Interactions amongst particles: Emergence in condensed matter

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See recent papers at the website
http://physics.ucsc.edu/~sriram/papers/

Quantum theory of many particles:

Interactions add spice

•Bosons and Fermions
•Atomic physics- beyond the Hydrogen atom- Hartree Fock theory- scaling-ideas
•Bosons: superfluidity- beyond the Bose condensation of ideal gas - probed by atomic traps and cold atom experiments.
•Solids:
    ★Bands and Hartree Fock theory
    ★Landau Fermi Liquid theory
    ★Magnetism
    ★Superconductivity
    ★Mott Physics
    ★Extreme correlations
*New models- exactly solvable type

Main thrust is on emergent properties of many particle systems.

Emergence = new and fundamental laws arising from assemblies of particles that are unexpected from single particle properties- e.g. superconductivity, superfluidity etc.

Emergence plays a big role in Condensed Matter Physics and also in Quantum field theory.

Wavefunctions for N particles

$$\psi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_N)$$

$$\psi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_N) = \pm\psi(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_N)$$

Bose
Fermi

Bosons in Nature:

- Elementary particles: pions,...all integer spin particles
- More useful idea: even number of Fermions act as bosons. Fermions are e,n,p,....
- ^{4}He (2 p+ 2n)-Kammerling Onnes 1908 liquefied and in 1938 Kapitza found superfluid property
- Atoms Rubidium-87, Sodium-23 .. Cooling by atomic traps and condensation (Nobel Prize 1997, 2001)

From Atoms to Solids: Hartree Fock theory and Landau Fermi Liquids

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j}\frac{e^2}{|r_i - r_j|} + \sum_i U(r_i)$$

U(r) is say Hydrogenic
atomic potential

Let the mean separation between electrons be called r_s

$$\frac{4\pi}{3}r_s^3 = \frac{V}{N} = \frac{1}{n}$$

We next scale all lengths by r_s

$$\nabla^2 \to \frac{1}{r_s^2}\hat{\nabla}^2$$

$$r_i \to r_s\ \hat{r}_i$$

$$H = \frac{1}{r_s^2}\left(\hat{T} + r_s\hat{V}\right) + U$$

Therefore as r_s becomes small we
can use perturbation theory around
kinetic energy. This is at the heart of the
Hartree Fock theory.

Question: What about Dirac eqn? (Graphene)

•Implications is that we can think of high density electrons as almost free electrons and treat potential energy as a perturbation.

•This is the reason why we can confidently predict the properties of heavy atoms with Z> 3, and also solids. For example in Aluminum, we have 10^{23} electrons per cc, excellent approximation is to use free electron theory- works like a charm,

•With Dirac type spectrum it is challenging though

Landau theory of Fermi liquids is a formalization of this idea.
In stat mech we learn that gas-> liquid are the same, liquids are more dense and hence more interacting. Similarly
Fermi gases <-> Fermi Liquids.... same and yet slightly different.
Fermi Liquids have certain standard set of properties, with “renormalization”

$$H_{eff} = \sum_{k\sigma} \varepsilon_k \delta n_{k\sigma} + \frac{1}{N} \sum_{kk'} f(k\sigma, k'\sigma') \delta n_{k\sigma} \delta n_{k'\sigma'}$$

Set of properties are controlled by Landau theory:

- Specific Heat $C_v = \gamma T + O(T^3)$
- Susceptibility χ= constant
- Compressibility κ
- Quasi particle lifetimes $1/\tau \sim (T^2 + \omega^2)$
- Resitivity ρ = const + T^2

Large number of systems Liquid ^{3}He, many metals satisfy the diktats of Landau Fermi Liquid theory.
Probed by many means, including calorimetry,magnetic measurements, ARPES, transport.

