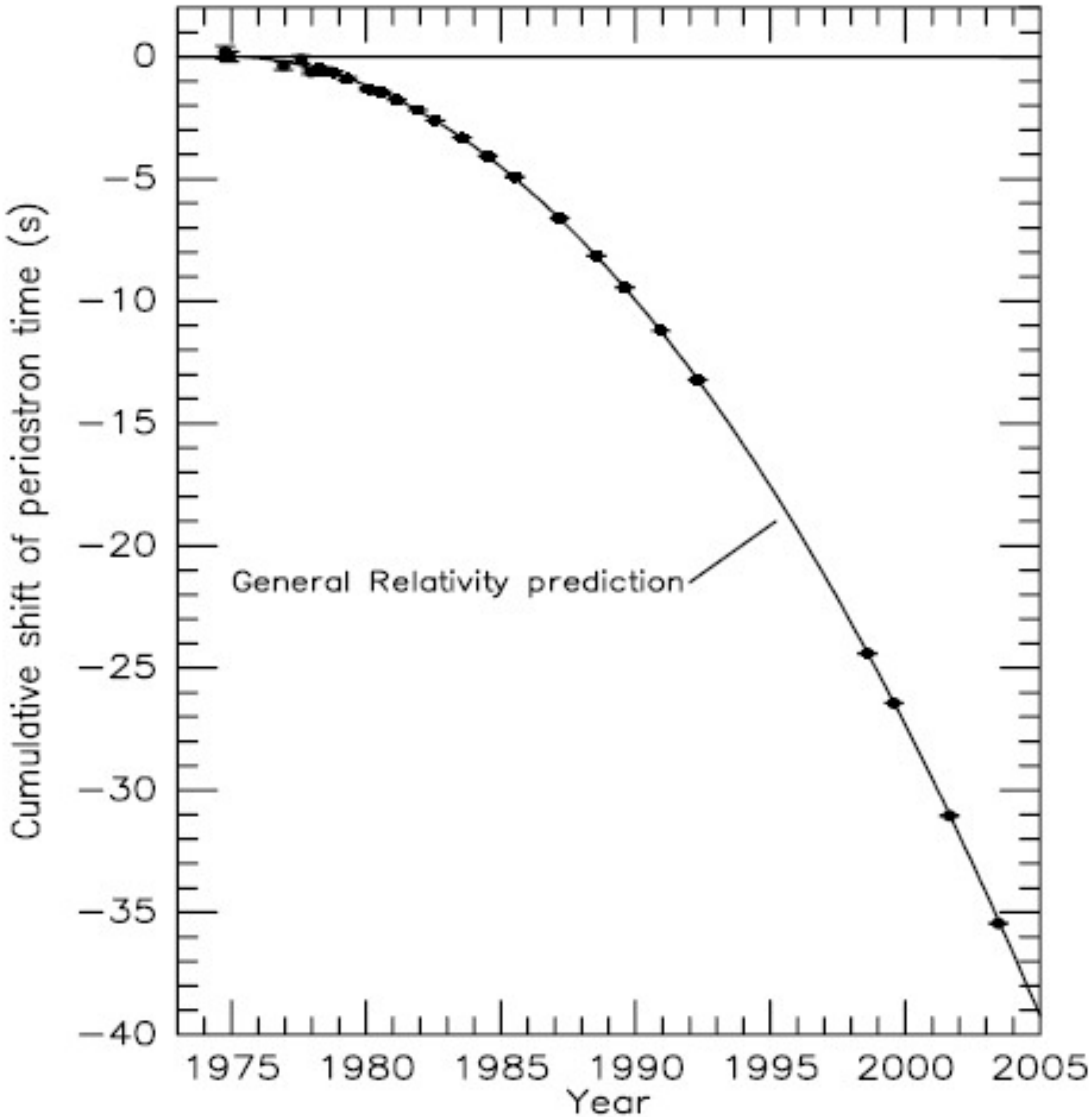
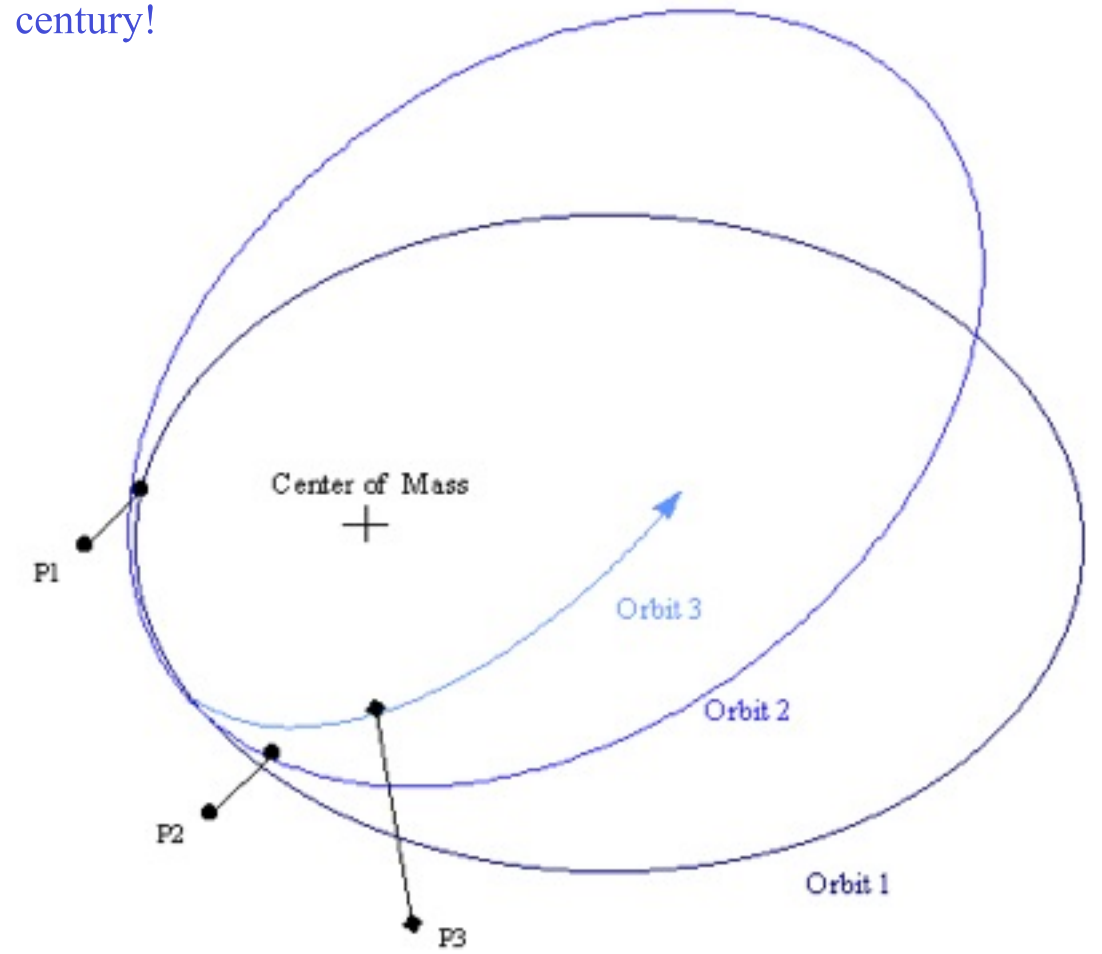


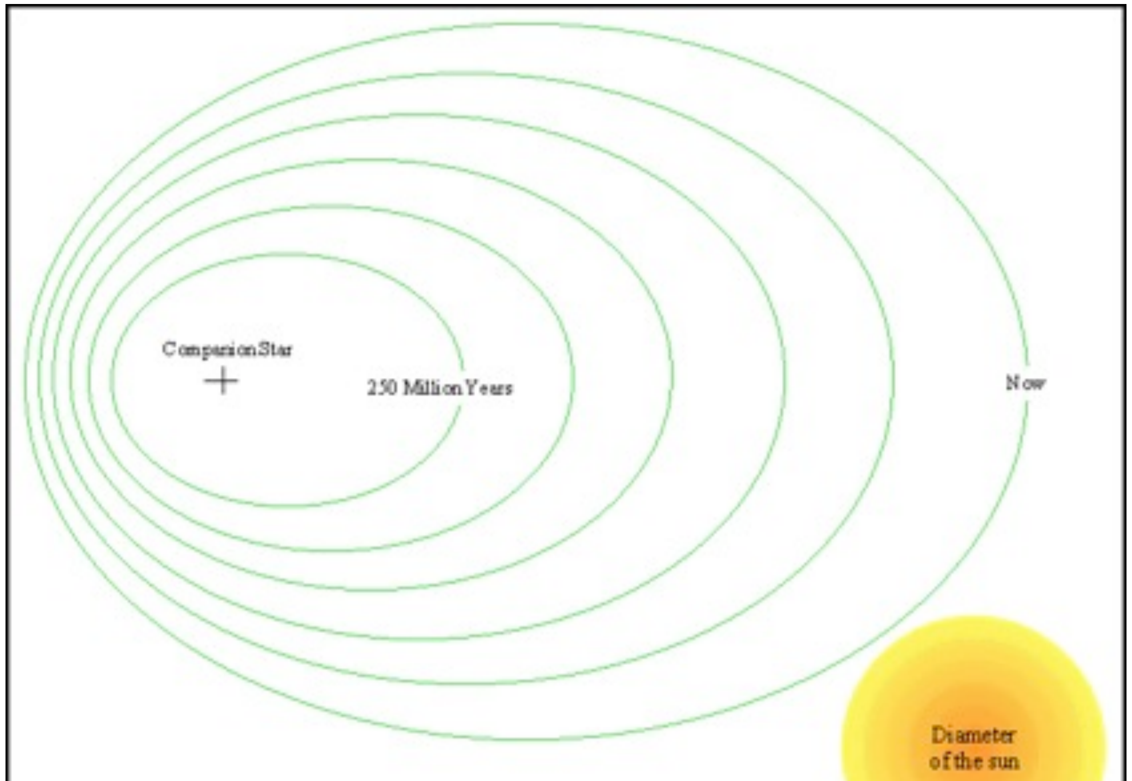
Figure 7: *Plot of the cumulative shift of the periastron time from 1975–2005. The points are data, the curve is the GR prediction. The gap during the middle 1990s was caused by a closure of Arecibo for upgrading [243].*

Data on the PSR B1913+16 system:

Right ascension	19h13m12.4655s
Declination	+16°01'08.189"
Distance	21,000 light years
Mass of detected pulsar	1.441 M_{Sun}
Mass of companion	1.387 M_{Sun}
Rotational period of detected pulsar	59.02999792988 sec
Diameter of each neutron star	20 km
Orbital period	7.751939106 hr
Eccentricity	0.617131
Semimajor axis	1,950,100 km
Periastron separation	746,600 km
Apastron separation	3,153,600 km
Orbital velocity of stars at periastron	300 km/sec
Orbital velocity of stars at apastron	75 km/sec
Rate of decrease of orbital period	0.0000765 sec per year
Rate of decrease of semimajor axis	3.5 meters per year
Calculated lifetime (to final inspiral)	300,000,000 years

Periastron advance per day = Mercury perihelion advance per century!

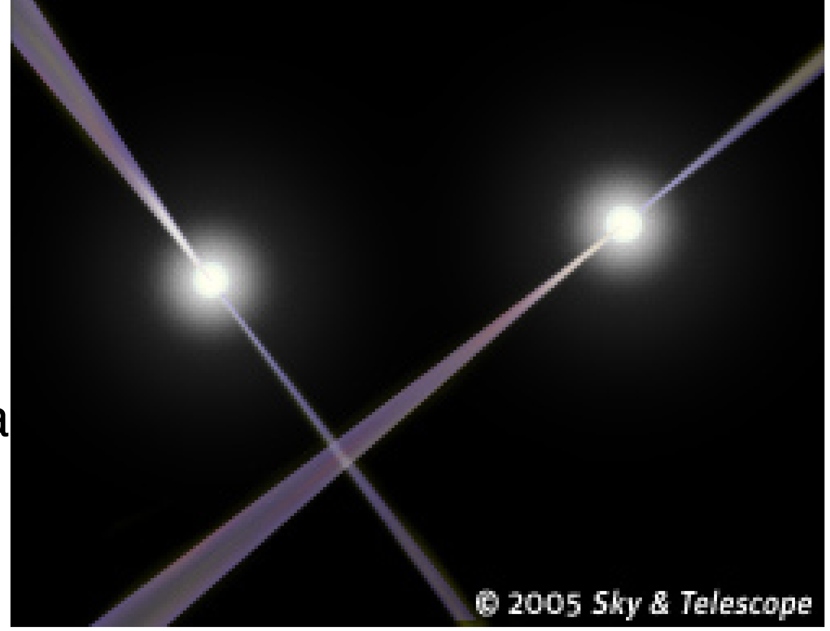




Weisberg et al. *Sci Am* Oct 1981

Einstein Passes New Tests

Sky & Telescope, March 3, 2005, by Robert Naeye



A binary pulsar system provides an excellent laboratory for testing some of the most bizarre predictions of general relativity. The two pulsars in the J0737-3039 system are actually very far apart compared to their sizes. In a true scale model, if the pulsars were the sizes of marbles, they would be about 750 feet (225 meters) apart.

Albert Einstein's 90-year-old general theory of relativity has just been put through a series of some of its most stringent tests yet, and it has passed each one with flying colors. Radio observations show that a recently discovered binary pulsar is behaving in lockstep accordance with Einstein's theory of gravity in at least four different ways, including the emission of gravitational waves and bizarre effects that occur when massive objects slow down the passage of time.

An international team led by Marta Burgay (University of Bologna, Italy) discovered the binary pulsar, known as J0737–3039 for its celestial coordinates, in late 2003 using the 64-meter Parkes radio telescope in Australia. Astronomers instantly recognized the importance of this system, because the two neutron stars are separated by only 800,000 kilometers (500,000 miles), which is only about twice the Earth–Moon distance. At that small distance, the two 1.3-solar-mass objects whirl around each other at a breakneck 300 kilometers per second (670,000 miles per hour), completing an orbit every 2.4 hours.

General relativity predicts that two stars orbiting so closely will throw off gravitational waves — ripples in the fabric of space-time generated by the motions of massive objects. By doing so, they will lose orbital energy and inch closer together. Radio observations from Australia, Germany, England, and the United States show that the system is doing exactly what Einstein's theory predicts. "The orbit shrinks by 7 millimeters per day, which is in accordance with general relativity," says Michael Kramer (University of Manchester, England), a member of the observing team.

Chin. J. Astron. Astrophys. Vol. 6 (2006), **Suppl. 2**, 162–168

A Review of The Double Pulsar - PSR J0737−3039

A. G. Lyne *

Table 1 Basic Observed Parameters of PSRs J0737−3039A and B

Pulsar	PSR J0737−3039A	PSR J0737−3039B
Pulse period P	22.7 ms	2.77 s
Period derivative $\dot{P}$	1.7×10^{-18}	0.88×10^{-15}
Orbital period $P_{\rm b}$	2.45 hours	
Eccentricity e	0.088	
Orbital inclination	∼88 deg	
Projected semi-major axis x	1.42 sec	1.51 sec
Stellar mass M	1.337(5) $M_\odot$	1.250(5) $M_\odot$
Mean orbital velocity $V_{\rm orb}$	301 km $\rm s^{-1}$	323 km $\rm s^{-1}$
Characteristic age τ	210 Myr	50 Myr
Magnetic field at surface B	6.3×10^9 G	1.2×10^{12} G
Radius of Light cylinder $R_{\rm LC}$	1080 km	132 000 km
Spin-down luminosity $\dot{E}$	6000×10^{30} erg $\rm s^{-1}$	1.6×10^{30} erg $\rm s^{-1}$

2 TESTS OF GRAVITATIONAL THEORY USING BINARY SYSTEMS

Non-relativistic binary systems are usually precisely described by the five Keplerian parameters, $P_{\rm orb}$, $a\sin i$, e, ω and T_o and these are all accurately measured when one object is a pulsar. However, a number of general relativistic corrections to this classical description of the orbit - the so-called post-Keplerian (PK) parameters - are needed if the gravitational fields are sufficiently strong. In only a few months, using the Parkes Telescope, the Lovell Telescope at Jodrell Bank and the Green Bank Telescope, it was possible to measure several general relativistic effects in 6 months that took years to measure with the Hulse-Taylor binary pulsar, PSR B1913+16.

The following five PK relativistic parameters have already been measured in A, all causing small, but highly significant, modifications to the arrival times of the pulsars' radio pulses:

Relativistic periastron advance, $\dot{\omega}$. This is the rotation of the line connecting the two pulsars at their closest approach to one another. It arises from the distortion of space-time caused by the two stars, but can also be understood as the result of the finite time needed for the gravitational influence of one star to travel to another. This causes a time delay, during which the stars move so that the attractive force is no longer radial.

