Physics 129 LECTURE 8 January 30, 2014

- Neutrino Masses and Oscillations
  - Simplest case: 2-flavor oscillations
  - Results of experiments: Δm_{12}^2, Δm_{23}^2 and mixing angles θ
  - See-saw model for neutrino masses
- Supersymmetry
- Introduction to Cosmology
  - The Expanding Universe
  - The Friedmann Equation
  - The Age of the Universe

Neutrino Masses and Flavor Oscillations

The fact that electron neutrino beams interact with matter to produce electrons, muon neutrinos produce muons, and tau neutrinos produce tau leptons suggested that all three lepton flavor numbers are conserved in the weak interactions. But neutrino oscillations violate such flavor conservation just as the mixing of the *d, s,* and *b* quarks in the weak doublets into *d', s'*, and *b'* allows weak violation of quark flavor conservation.

In order for the sun to fuse four protons into a helium nucleus, two weak transformations of protons into neutrons are required: $p \longrightarrow n + e^+ + \nu_e$, which requires the emission of two electron-type neutrinos. Neutrino flavor oscillations explain the experimental measurement that the sun emits only about a third of the number of electron-type neutrinos. Such oscillations have now been measured involving all three types of neutrinos. Further experimental evidence has allowed mesurement of the differences of the squared neutrino masses, but not yet their actual masses.

Neutrinos are produced as flavor eigenstates ν_e, ν_μ, or ν_τ, but these are mixtures of the mass eigenstates ν_1, ν_2, ν_3. Since the masses differ, the superposition that corresponds to any particular flavor eigenstate will oscillate into a mixture of the other flavor eigenstates.

It is simplest to discuss just two types of neutrinos, for example ν_μ and ν_τ, which for simplicity we can regard as mixtures of ν_2 and ν_3. This is a pretty good description of atmospheric neutrinos. Pions are abundantly produced by cosmic rays hitting air molecules the upper atmosphere, and the charged pions mainly decay to muons: e.g, $\pi^+ \longrightarrow \mu^+ + \nu_\mu$. Then the muons decay to muon and electron neutrinos: $\mu^+ \longrightarrow e^+ + \nu_e + \overline{\nu}_\mu$. The result is that we expect two ν_μ for every ν_e (and similarly for antineutrinos). This is true for downward going ν_μ, but the neutrinos coming from larger zenith angles or coming up through the earth have a lower ν_μ/ν_e ratio because of these atmospheric neutrino oscillations.

Neutrino Masses and Flavor Oscillations

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The correspoding neutrino mixing is described by

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

and the neutrino mass eigenstates will propagate as

$$\nu_2(t) = \nu_2(0)\exp(-iE_2 t)$$

$$\nu_3(t) = \nu_3(0)\exp(-iE_3 t)$$

It is always a good approximation the write $E_i = p_i\,(1+m_i^2/p^2)^{1/2} = p + m_i^2/2p$ since the neutrino masses m_i are are so much smaller than the neutrino energies.

Neutrino Masses and Flavor Oscillations

Recall that

$$\nu_2(t) = \nu_2(0)\exp(-iE_2 t)$$

$$\nu_3(t) = \nu_3(0)\exp(-iE_3 t)$$

If we start off with muon neutrinos, i.e. $\nu_\mu(0) = 1$, then

$$\nu_2(0) = \nu_\mu(0)\ \cos\theta$$

$$\nu_3(0) = \nu_\mu(0)\ \sin\theta$$

and

$$\nu_\mu(t) = \nu_2(t)\ \cos\theta + \nu_3(t)\ \sin\theta$$

The time dependence of the muon neutrino amplitude becomes

$$A_\mu(t) = \frac{\nu_\mu(t)}{\nu_\mu(0)} = \cos^2\theta\ \exp(-iE_2 t) + \sin^2\theta\ \exp(-iE_3 t)$$

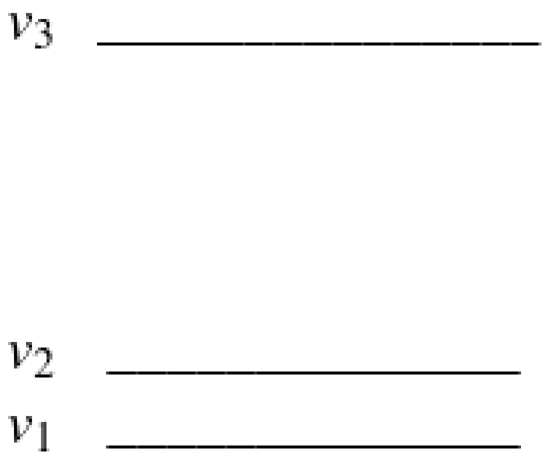
and the corresponding intensity is

$$\frac{I_\mu(t)}{I_\mu(0)} = AA^* = 1 - \sin^2 2\theta\ \sin^2\left[\frac{(E_3 - E_2)\,t}{2}\right] = 1 - \sin^2 2\theta \cdot \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right)$$

where we define $\Delta m_{23}{}^2 \equiv m_3{}^2 - m_2{}^2$ (and assume for definiteness that $m_3 > m_2$) and where L is in km, E is in GeV and Δm^2 is in $(\mathrm{eV})^2$ (see homework 3 problem 6).

Neutrino Masses and Flavor Oscillations

The mixing angles θ_{ij} and squared mass differences $\Delta m_{ij}{}^2$ are experimentally found to be



$\leftarrow$ $\sin^2(2\theta_{23}) > 0.95$
$\Delta m^2_{32} = (2.32^{+0.12}_{-0.08}) \times 10^{-3}\ \mathrm{eV}^2$

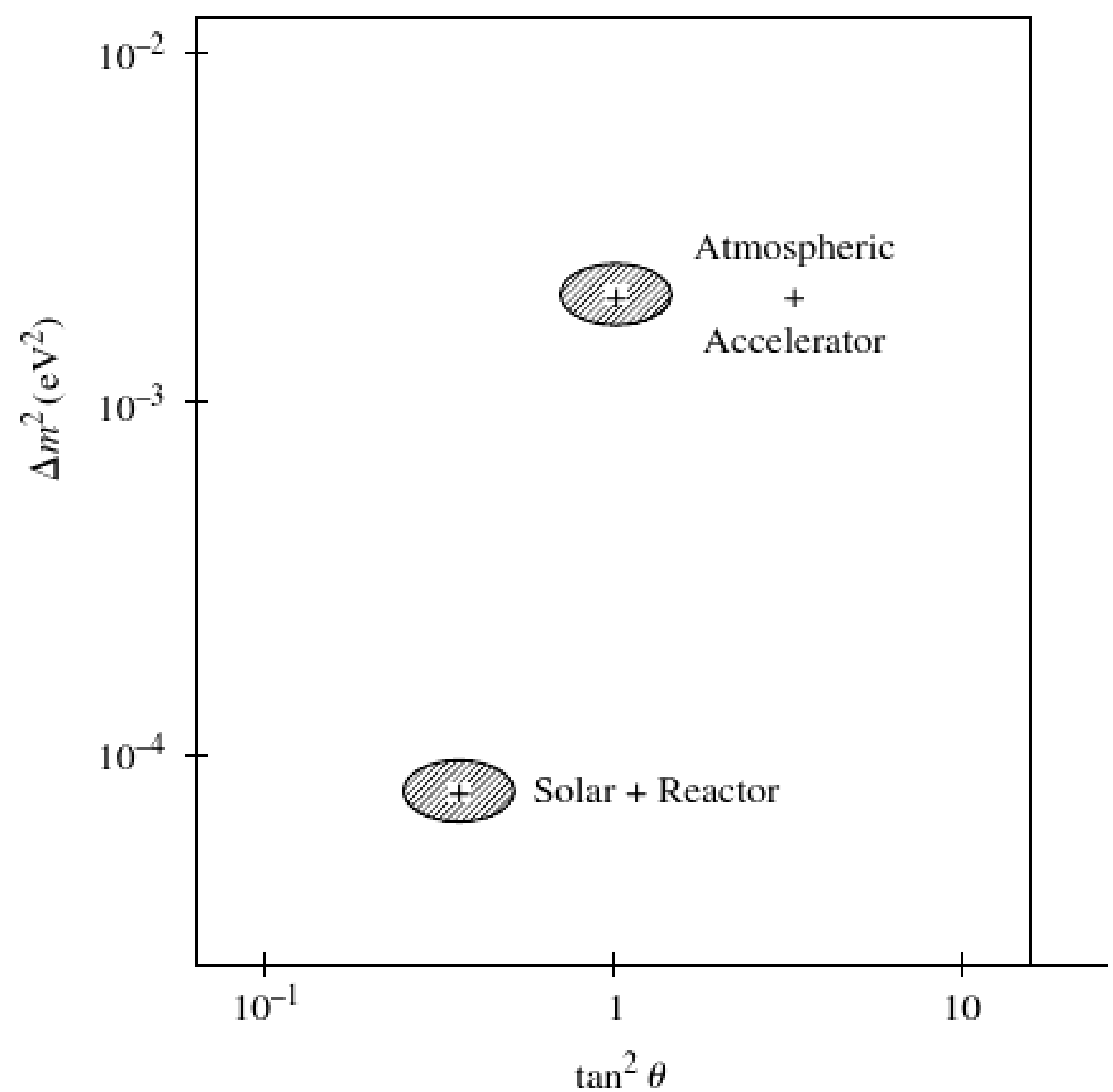
$\leftarrow$ $\sin^2(2\theta_{12}) = 0.857 \pm 0.024$
$\Delta m^2_{21} = (7.50 \pm 0.20) \times 10^{-5}\ \mathrm{eV}^2$

