

Physics 5I FIRST LECTURE September 23, 2011

Physics 5I Introduction to Physics I – HONORS SECTION Spring 2010

Class meets: Fridays 11:00 – 12:10 in ISB 231 (or possibly on some Mondays)

Instructor: Joel Primack– office hours: Wed 2:00-3:00 pm or by appointment
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Website: <http://physics.ucsc.edu/~joel/Phys5I>

Brief Course Description: Weekly 70-minute 2 unit class covering advanced aspects of classical mechanics and related aspects of modern physics. Concurrent enrollment in course 5A is required. Grades will be based on ~5 homework assignments, a midterm exam on November 18, and class participation. Physics 5I will include some history of the physics of Galileo and Newton and their successors, try to develop a deeper understanding of classical physics, and introduce some relevant aspects of relativity and other modern physics. **The goal is to help students understand some of the most interesting and challenging aspects of physics, and have fun doing so.**

Topics to be covered will include many of the following:

History of physics, and of the mathematics and technology that made it possible.

Philosophy of science, and especially of mechanics.

The significance of the concepts of mass and force.

The deep origin and significance energy, momentum, and angular momentum conservation.

Tricky mechanics problems and how to solve them.

Special relativity, important for motion with high velocity, with applications to particles emitted by radioactive materials and to cosmic rays.

General relativity, our modern theory of space, time, and gravity.

Black holes in the universe.

Galaxies and cosmology, including the role of dark matter.

This Lecture

The Scientific Revolution

CLOCKS: From Frere Jacques to Your Wristwatch

Encompassing Scientific Revolutions

Galileo's Inclined Plane Experiment

J. W. Gibbs and Vector Calculus

The Scientific Revolution

Medieval geocentric cosmology was consistent with observation, because

The earth doesn't seem to move.

The stars pass overhead every night, the sun every day.

The explanation was based on ancient tradition.

Copernicus's heliocentric model overthrew common sense.

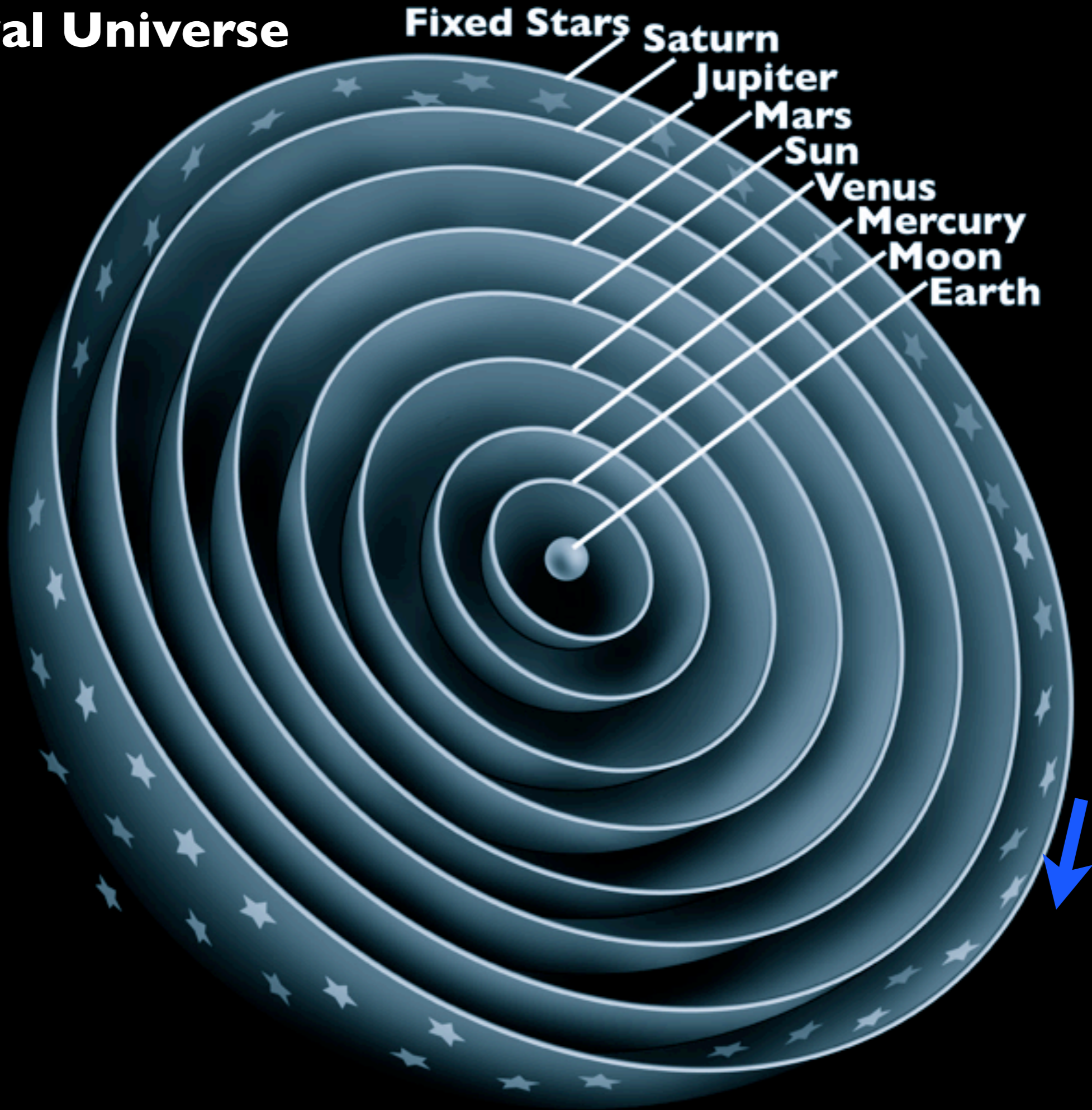
A harmonious theoretical system was more important to Copernicus than arbitrary explanations to uphold common sense.