Gravitational redshift and time dilation, γ. The redshift results in clocks appearing to run slowly in a gravitational potential well and time dilation is the special relativistic effect which results in moving clocks appearing to run slowly. Both effects cause clocks close to a neutron star to tick more slowly than those further away. In other words the apparent pulse rate for A will slow down when it is close to B, and vice versa.

Shapiro delay, r and s. Radiation passing close to a massive body is delayed because its path length is increased by the curvature of space-time, an effect that Einstein overlooked but that was discovered in 1964 by Irwin Shapiro (Shapiro 1964) from radar measurements in the Solar System. Signals from A are measured after they have passed through the distorted space-time of B (in principle the effect could also be measured for the signals from B but its pulses are much broader and do not provide sufficient temporal resolution). The signal delay is essentially a function of two parameters: s, the shape, and r, the range, of the delay experienced by the pulses (with s being dependent on the inclination of the orbital plane and r on the mass of B).

Gravitational radiation and orbital decay, dP_b/dt**.** Almost every theory of gravitation predicts that the movement of massive bodies around one another in a binary system will result in the emission of gravitational waves. This emission causes the bodies to lose energy and hence to spiral into one another, so that they will eventually merge, creating a burst of gravitational waves when they do so. The rate of decrease of the orbital period, dP_b/dt, indicates that orbits of the pulsars are currently shrinking by about 7 mm per day.

5 CONCLUSION

Several fortunate circumstances have come together to make these studies possible. Not only is this a double-neutron-star system, but

- It has a very compact orbit, giving rise to intense gravitational fields and accelerations and hence abundant post-Keplerian gravitational effects
- One pulsar is a millisecond pulsar which enables these effects to be measured with high precision
- Both neutron stars are visible, allowing the mass-ratio to be determined
- Both pulsars have large flux densities, giving high-precision measurements
- The orbit is nearly edge on, so that the Shapiro delay can be measured with high precision.

All these properties enhance the quality and speed of the tests of gravitation theories in the strong-field regime. Furthermore, the last three also enable the investigations of the interactions between the stars and the probing of the magnetospheric properties.

Future observations of binary systems like PSR J0737−3039 promise to greatly increase our knowledge of strong-field gravity, but finding these systems will be a challenge. This is because double pulsars are extremely rare and, more importantly, because the Doppler effect causes their pulse periods to vary rapidly even during a short observation. It therefore becomes more difficult to detect the pulsars' periodicity using normal Fourier techniques and more sophisticated and computationally challenging search algorithms will have to be employed to uncover them.

Conclusions

We find that general relativity has held up under extensive experimental scrutiny. The question then arises, why bother to continue to test it? One reason is that gravity is a fundamental interaction of nature, and as such requires the most solid empirical underpinning we can provide. Another is that all attempts to quantize gravity and to unify it with the other forces suggest that the standard general relativity of Einstein is not likely to be the last word. Furthermore, the predictions of general relativity are fixed; the theory contains no adjustable constants so nothing can be changed. Thus every test of the theory is either a potentially deadly test or a possible probe for new physics. Although it is remarkable that this theory, born 90 years ago out of almost pure thought, has managed to survive every test, the possibility of finding a discrepancy will continue to drive experiments for years to come.

Clifford Will, Living Reviews, 2006

How the GPS System Works

There are ~30 GPS satellites in orbits such that 4 or more are visible at any time from almost any point on earth. Each GPS satellite carries very accurate atomic clocks, and it corrects the time so that it is what a stationary clock would read. A GPS receiver uses the times it receives to correct its own much less accurate clock. It knows the location of each GPS satellite, and it then triangulates to determine its position: i.e. it solves the (Euclidean space) equations $c^2(t - t_i)^2 = |\boldsymbol{r} - \boldsymbol{r}_j|^2$. In 3D there is only one solution, which determines the location of the GPS receiver. Since $c \approx 30$ cm/ns, it is essential to have the GPS receiver clock correct to ~ 1 ns to get accurate locations. If the GPS satellites didn't correct their time signals for SR and GR effects, the errors would be huge!

General Relativity

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

MATTER TELLS SPACE HOW TO CURVE

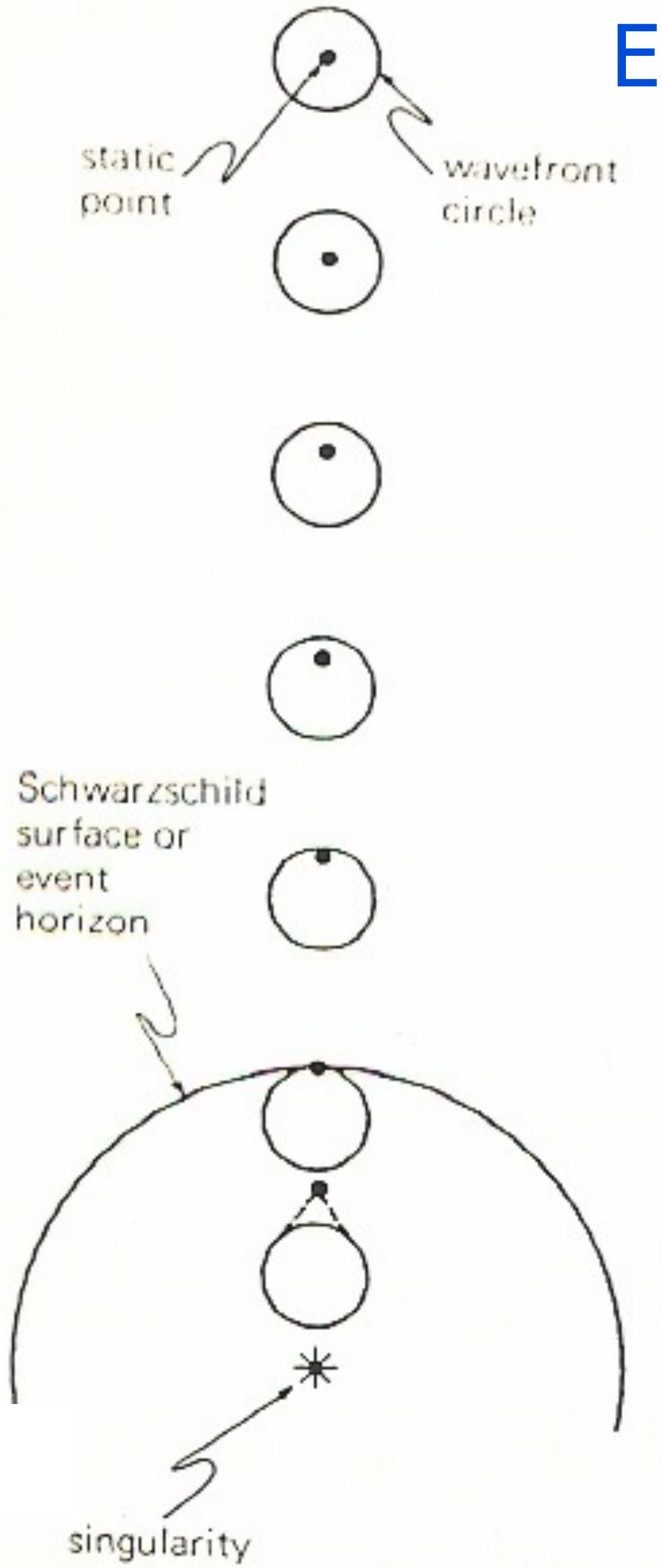
Einstein Field Equations

$$G^{\mu\nu} \equiv R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} = -\,8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

Curved spacetime is not just an arena within which things happen, spacetime is dynamic. Curvature can even cause horizons, beyond which information cannot be sent.

There are event horizons around black holes and we are also surrounded by both particle and event horizons.

EFFECTS OF CURVATURE NEAR A BLACK HOLE



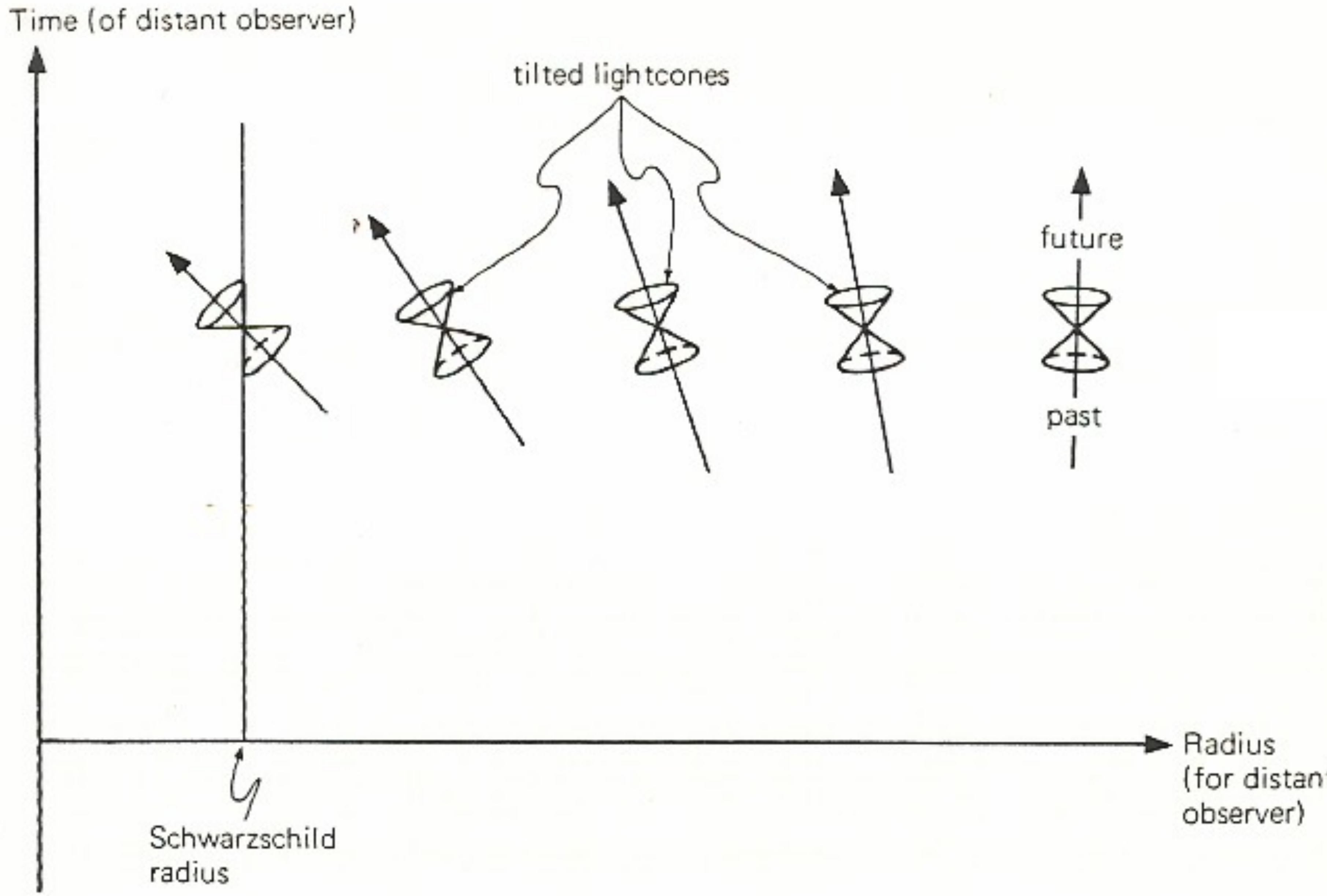
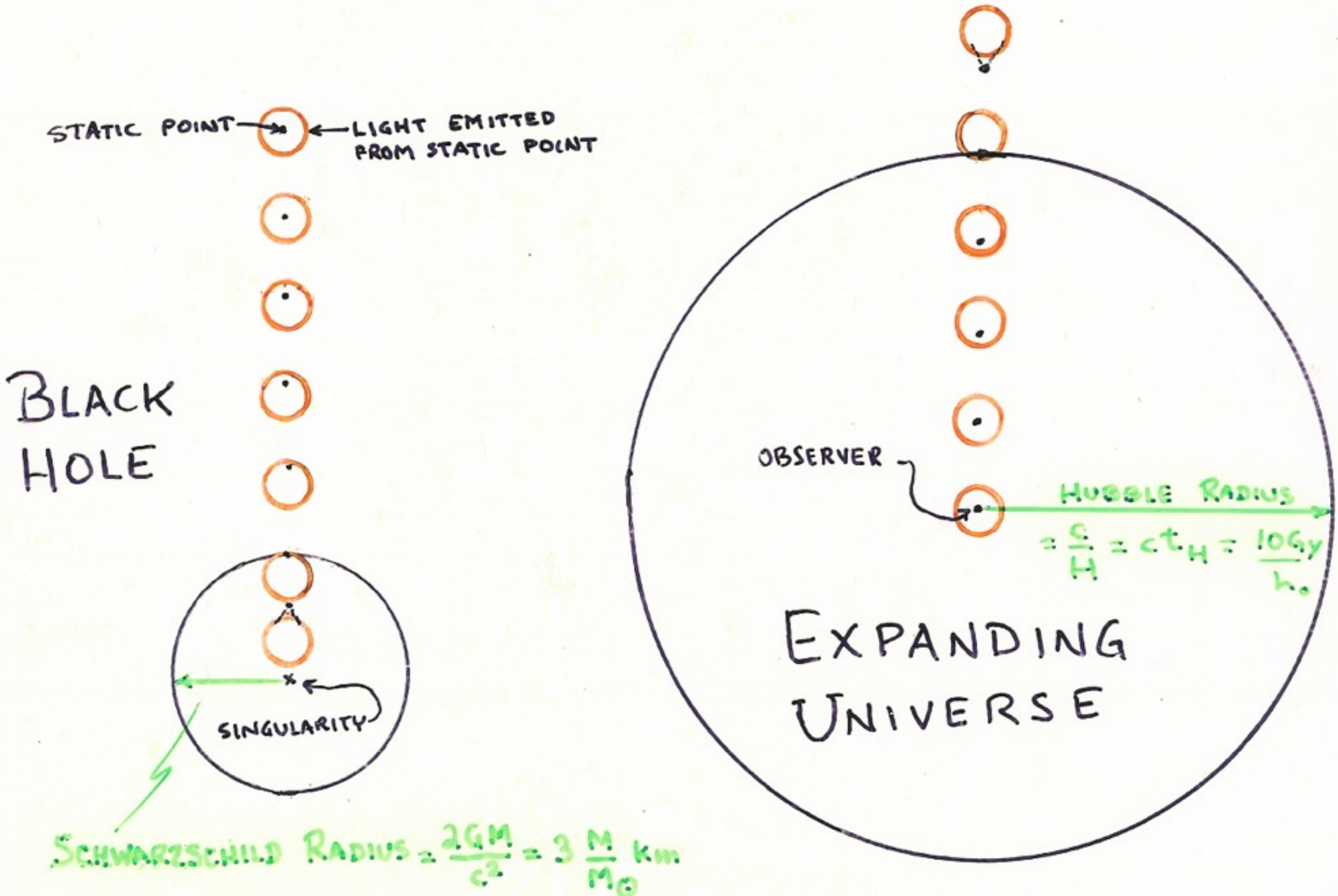
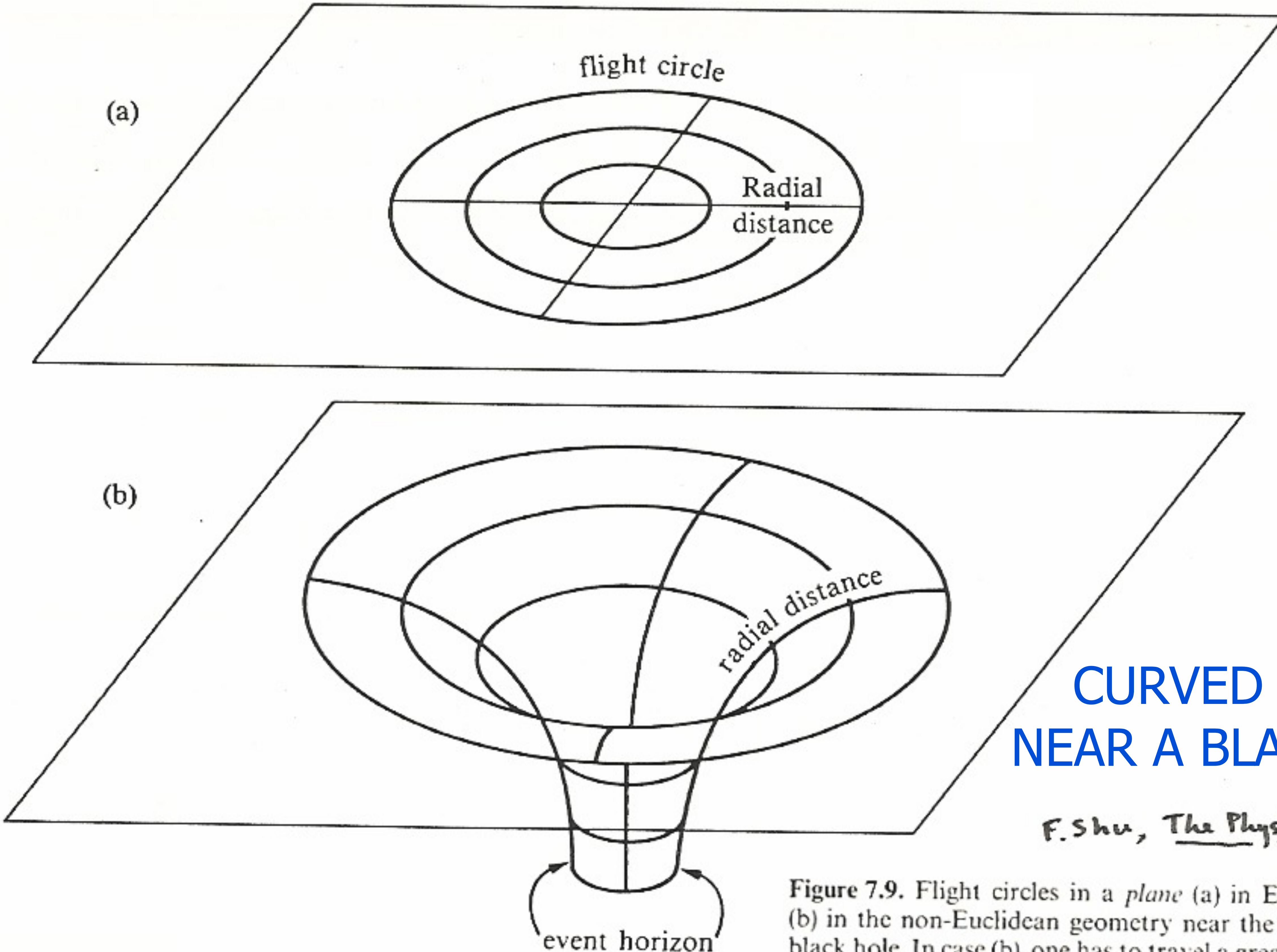