These are from the latest Particle Data Group summary table, which also says that $\sin^2(2\theta_{13}) = 0.095 \pm 0.010$

The plot at the right shows $\Delta m_{23}{}^2$ and $\Delta m_{12}{}^2$ vs. the corresponding $\tan^2\theta_{ij}$. If the neutrino masses are hierarchical rather than nearly degenerate, then

$m_3 \sim (2.3\times10^{-3}\ \mathrm{eV}^2)^{1/2} = 0.05\ \mathrm{eV}$
$m_2 \sim (7.5\times10^{-5}\ \mathrm{eV}^2)^{1/2} = 0.007\ \mathrm{eV}$

As already meantioned, cosmological data shows that $m_1 + m_2 + m_2 < 0.23$ eV.



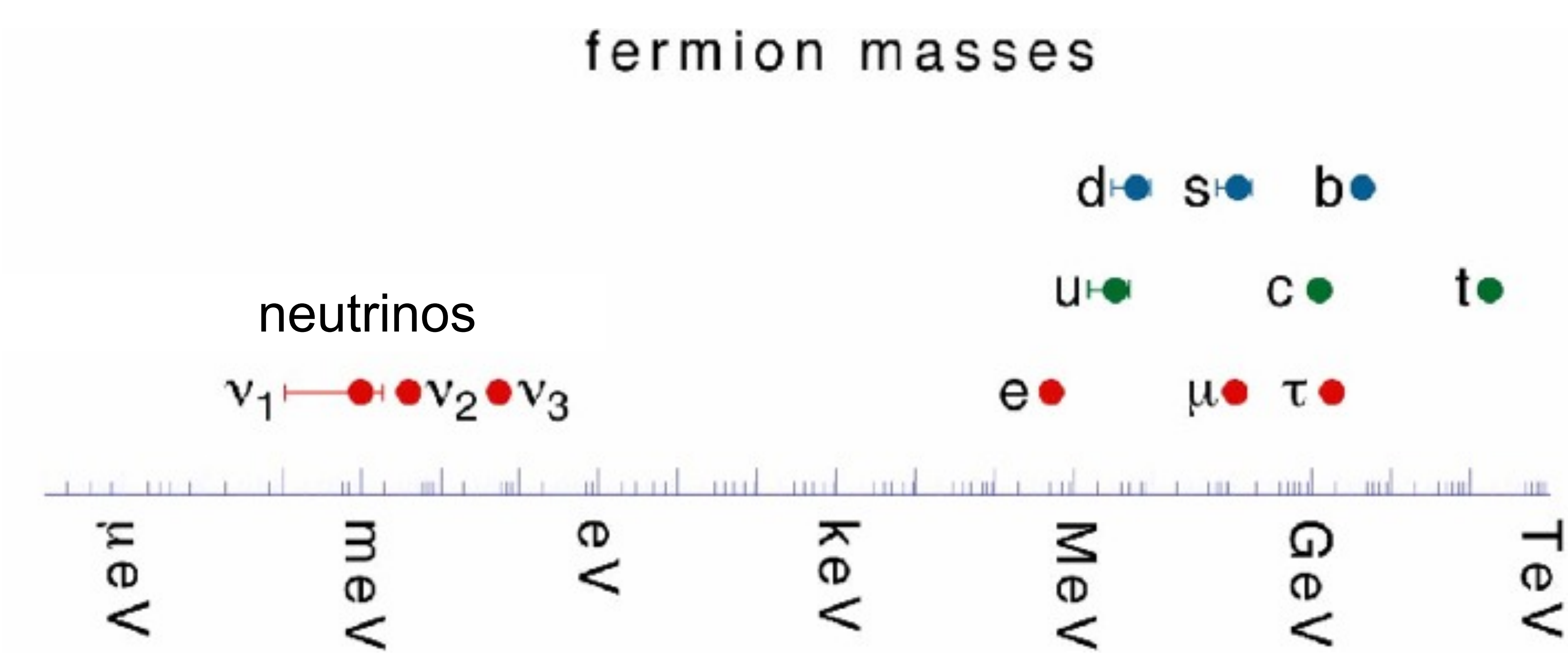
THE ATMOSPHERIC-NEUTRINO DATA from the Super-Kamiokande underground neutrino detector in Japan provide strong evidence of muon to tau neutrino oscillations, and therefore that these neutrinos have nonzero mass (see the article by John Learned in the Winter 1999 *Beam Line*, Vol. 29, No. 3). This result is now being confirmed by results from the K2K experiment, in which a muon neutrino beam from the KEK accelerator is directed toward Super-Kamiokande and the number of muon neutrinos detected is about as expected from the atmospheric-neutrino data (see article by Jeffrey Wilkes and Koichiro Nishikawa, this issue).

But oscillation experiments cannot measure neutrino masses directly, only the squared mass difference $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$ between the oscillating species. The Super-Kamiokande atmospheric neutrino data imply that $1.7\times10^{-4} < \Delta m_{\tau\mu}^2 < 4\times10^{-3}$ eV² (90 percent confidence), with a central value $\Delta m_{\tau\mu}^2 = 2.5\times10^{-3}$ eV². If the neutrinos have a hierarchical mass pattern $m_{\nu_e} << m_{\nu_\mu} << m_{\nu_\tau}$ like the quarks and charged leptons, then this implies that $\Delta m_{\tau\mu}^2 \cong m_{\nu_\tau}^2$ so $m_{\nu_\tau} \sim 0.05$ eV.