These are the "Normal" systems

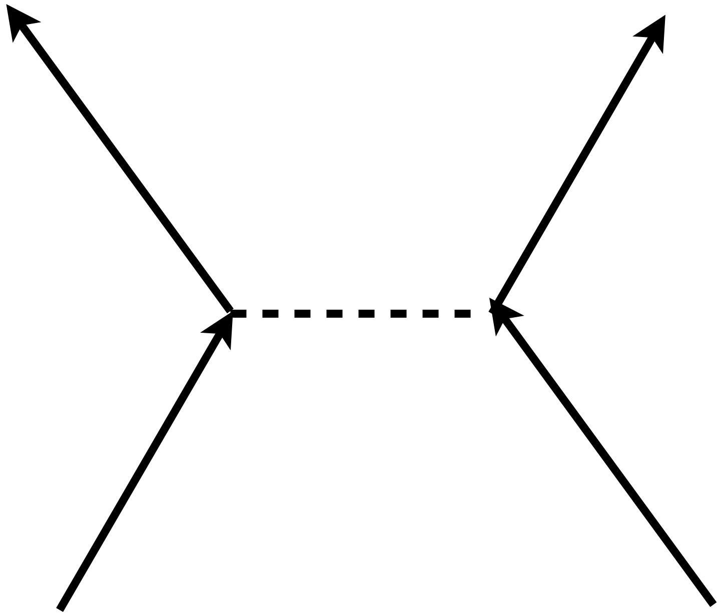
Now for the exceptions:

Liquids become solids at low temperature due to "broken symmetry" + interactions (Translation invariance in this case)
Fermi liquids become either magnets (of various denomination), or superconductors by breaking spin-rotation symmetry or gauge symmetry at low enough temperatures.

Superconductivity

$$H = KineticEnergy\ - |U_{eff}| \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$U_{eff} = U_{electron-phonon} - U_{Coulomb}$$



$$H = KE - |U_{eff}| \sum_{k,p} b_k^\dagger b_p;$$

$$b_k = C_{k\downarrow} C_{-k\uparrow}$$

$$\sum_k < b_k >= \Delta$$

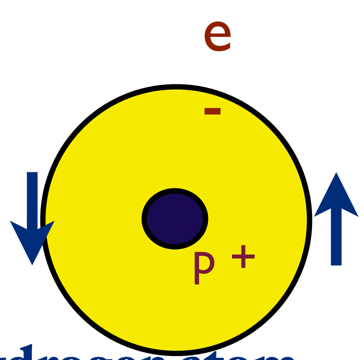
Violates gauge symmetry.
BCS theory of superconductors follows.
Attraction does not just create simple molecules- it can do a lot more. Macroscopic QM
Non perturbative in U! One of the most comprehensive theories in Physics with detailed predictions for many measurements.