Figure13.6. *The effect of spacetime curvature near a black hole. Lightcones are tilted in such a way that the future-pointing lightcone tips toward the black hole and the past-pointing lightcone tips away from the black hole. At the surface of the black hole (the Schwarzschild surface), all rays emitted in the future direction fall into the black hole, and no rays from the past are received from the black hole. A person passing into a black hole therefore receives no information of what lies ahead.*

E. Harrison, Cosmology

CURVED SPACE-TIME IS NOT JUST AN ARENA IN WHICH THINGS MOVE, IT IS DYNAMIC. CURVATURE CAN CAUSE HORIZONS, BEYOND WHICH INFORMATION CANNOT BE SENT.



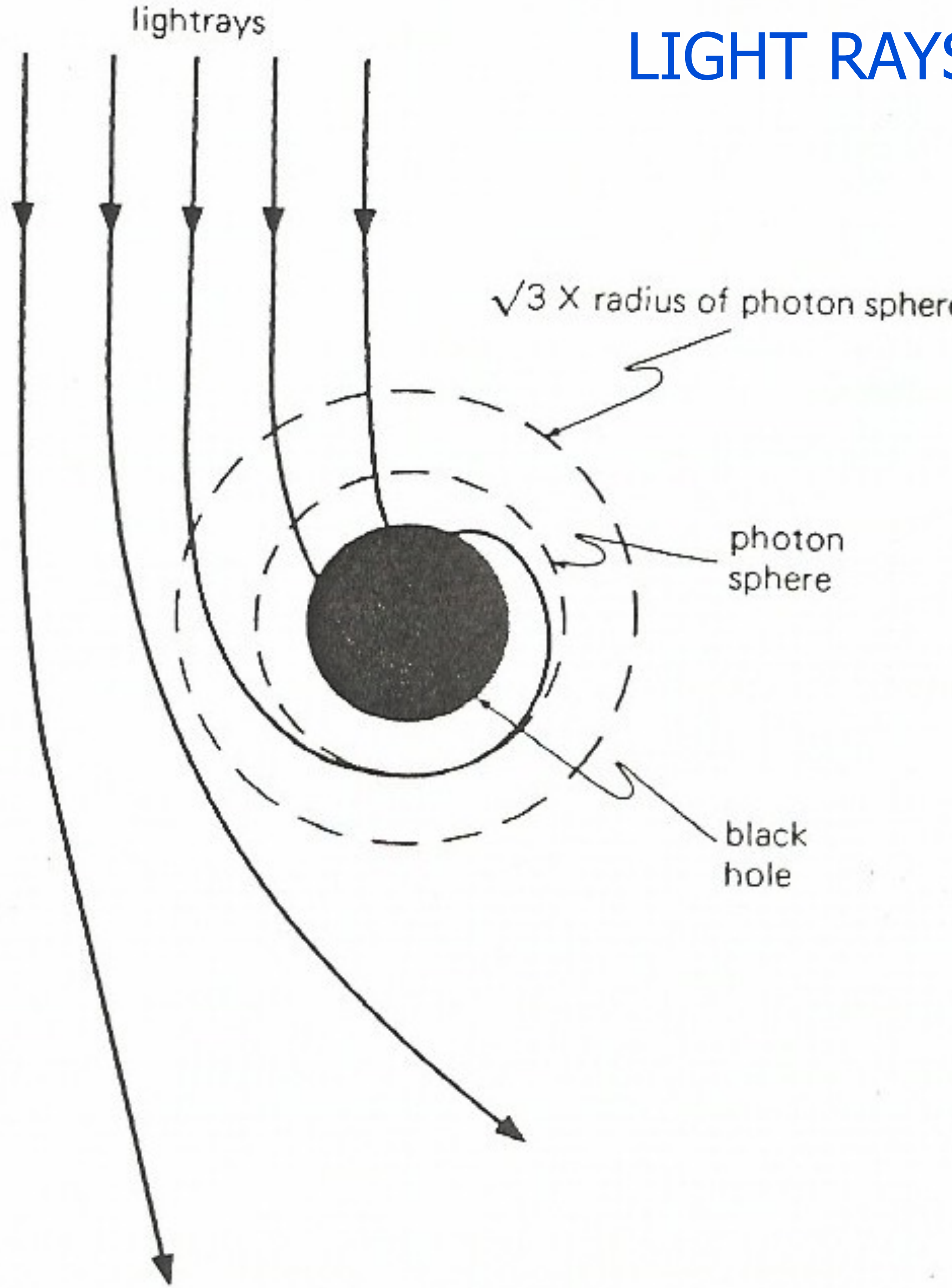


CURVED SPACE NEAR A BLACK HOLE

F. Shu, The Physical Universe

Figure 7.9. Flight circles in a *plane* (a) in Euclidean geometry, (b) in the non-Euclidean geometry near the event horizon of a black hole. In case (b), one has to travel a greater distance inward than in case (a) to have a flight circle of given smaller circumference. The radial direction in both cases is as indicated. At great distances from the event horizon (not drawn), the "curvature" of our embedding diagram becomes negligibly small, and the flight circles of case (b) have nearly the same geometry as case (a).

LIGHT RAYS NEAR A BLACK HOLE



. Deflection of lightrays by a black hole. Rays approaching closer than √3 times the radius of the photon sphere are captured.

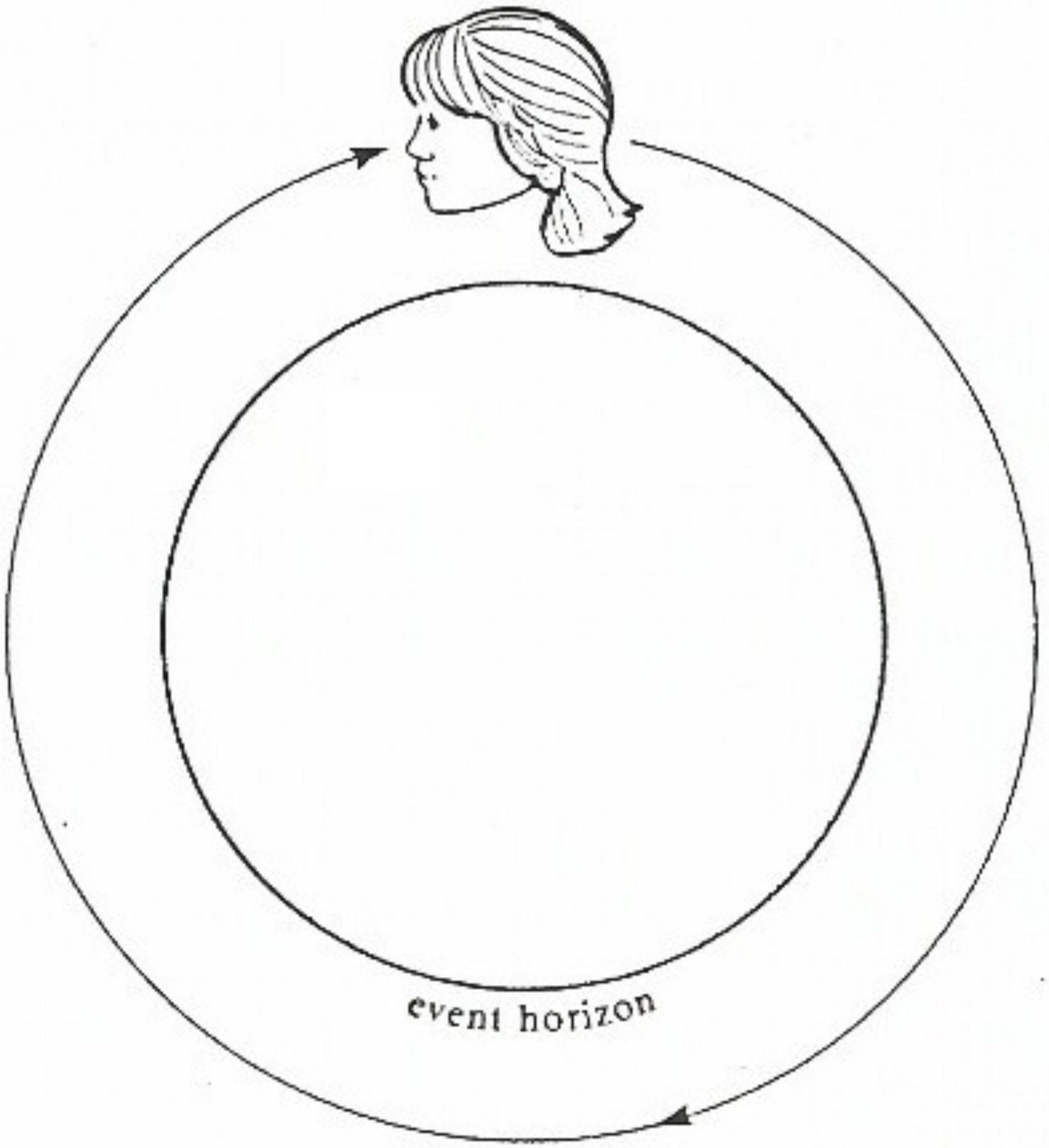
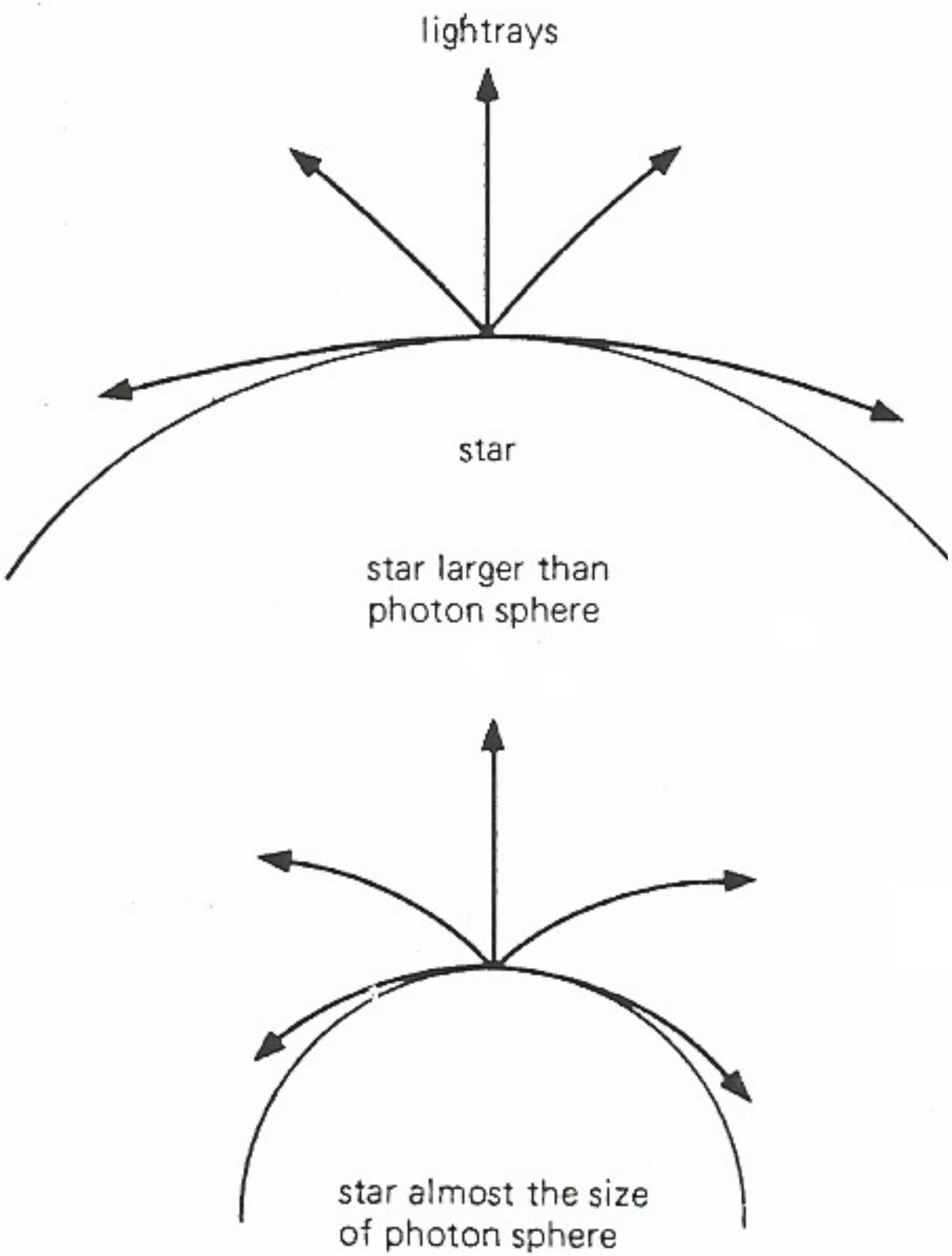
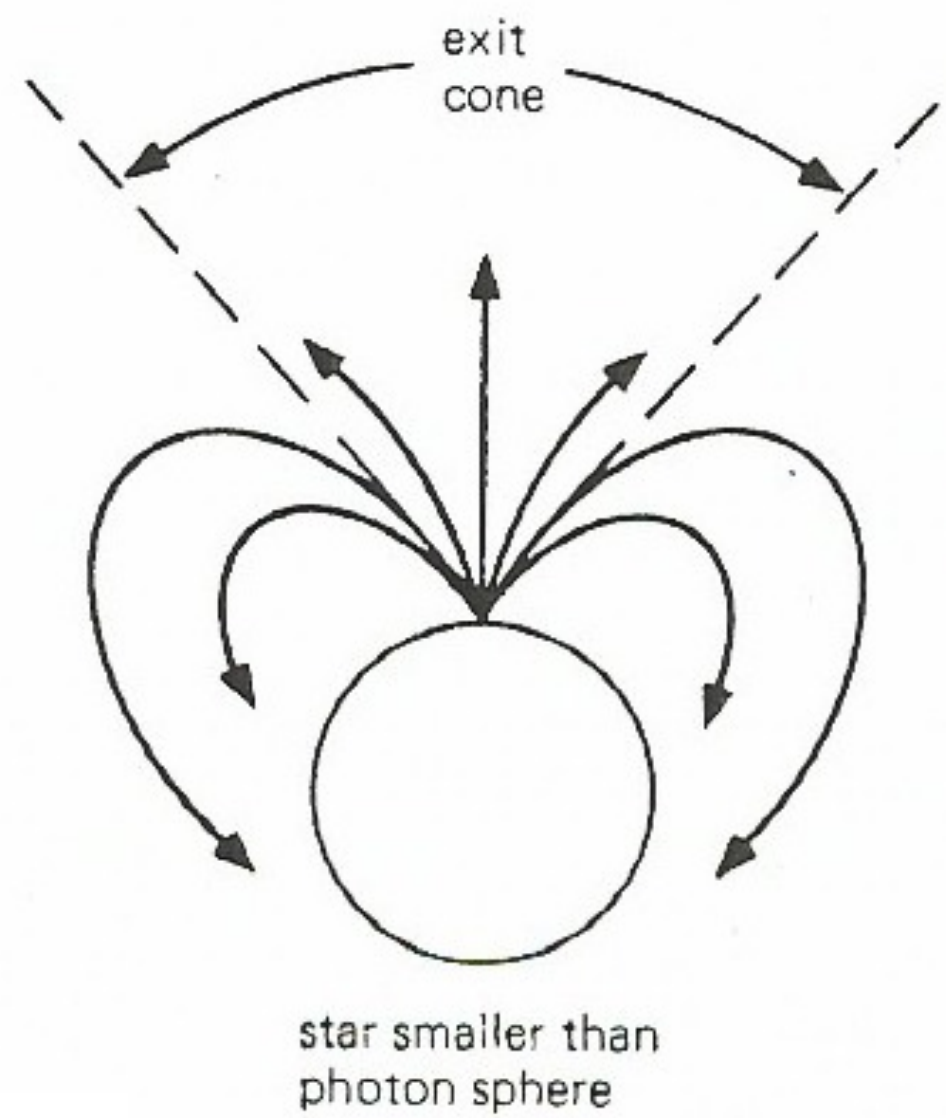


Figure 7.10. When at a circumference equal to 1.5 times the circumference of the event horizon of a black hole, a suitably suspended astronaut can see the back of her own head without the benefit of any mirrors.