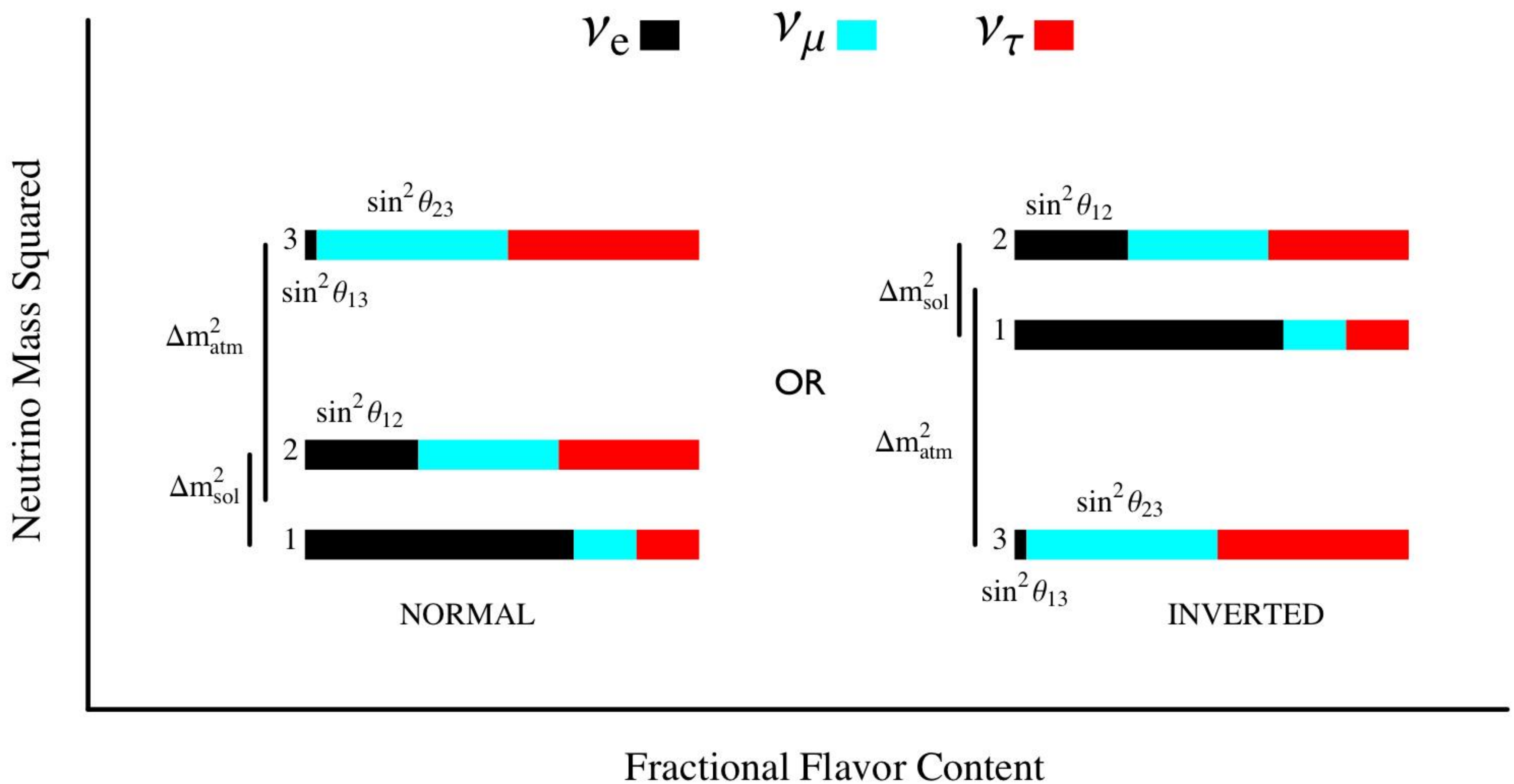
These data then imply a lower limit on the HDM (or light neutrino) contribution to the cosmological matter density of $\Omega_\nu > 0.001$—almost as much as that of all the stars in the disks of galaxies. There is a connection between neutrino mass and the corresponding contribution to the cosmological density, because the thermodynamics of the early Universe specifies the abundance of neutrinos to be about 112 per cubic centimeter for each of the three species (including both neutrinos and antineutrinos). It follows that the density Ω_ν contributed by neutrinos is $\Omega_\nu = m(\nu)/(93\ h^2\ \text{eV})$, where $m(\nu)$ is the sum of the masses of all three neutrinos. Since $h^2 \sim 0.5$, $m_{\nu_\tau} \sim 0.05$ eV corresponds to $\Omega_\nu \sim 10^{-3}$.

This is however a lower limit, since in the alternative case where the oscillating neutrino species have nearly equal masses, the values of the individual masses could be much larger. The only other laboratory approaches to measuring neutrino masses are attempts to detect neutrinoless double beta decay, which are sensitive to a possible Majorana component of the electron neutrino mass, and measurements of the endpoint of the tritium beta-decay spectrum. The latter gives an upper limit on the electron neutrino mass, currently taken to be 3 eV. Because of the small values of both squared-mass differences, this tritium limit becomes an upper limit on all three neutrino masses, corresponding to $m(\nu) < 9$ eV. A bit surprisingly, cosmology already provides a stronger constraint on neutrino mass than laboratory measurements, based on the effects of neutrinos on large-scale structure formation.

Joel Primack, *Beam Line*, Fall 2001



Hitoshi Murayama



What is the neutrino mass hierarchy?

Two of the three neutrinos of definite mass, ν_1 and ν_2, have squared masses differing by $\Delta m^2_{sol} \cong +7.6\times 10^{-5}\,eV^2$. The third, ν_3, is separated from the $\nu_1 - \nu_2$ pair by a splitting that is thirty times larger: $\left|\Delta m^2_{atm}\right| \cong 2.4\times 10^{-3}\,eV^2$. These Δm^2 were first determined by solar and atmospheric neutrino experiments, respectively. We do not know whether the closely-spaced $\nu_1 - \nu_2$ pair is at the bottom of the spectrum, as on the left of the figure below, or at the top, as on the right. If the closely-spaced pair is at the bottom, then the neutrino spectrum resembles the charged lepton and quark spectra, and for this reason would be called a normal hierarchy. If this pair is at the top, the spectrum would be referred to as an inverted hierarchy.

Just as each neutrino of definite flavor, such as ν_e, is a superposition of the neutrinos of definite mass, so each of the latter is a superposition of the neutrinos of definite flavor. In the figure, we indicate what is known experimentally about the flavor content of each neutrino of definite mass by color coding, showing the ν_e fraction in black, the ν_μ fraction in cyan, and the ν_τ fraction in red. The indicated small ν_e fraction of the isolated member of the spectrum, ν_3, is just an illustration; at present we know only that this fraction is no larger than 3% of this neutrino. We see from the figure that no neutrino of definite mass is anywhere near being just a neutrino of a single flavor. That is, neutrino mixing is large, in striking contrast to quark mixing, which is present, to be sure, but is quite small.

Why is the mass hierarchy important?

The Grand Unified Theories (GUTs) that unify the weak, electromagnetic, and strong interactions lead us to expect – at first – that the neutrino spectrum will resemble the charged lepton and quark spectra. The reason is simply that in a GUT the neutrinos, charged leptons, and quarks are all related; they belong to common multiplets of the theory. On the other hand, the neutrinos can have Majorana masses, but the charged leptons and quarks cannot. A Majorana mass mixes a particle with its antiparticle, and such mixing violates electric charge conservation if the particle is charged. Thus, the possibility of Majorana masses distinguishes the neutrinos from the other constituents of matter, and Majorana masses can readily turn a normal, quark-like neutrino spectrum into an inverted one. In addition, some classes of string theories lead one to expect an inverted neutrino spectrum. Clearly, in working toward an understanding of the origin of neutrino mass, we would like to know whether the mass spectrum is normal or inverted.

Article by Kayser & Parke, posted at http://physics.ucsc.edu/~joel/Phys129/Kayser&Parke-Neutrino-overview.pdf

See-saw Mechanism for Neutrino Masses

It is puzzling that the neutrino masses, ~0.1 eV or less, are so much smaller than the other fermion masses. A plausible explanation is the "see-saw" mechanism, in which neutrino masses are a mixture of Majorana masses m_L and m_R, which are separate for left- and right-handed neutrinos, and a Dirac mass, which mixes L and R. The corresponding mass matrix is

$$\begin{vmatrix} m_L & m_D \\ m_D & m_R \end{vmatrix}$$

Diagonalizing this matrix gives

$$m_{1,2} = \frac{1}{2}\left[(m_R + m_L) \pm \sqrt{(m_R - m_L)^2 + 4m_D^2}\right]$$

If m_L is so small that we can neglect it, and $M \equiv m_R >> m_D$, then

$$m_1 \approx \frac{(m_D)^2}{M}, \quad m_2 \approx M$$

If we take $m_D \sim 10$ GeV, then $M \sim 10^{12}$ GeV gives $m_1 \sim 0.1$ eV, as required. This is another indication, besides Grand Unification, that there might be interesting new physics at high mass scales. Decay of these hypothetical very massive right-handed neutrinos is also a plausible mechanism to help explain the cosmic asymmetry between matter and antimatter.

Supersymmetry

When the British physicist Paul Dirac first combined Special Relativity with quantum mechanics, he found that this predicted that for every ordinary particle like the electron, there must be another particle with the opposite electric charge – the anti-electron (positron). Similarly, corresponding to the proton there must be an anti-proton. Supersymmetry appears to be required to combine General Relativity (our modern theory of space, time, and gravity) with the other forces of nature (the electromagnetic, weak, and strong interactions). The consequence is another doubling of the number of particles, since supersymmetry predicts that for every particle that we now know, including the antiparticles, there must be another, thus far undiscovered particle with the same electric charge but with *spin* differing by half a unit.