Galileo's observations of the moons of Jupiter, moon's rough landscape, phases of Venus, etc., convinced nonscientists too of the failure of Ptolemy's geocentric system, but did not actually prove the earth moved. "Philosophy is written in this grand book, the universe, ...in the language of mathematics." Galileo timed the motion of balls down inclined planes, and developed the mathematical description of accelerated motion.

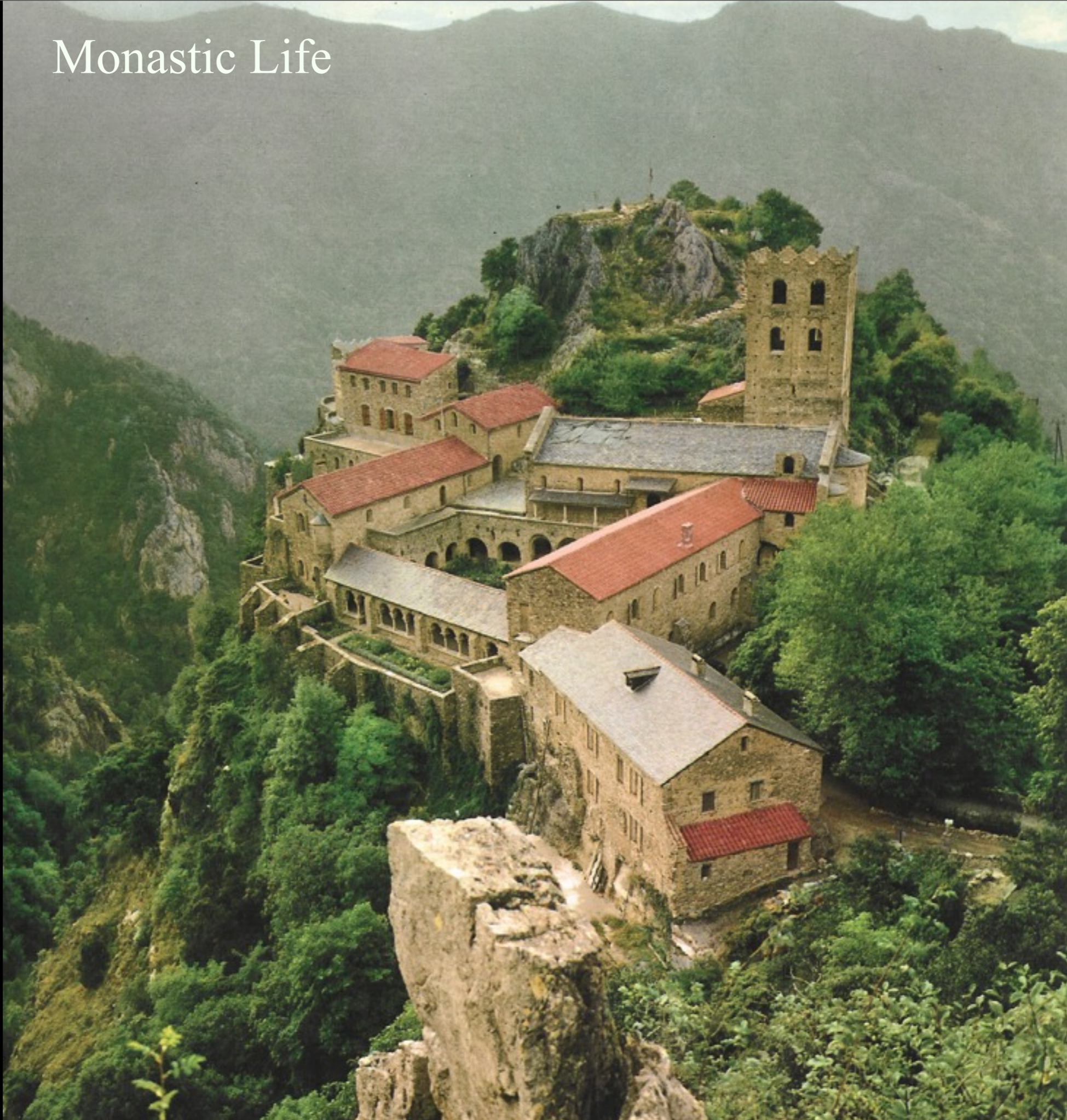
For Kepler, mathematical regularity was an explanation because God spoke in mathematics. His laws of planetary motion summarized careful measurements.

Newton completed the scientific revolution begun by Copernicus by creating the concept of gravity and tying together the orbits of the planets with the motions of particles and the falling of apples -- quantitatively.

Medieval Universe



Monastic Life





Medieval Monk's Contemplation

Imagine that it is the year 1200 CE, and you are a monk in a monastery somewhere in Europe. You have just awakened in your cell. It is pitch black and very cold. You wrap yourself tightly in your woolen habit and fling open the window. The moon has not yet set. The world outside is silent and the sky sparkles with stars. You shiver, not only with the cold but with the awesome beauty above.

Everything in creation has a place that God has decreed for it, and it moves toward that place by its own desire, because it loves God and wants to fulfill His will. God has given every object this tendency, to keep the universe orderly. Earth is at the center of the universe because it is the heaviest of all elements. The waters lie above it, the air above that, and fire soars upwards to the heavens. How nice a fire would be now. You try to imagine the heat and the reddish glow of earthly impurities burning off. Real fire, of course, is invisible and flies always towards its proper home just beneath the sphere of the moon. But beyond the moon – there lies perfection! The crystal spheres, made of quintessence, all turning at different speeds, not a hairsbreadth of empty space between them yet absolutely frictionless – only God could have engineered such precision. You rarely have time to contemplate the stars, but right now you hold your breath and listen very carefully – perhaps you can just make out the ethereal music of the spheres as they revolve!

You look up at Jupiter, gleaming brightly and riding ever so regally upon its sphere below the stars. Everyone knows kingly Jupiter brings great fortune, but he is following on the heels of Saturn, who brings great misfortune. What can this portend? Of course, the planets are not gods. The Romans were unredeemed pagans, and their gods' names on the planets mean nothing. The planets simply ride their crystal spheres, tracing perfect circles forever around the earth. Still, their influence is powerful.