E. R. Harrison, *Cosmology*





Lightrays leaving a gravitating body are curved as shown. As the body shrinks in size the rays become more curved. When the radius is less than 1.5 times the Schwarzschild radius, which is the radius of the photon sphere, the exit cone begins to close. Rays emitted within the exit cone escape, but those outside are trapped and fall back.

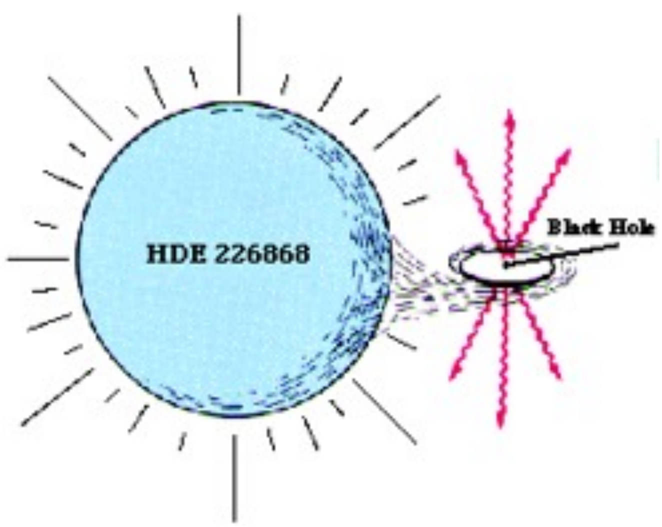
E. R. Harrison, *Cosmology*

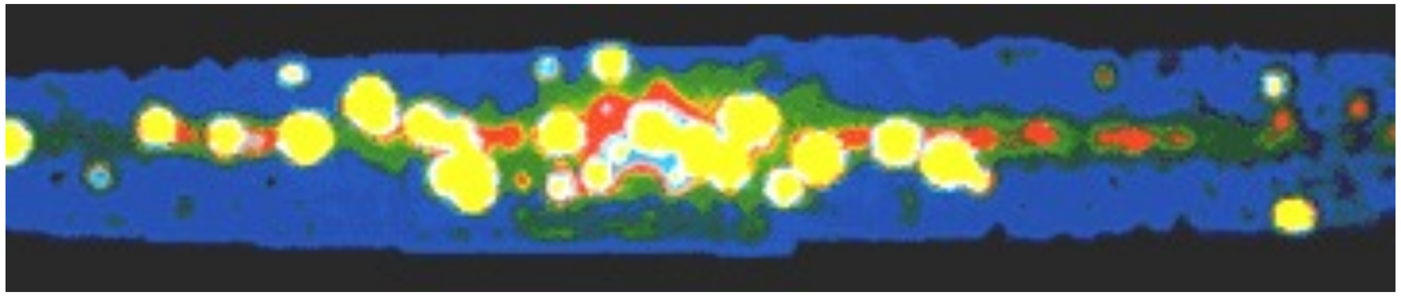
More About Black Holes

- Black Holes
  - Schwarzschild Radius, Photon Sphere, etc.
  - Hawking Radiation (a quantum effect)
- Planck Length – Gravitation Meets Quantum Physics
- Black Holes from Stellar Collapse
- Black Hole at the Center of Our Galaxy
- Black Holes at the Centers of Galaxies
- Intermediate Mass Black Holes?
- Evaporating Black Holes?

BLACK HOLES FROM STELLAR COLLAPSE

If a large number of stars form, about 10% of the mass turns into a small number of stars more massive than 8 solar masses. Such high mass stars are rare, only about 0.2% of all stars. But they are at least 100,000 times as bright as the sun, and they fuse all the available fuel in their centers within a few million years. They then collapse into either neutron stars or black holes. If the black hole is a member of a binary star system, we can see its effects. Sometimes the black hole attracts matter to it from the other star, and as this matter (mostly hydrogen) falls into the black hole's event horizon it is heated tremendously and it radiates X-rays, which we can detect. One of these binary systems, called GRO J1655-40, was discovered in 2002 to be moving roughly toward us at about 110 kilometers per second. Such accreting black holes acquire angular momentum from the accreted material, so they should be spinning. Evidence that some black holes spin has been found in "quasi-periodic oscillations" of their X-rays at frequencies too high to come from non-spinning black holes.



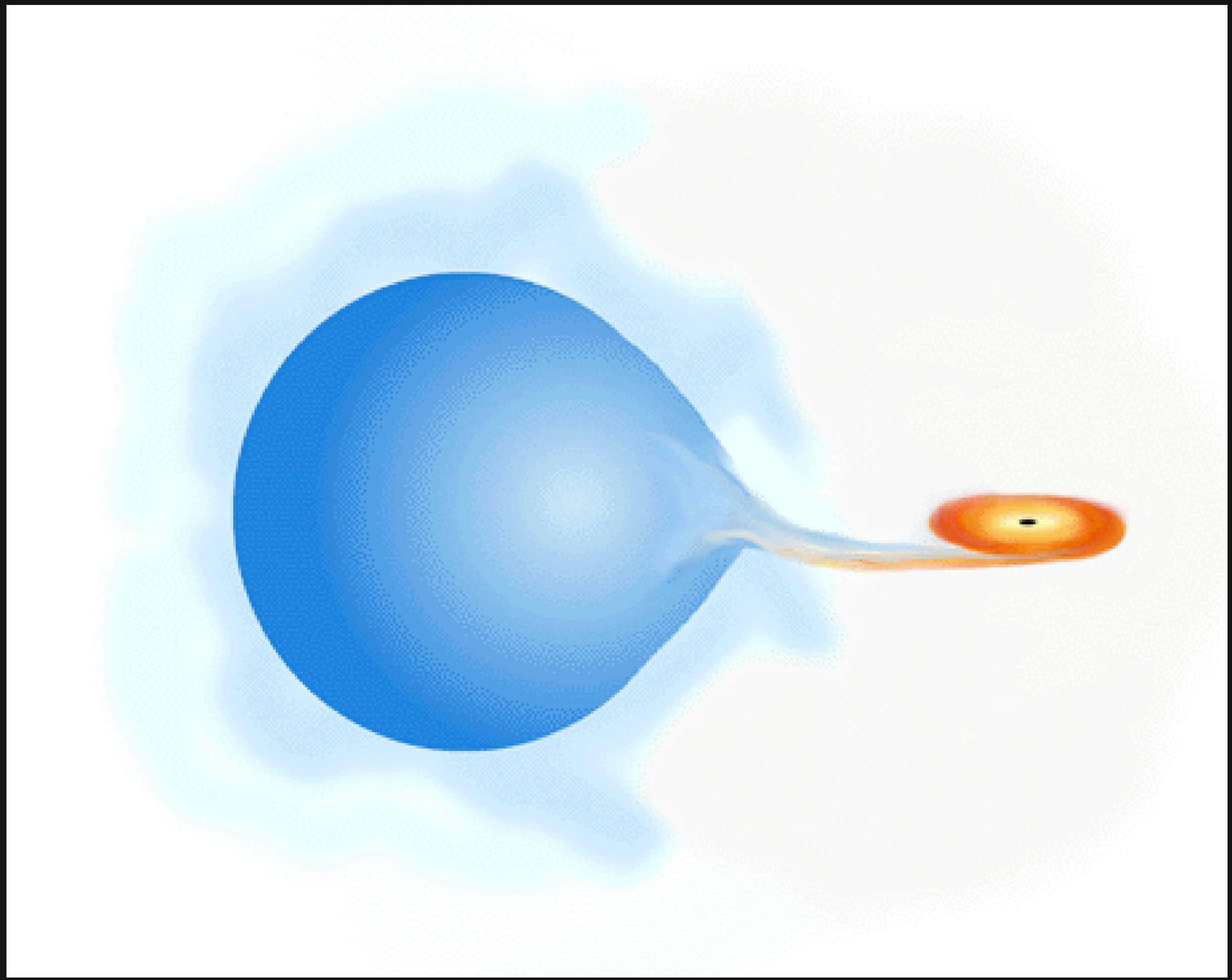


X-ray Binaries (in yellow) near the Galactic Center

Interesting links: Fantasy trip to and around a black hole - http://antwrp.gsfc.nasa.gov/htmltest/rjn_bht.html
See also http://ircamera.as.arizona.edu/NatSci102/lectures/blackhole.htm

Black Holes in Binary Star Systems

- Black holes are often part of a binary star system - two stars revolving around each other.
- What we see from Earth is a visible star orbiting around what appears to be nothing.
- We can infer the mass of the black hole by the way the visible star is orbiting around it.
- The larger the black hole, the greater the gravitational pull, and the greater the effect on the visible star.

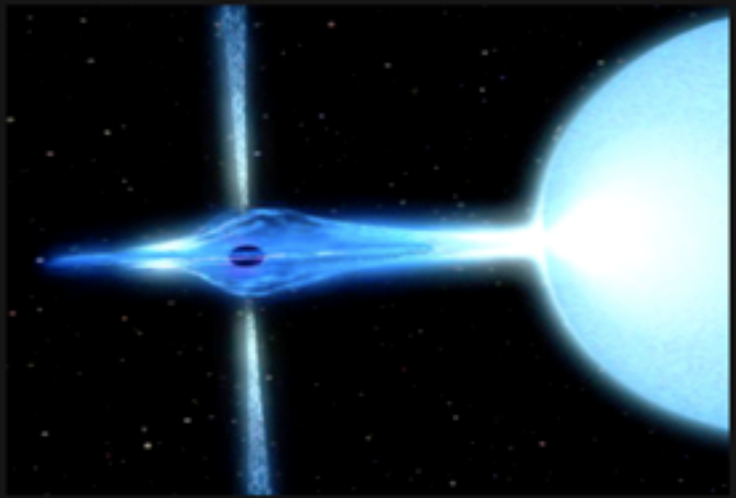


Chandra illustration

BLACK HOLES FROM STELLAR COLLAPSE

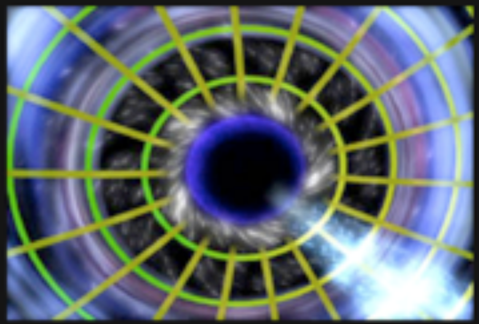
X-rays from Black Holes

In close binary systems, material flows from normal star to black hole. X-rays are emitted from disk of hot gas swirling around the black hole.

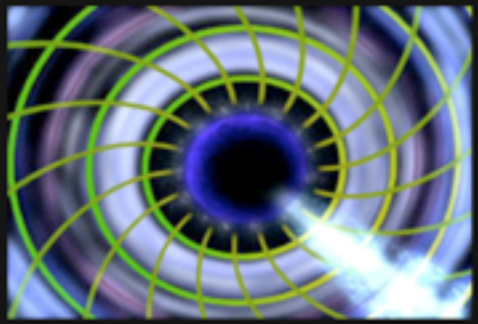


X-ray: A Rotating Black Hole

We expect everything in the Universe to rotate. Non-rotating black holes are different from rotating ones.



Non-rotating black hole

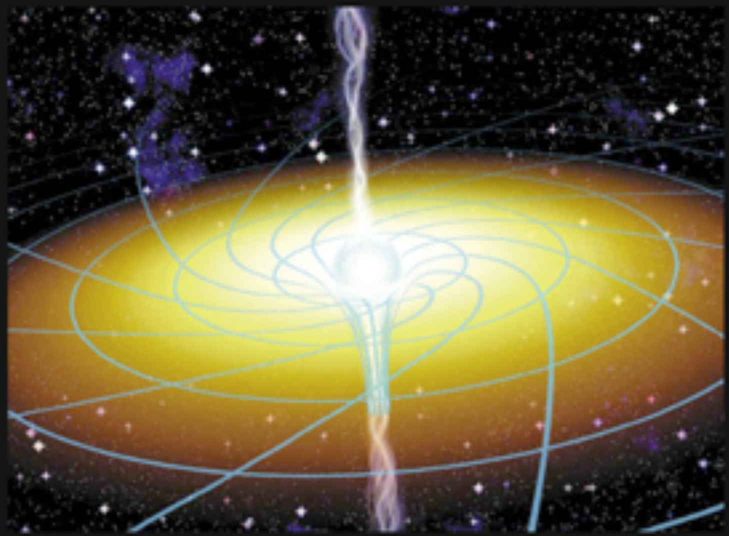


Rotating black hole

In GRO J1655-40, a 2.2 ms period was discovered. This implies an orbit that is too small to be around a non-rotating black hole. This means the black hole is rotating.

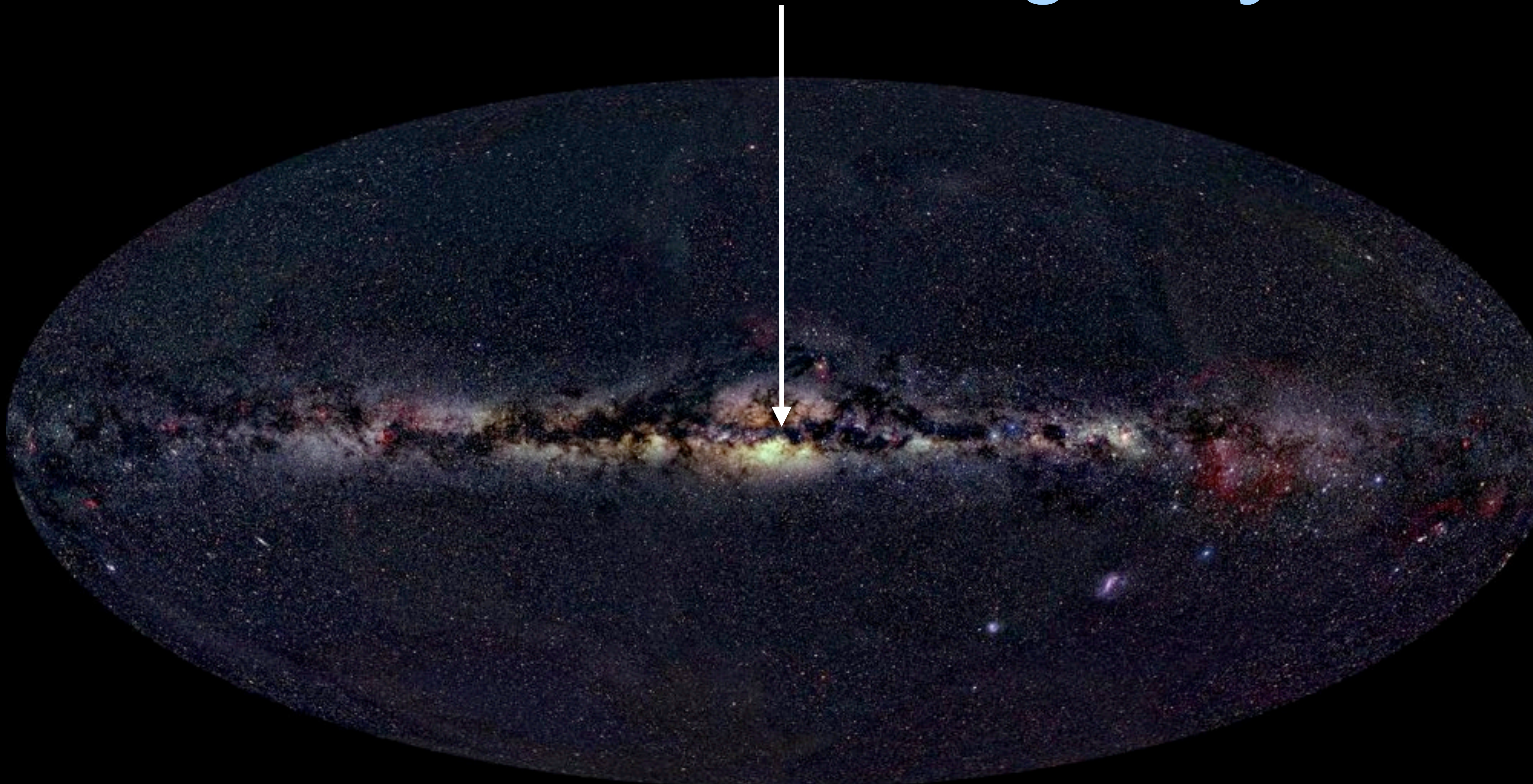
X-ray: Frame Dragging

- Detection of a period in GRO J1655-40 due to precession of the disk.
- This precession period matches that expected for frame dragging of space-time around the black hole.