Spin	**Matter** (fermions)	**Forces** (bosons)
2		graviton
1		photon, $W^{\pm}$, Z^0 gluons
1/2	quarks u,d,... leptons $e, \nu_e, \ldots$	
0		Higgs bosons axion

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after doubling

Spin	Matter (fermions)	Forces (bosons)	Hypothetical Superpartners	Spin
2		graviton	gravitino	3/2
1		photon, $W^{\pm}$, Z^0 gluons	photino, winos, zino, gluinos	1/2
1/2	quarks u,d,... leptons e, ν_e, ...		squarks $\tilde{u}$, $\tilde{d}$, ... sleptons $\tilde{e}$, $\tilde{\nu}_e$, ...	0
0		Higgs bosons axion	Higgsinos axinos	1/2

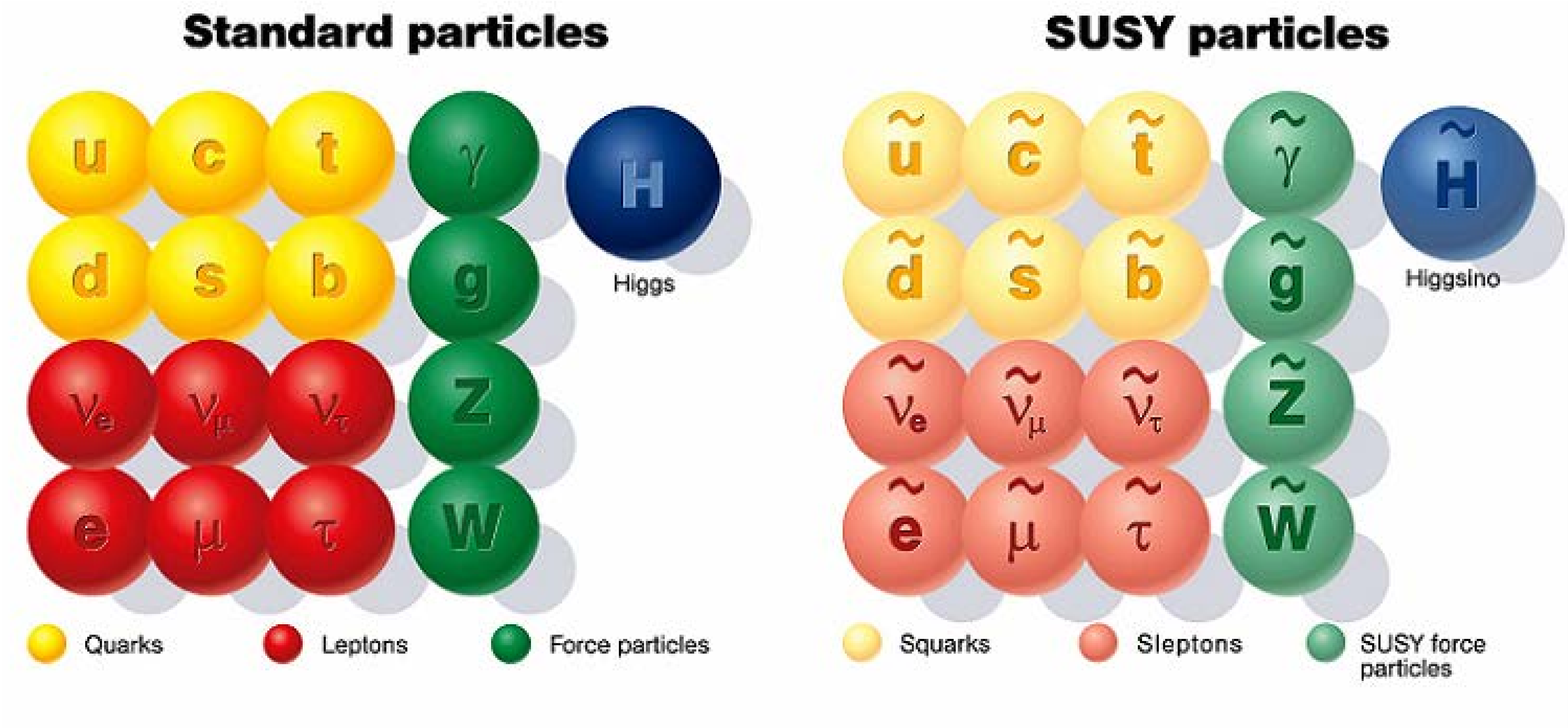
Note: Supersymmetric cold dark matter candidate particles are underlined.

Supersymmetric WIMPs

Spin is a fundamental property of elementary particles. Matter particles like electrons and quarks (protons and neutrons are each made up of three quarks) have spin ½, while force particles like photons, W,Z, and gluons have spin 1. The supersymmetric partners of electrons and quarks are called selectrons and squarks, and are bosons of spin 0. The supersymmetric partners of the force particles are called the photino, Winos, Zino, and gluinos, and they have spin ½, so these fermions might be matter particles. The lightest of these particles might be the photino. Whichever is lightest should be stable, so it is a natural candidate to be the dark matter WIMP, as first suggested by Pagels & Primack 1982. A supersymmetric WIMP also naturally has about the observed dark matter density. Its mass is not predicted by supersymmetry, but it will be produced soon at the LHC if it exists and its mass is not above ~1 TeV!

Supersymmetry thus helps unify gravity with the other forces, and it provides a natural candidate for the dark matter particle. The boson-fermion cancellation built into supersymmetry also helps to control the vacuum energy (related to the cosmological constant) and to explain the "gauge hierarchy problem" (why the Electroweak scale is so much less than the GUT or Planck scales).

Supersymmetry



For a review, see H.E. Haber, *Supersymmetry Theory*, in the 2013 partial update for the 2014 edition of the *Review of Particle Physics*, to be published by the Particle Data Group [http://pdg.lbl.gov/2013/reviews/rpp2013-rev-susy-1-theory.pdf].

The Expanding Universe

Edwin Hubble discovered the expansion of the universe by discovering a linear relation between the expansion velocity v of a galaxy and its distance D:

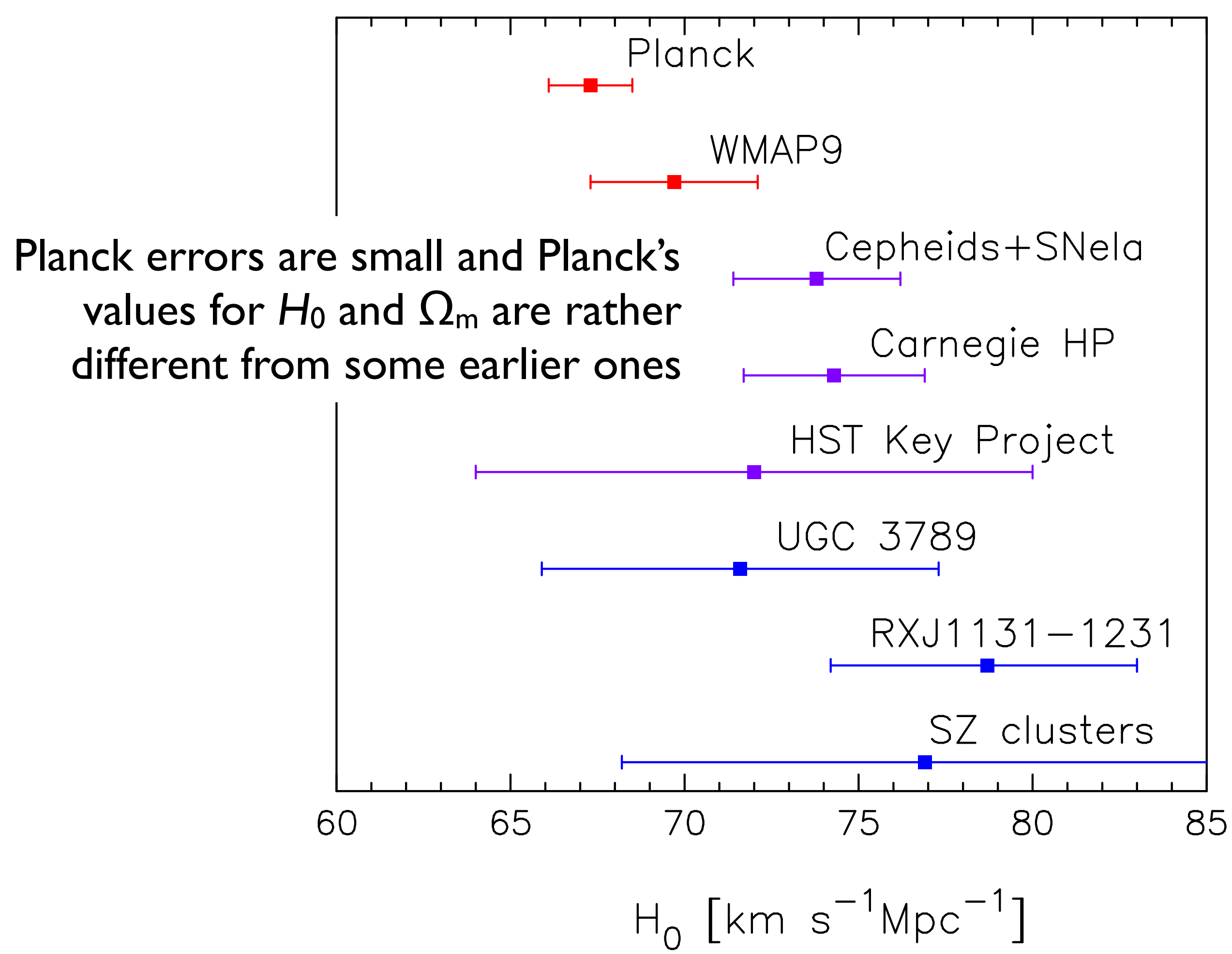
$$v = H_0 D$$

where the constant of proportionality H_0, called the Hubble constant or Hubble parameter, has the value (according to Perkins)

$$H_0 = 72 \pm 3 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Actually, the latest value for H_0, from the Planck satellite data plus much other astronomical data, is 67.80 ± 0.77 .

Note that measuring galaxy redshifts is easy, but measuring their distances is hard. Milton Humason and others had measured a number of galaxy redshifts, but Hubble figured out how to measure distances to galaxies using Cepheid variable stars. He got the relative distances more or less right, although his distance scale was later recalibrated as Cepheid variables were better understood.



General Relativity

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

MATTER TELLS SPACE HOW TO CURVE

Einstein Field Equations

$$G^{\mu\nu} \equiv R^{\mu\nu} - \tfrac{1}{2}Rg^{\mu\nu} = -\,8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

Here u^α is the velocity 4-vector of a particle. The Ricci curvature tensor $R_{\mu\nu} \equiv R_{\lambda\mu\sigma\nu} g^{\lambda\sigma}$, the Riemann curvature tensor $R^\lambda{}_{\mu\sigma\nu}$, and the affine connection $\Gamma^\mu{}_{\alpha\beta}$ can be calculated from the metric tensor $g_{\lambda\sigma}$. If the metric is just that of flat space, then $\Gamma^\mu{}_{\alpha\beta} = 0$ and the first equation above just says that the particle is unaccelerated -- i.e., it satisfies the law of inertia (Newton's 1st law).

General Relativity and Cosmology

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

MATTER TELLS SPACE HOW TO CURVE

Einstein Field Equations

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Einstein's Cosmological Principle: on large scales, space is uniform and isotropic.

COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\,[dr^2\,(1 - kr^2)^{-1} + r^2\, d\Omega^2]$$

with curvature constant k = –1, 0, or +1. Substituting this metric into the Einstein equations above, we get the Friedmann equations.

Friedmann-Robertson-Walker Metric

(homogeneous, isotropic universe)

$$\text{FRW E(00)} \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$ ⟵ **Friedmann equation**

$$\text{FRW E}(ii) \quad \frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$$

$$H_0 \equiv 100h\,\mathrm{km\,s^{-1}Mpc^{-2}} \equiv 70h_{70}\,\mathrm{km\,s^{-1}Mpc^{-2}}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_\odot \mathrm{Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi G p + \frac{2}{3}\Lambda$$

$$\text{Divide by } 2E(00) \Rightarrow \quad q_0 \equiv -\left(\frac{\ddot{a}}{a}\frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$ ⟵ **deceleration parameter**

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{-\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{da}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \qquad H_0^{-1} = 9.78h^{-1}\mathrm{Gyr} \qquad f(1,0) = \tfrac{2}{3}$$

age of the universe

$$f(0,0) = 1$$
$$f(0,1) = \infty$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

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$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \qquad H_0^{-1} = 9.78 h^{-1}\text{Gyr} \qquad \begin{array}{l} f(1,0) = \frac{2}{3} \\ f(0,0) = 1 \\ f(0,1) = \infty \end{array}$$

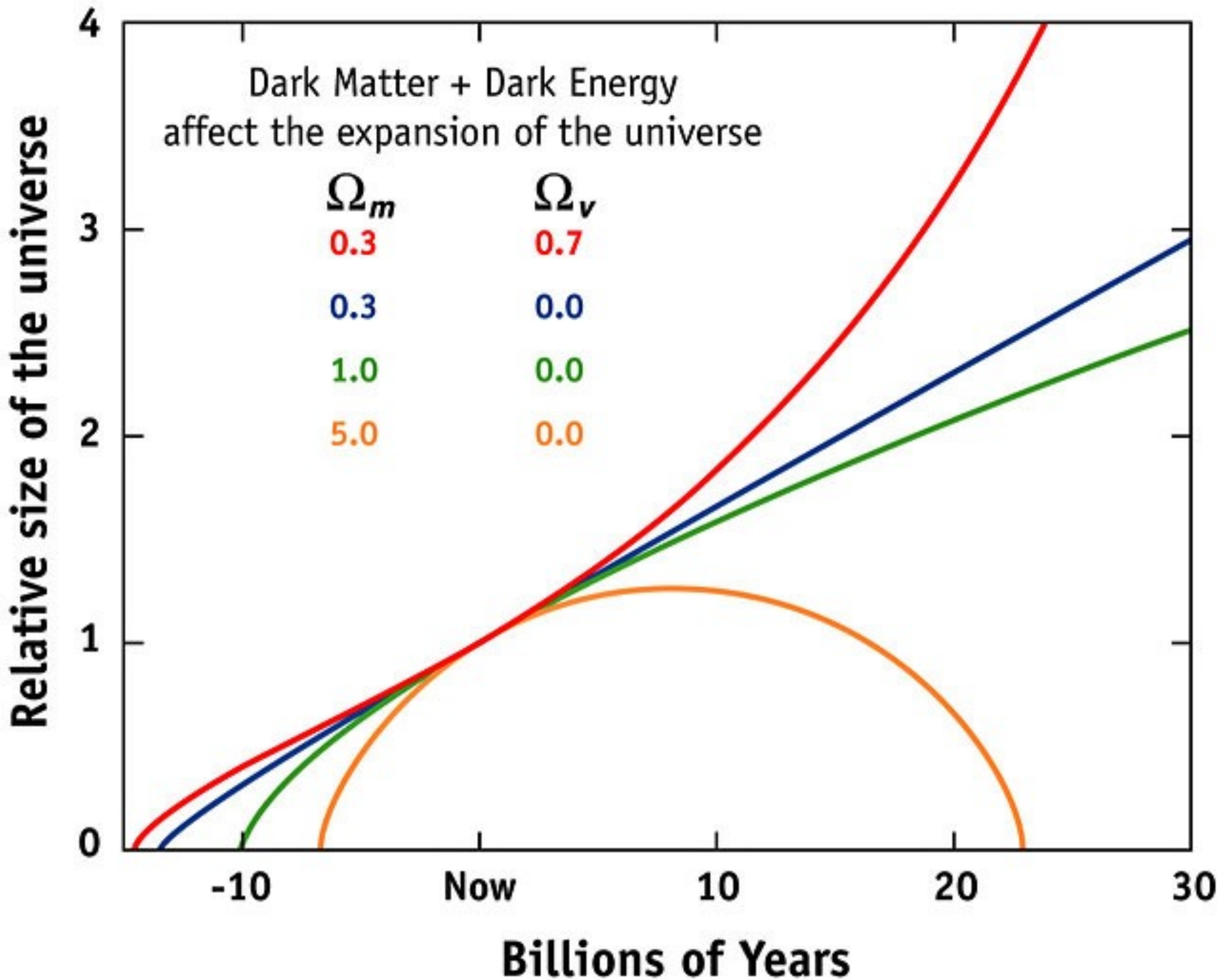
$[E(00)a^3]'$ vs. $E(ii) \Rightarrow \quad \frac{\partial}{\partial a}(\rho a^3) = -3pa^2$ ("continuity")