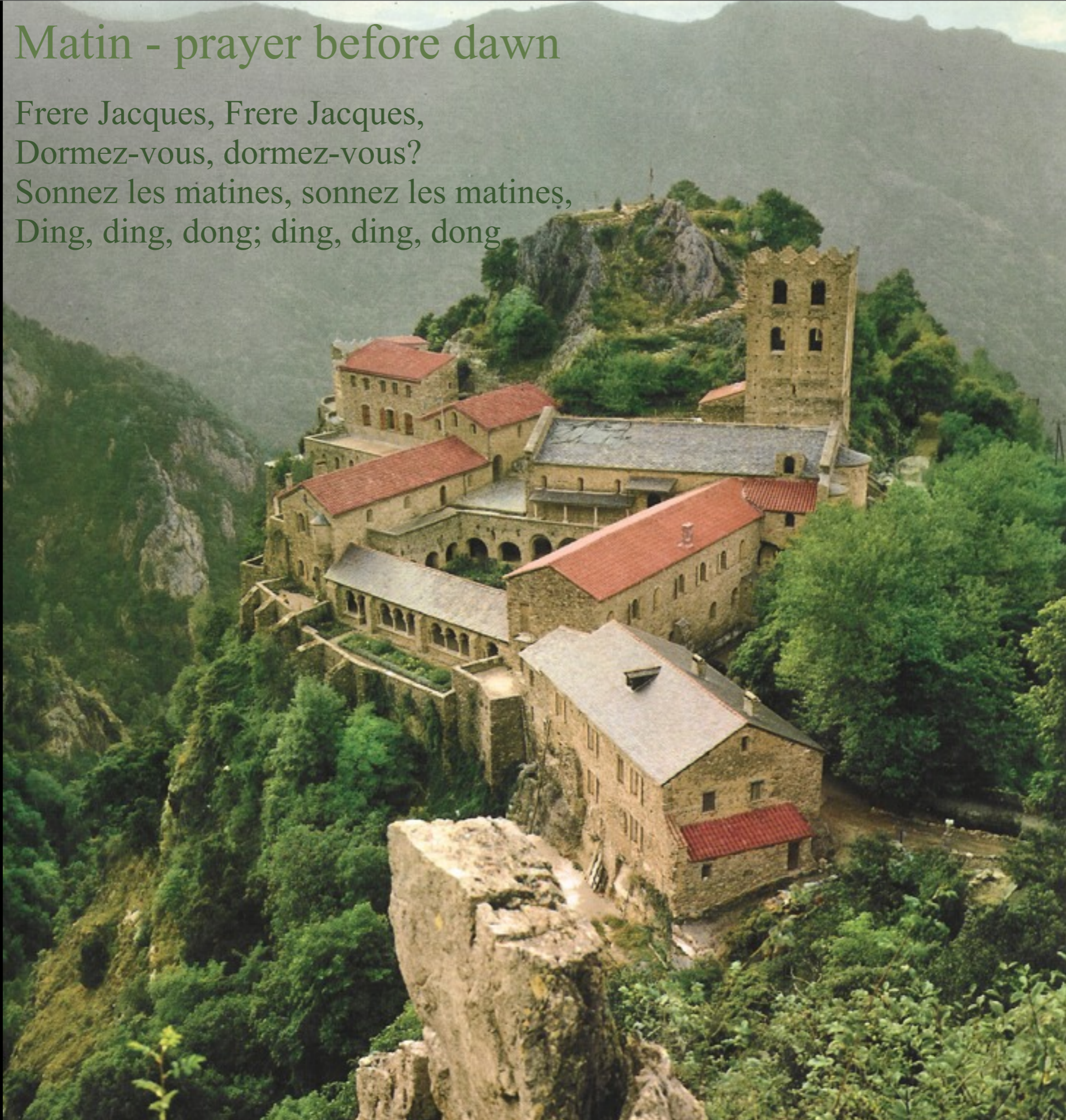
The stars are bright and clear on their crystal sphere. You know the sun right now is still on the other side of the earth, so that the monastery is in earth's shadow. You try to imagine the way it looks from outside, and you can visualize the cone of darkness extending out from earth opposite the sun. Your monastery and the whole village are in that cone of darkness. But high up beyond earth's shadow, all is bright, all is lit by the sun.

