
Midterm Exam

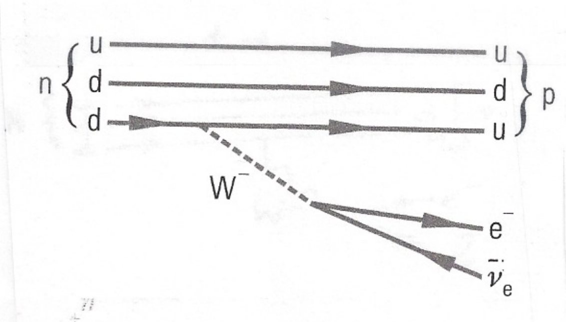
Thursday February 13 – Open Book and Notes

1. For each of the terms below, (a) give one example of a particle of this type, and (b) briefly explain the meaning of the term:

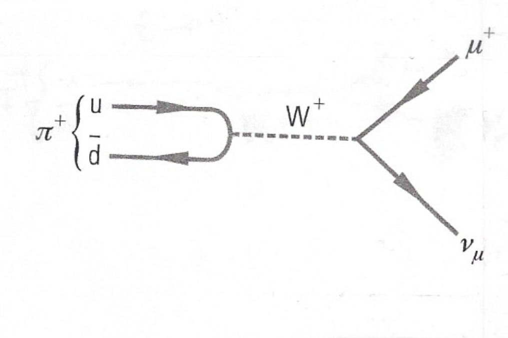
	Examples	Meaning
Fermion	e, p, n, quarks	particles with spin $\frac{1}{2}, \frac{3}{2}, \ldots$
Boson	π, Higgs, γ, Z, W	particles with spin 0, 1, 2, ...
Hadron	p, n, quarks, gluon	particles with Strong Interaction
Lepton	$e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$	particles with Weak but not Strong Interactions
Baryon	$p, n, \Lambda, \Sigma, \Xi, \Omega, \Delta$	hadrons made of 3 quarks

2. Draw a Feynman diagram including quark lines for

(a) neutron beta decay, and



(b) the main π^+ decay mode.



3. The muon mass $m_\mu = 106$ MeV and its lifetime $\tau_\mu = 2.2 \times 10^{-6}$ s. Muons with an energy of 10.6 GeV are produced by cosmic rays at an altitude of 30 km. Calculate the fraction of these muons headed straight downward that reach you at sea level.

$E_\mu = \gamma m_\mu c^2$ so $\gamma = 100$ and the muon's effective lifetime is 2.2×10^{-4} s. These relativistic muons travel at $v \simeq c$, so they take $\frac{30\text{ km}}{300{,}000\text{ km/s}} = 10^{-4}$ s to reach the ground at sea level.

The fraction that have not decayed is $e^{-t/\gamma\tau} = e^{-10^{-4}/2.2\times10^{-4}}$ $= e^{-1/2.2} = e^{-0.45} = \underline{0.63}$.

4. Explain briefly the evidence that muon neutrinos produced in cosmic ray interactions in the upper atmosphere oscillate into other neutrino types.

The neutrinos are produced in $\pi^+ \to \mu^+ + \nu_\mu$ followed by $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ (and $\pi^- \to \mu^- + \bar{\nu}_\mu$, $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$) so we expect 2 muon-type neutrinos for each electron-type neutrino. This is seen for downward going neutrinos but at larger zenith angles including up through the earth the ratio of electron to muon neutrinos is ~~small~~ larger.

5. For each of the following decays, state a conservation law that forbids it:

$n \to p + e^-$	angular momentum, lepton number
$n \to \pi^+ + e^-$	baryon number and lepton number
$n \to p + \pi^+$	energy
$n \to p + \gamma$	electric charge

6. For each of the following high-energy interactions or decays state (a) whether it is allowed or forbidden, (b) reason if forbidden, and (c) type of interaction (S, W, or E) if allowed:

$\pi^- + p \to \pi^0 + n$	allowed, strong interaction
$\pi^0 \to \gamma + \gamma + \gamma$	C, since π^0 has $C=+1$, γ has $C=-1$
$\pi^0 \to \gamma + \gamma$	allowed, electromagnetic interaction
$\pi^+ \to \mu^+ + \nu_\mu$	allowed, Weak
$\pi^+ \to \mu^+ + \bar{\nu}_\mu$	μ-type lepton number
$K^- \to \pi^0 + e^-$	lepton number
$p + \bar{p} \to \Lambda^0 + \Lambda^0$	baryon number
$\Lambda^0 \to K^0 + \pi^0$	baryon number

7. There are no known mesons of electric charge +2, but there are baryons with electric charge +2. Give a simple explanation for this.