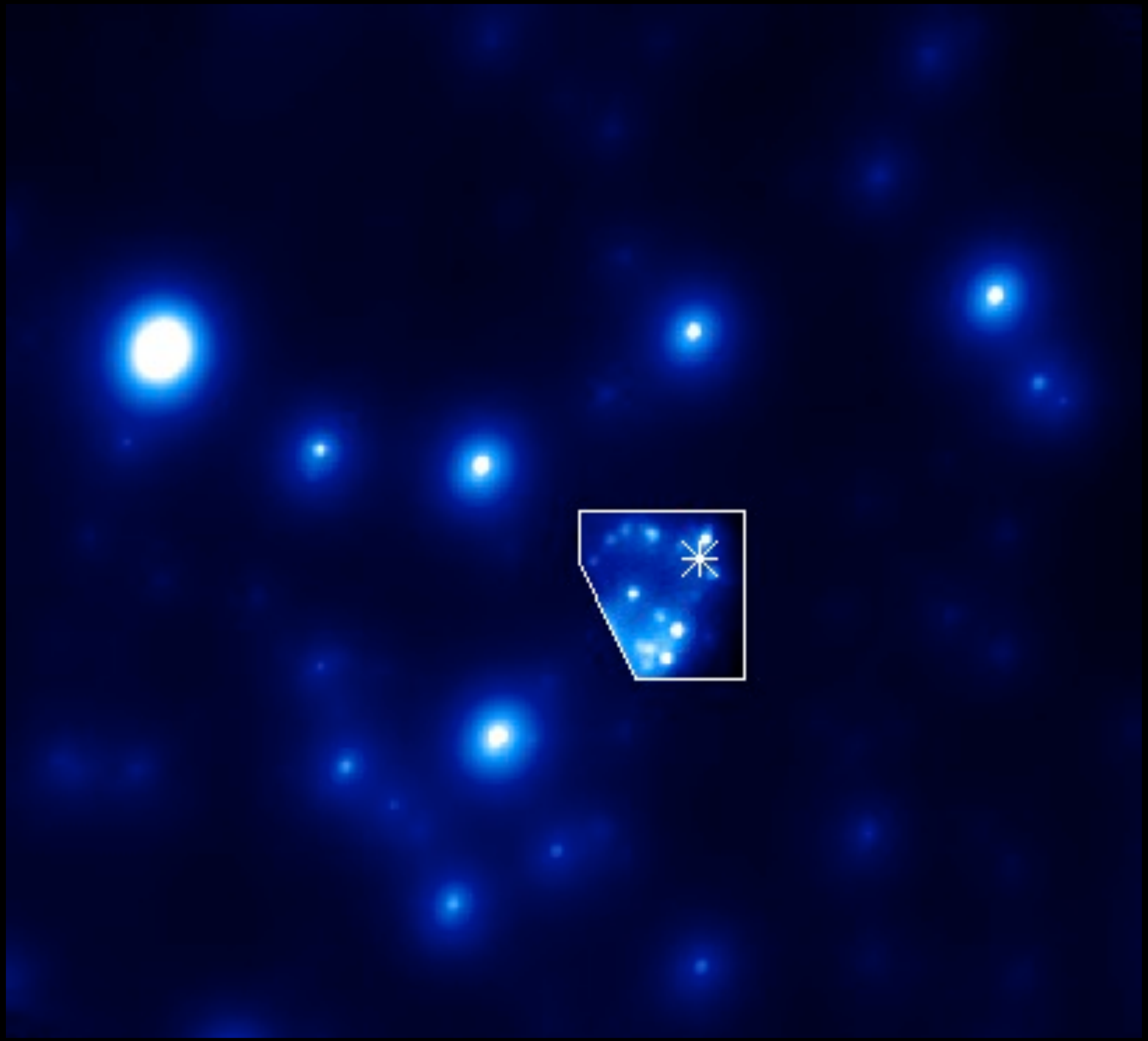
Credit: J. Bergeron, Sky & Telescope Magazine

The center of our galaxy



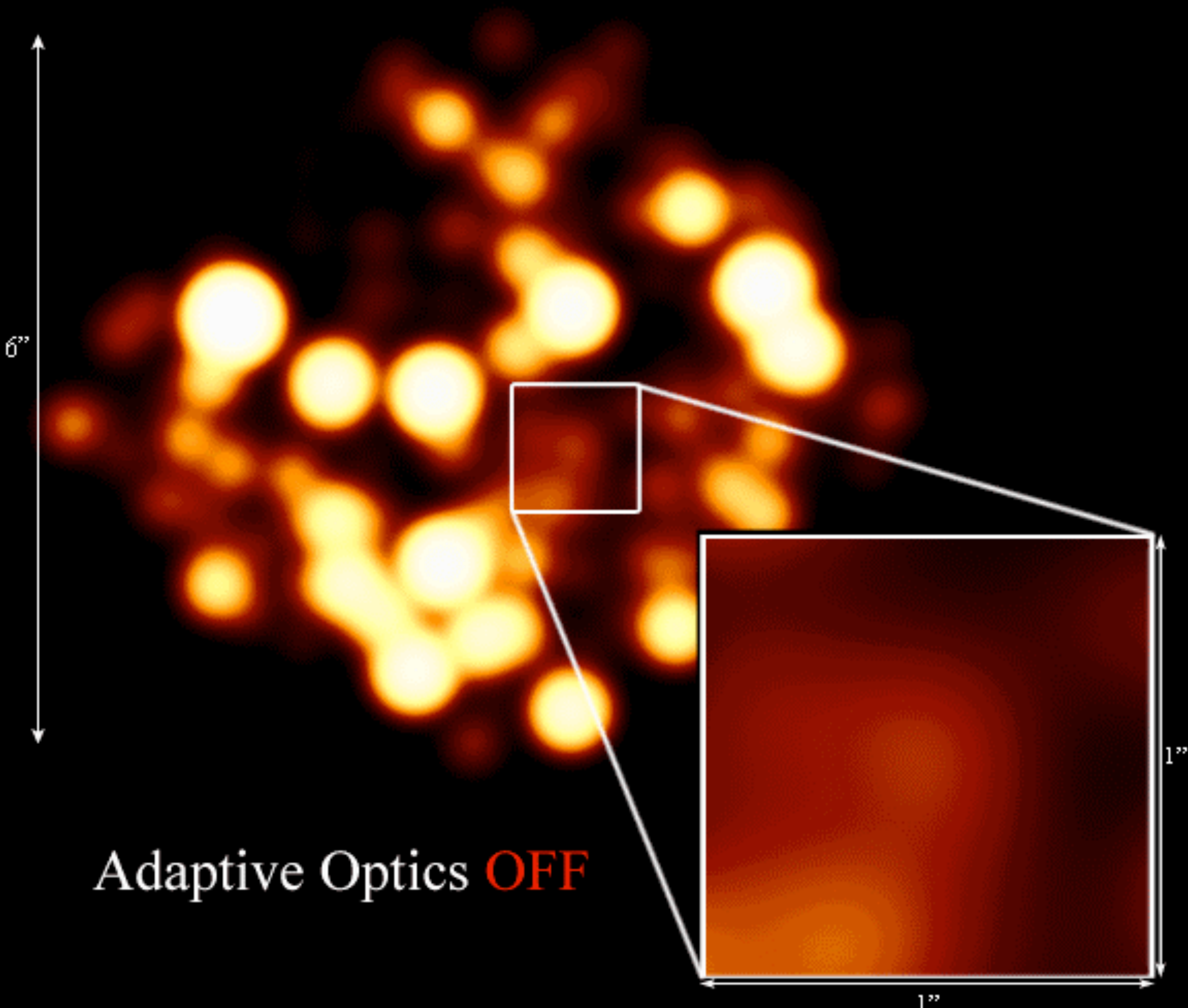
There's a supermassive black hole at the center of our galaxy…

- Modern large telescopes can track individual stars at galactic center
- Need infrared (to penetrate dust)
- Need very good resolution -- use adaptive optics



Keck, 2 μm *Ghez, et al.*

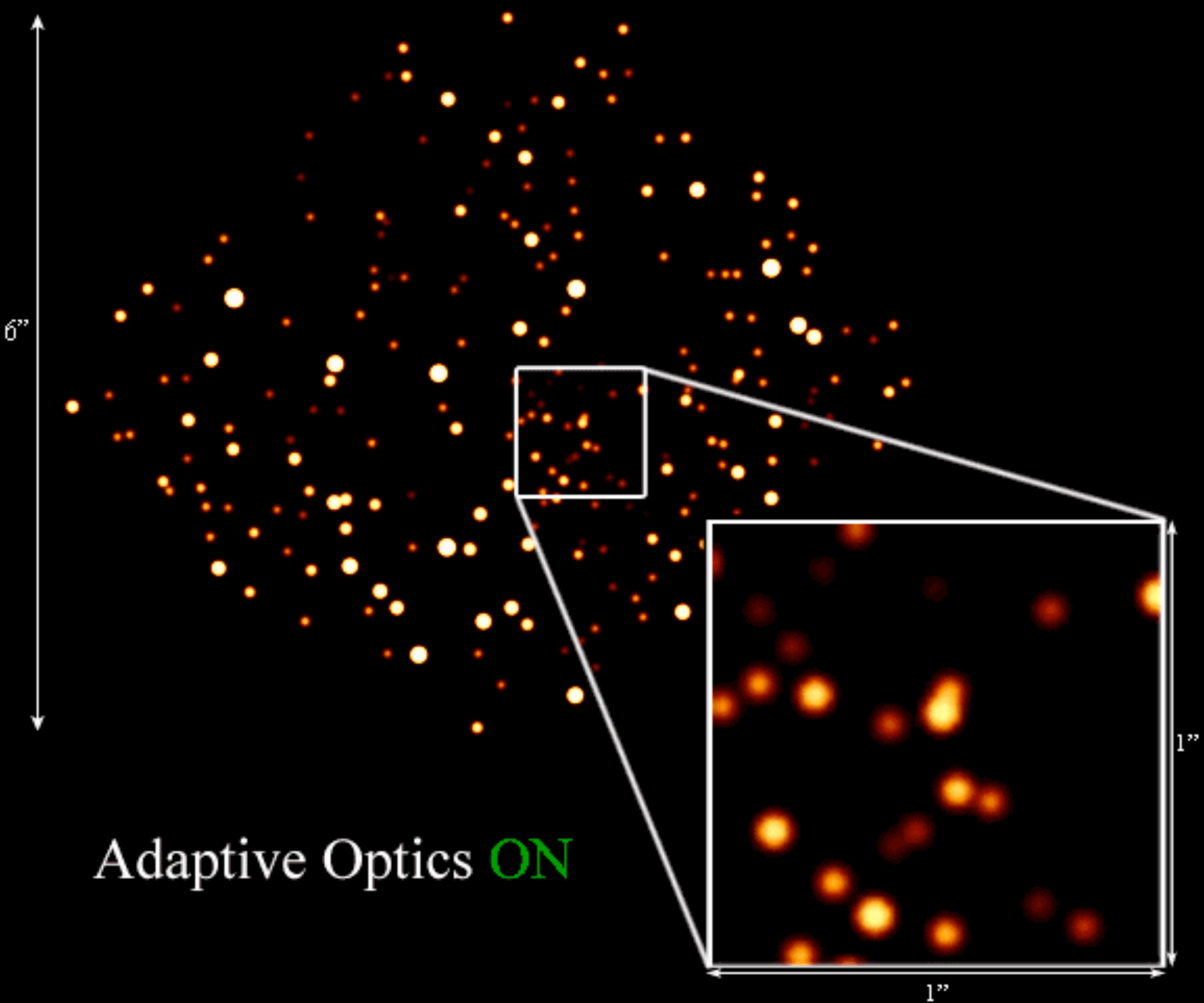
The Galactic Center at 2.2 microns



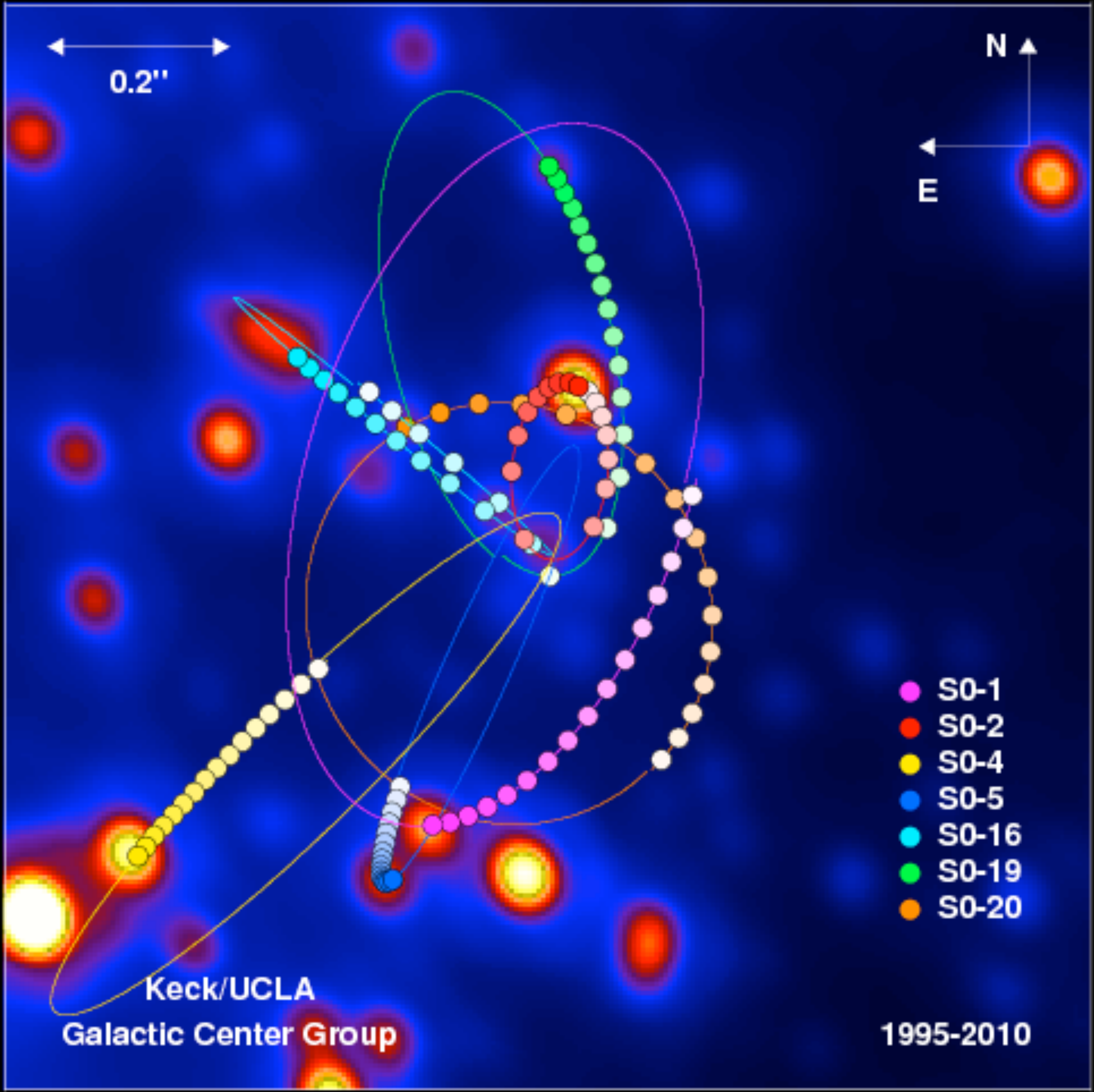
Adaptive Optics OFF

Ghez, et al.