Mott Physics and Extreme Correlations

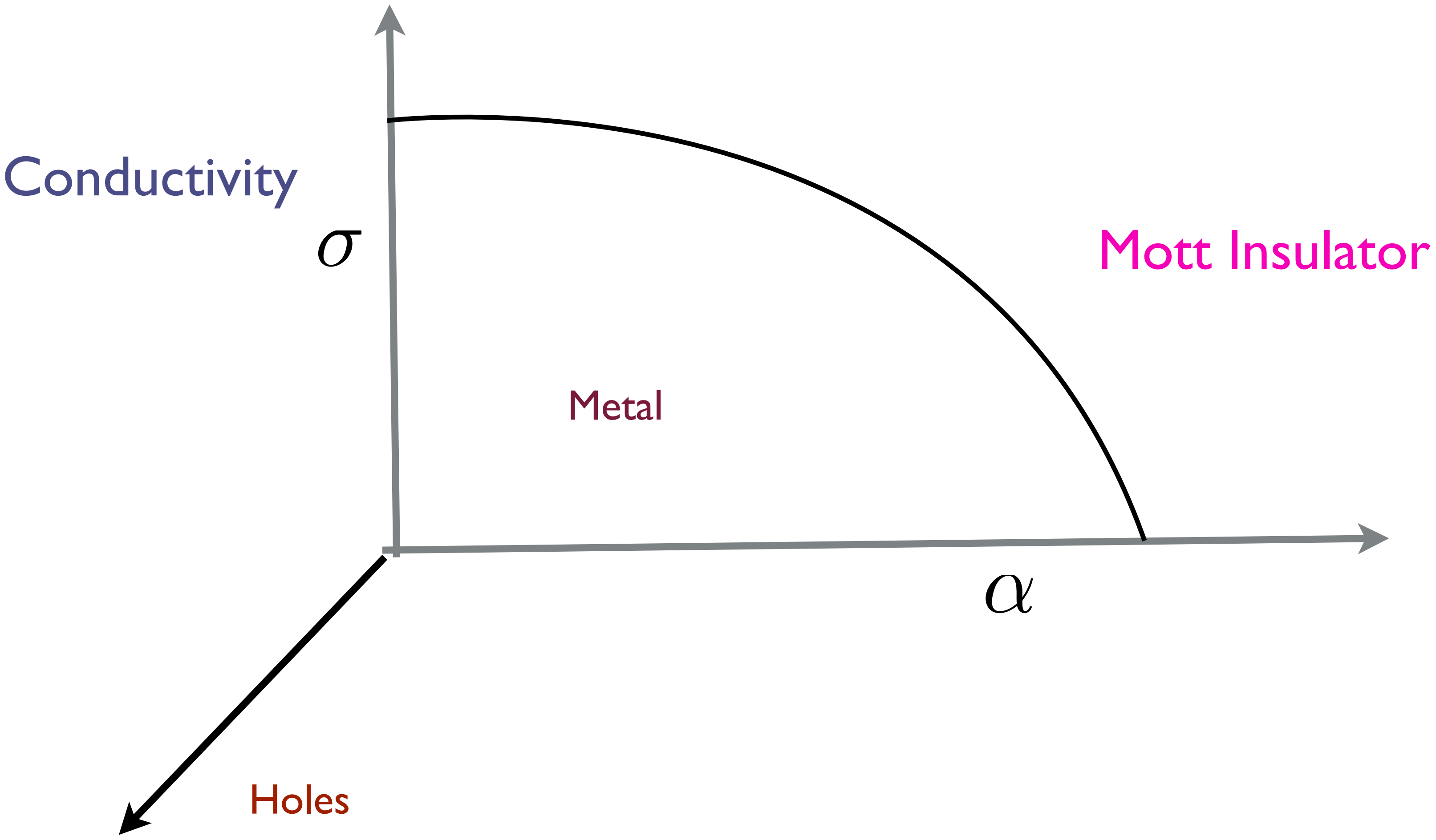
$$H = \text{Kinetic Energy} \; + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

When U becomes larger than the range of the kinetic energy, we get a new possibility that was described by Mott. At exactly 1 electron per atom, we get an insulator at large U.

Array of Hydrogen Atoms with separation α



Hydrogen atom
The 1-s level may contain
0,1 or 2 electrons



Description of the doped Mott Insulator is expected to lead to
an understanding of High Tc systems- the holy grail!

The Physics of Extreme Correlations in Cuprates and its implications for ARPES

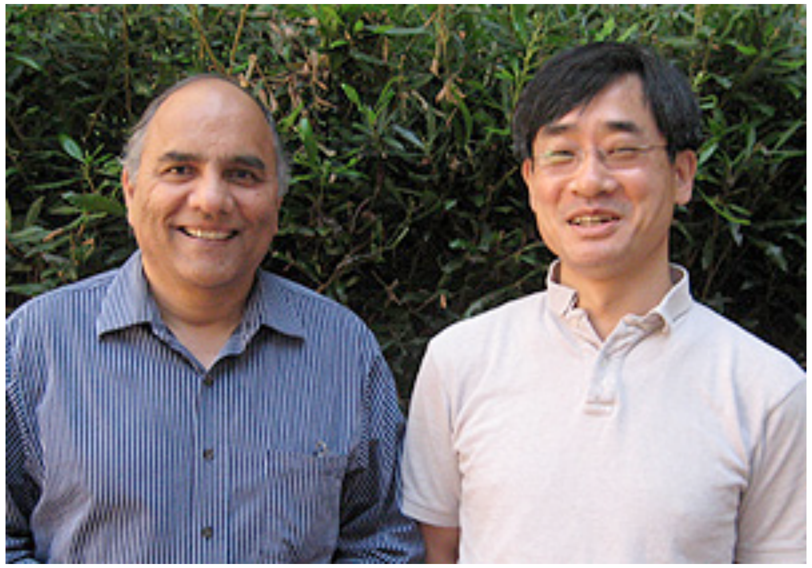
How I Learned to Stop Worrying and Love the Limit of Infinite U

[1] Extremely Correlated Fermi Liquids, Phys. Rev. Letts. 107, 056403 (2011).

