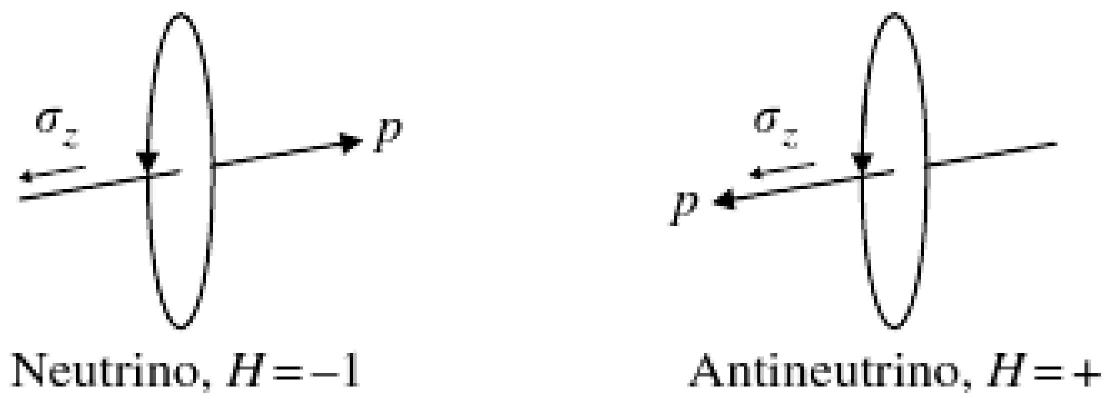
Physics 129 LECTURE 6 January 23, 2014

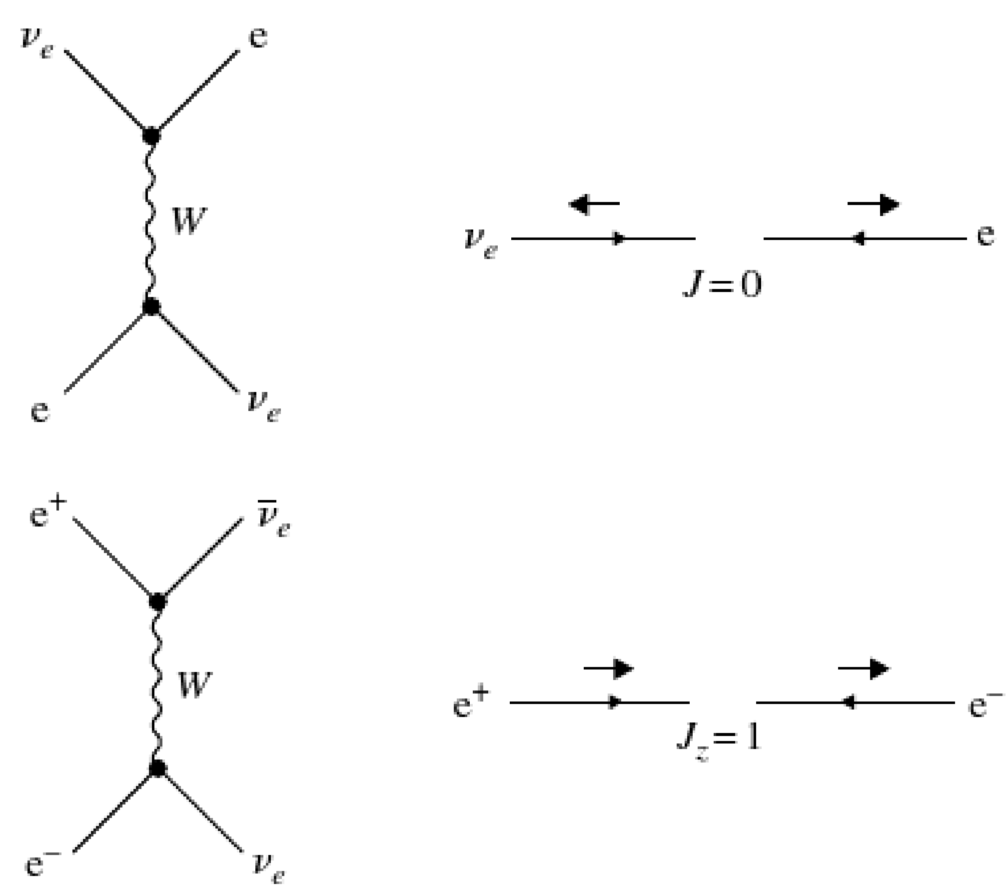
- Particle Physics Symmetries (Perkins Chapter 3)
  - Lagrangian Deductions
  - Rotations
  - Parity
  - Charge Conjugation
- Gauge Invariance and Charge Conservation
- The Higgs Mechanism
- The Parton Model of Deep Inelastic Scattering
- Running Coupling Constants
- Grand Unification and Proton Decay (Perkins Ch 4)

Parity Nonconservation

Parity is not conserved in the Weak interactions; on the contrary, the $W^{\pm}$ interacts only with "left-handed" particles. In the Standard Model, neutrinos are massless, which means that they are eigenstates of helicity $H = \boldsymbol{\sigma} \cdot \mathbf{p}/|p|$ with $H = \pm 1$. While neutrinos are not actually massless, their masses, $\lesssim 0.1$ eV, are so small that they can be neglected in high-energy processes. neutrinos as having helicity −1 ("left handed") and anti-neutrinos as having helicity +1 ("right handed"), as shown at right.



Parity can be important in determining the values of cross sections. Perkins shows, for example, that although the two processes shown in the diagrams at right are very similar, the cross section for e+e− annihilation to neutrinos is only one-sixth that of the electron-neutrino scattering process, taking into account the effects of parity and identical particles in the final state. The e^+e^- annihilation to neutrinos is extremely important in core-collapse supernovae and also in the early universe.



Charge Conjugation Invariance

The charge conjugation operation C turns particles into their antiparticles (for example, $C\,e^- = e^+$, and $C\,p \longrightarrow \bar{p}$), so it reverses the electric charge and other additively conserved quantum numbers such as baryon and lepton numbers. It thus also reverses the sign of the electric current and the magnetic moment.

The Strong and Electroweak interactions are invariant under C, but the Weak Interaction is not. For example, a neutrino has helicity $H = -1$ (we say it is left-handed), and an anti-neutrino has helicity $H = +1$. Charge conjugation would turn a left-handed neutrino into a left-handed antineutrino, a state that doesn't exist.

However, the combined parity and charge conjugation operation CP would change a right-handed neutrino into a left-handed antineutrino, which does exist. It was expected therefore that the Weak Interactions would likely be invariant under CP. However as we will see, this is not the case, as was first discovered in neutral K decay -- and this has very important implications in the early unverse.