The Galactic Center at 2.2 microns



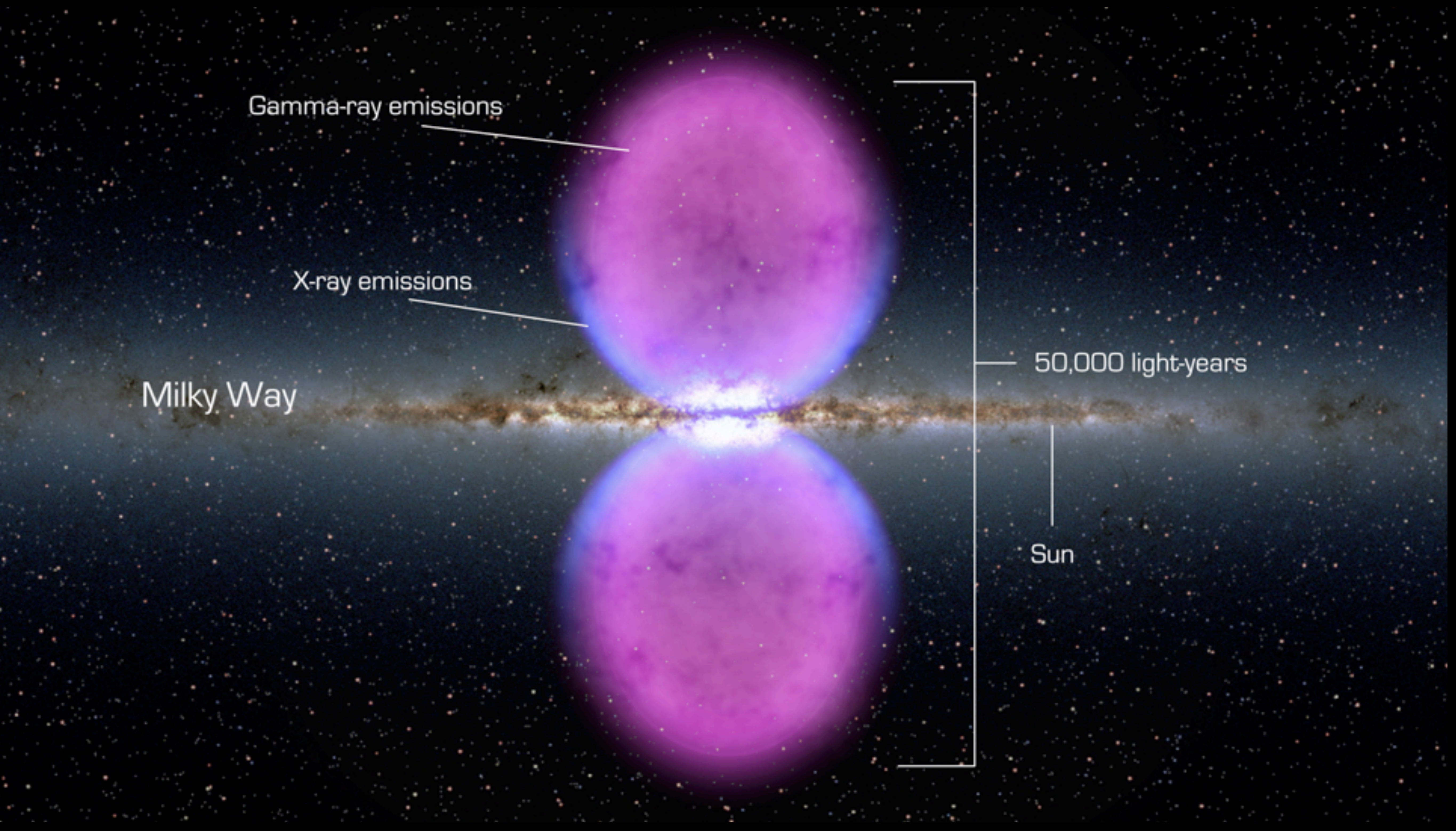
Ghez, et al.



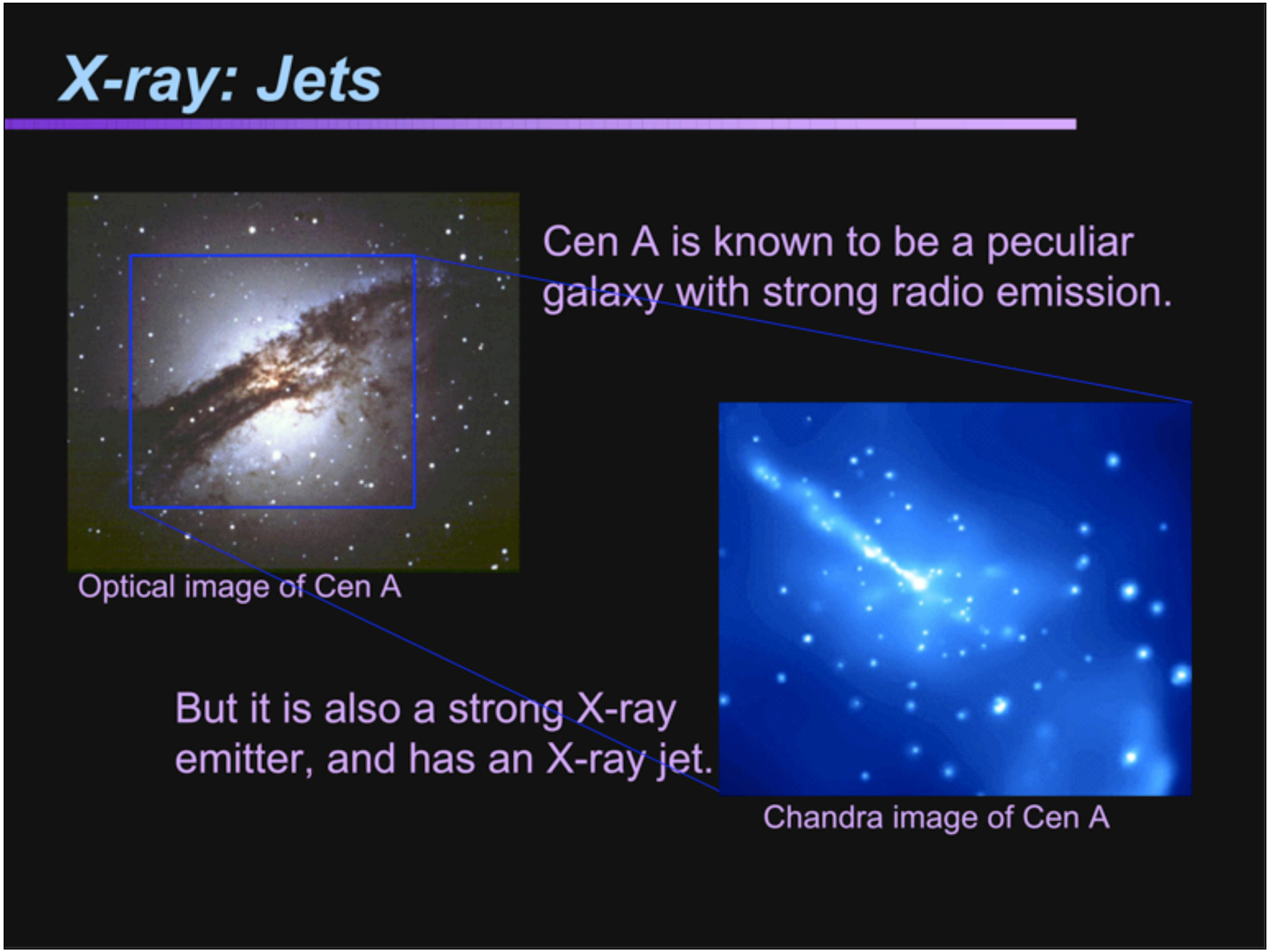
Motions of stars consistent with large, dark mass located at Sgr A...*

Ghez, et al.

- **The central object at the center of the Milky Way is…**
  - Very massive (~4 million solar masses).
  - Must be very compact (star S0-2 gets within 17 light hours of the center).
  - Now seen to flare in X-rays and IR, in the past in Gamma rays.

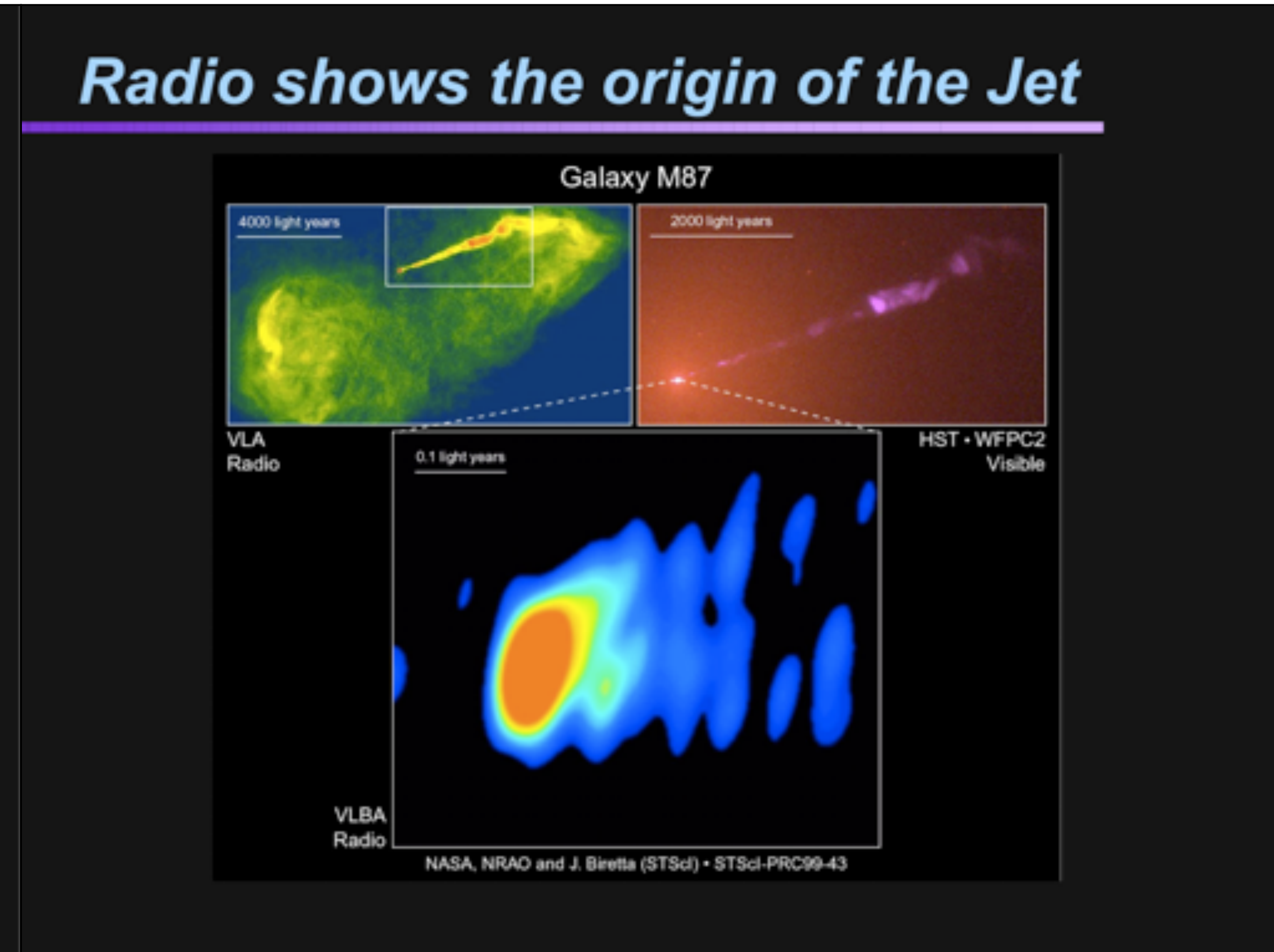


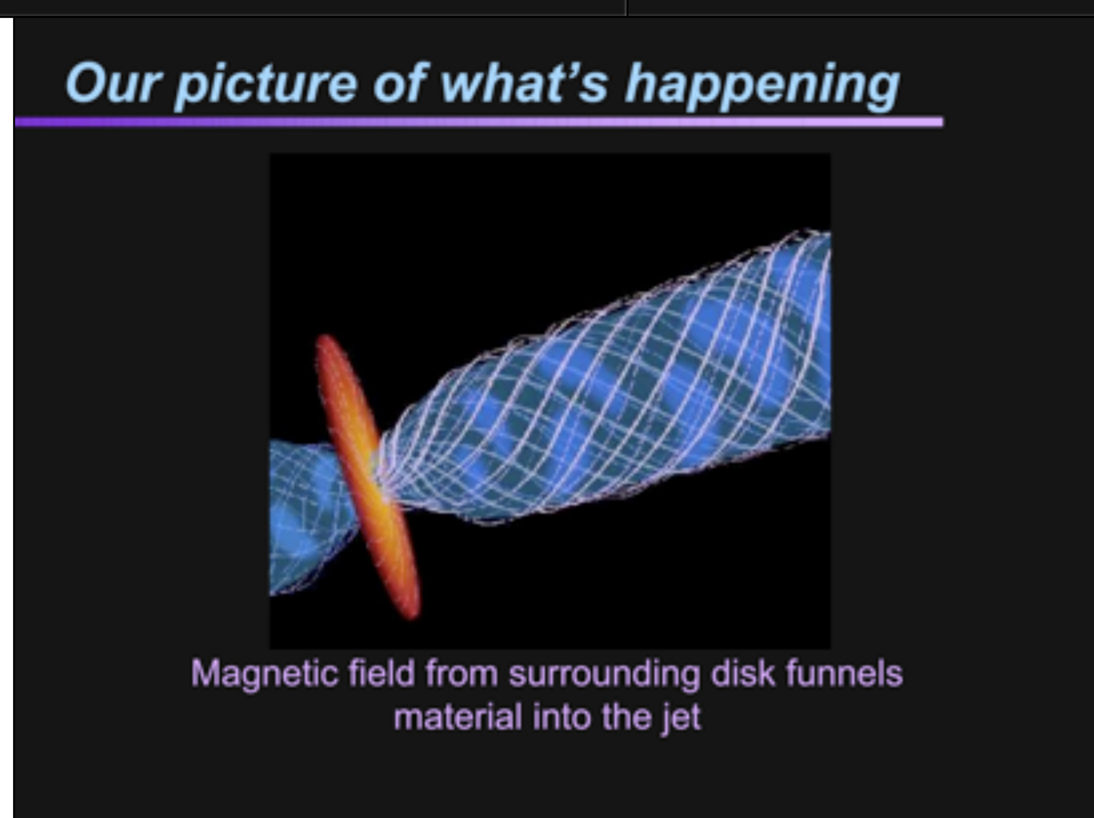
BLACK HOLES AT CENTERS OF GALAXIES



The mass of the black holes in galaxy centers is about 1/1000 the mass of the central spheroid of stars.

