Homework Set 3

DUE: Tuesday February 4

1. (parts a-b-c: 10 points) Coupling constants are not really constant, but "flow" with the energy scale. This flow of coupling constants obeys certain differential equations. At leading log order this equation has a particularly simple form:

$$\frac{dg(\mu)}{d(\ln\mu)} = -b_0 \frac{g^3(\mu)}{16\pi^2}$$

Here b_0 is the one-loop beta coefficient that depends on the number of light degrees of fredom (light compared to the energy scale μ).

(a) Show that if you define $\tau(\mu) = \frac{8\pi^2}{g^2(\mu)} = \frac{2\pi}{\alpha(\mu)}$, then the equation $\frac{d\tau}{d(\ln\mu)} = b_0$ is equivalent to the above equation.

(b) Using any of the above differential equations derive the following equation:

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(M)} + \frac{b_0}{2\pi} \ln\left(\frac{\mu}{M}\right)$$

If we know the α at scale M and b_0, we can figure out the α at scale μ.

(c) In grand unified theory (GUT), coupling constants are postulated to unify (all become equal) at some high energy. Let $\alpha_1(M_z) \approx 1/60$, $\alpha_2(M_z) \approx 1/29.5$, and $\alpha_3(M_z) \approx 0.23$ be the coupling strength for electromagnetism (to be precise, it is the $U(1)$ electroweak coupling), weak and strong forces respectively at the energy scale of Z mass M_z. Also take $b_0^1 = -41/10$, $b_0^2 = 19/6$, $b_0^3 = 7$. Find the energy scale where $\alpha_1 = \alpha_2$. What is the value of α_3 at that energy scale?

(d) (10 points additional credit) As you can see, these three couplings don't unify! In Supersymmetric models new particles are introduced at some higher energy, thus modifying the b_0's. In some of these models the b_0 changes in such a way that the couplings unify. In one such model new particles are introduced at energy ~ 1000 GeV, which changes the b_0^i to:

$$b_0^3 = 3, \qquad b_0^2 = -1, \qquad b_0^1 = -33/5$$

Create a plot similar to Fig. 4.6 using your favorite computer program.

3. (5 points) Perkins Problem 3.4.

4. (5 points) Perkins Problem 3.8 (worked in the back of the book).

5. (10 points) Perkins Problem 3.9.

6. (10 points) Consider making making mesons and baryons out of quarks.

(a) A quark and an antiquark are bound together, in a state of zero orbital angular momentum, to form a meson. What are the possible values of the meson's spin?

(b) Suppose you combine three quarks in a state of zero orbital angular momentum. What are the possible spins of the resulting baryon?

7. (10 points) Show that Perkins Eq. (4.11) follows from the equations above it.

8. (10 points) Perkins Problem 4.1.