[2] Extremely Correlated Fermi Liquid Description of Normal State ARPES in Cuprates, Phys. Rev. Letts. 107, 056404 (2011). (G.-H. Gweon, B. S. Shastry and G. D. Gu,)

[3] " Anatomy of the self energy", Phys. Rev. B (2011)

[4] " Dynamical Particle Hole Asymmetry in Cuprate Superconductors", arXiv:1110.1032



Gey Hong

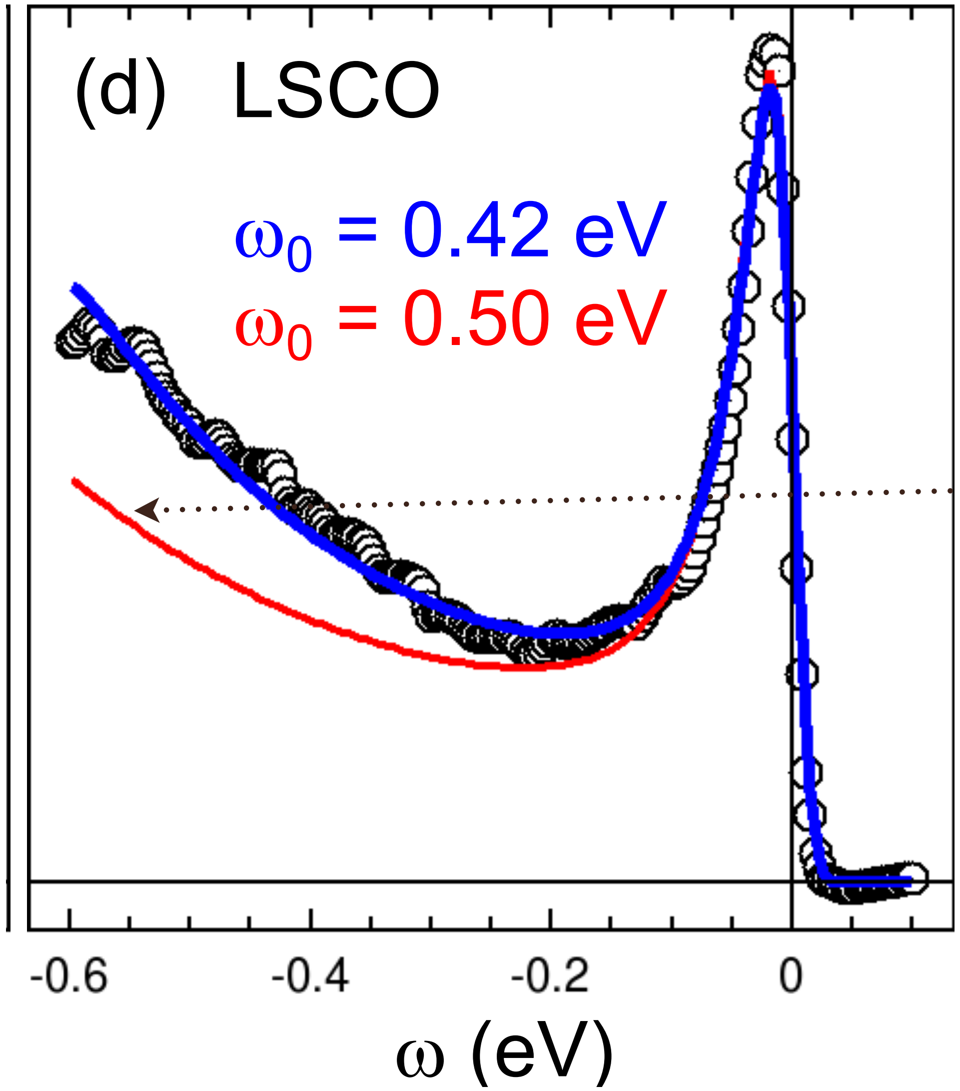
ARPES = angle resolved photo emission
Significant probe since it measures the single electron propagator using the photoemission theory.