BLACK HOLES AT CENTERS OF GALAXIES

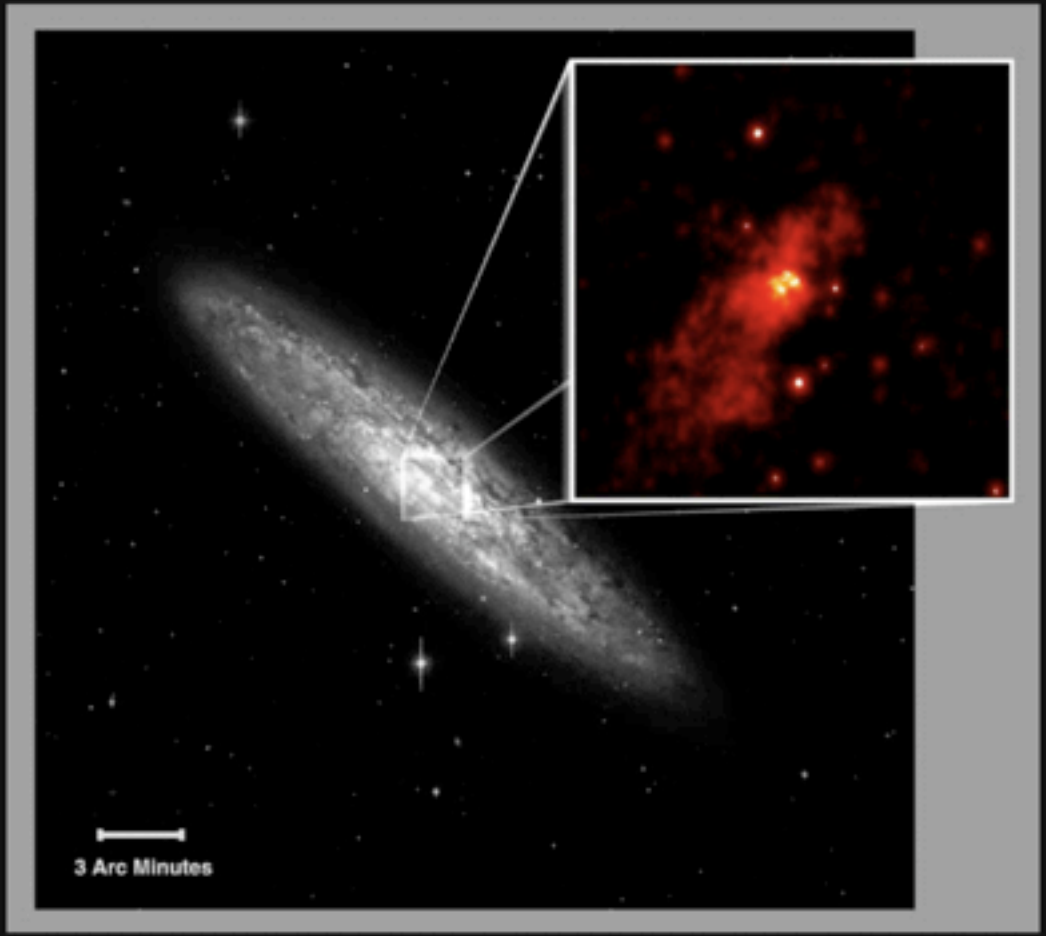




INTERMEDIATE MASS BLACK HOLES

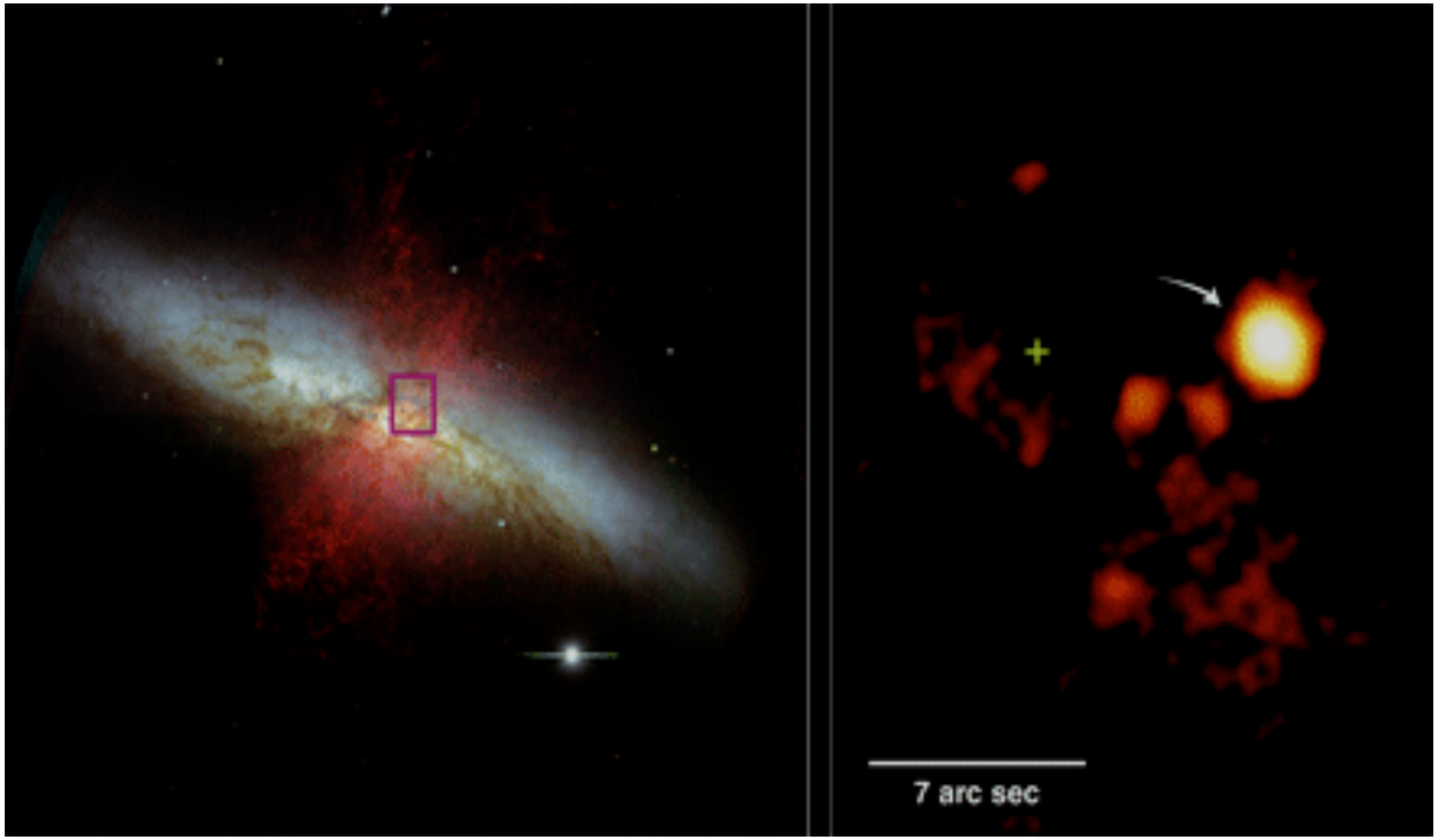
X-ray: Mid mass black holes

- Black Holes with masses a few hundred to a few thousand times the mass of the sun have been found outside the central regions of a number of galaxies.
- Often found in Starburst galaxies.
- May be precursors to Active Galaxies.



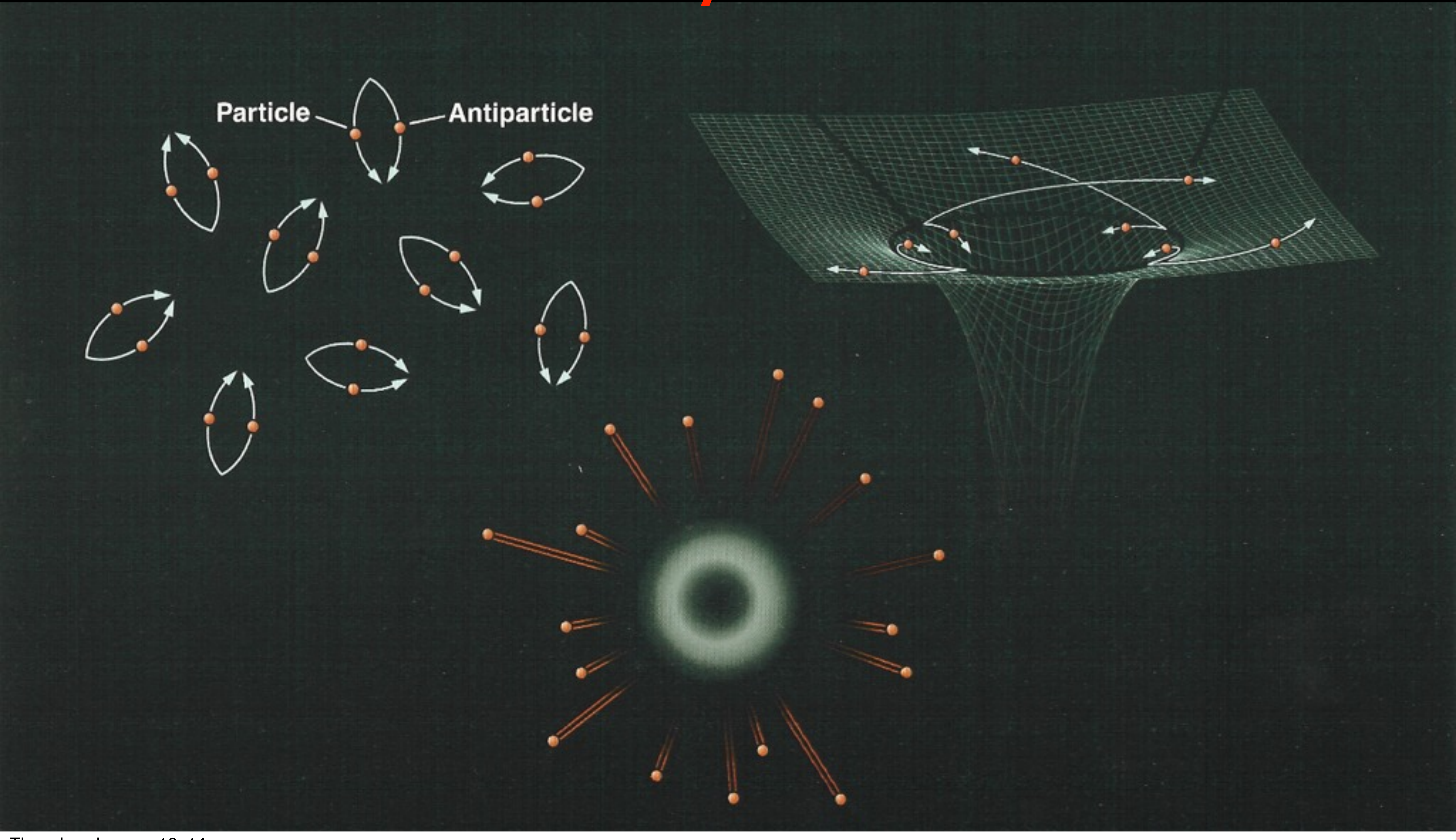
Optical and X-ray images of NGC 253

INTERMEDIATE MASS BLACK HOLES



The best candidate for an intermediate-mass black hole. Optical (**left**) and Chandra x-ray (**right**) images of the M82 galaxy. The arrow points to the location of the ultraluminous x-ray source that is likely to be an intermediate-mass black hole. The area covered by the right image lies within the rectangle at the center of the left image. The green cross (right image) is the galaxy nucleus. CREDIT: LEFT PANEL: SUBARU TELESCOPE, NAO JAPAN; RIGHT PANEL: NASA/SAO/CXC

HAWKING RADIATION
from tiny black holes



HAWKING RADIATION

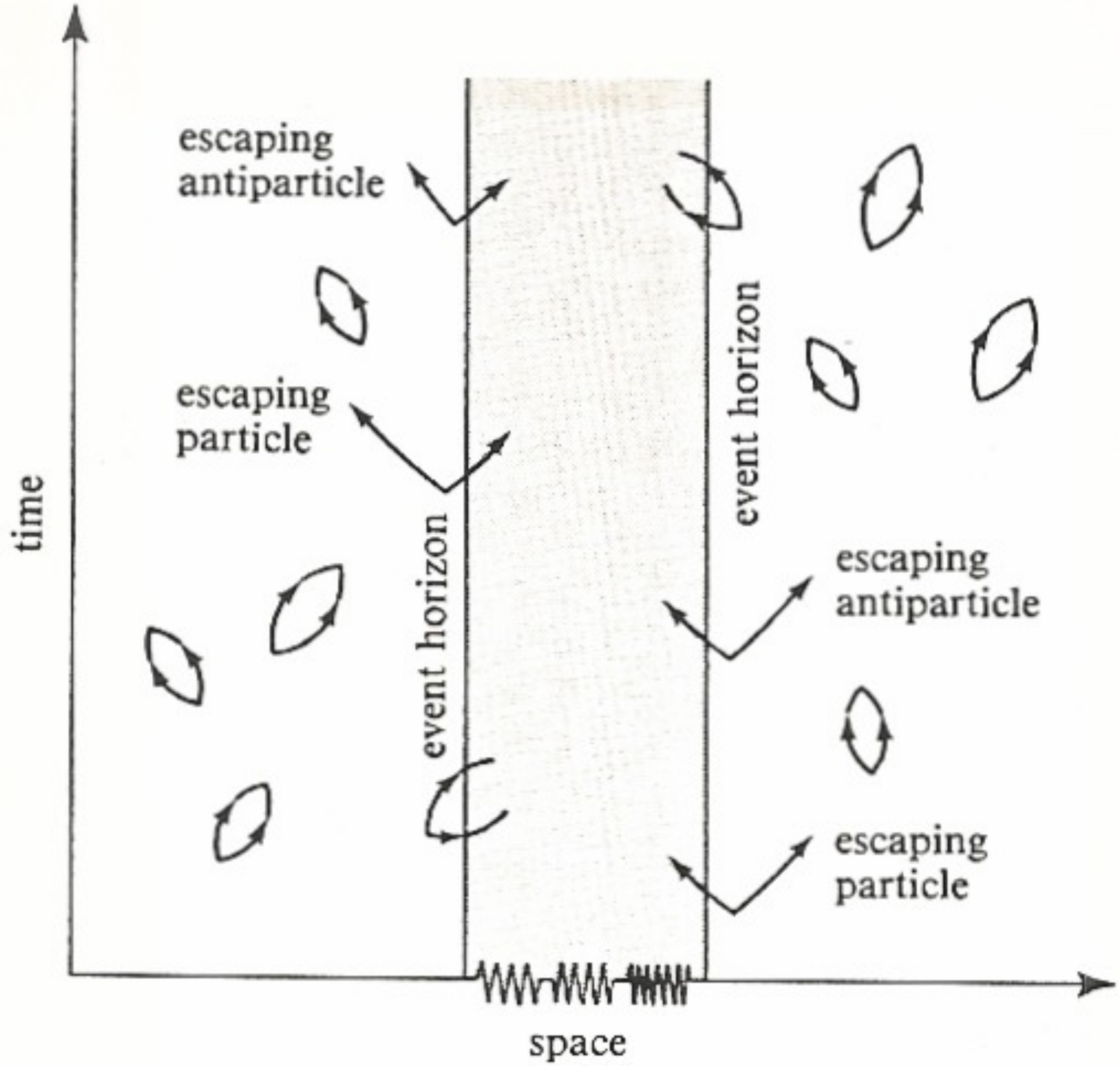
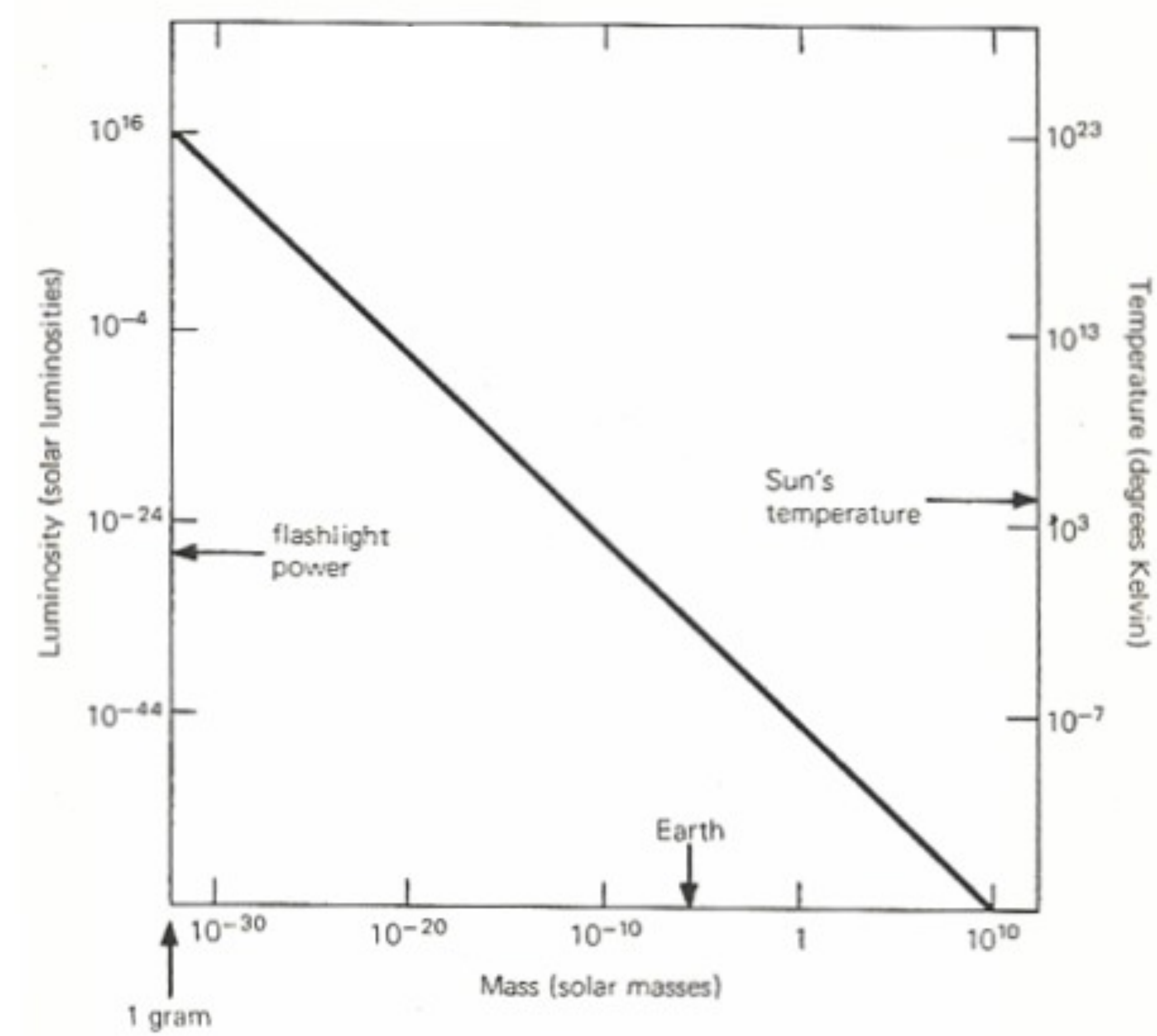


Figure 7.13. Hawking process near the event horizon of a black hole. The creation of particle-antiparticle pairs usually ends with their destruction a short time later. Occasionally, however, the tidal forces of the black hole can cause one of the pair to fall into the black hole, enabling its liberated partner to "tunnel" safely to infinity and escape. Notice the assumption that the black hole is (quasi) stationary implies that the two sides of the event horizon remain fixed in space. Actually, this diagram is even more schematic than usual, because the actual spacetime near a black hole is highly curved.

