

Homework #4 – Due Friday November 4

This homework assignment is again based on special relativity. Recall that if an observer at rest in an inertial reference frame sees a clock moving in the x -direction at speed v , the clock appears to run slow by the factor $1/\gamma$ where $\gamma = (1-v^2/c^2)^{-1/2}$ and its length in the x -direction appears shorter by the same factor. These special relativistic effects are known as time dilation and length contraction. If mass is converted into energy, the energy available is given by Einstein's famous formula $E = mc^2$.

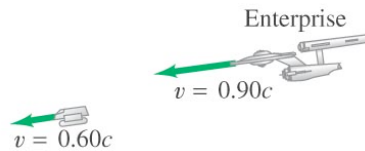
1. At what speed do the relativistic formulas for (a) length and (b) time intervals differ from classical values by 1.00%? (This is a reasonable way to estimate when to do relativistic calculations rather than classical.)

2. The nearest star to Earth is Proxima Centauri, 4.3 light-years away.

(a) At what constant velocity must a spacecraft travel from Earth if it is to reach the star in 4.6 years, as measured by travelers on the spacecraft?

(b) How long does the trip take according to Earth observers?

3. An observer on Earth sees an alien vessel approach at a speed of $0.60c$. The *Enterprise* comes to the rescue overtaking the aliens while moving directly toward Earth at a speed of $0.90c$ relative to Earth.



What is the relative speed of one vessel as seen by the other?

4. The total energy of a particle of rest mass m moving at velocity v is $E = m\gamma c^2$ and its kinetic energy K is its total energy minus its rest energy: $K = m\gamma c^2 - mc^2 = mc^2(\gamma - 1)$.

(a) Show that in the limit of slow speeds $v \ll c$, this reduces to the non-relativistic expression $K = \frac{1}{2} mv^2$.

(b) What is the speed of a particle when its kinetic energy equals its rest energy?

(c) Show that the kinetic energy K of a particle of mass m is related to its momentum p by the equation $p = \sqrt{K^2 + 2Kmc^2} / c$.

5. The Sun radiates energy at a rate of about 4×10^{26} W.

(a) At what rate is the Sun's mass decreasing?

(b) How long does it take for the Sun to lose a mass equal to that of Earth?

(c) Estimate how long the Sun could last if it radiated constantly at this rate.

6. A farm boy studying physics believes that he can fit a 12.0-m long pole into a 10.0-m long barn if the pole moves fast enough.

(a) How fast would the pole have to move for this to be possible?

(b) How does this fit with the idea that in the frame of the moving pole, the barn looks even shorter than 10.0 m?

(*Hint:* To answer (b), you have to consider whether, in the frame in which the pole is at rest, the pole is ever entirely within the barn. The key is that two events at different locations that appear simultaneous in one inertial frame are not simultaneous in a frame moving with respect to this one.)