Since C reverses electric charge, no charged particle state can be an eigenfunction of C. But neutral particles such as the π^0 can be. Since $C^2 = 1$, the eigenvalues of C can only be ±1. Since the Electromagnetic interaction $J \bullet A = J^\mu A_\mu$ is invariant under C, where J is the electric current 4-vector $J = (\rho, \boldsymbol{J})$ and A is the electromagetic potential 4-vector $A = (\varphi, \boldsymbol{A})$, and J^μ changes sign under C, so must A_μ. But in quantum field theory A_μ is the operator that creates and destroys photons, so the photon must also have $C = -1$. The main decay of the π^0 is $\pi^0 \longrightarrow \gamma\gamma$, which has $C = (-1)^2 = +1$, so π^0 also has $C = +1$. Thus the decay $\pi^0 \longrightarrow \gamma\gamma\gamma$ is forbidden by C conservation.

Gauge Invariance

Global gauge invariance is the invariance of physical quantities when charged fields Ψ are multiplied by a constant phase $e^{i\varphi}$, i.e. $\Psi \longrightarrow e^{i\varphi}\,\Psi$. Since a physical quantity like probability density $\Psi^*\,\Psi$ or momentum $P_\mu = -i\Psi^*\,\partial_\mu\,\Psi$ is not affected by this since the constant phase cancels out between Ψ^* and Ψ. (Note that $\partial_\mu = (\partial/\partial t, \partial/\partial\mathbf{x})$, and that the electric and magnetic fields are determined by the 4-vector electromagnetic potential by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, which is equivalent to $\boldsymbol{E} = \nabla\varphi - \partial\boldsymbol{A}/\partial t$, $\boldsymbol{B} = \nabla\times\boldsymbol{A}$.)

The conserved current associated with global gauge invariance is the electric current, i.e. $\partial_\mu J^\mu = 0$ or $\partial\rho/\partial t - \nabla\cdot\boldsymbol{J} = 0$, which means that electric charge is conserved. Thus integrating $\partial\rho/\partial t = \nabla\cdot\boldsymbol{J}$ over a volume V, the l.h.s. is the time derivative of the electric charge in V and the r.h.s. is, by Gauss's theorem, the integral of the electric current over the surface = the charge crossing the surface.

E.g., the Lagrangian density of a scalar field Φ is $\mathscr{L} = \mathcal{T} - \mathcal{V} = \left(\frac{1}{2}\right)\left(\frac{\partial\Phi}{\partial x_\mu}\right)^2 - \frac{\mu^2\Phi^2}{2}$

The corresponding Euler-Lagrange equation is the Klein-Gordon equation with mass μ:

$\left(\frac{\partial^2}{\partial\mathbf{r}^2} - \frac{\partial^2}{\partial t^2} - \mu^2\right)\Phi = 0$. With $E = -i\partial/\partial t$ and $\boldsymbol{p} = -i\partial/\partial\mathbf{x}$, this is just $-\boldsymbol{p}^2 + E^2 - \mu^2 = 0$.

(The Klein-Gordon equation, the relativistic equation for a scalar field, is discussed in Perkins p. 76 and Appendix B and in books on quantum mechanics. We will return to it when we discuss the Higgs boson.)

Perkins p. 70 gives the following derivation of current conservation from global gauge invariance, i.e. invariance of the Lagrangian (density) *L* under $\Psi \longrightarrow e^{i\alpha}\Psi = \Psi(1+i\alpha)$, working to first order in α and using the Euler-Lagrange equations

$$\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \psi'}\right) - \frac{\partial L}{\partial \psi} = 0$$

In the first term Ψ' is shorthand for $\partial_\mu \Psi \equiv \Psi_{,\mu}$, $\partial/\partial x$ is shorthand for ∂_μ, and there is an implicit sum over μ. That is, the first term is $\partial_\mu (\partial L/\partial \Psi_{,\mu})$. If *L* is invariant under $\Psi \longrightarrow e^{i\alpha}\Psi = \Psi(1+i\alpha)$, then

$$\delta L = 0 = i\alpha\psi\left(\frac{\partial L}{\partial \psi}\right) + i\alpha\psi'\left(\frac{\partial L}{\partial \psi'}\right)$$

and since

$$i\alpha\frac{\partial}{\partial x}\left(\psi\frac{\partial L}{\partial \psi'}\right) = i\alpha\psi'\frac{\partial L}{\partial \psi'} + i\alpha\psi\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \psi'}\right)$$

then

$$\delta L = 0 = i\alpha\psi\left(\frac{\partial L}{\partial \psi} - \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \psi'}\right)\right) + i\alpha\frac{\partial}{\partial x}\left(\psi\frac{\partial L}{\partial \psi'}\right)$$

The first term on the r.h.s. vanishes by the Euler-Lagrange equation above, and so this equation just says that $\partial_\mu J^\mu = 0$, where $J^\mu = \Psi(\partial L/\partial \Psi_{,\mu})$. This is actually the probability current; the electric current is just charge *e* times this. Thus charge conservation follows from global gauge invariance, i.e. invariance under $\Psi \longrightarrow e^{i\alpha}\Psi$.

Local gauge invariance means that the complex phase $\alpha(x)$ is a function of **x** and t. The local gauge transformation is

$\Psi \longrightarrow e^{ie\alpha(x)}\,\Psi$ for all charged fields Ψ, together with the transformation $A_\mu \longrightarrow A_\mu - i\partial_\mu\alpha$.

Then the "covariant derivative" $D_\mu \equiv \partial_\mu - ieA_\mu$ applied to a charged field Ψ is indeed covariant (i.e. invariant) under the gauge transformation, since the term $+ie\partial_\mu\alpha$ from $\partial_\mu\Psi$ is cancelled by the $-ie\partial_\mu\alpha$ from $A_\mu \longrightarrow A_\mu - i\partial_\mu\alpha$. The covariant derivative D_μ is used in quantum electrodynamics (QED), the relativistic quantum field theory of a charged fermion field interacting with the electromagnetic field. The nonrelativistic Schroedinger equation for a free charged particle represented by wavefunction Ψ is $-i\partial\Psi/\partial t = H\Psi$, where $H = (\boldsymbol{p} - ie\boldsymbol{A})^2/(2m)$.