F. Shu, *The Physical Universe*



Luminosity and temperature of a black hole of given mass.

E. R. Harrison, *Cosmology*

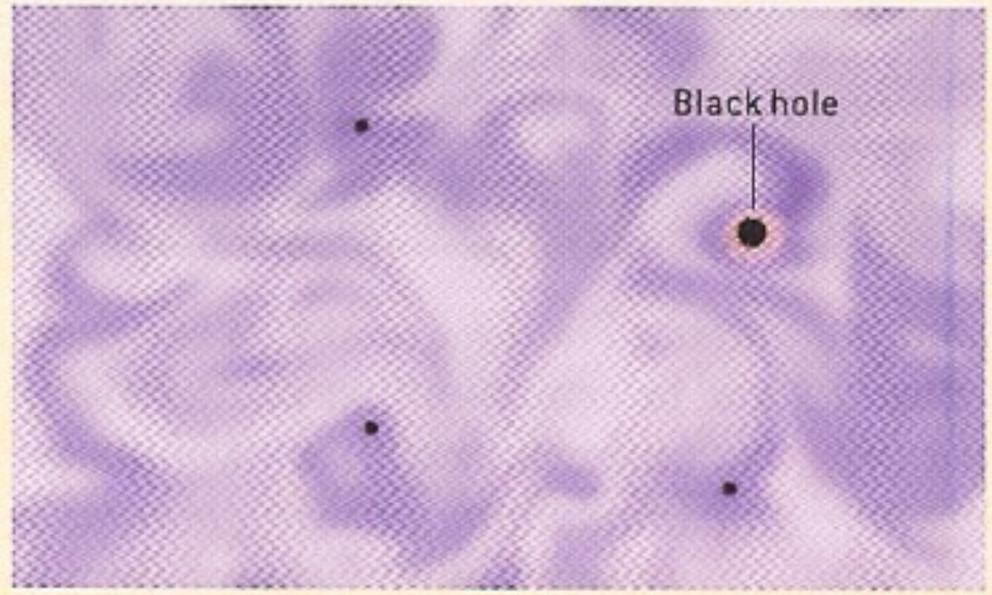
$$T_{BH} = 10^{-7}\ M_{sun}/M$$

$$T_{evap} = 10^{62}\ (M_{sun}/M)^3\ \text{yr}$$

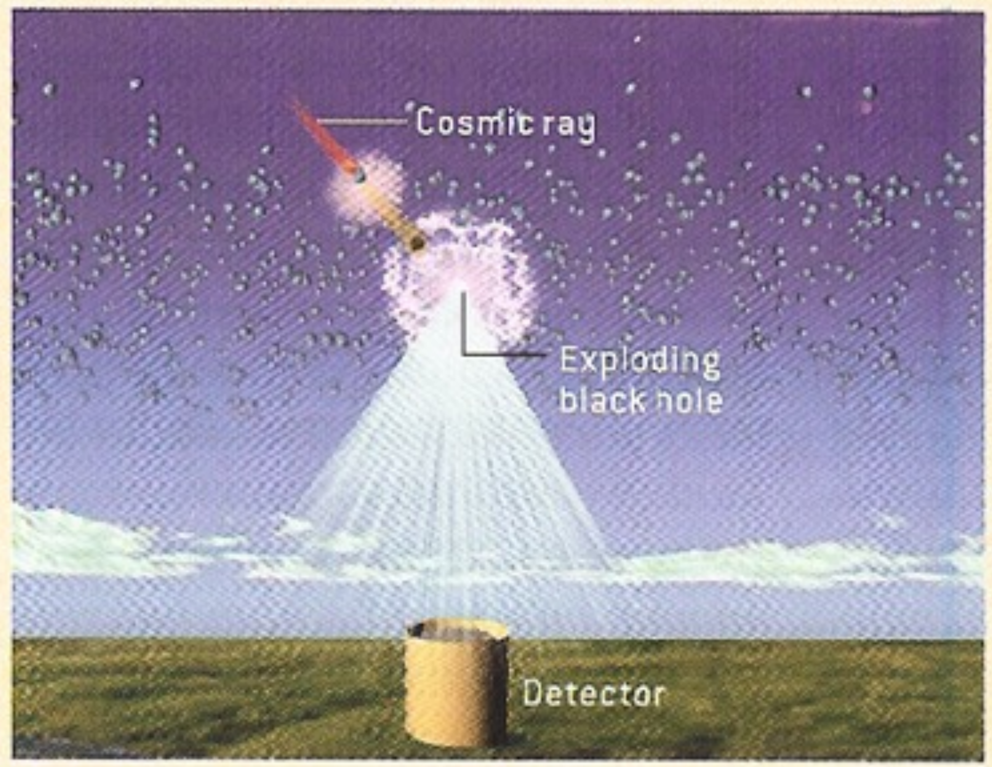
Making Evaporating Black Holes?

See "Quantum Black Holes" by Steve Giddings and Bernard Carr in the May 2005 *Scientific American*

WAYS TO MAKE A MINI BLACK HOLE



PRIMORDIAL DENSITY FLUCTUATIONS
Early in the history of our universe, space was filled with hot, dense plasma. The density varied from place to place, and in locations where the relative density was sufficiently high, the plasma could collapse into a black hole.

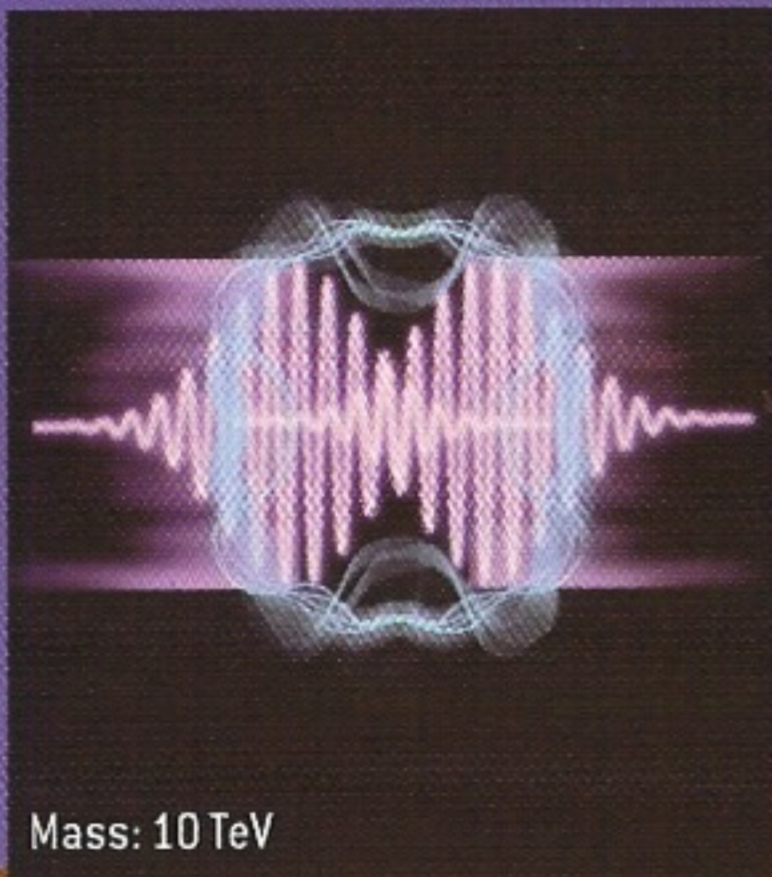


COSMIC-RAY COLLISIONS
Cosmic rays—highly energetic particles from celestial sources—could smack into Earth's atmosphere and form black holes. They would explode in a shower of radiation and secondary particles that could be detected on the ground.

PARTICLE ACCELERATOR
An accelerator such as the LHC could crash two particles together at such an energy that they would collapse into a black hole. Detectors would register the subsequent decay of the hole.

THE RISE AND DEMISE OF A QUANTUM BLACK HOLE

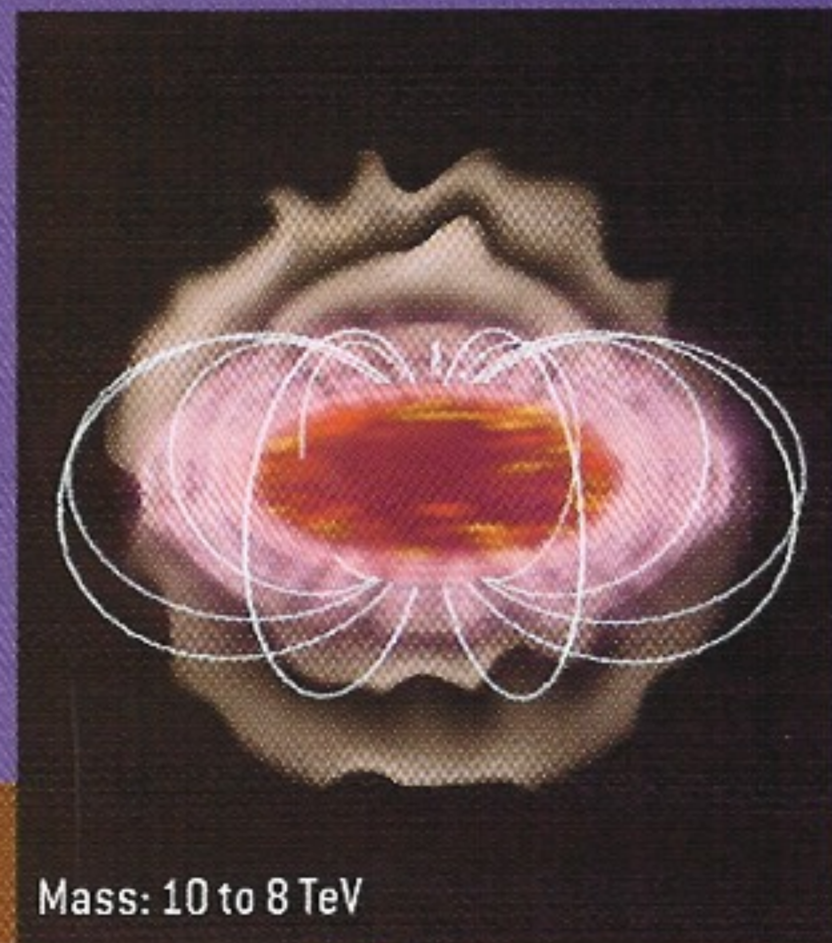
BIRTH



TIME 0

If conditions are right, two particles (shown here as wave packets) can collide to create a black hole. The newborn hole is asymmetrical. It can be rotating, vibrating and electrically charged. (Times and masses are approximate; 1 TeV is the energy equivalent of about 10^{-24} kilogram.)

BALDING PHASE



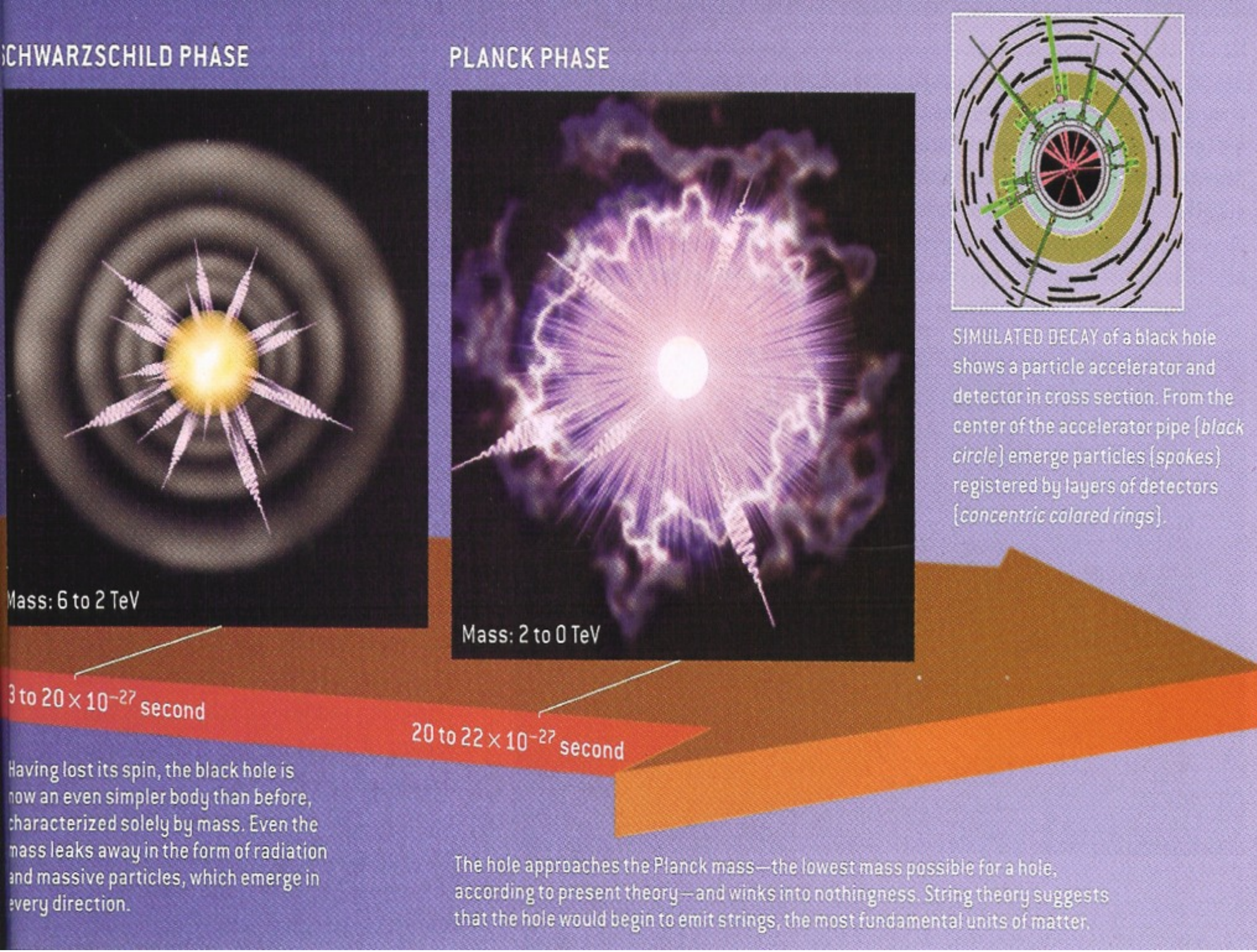
0 to 1×10^{-27} second

As it settles down, the black hole emits gravitational and electromagnetic waves. To paraphrase physicist John A. Wheeler, the hole loses its hair—it becomes an almost featureless body, characterized solely by charge, spin and mass. Even the charge quickly leaks away as the hole gives off charged particles.

SPIN-DOWN PHASE

1 to 3×10^{-27} second

The black hole is no longer black: it radiates. At first, the emission comes at the expense of spin, so the hole slows down and relaxes into a spherical shape. The radiation emerges mainly along the equatorial plane of the black hole.


CHWARZSCHILD PHASE
PLANCK PHASE
Mass: 6 to 2 TeV
Mass: 2 to 0 TeV
3 to 20 × 10^{-27} second
20 to 22 × 10^{-27} second
Having lost its spin, the black hole is now an even simpler body than before, characterized solely by mass. Even the mass leaks away in the form of radiation and massive particles, which emerge in every direction.
The hole approaches the Planck mass—the lowest mass possible for a hole, according to present theory—and winks into nothingness. String theory suggests that the hole would begin to emit strings, the most fundamental units of matter.
SIMULATED DECAY of a black hole shows a particle accelerator and detector in cross section. From the center of the accelerator pipe (*black circle*) emerge particles (*spokes*) registered by layers of detectors (*concentric colored rings*).