Mesons are made of quark + antiquark, so the maximum electric charge is $u\bar{d} = +\frac{2}{3} + \frac{1}{3} = +1$.
Baryons are made of 3 quarks so the maximum electric charge is $uuu = 3(\frac{2}{3}) = +2$, for example the Δ^+.

8. The Σ^{*+} is an unstable baryon with mass 1385 MeV and decay width $\Gamma = 35$ MeV. It is produced in the reaction $K^- + p \rightarrow \pi^- + \Sigma^{*+}$ and mostly decays via $\Sigma^{*+} \rightarrow \pi^+ + \Lambda^0$. From these facts (a) determine the strangeness of the Σ^{*+} and (b) determine whether its decay $\Sigma^{*+} \rightarrow \pi^+ + \Lambda^0$ is via the strong or the weak interaction.

(a) $K^- + p \rightarrow \pi^- + \Sigma^{*+}$ is a strong Interaction, which conserves strangeness – so Σ^{*+} must have $S = -1$ like K^-.

(b) The decay width $\Gamma = 35$ MeV corresponds to a lifetime $\frac{6.6\times10^{-22}\text{ MeV-s}}{35\text{ MeV}} = 1.9\times10^{-23}\text{ s}$, typical of a Strong Interaction. The decay does not violate strangeness since Λ^0 also has $S = -1$.

9. Briefly describe the difference between direct and indirect CP violation in neutral kaon decay.

The CP eigenstates are $(K^0 + \bar{K}^0)/\sqrt{2}$ $C = +1$, and $(K^0 - \bar{K}^0)/\sqrt{2}$ $C = -1$. The long-lived K sometimes decays into 2 pions, violating CP. This occurs mainly because of a small admixture of the $CP = +1$ state in the K_L, called indirect CP violation. Direct CP violation occurs because the decay violates CP via the complex phase in the CKM matrix.

10. (a) Briefly state the main motivations for supersymmetry, and (b) briefly describe the new particles that supersymmetry requires.

(a) Motivations:

Combine gravity with the other 3 forces

Better control of vacuum energy because of boson-fermion cancellation

Grand unification – α_1, α_2, and α_3 are all equal at $\sim 10^{16}$ GeV

Dark matter candidate partice = lightest superpartner

(b) For every fundamental fermion, a boson partener: for leptons, sleptons; for quarks, squarks.
For every fundamental boson, a fermion partner: for photon, photino; for W, wino; for Z, Zino; for gluon, gluino; for Higgs, Higgsino; for graviton, gravitino (spin 3/2)

11. Describe the two types of black holes that are known to exist, including their masses, typical locations, and how we observe their effects.

(a) Stellar mass black holes, masses several times the mass of the sun $M_\odot$, found throughout the galaxy, observed via high-energy radiation (X-rays) emitted as black holes accrete from stellar companions,

(b) supermassive black holes, $m \approx 10^6 - 10^9\ M_\odot$, found in galactic centers, observed via very-high-energy radiation emitted as they accrete matter.

12. Show that during the early universe, when the energy density of the universe was mainly in the form of relativistic particles, the scale factor satisfies $a \propto t^{1/2}$.

Friedmann eq. $H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho \cancel{=} \frac{\Omega_{r,0}}{a^4}$ in the early universe

$$dt = \frac{dt}{da}da = \frac{da}{\dot a} = \frac{da}{(\dot a/a)\,a} = \frac{da}{Ha} \propto \frac{da}{\frac{1}{a^2}a} = a\,da$$

So $t \propto a^2$ or $\underline{a \propto t^{1/2}}$

13. A galaxy is observed to have an angular diameter of 1 arc sec. How large is its diameter in kiloparsecs (kpc) if its redshift is (a) 0.5, (b) 2, (c) 4.

Use the slide in

~~14~~ 13. Show that the distance to the particle horizon in the Einstein-de Sitter cosmology ($\Omega_m = 1$, $\Omega_\Lambda = 0$) is $d_P = 2c/H_0$, determine this distance in light years if $H_0 = 70$ km/s/Mpc, and explain briefly why the particle horizon is farther away than light could travel in the age of the universe.

$ds^2 = 0 = dt^2 - a^2 dr^2$ so $dr = dt/a$

$$d_p = \int_0^{d_p} dr = \int_0^{t_0} \frac{dt}{a} = \int_0^1 \frac{1}{a}\frac{dt}{da}\,da = \int_0^1 \frac{da}{a\dot a} = \int_0^1 \frac{da}{a^2 H}, \quad H^2 = H_0^2 \frac{1}{a^3}$$

$$= \int_0^1 \frac{da}{a^2 H_0 a^{-3/2}} = \frac{1}{H_0}\int_0^1 a^{-1/2}\,da = \frac{2a^{1/2}}{H_0}\Big|_0^1 = \frac{2}{H_0}$$