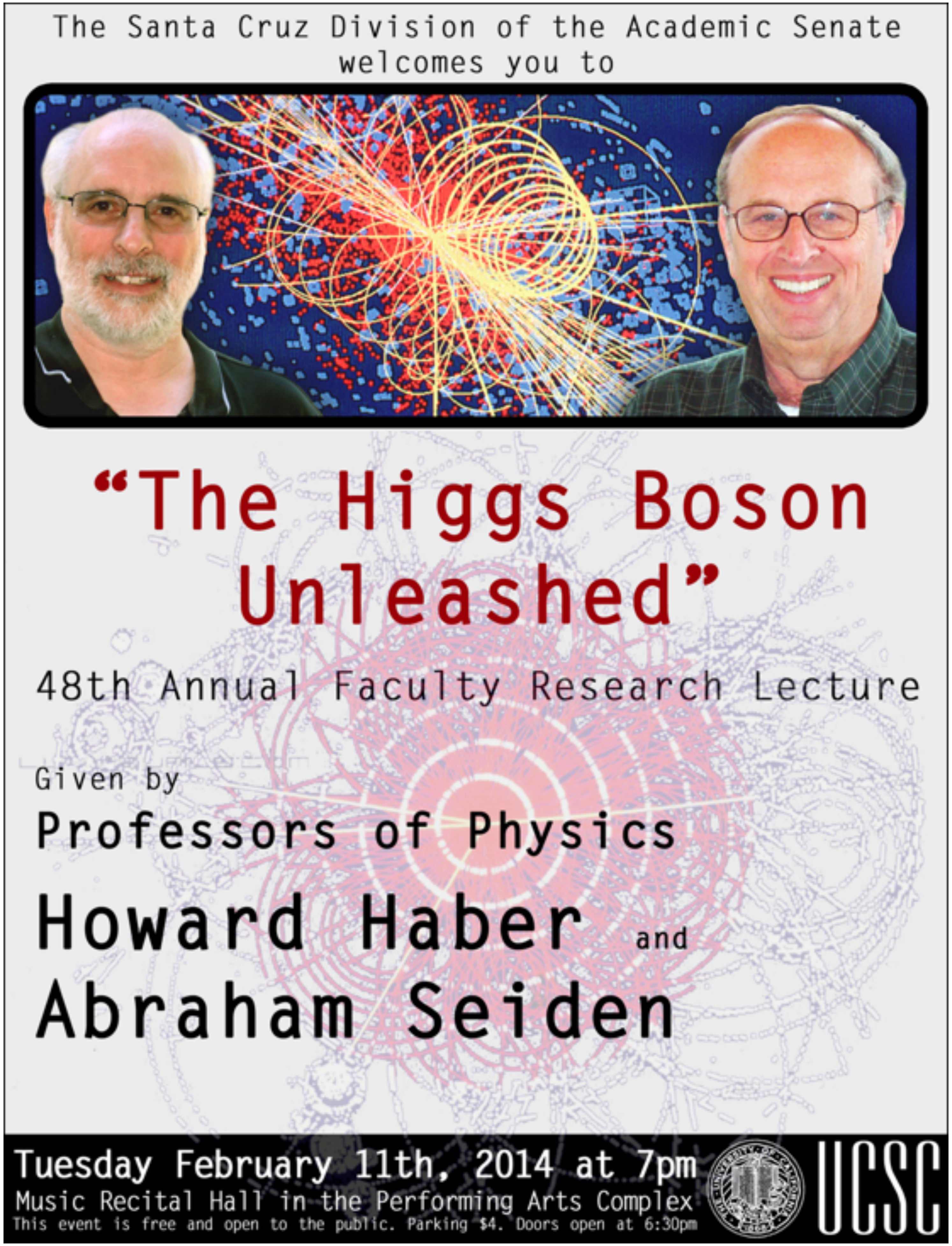
Local gauge invariance implies that the photon is massless, and local gauge invariance is also essential for renormalization to work. QED makes predictions that are extremely accurate, for example

Table 3.1 Anomalous magnetic moments of electron and muon $(g-2)/2 \times 10^{10}$

	Predicted	Observed
Electron	$11,596,524 \pm 4$	$11,596,521.9 \pm 0.1$
Muon	$11,659,180 \pm 100$	$11,659,230 \pm 80$

However, electrodynamics was generalized to the Electroweak theory, with massive gauge bosons $W^\pm$ and Z^0. The Brout-Englert-Higgs phenomenon was necessary in order to keep local gauge invariance despite the gauge bosons being massive.

The Higgs Boson



UCSC Physics Professors Howie Haber and Abe Seiden are giving the UCSC Faculty Research Lecture on the discovery of the Higgs Boson, in which they both played crucial roles, in the Music Receital Hall at 7:00 pm on Tuesday night February 11. I strongly suggest that you attend. I'll be there.

The Higgs Mechanism

The Electroweak theory generalizes QED by introducing, in addition to a gauge field B_μ a triplet of vector fields $\boldsymbol{W}_\mu = (W^-_\mu, W^0_\mu, W^+_\mu)$. Then the photon is one linear combination of B_μ and W^0_μ, and the Z^0 boson is the other linear combination of B_μ and W^0_μ. The gauge transformation involving the ***W*** field involves the **τ** (tau) matrices (same as the three sigma matrices for spin-1/2) acting on two-component fields Ψ or Φ:

$$\Psi \longrightarrow \exp[ie\boldsymbol{\alpha}(x)\boldsymbol{\cdot}\boldsymbol{\tau}/2]\,\Psi$$

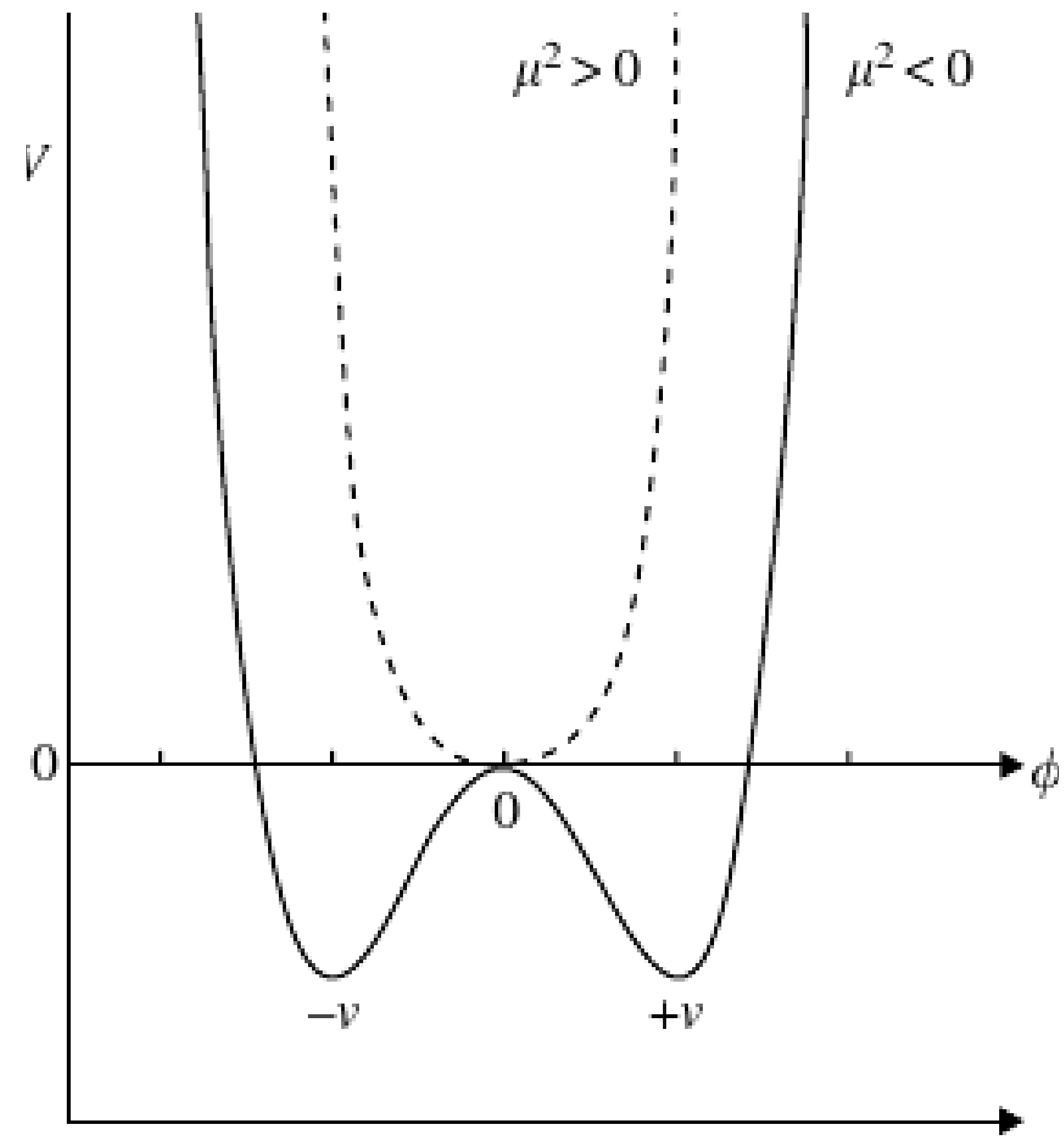
The corresponding gauge transformation of the W field is more complicated than before, because the **τ** matrices don't commute -- we say that the corresponding symmetry is non-abelian. You are already familiar with the fact that the three components of angular momentum don't commute; the commutation relations of the **τ**/2 matrices are just the same SU(2). Including the gauge field B_μ, the full gauge group is SU(2)xU(1).