ECFL== Extremely Correlated Fermi liquid solution of tJ model.

$$\boxed{\begin{aligned} G(k,z) &= \frac{a_G}{z-\hat{E}_k-\Sigma(k,z)}, \quad \text{Dyson} \\ &= \frac{a_G+\Psi(k,z)}{z-\hat{E}_k-\Phi(k,z)}. \quad \text{ECFL} \end{aligned}}$$

$a_G = 1-n/2$, n= density

extreme correlation model is best described in terms of ``two self energies''.

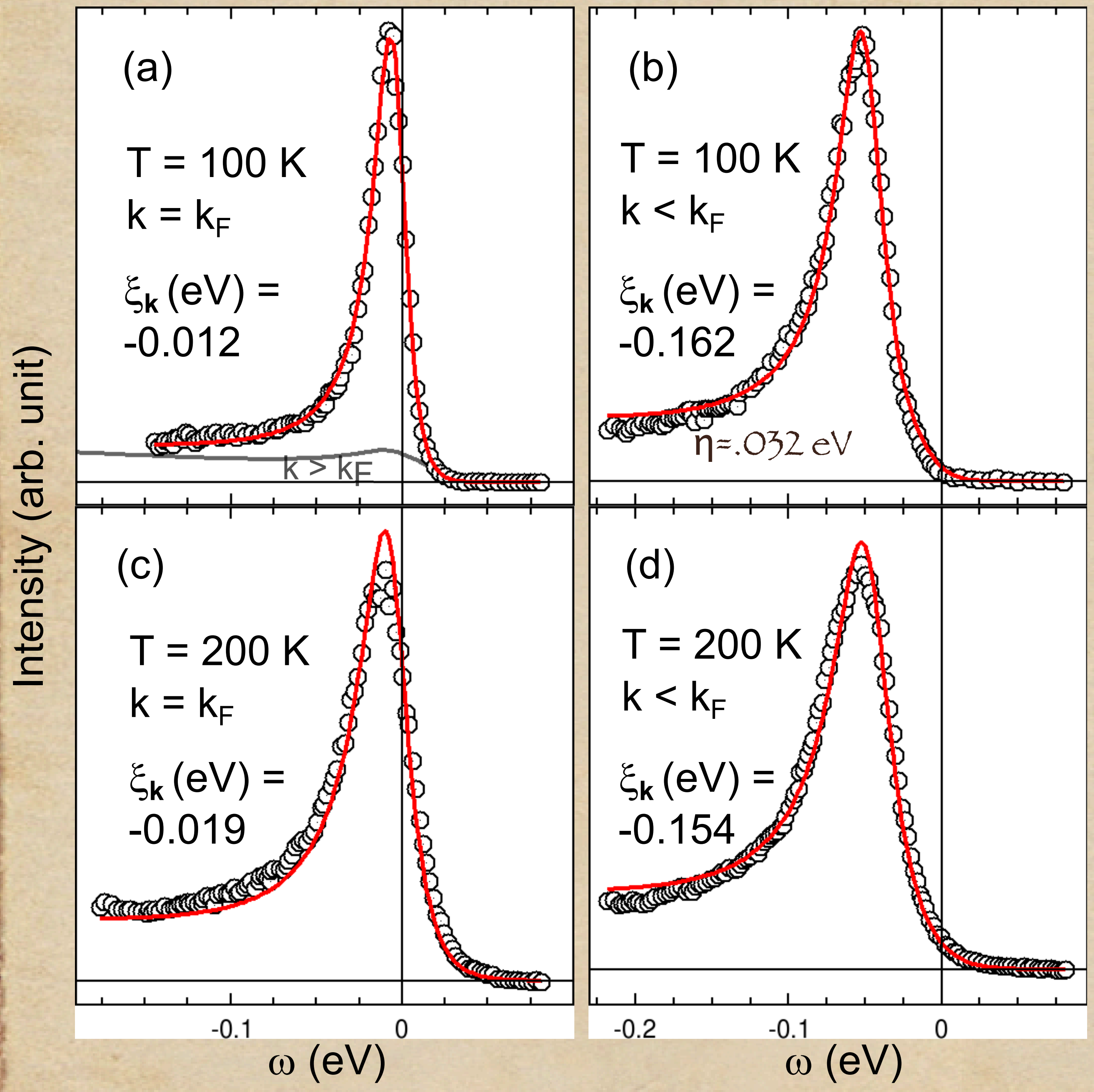


Smoking gun
Linear rise of intensity
for occupied states.