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

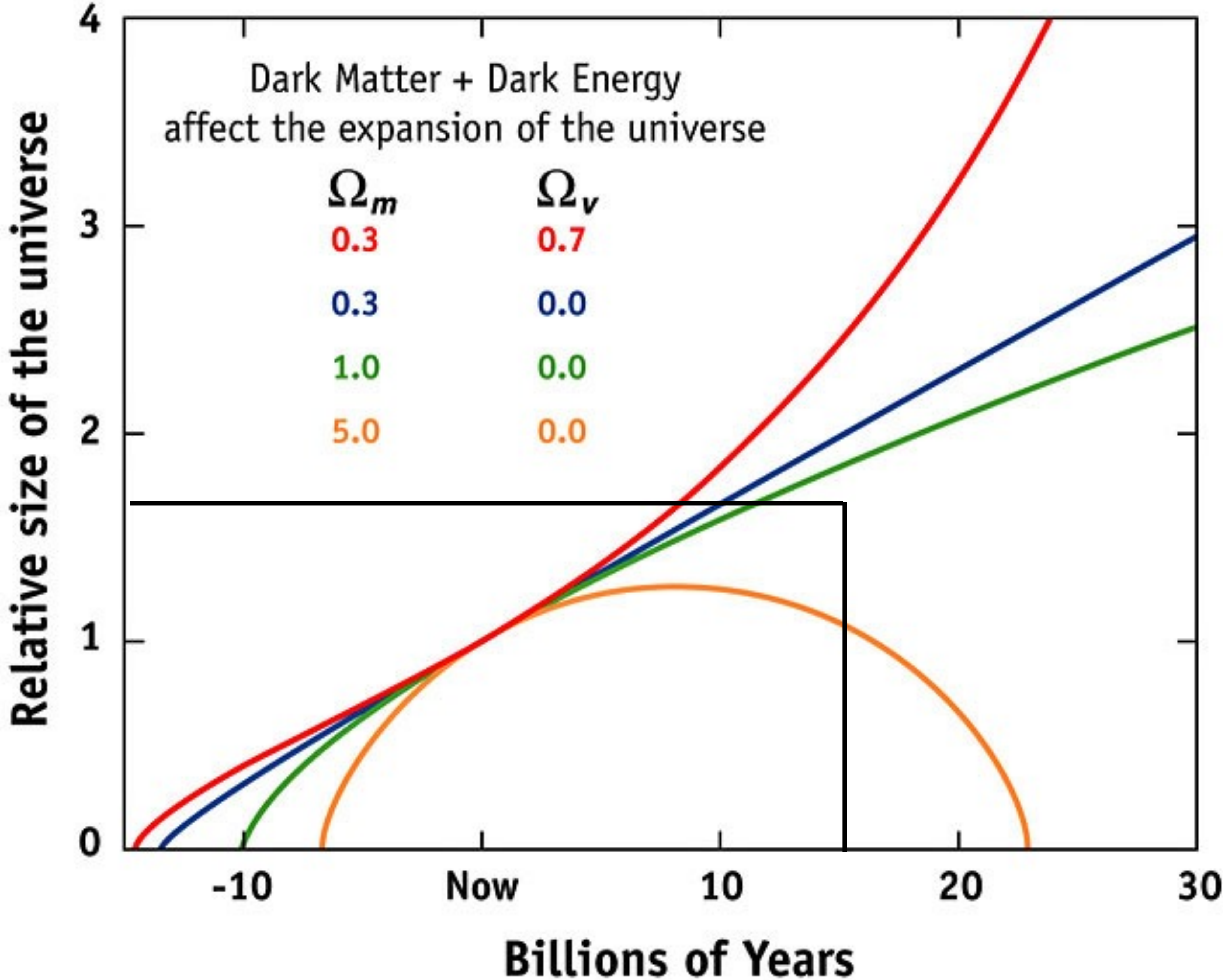
Examples:

$$\begin{aligned} p &= 0 \Rightarrow \rho = \rho_0 a^{-3} \text{ (assumed above in } q_0,\ t_0 \text{ eqs.)} \\ p &= \frac{\rho}{3},\ k = 0 \Rightarrow \rho \propto a^{-4} \end{aligned}$$

History of Cosmic Expansion for General Ω_M & Ω_Λ



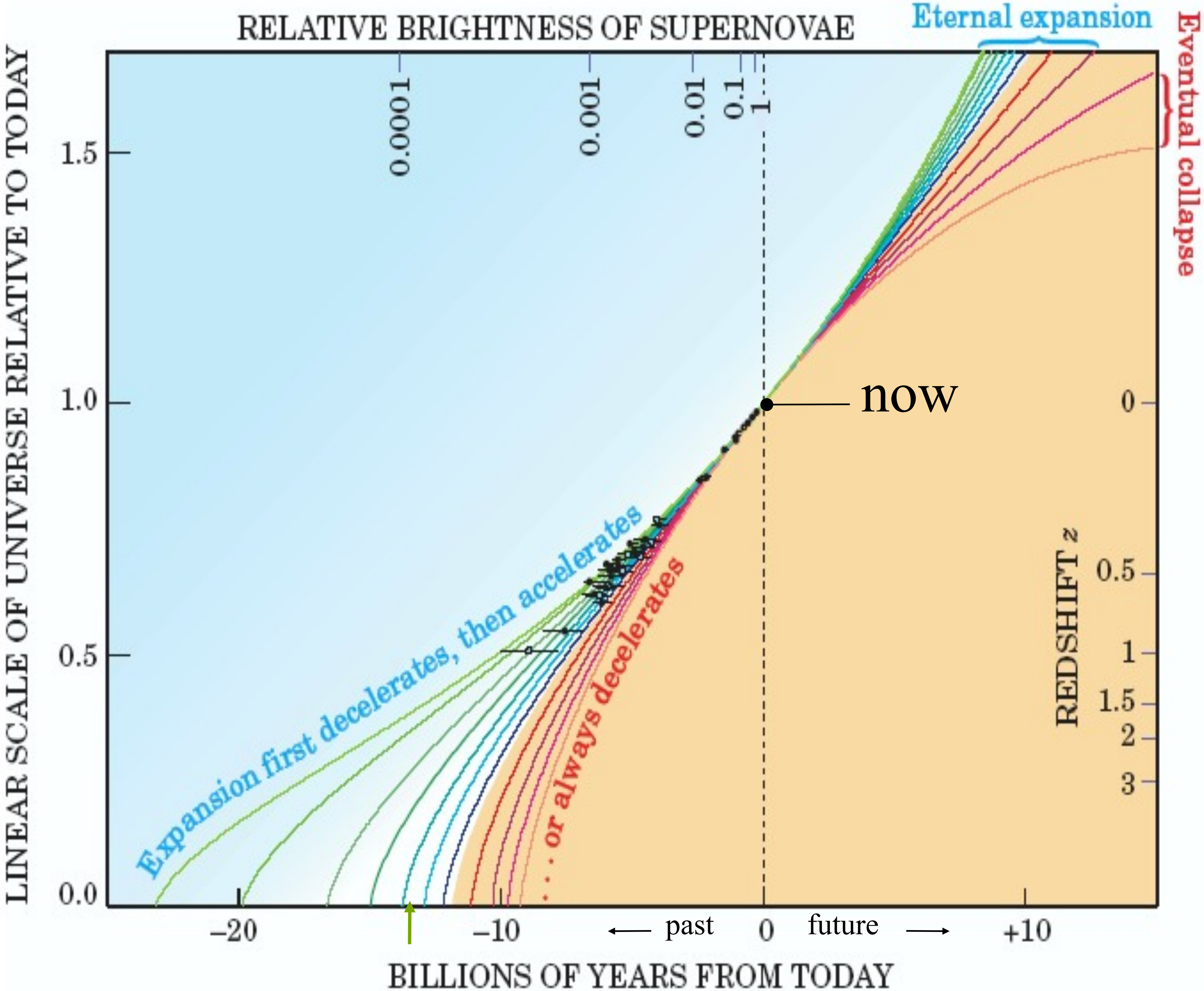
History of Cosmic Expansion for General Ω_M & Ω_Λ



History of Cosmic Expansion for $\Omega_\Lambda = 1 - \Omega_M$

With Ω_Λ = 0 the age of the decelerating universe would be only 9 Gyr, but Ω_Λ = 0.7, Ω_m = 0.3 gives an age of 14 Gyr, consistent with stellar and radioactive decay ages

Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor R of the universe is taken to be 1 at present, so it equals $1/(1 + z)$. The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from 0.95 ρ_c (top curve) down to 0.4 ρ_c. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from 0.8 ρ_c up to 1.4 ρ_c. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.