Soon it will be time to awaken the other monks for morning prayers. If you all pray at exactly the same time as all the other monks at all the other monasteries, God will surely hear you. It's a good thing, too, because if you had to send prayers to God by mule, and the mule walked 40 miles a day straight up, right past Saturn, the Seventh Heaven, it wouldn't even get to the sphere of the fixed stars for 8000 years. What a huge sphere -- so big, in fact, you can hardly imagine how it can turn all the way around the earth every day, even for the love of God. And God is even farther away! The thought of being that high makes you dizzy. Whenever you walk into the Cathedral, though, and look way, way up at the angels on the ceiling, you feel grandeur and you do understand, at least a little. No matter how high the sphere of the heavens is, God and the angels are always there, looking down at you.

The heavens are so much more beautiful than the corrupt earth. But we each have to live out our time here first, in whatever place God put us, and he has placed us here in this cold world full of suffering, far from heaven. You look longingly at the realms of the heavenly spheres. They are filled with angels and other ethereal beings, all overflowing with divine love. The moon is just setting. It is time to awaken the monks for morning prayers. This is your lot in life. Yours is an important role, and you are grateful for it. You close the window and begin your day's work

Matin - prayer before dawn

Frere Jacques, Frere Jacques,
Dormez-vous, dormez-vous?
Sonnez les matines, sonnez les matines,
Ding, ding, dong; ding, ding, dong



CLOCKS: From Frere Jacques to Your Wristwatch

Need: synchronicity of prayer

Seven Canonical Hours: Matins (before dawn), Prime (sunrise), Tierce (3 hours after sunrise), Sext (6), Nones (9), Vespers (11), Compline (after sunset).

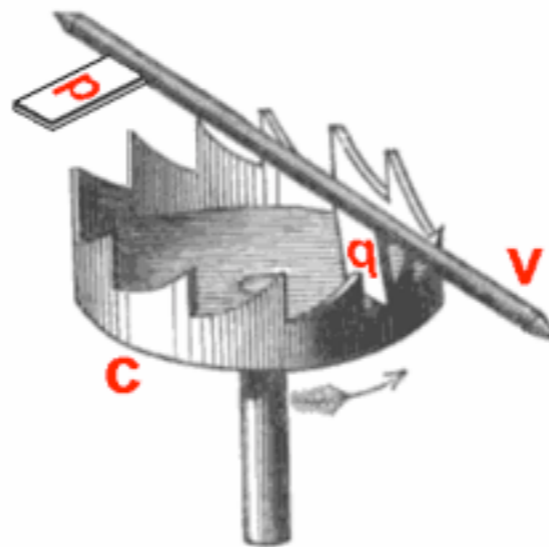
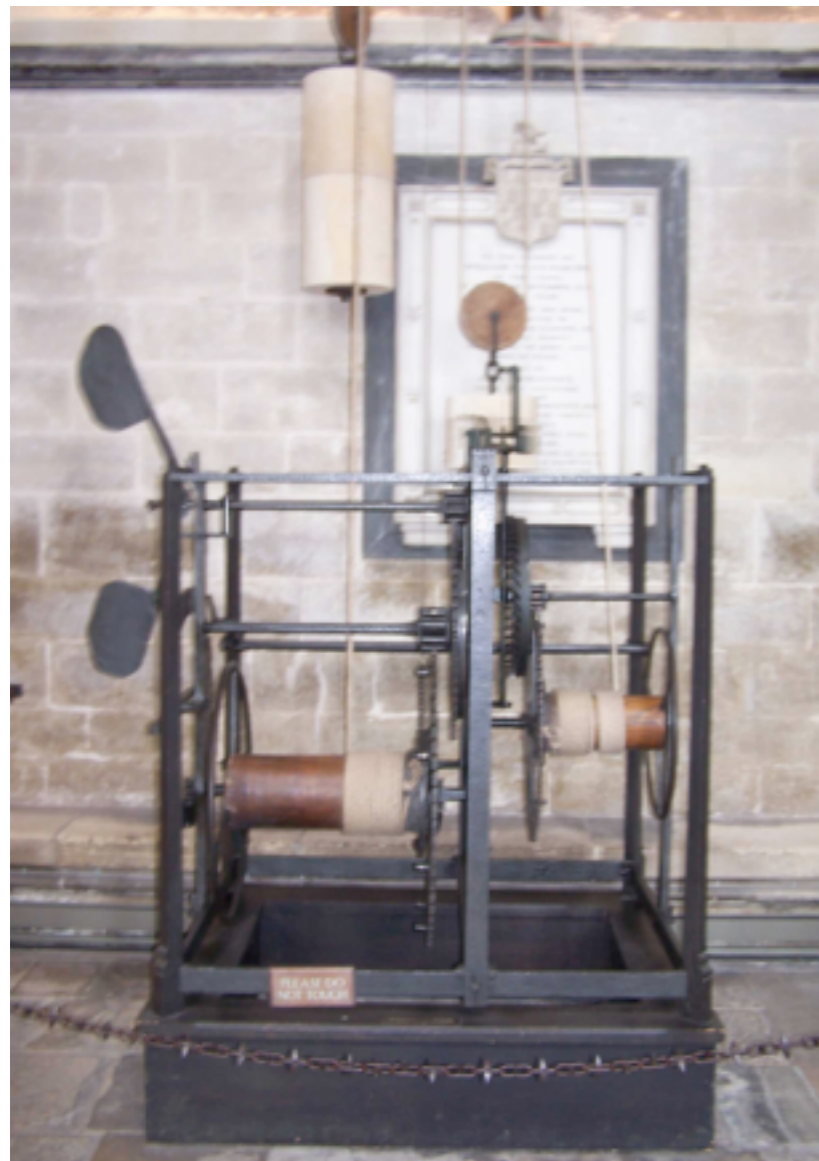
Salisbury Cathedral
1258