$$A(\vec{k},\omega) = A_{FL}(\vec{k},\omega)\left(1-\frac{n}{2}+\frac{n^2}{4}\cdot\frac{\xi_{\vec{k}}-\omega}{\Delta_0}\right)_+$$

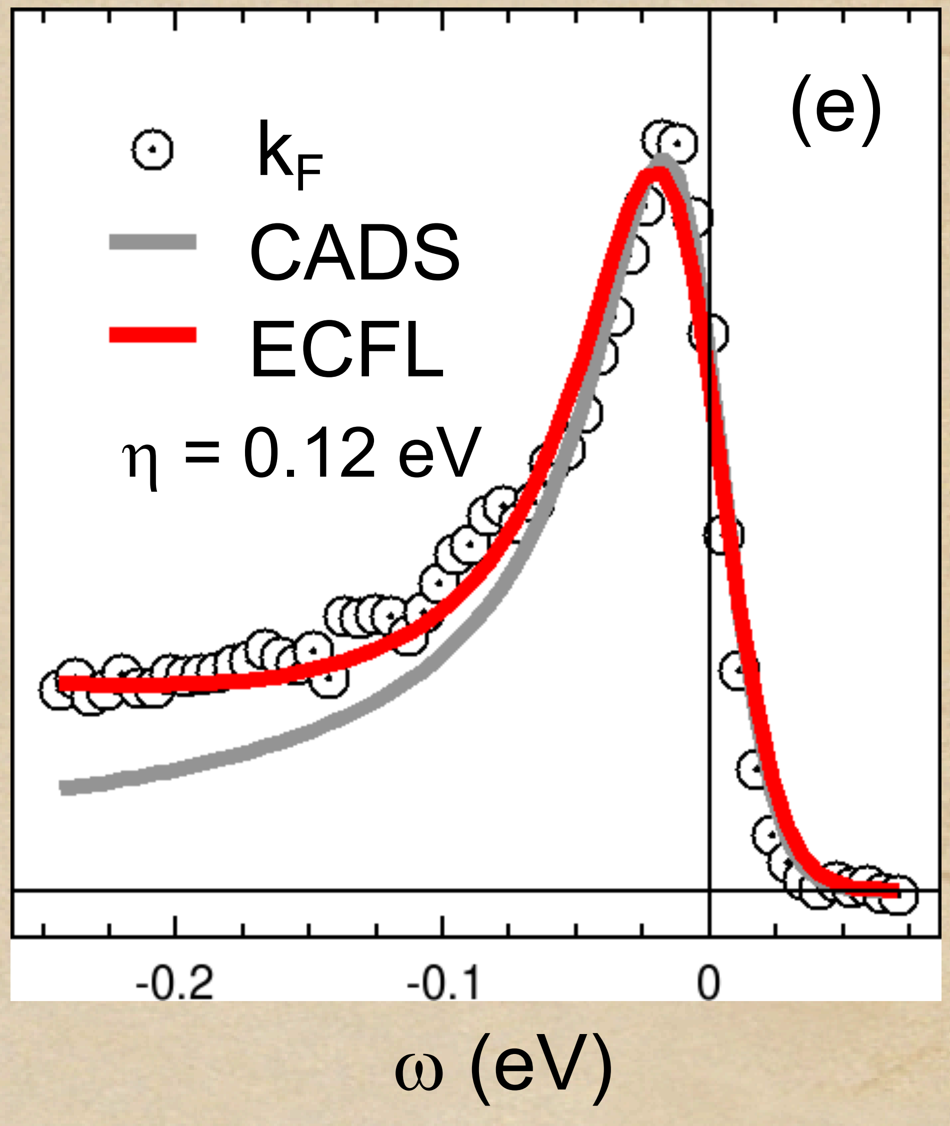
Laser ARPES BISCO 2212

Synchrotron ARPES BISCO 2212



η≈.032 eV

Same physical parameters only η different



η arises from impurities and sample penetration of ARPES.