The quark and lepton fields are doublets, for example (u,d) and (ν, e^-). The Higgs fields Φ are a complex doublet, i.e. each component is complex, equal to a real and imaginary field, so there are a total of 4 fields. Three of them become absorbed into the massive gauge fields $W^\pm$ and Z^0, and the remaining electrically neutral field is the physical Higgs boson. Because the underlying theory is invariant under gauge transformations, it remains renormalizable -- but three of the vector bosons have acquired masses. A massless vector field has only two independent components (the relativistically covariant gauge condition $\partial\mu A^\mu = 0$ removes one component, and an additional gauge condition such as $A_0 = 0$ (radiation gauge) removes another. Massive vector fields such as $W^\pm$ and Z^0 have three independent components.

The Higgs Mechanism

Perkins discusses a simple version of the Higgs mechanism, with only a one-component Higgs particle, using the Klein-Gordon Lagrangian (density) with an added λ term:

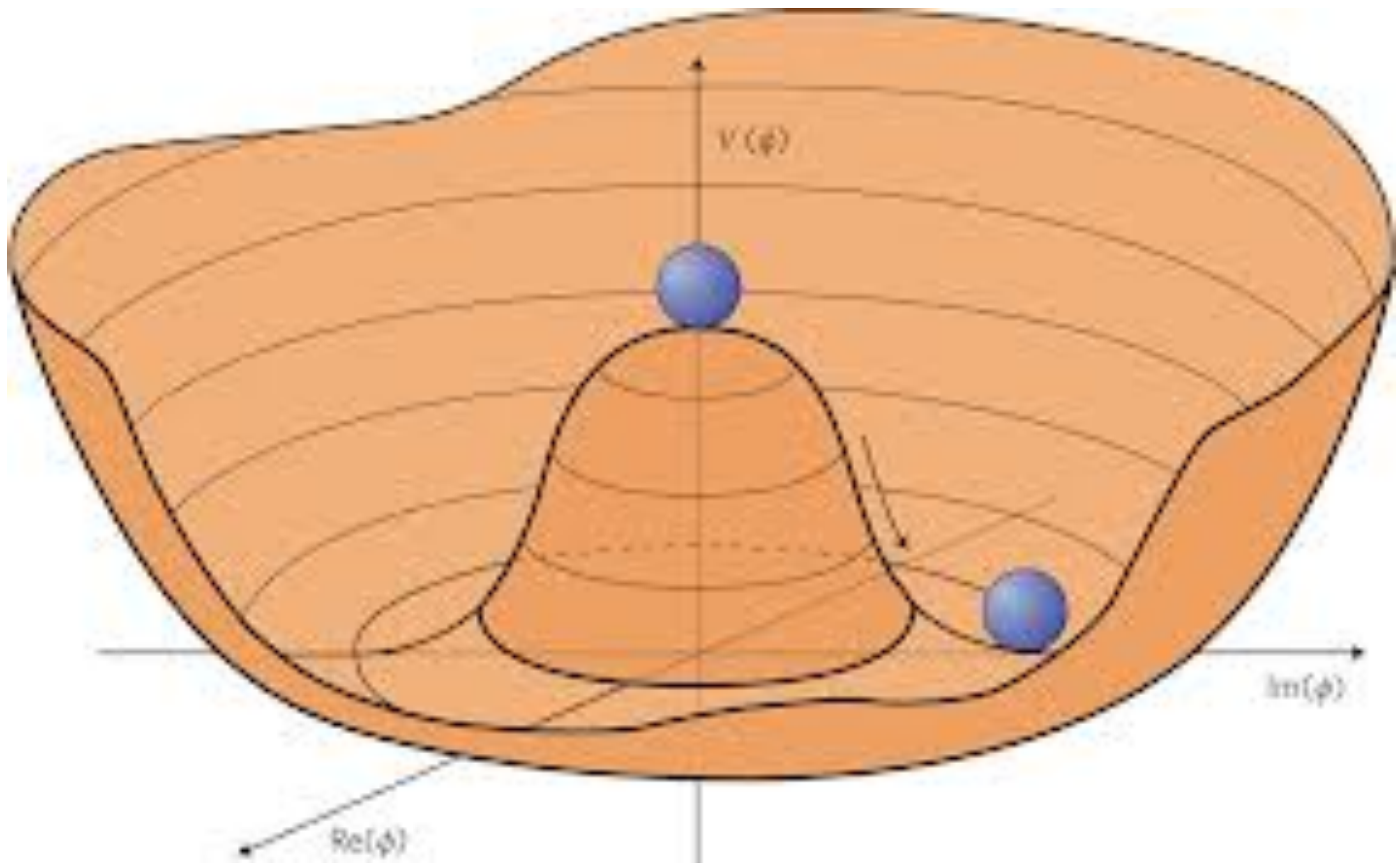
$$L = \left(\frac{1}{2}\right)\left(\frac{\partial\Phi}{\partial x_\mu}\right)^2 \underbrace{- \left(\frac{1}{2}\right)\mu^2\Phi^2 - \left(\frac{1}{4}\right)\lambda\Phi^4}_{= -V(\Phi)}$$



If we take the mass term μ^2 to be positive, $\mu^2 < 0$, the potential *V,* shown by the dashed line, will be minimized for $\Phi = 0$. But if take the mass term μ^2 to be negative, $\mu^2 < 0$, which corresponds to the potential shown by the solid line, then the minimum is for nonzero values of Φ. Then we we say that Φ gets a nonzero expectation value Φ_0. We expand about this nonzero value to describe quantum phenomena, and the mass of the Higgs field corresponds to the curvature of the parabola centered on this nonzero value. If we add other fields Ψ and have their interactions with the Higgs field of the standard form $\beta_i\Psi_i{}^*\Psi_i\Phi$, this will result in their getting masses $\beta_i\Phi_0$.