Saul Perlmutter, ***Physics Today***, Apr 2003

LCDM Benchmark Cosmological Model: Ingredients & Epochs

	List of Ingredients
photons:	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos:	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation:	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic matter:	$\Omega_{\mathrm{bary},0} = 0.04$
nonbaryonic dark matter:	$\Omega_{\mathrm{dm},0} = 0.26$
total matter:	$\Omega_{m,0} = 0.30$
cosmological constant:	$\Omega_{\Lambda,0} \approx 0.70$

Important Epochs

radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^{4}$ yr
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.5$ Gyr

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Benchmark Model: Scale Factor vs. Time

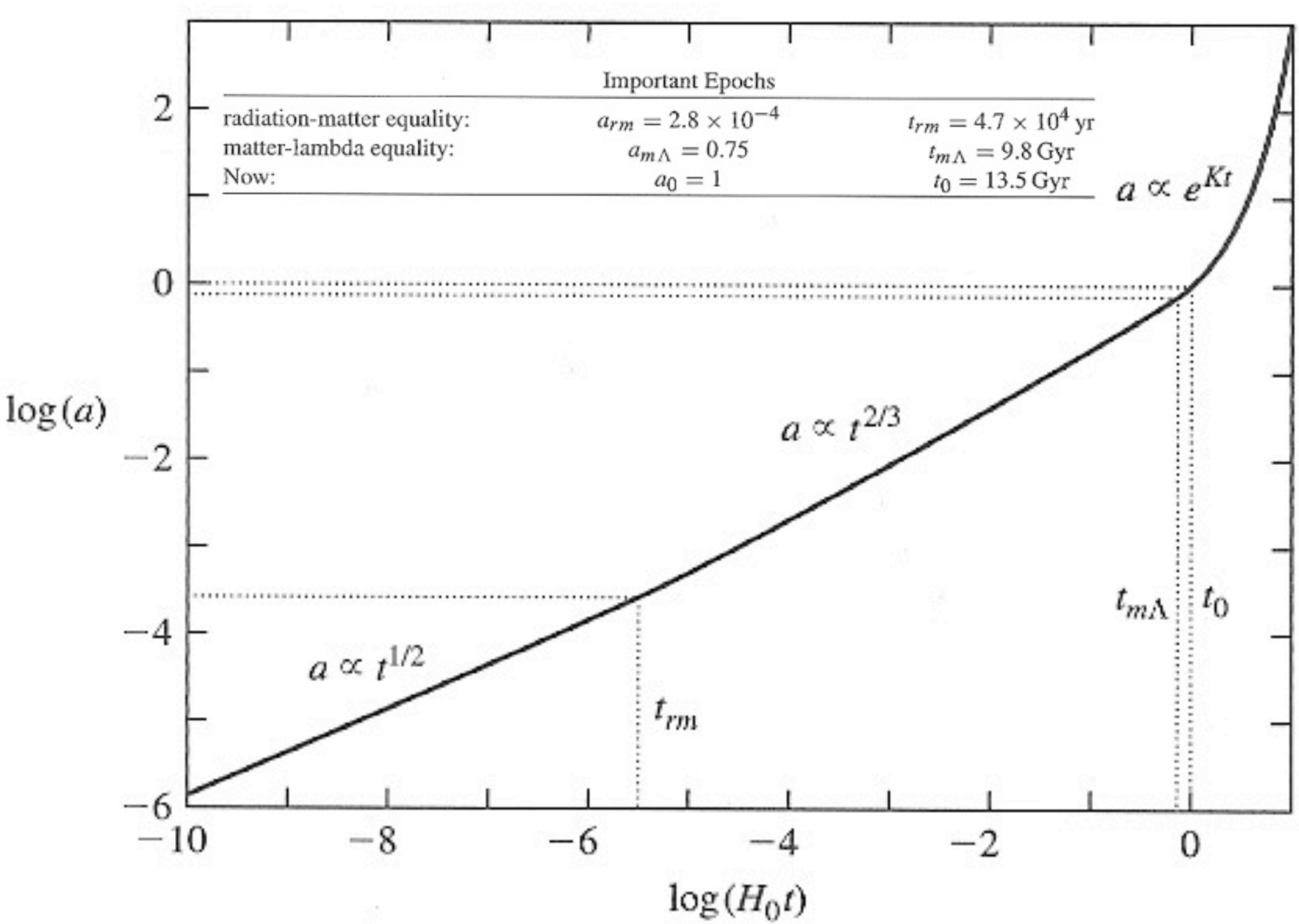


FIGURE 6.5 The scale factor a as a function of time t (measured in units of the Hubble time), computed for the Benchmark Model. The dotted lines indicate the time of radiation-matter equality, $a_{rm} = 2.8 \times 10^{-4}$, the time of matter-lambda equality, $a_{m\Lambda} = 0.75$, and the present moment, $a_0 = 1$.