The Higgs Mechanism

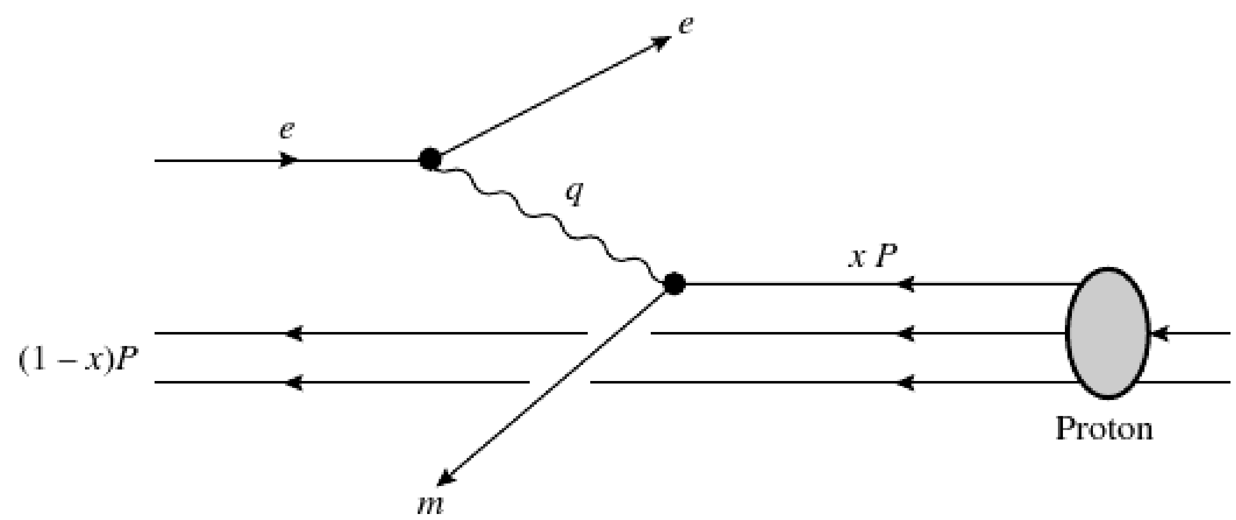
Perkins discusses a simple version of the Higgs mechanism, with only a one-component Higgs particle, using the Klein-Gordon Lagrangian (density) with an added λ term. If instead we had considered the full complex Higgs field, the potential looks like a Mexican hat:



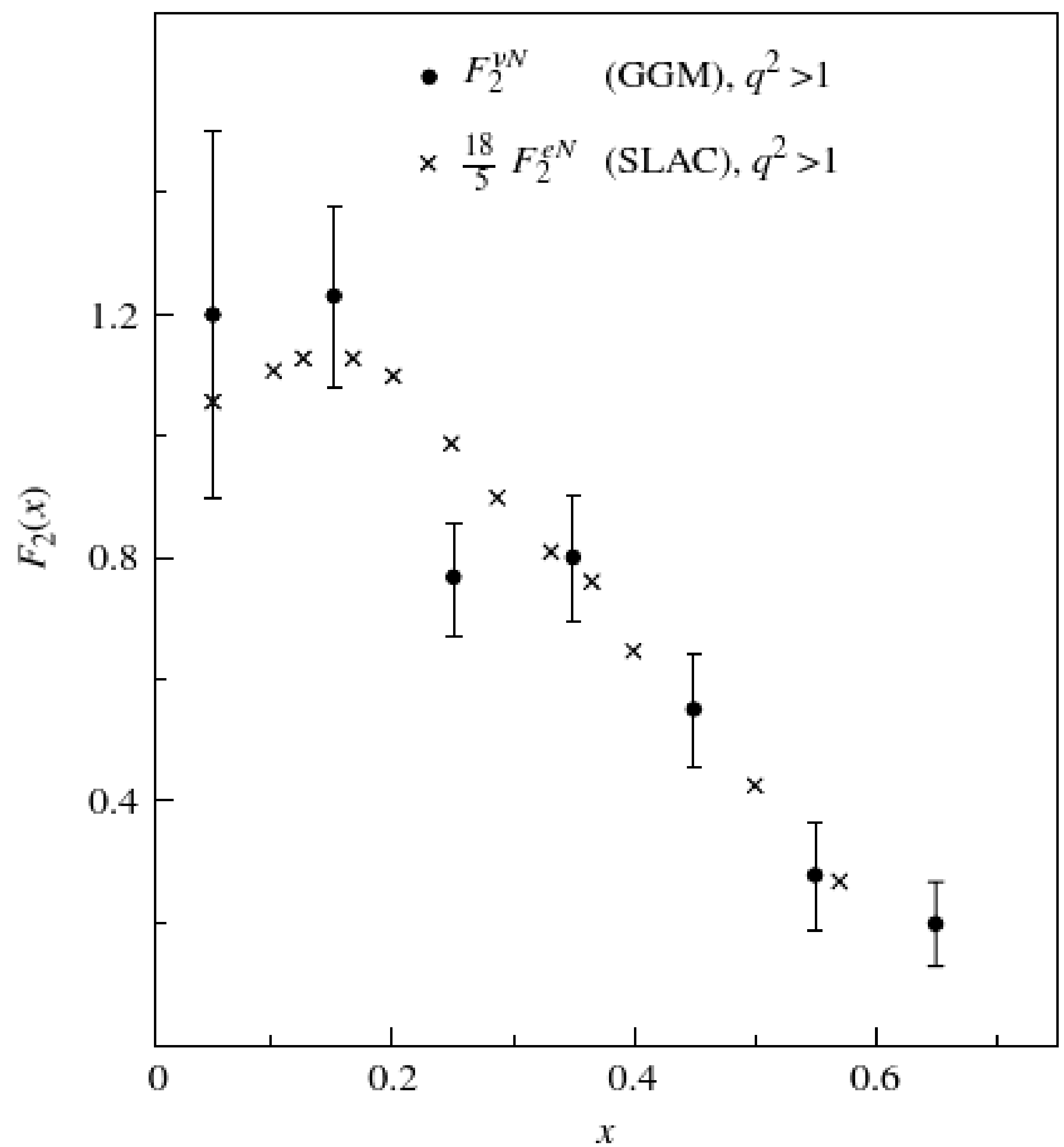
As before, the potential $V(\Phi)$ Ihas a minimum for $\Phi \neq 0$. Again, the physical Higgs field corresponds to fluctuations around this minimum.

The Parton Model

The diagram at right is of the collision of a high-energy electron with a “parton” in the proton. Richard Feynman had introduced the “parton” model to explain the observed behavior of deep (i.e., large momentum transfer q^2) inelastic electron scattering at SLAC; J. D. Bjorken had independently proposed similar ideas. The “partons” turned out to be quarks, and the diagram shows the electron scattering via photon exchange off a quark carrying a fraction *x* of the proton momentum.