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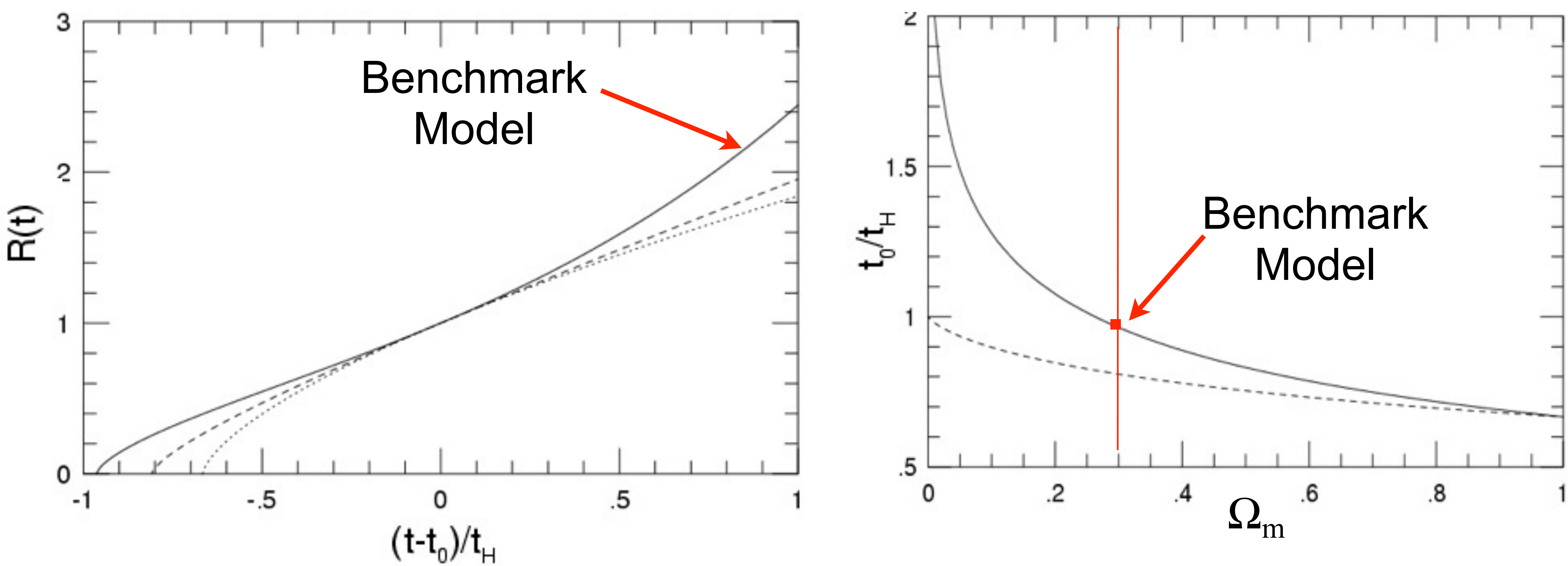
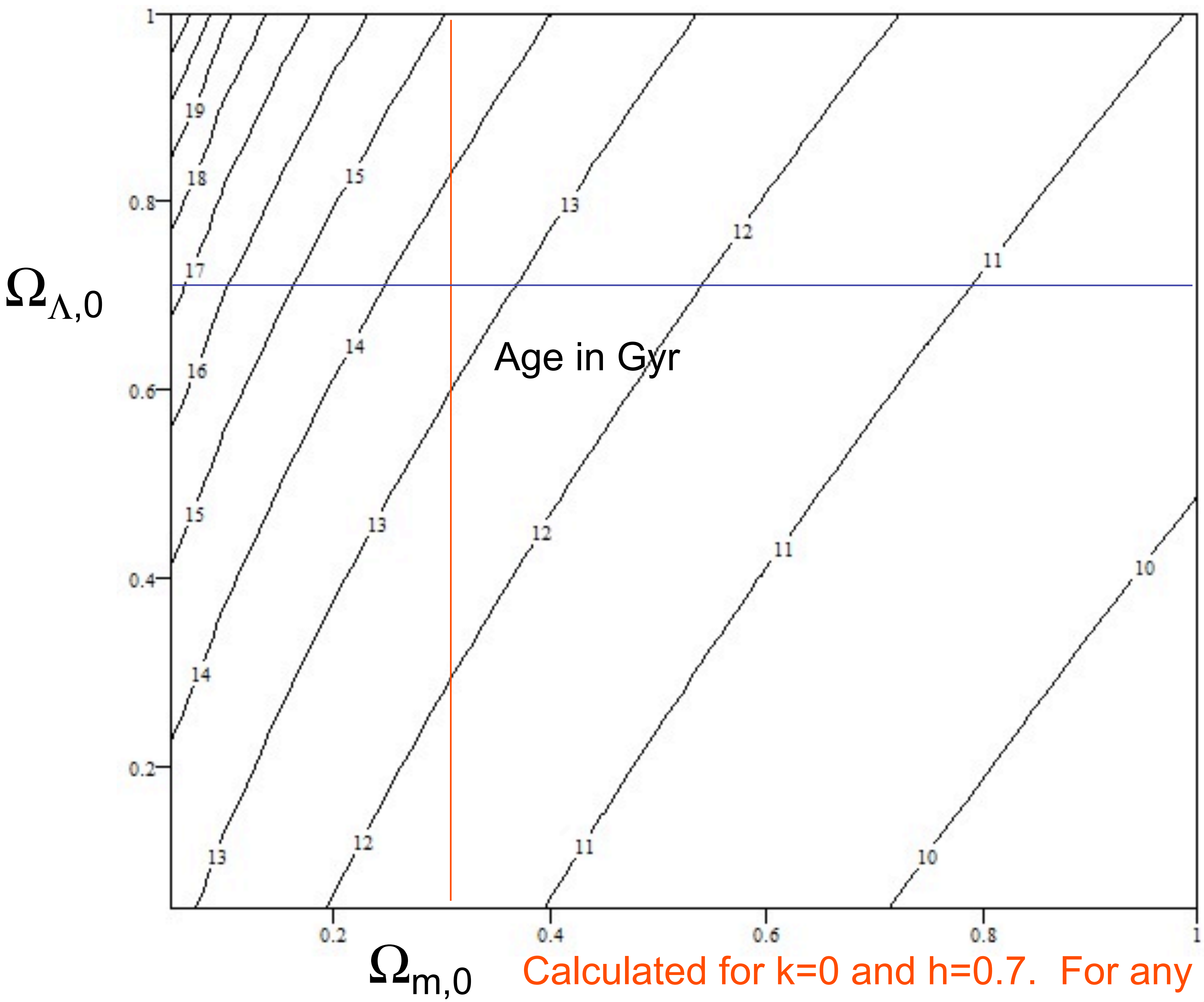
Age of the Universe t_0 in FRW Cosmologies



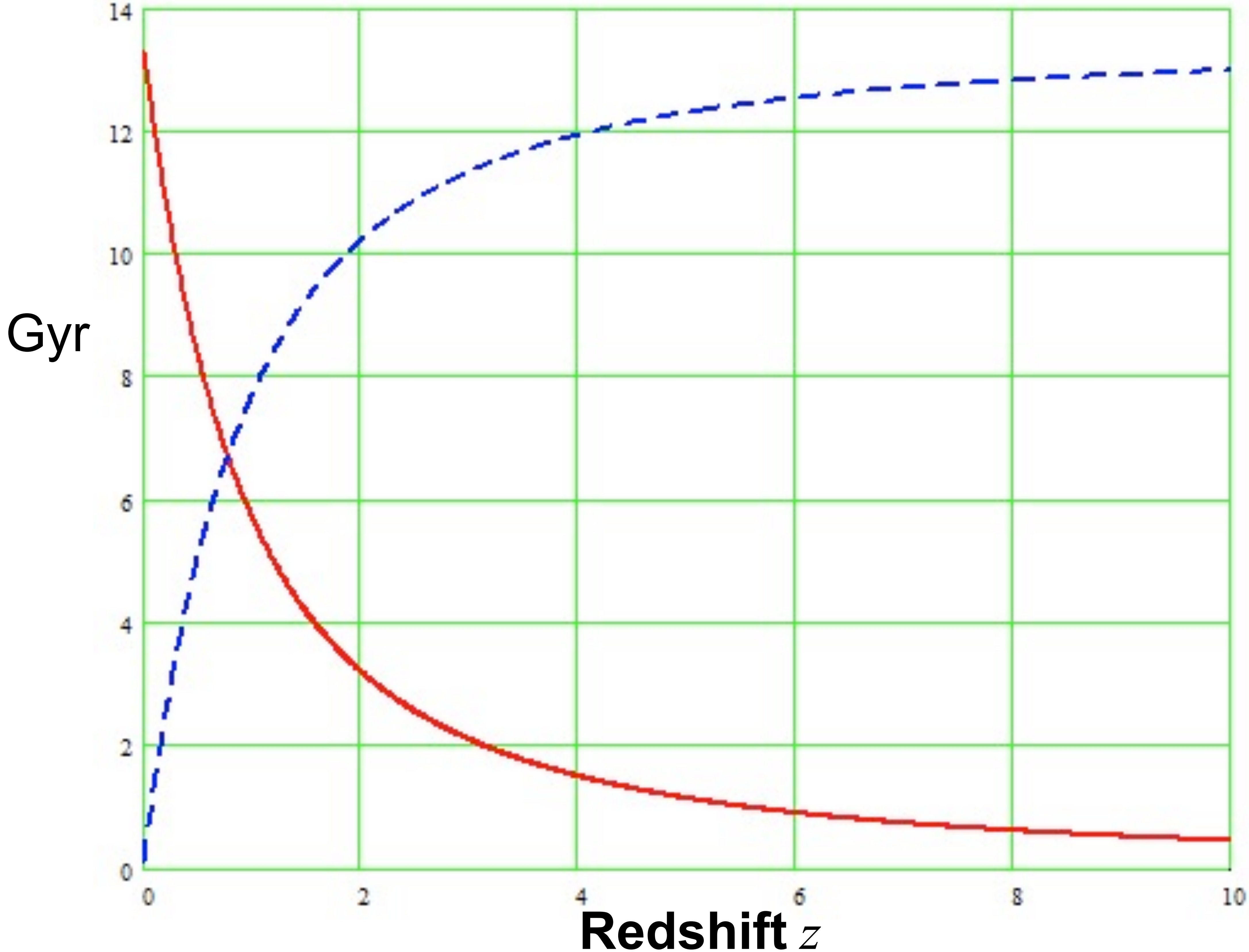
(a) Evolution of the scale factor $a(t)$ plotted vs. the time after the present $(t - t_0)$ in units of Hubble time $t_H \equiv H_0^{-1} = 9.78h^{-1}$ Gyr for three different cosmologies: Einstein-de Sitter ($\Omega_0 = 1, \Omega_\Lambda = 0$ dotted curve), negative curvature ($\Omega_0 = 0.3, \Omega_\Lambda = 0$: dashed curve), and low-Ω_0 flat ($\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$: solid curve). (b) Age of the universe today t_0 in units of Hubble time t_H as a function of Ω_0 for $\Lambda = 0$ (dashed curve) and flat $\Omega_0 + \Omega_\Lambda = 1$ (solid curve) cosmologies.

Age t_0 of the Double Dark Universe



Calculated for k=0 and h=0.7. For any other value of the Hubble parameter, multiply the age by (h/0.7).

Age of the Universe and Lookback Time



These are for the Benchmark Model $\Omega_{m,0}$=0.3, $\Omega_{\Lambda,0}$=0.7, h=0.7.