Similar deep inelastic scattering experiments with neutrinos were carried out at CERN. The graph at the right shows the electric-type structure function $F_2(x)$ measured in both experiments. It shows (a) that if the electron results are multiplied by the expected factor 18/5 they agree well with the neutrino resultsand (b) that the integral over *x* is about 1/2, showing that quarks carry about half the momentum of the proton. Scattering off nuclei involves about equal numbers of u and d quarks, and the origin of the factor is 5/18 = $[(2/3)^2 + (1/3)^2]/2$, confirming the quark charges.

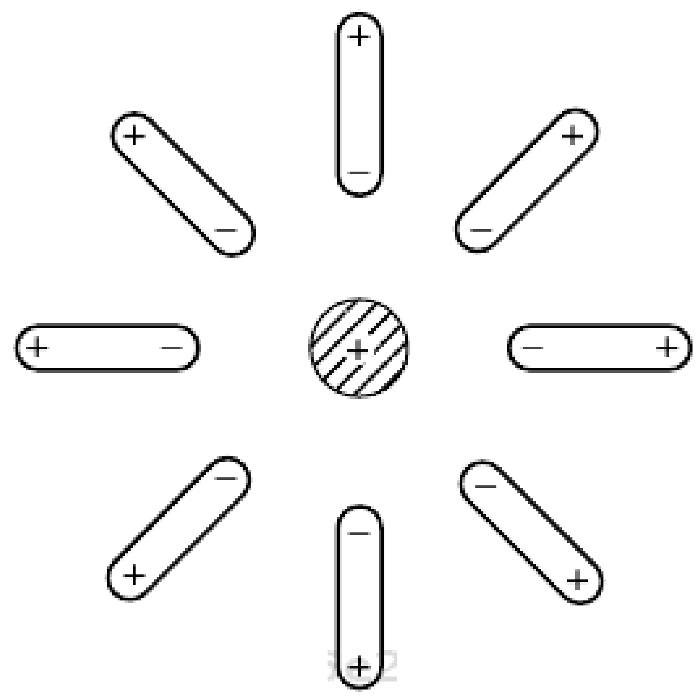


Running Coupling Constants

In the Standard Model of particle physics, the gauge theory SU(2)xU(1) describes the Electroweak interaction and "color" SU(3) describes the Strong interaction. It turns out to be possible to calculate the values of the effective couplings of these gauge fields as a function of the momentum transfer q^2 through the interaction. To first order in terms that are logarithmic in q^2, the electromagnetic coupling, which is $\alpha \approx 1/137$ at low q^2, becomes

$$\alpha\left(q^2\right) = \frac{\alpha\left(\mu^2\right)}{\left[1 - (1/\pi)\,\alpha\left(\mu^2\right)\ln\left(q^2/\mu^2\right)\right]}$$

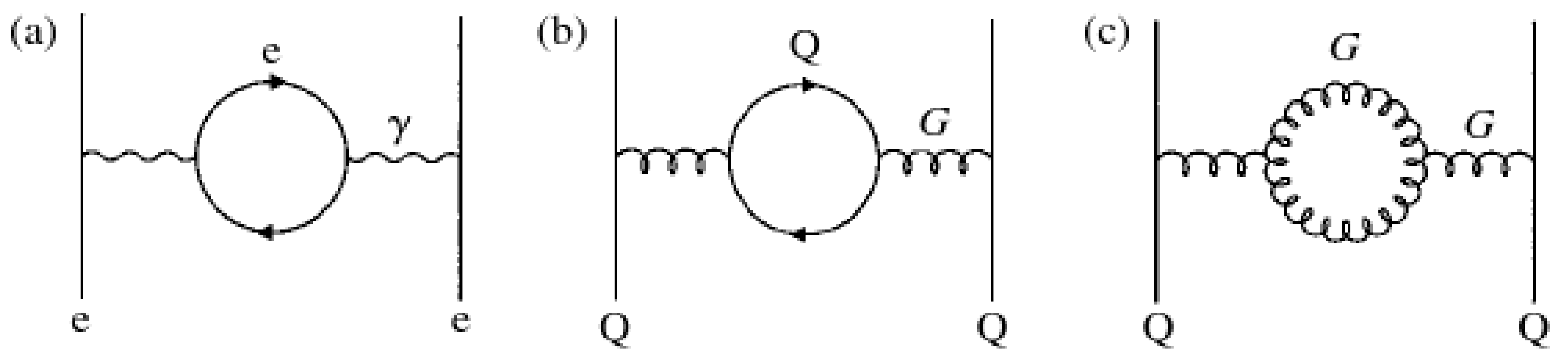
This relates the coupling at one momentum transfer q^2 to that at another μ^2. Note that as q^2 increases, the effective electro-magetic coupling constant $\alpha(q^2)$ increases. This is because higher q^2 corresponds to shorter distances and greater penetration of the cloud of e^+e^- pairs that shields the bare charge at larger distances.



Perkins' Example 3.3 uses the above formula to calculate the value of $1/\alpha(q^2)$ at two values of q^2, the electroweak scale $q \sim 100$ GeV, where $1/\alpha \sim 129$, and a grand unified theory (GUT) scale $q \sim 3\text{x}10^{14}$ GeV, where $1/\alpha \sim 111$.

Running Coupling Constants

In the Standard Model of particle physics, for the gauge theories SU(2) and "color" SU(3), the effective couplings of these nonabelian gauge fields decrease as a function of the momentum transfer q^2, unlike for the abelian U(1) electromagnetic case. The reason is that the photon interacts only with electrically charged particles but not with itself since it is uncharged, while in the nonabelian theories the gauge particles interact with themselves. The diagrams below are examples of how gluons interact with quarks (b) but also with themselves (c).



The leading log approximation is shown at right for the "running" of the Strong coupling $\alpha_S(q^2)$, which decreases as q^2 increases. This behaviour is known as "asymptotic freedom". The effective Weak coupling also decreases as q^2 increases.

$$\alpha_s\left(q^2\right) = \frac{\alpha_s\left(\mu^2\right)}{\left[1 + B\alpha_s\left(\mu^2\right)\ln\left(q^2/\mu^2\right)\right]} = \frac{1}{\left[B\ln\left(q^2/\Lambda^2\right)\right]}$$

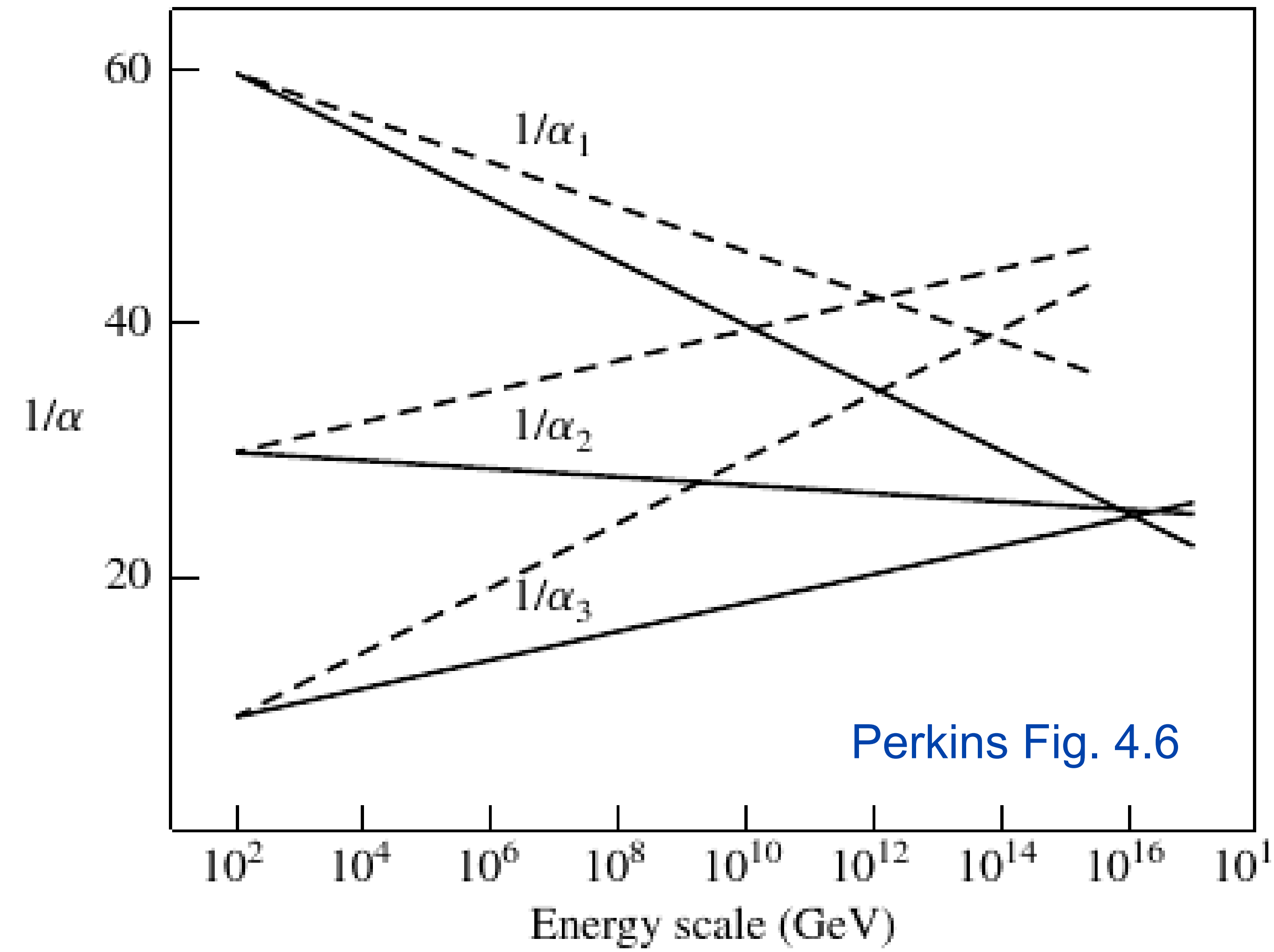
where $B = 7/4\pi$ and

$$\Lambda^2 = \mu^2 \exp\left[-1/B\alpha_s\left(\mu^2\right)\right].$$

Experiments have abundantly confirmed these predictions of the Standard Model, as discussed in Perkins Section 3.12.

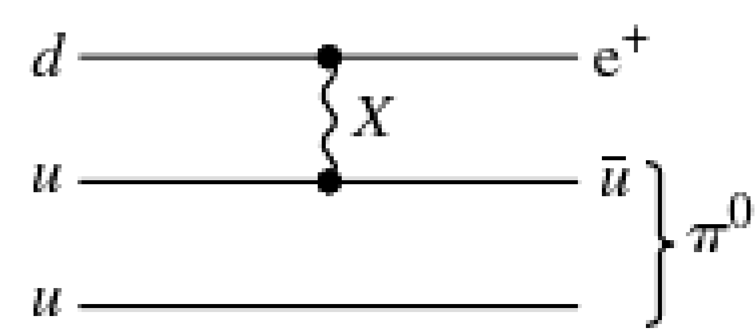
Grand Unified Theories (GUTs)

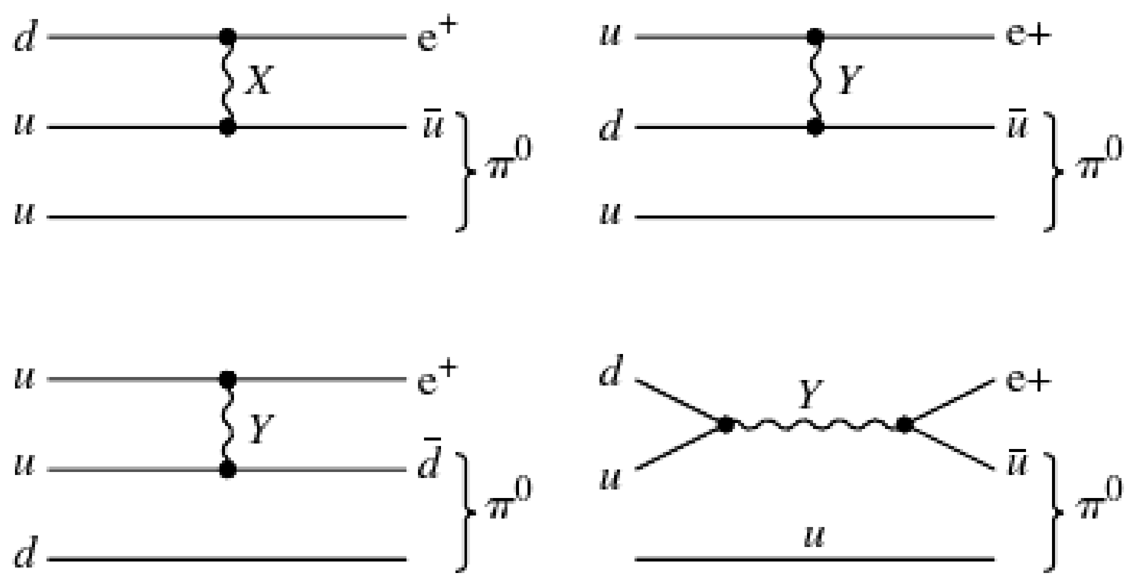
If the Electromagnetic, Weak and Strong interactions are "grand unified", then they will all meet at some value of q^2 when we run the coupling constants. The dashed lines at right show what happens in the Standard Model: they do not all meet. The solid lines show what happens in a supersymmetric version of the same model: they all meet at about $q = 10^{16}$ GeV. In this figure, α_1 is the Electromagnetic, α_2 the Weak, and α_3 the Strong coupling. We will discuss supersymmetry in the next lecture.



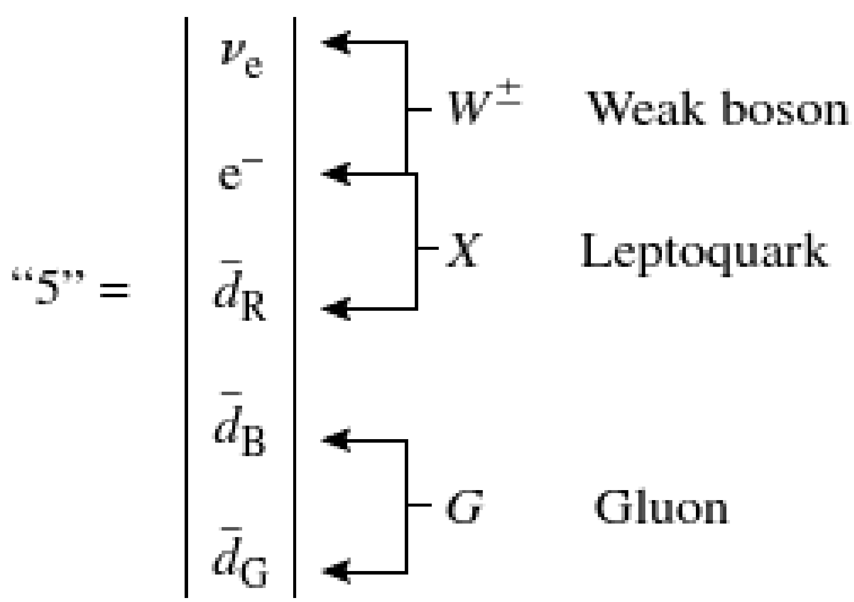
Perkins Fig. 4.6

In GUTs there are, besides the Standard Model gauge bosons (photon, $W^{\pm}$, Z^0, and gluons), there are additional leptoquark gauge bosons X and Y that transform leptons into quarks and vice versa. These bosons would have masses of order the unification energy scale, where the couplings meet. The first energy scale where the non-supersymmetric Standard Model couplings meet is $q \sim 10^{14.5}$ GeV. If quarks can turn into leptons, the proton could decay. The proton lifetime would be $\tau_p \sim M_X^4/(\alpha_S m_p^5)$, which for $M_X \sim 10^{14.5}$ GeV is $\tau_p \sim 10^{30}$ yr. However the SuperKamiokande experimental lower limit is $\tau_p > 8\text{x}10^{33}$ yr.

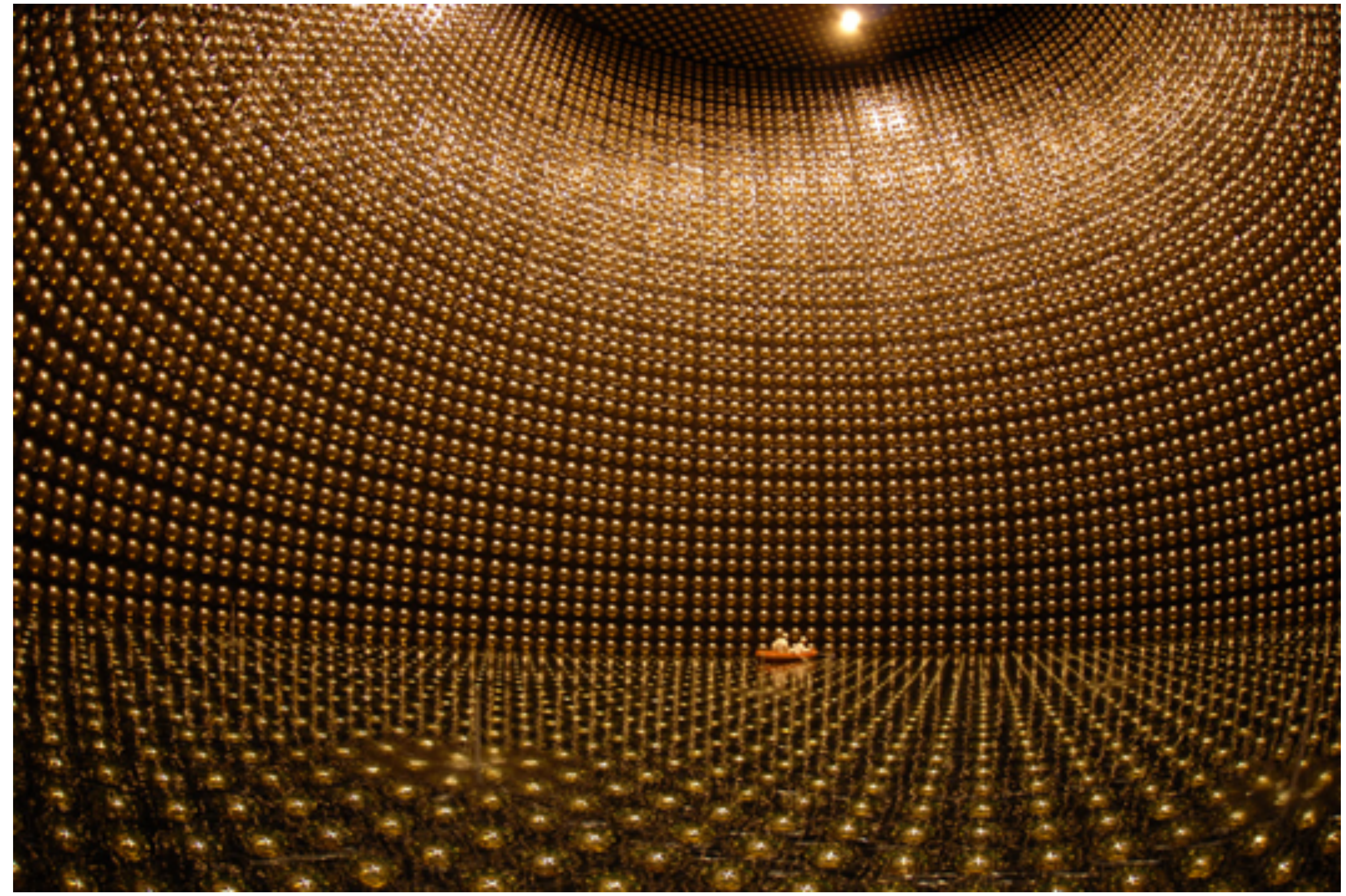




Feynman Diagrams for GUT Proton Decay



GUT "5" Multiplet of Leptons & Quarks



SuperKamiokande Neutrino and Proton Decay Detector
(see also Perkins Fig. 4.7 for more information)

Note that the possibility of "baryogenesis" -- making more quarks than antiquarks in the early universe -- is required in modern cosmology. If this is possible, proton decay should also be possible, which is a motivation to consider GUTs. If we do the same calculation of the proton lifetime but with $M_X \sim 10^{16}$ GeV, the proton lifetime is $\tau_p \sim 10^{36}$ yr, perfectly compatible with the current limits, and challenging to test!