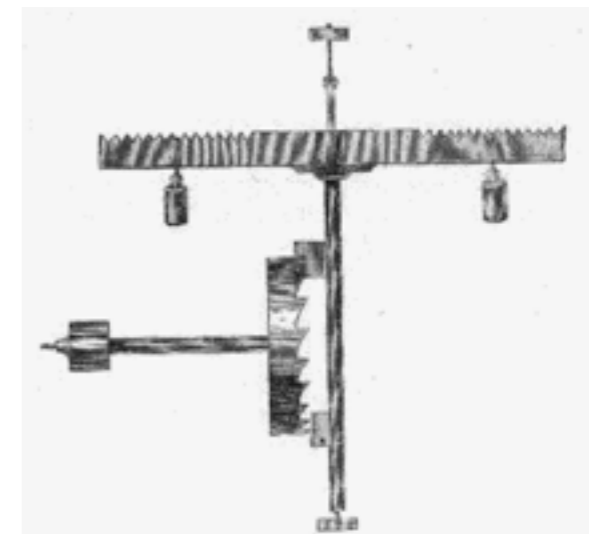
Invention of mechanical clock (~1285)

A clock in Salisbury Cathedral that struck the hours was mentioned in 1306. This was probably one of the precursors of the 1386 clock illustrated at left, the oldest of the few working medieval mechanical clocks that survive. Its main purpose was to strike a bell at precise times. It did not have a dial. The wheels and gears are mounted in an open box-like iron frame about 1.2m square. The framework was not held together with nuts and bolts (which had not been invented), but rather with metal dowels and pegs.

The escapement was the [verge and foliot](#) type, standard for clocks of this age. The power was supplied by two large stones hanging from pulleys. As the weights fall, ropes unwind from the wooden barrels. The barrel on the right drives the main wheel which is regulated by the escapement, the barrel on the left drives the striking mechanism.



Verge escapement showing (c) crown wheel, (v) verge, (p,q) pallets



Verge and foliot escapement from De Vick clock, built Paris, 1379, by Henri de Vick

CLOCKS: From Frere Jacques to Your Wristwatch

Religious need: synchronicity of prayer

Canonical Hours: Matins (before dawn), Prime (sunrise), Tierce (3 hours after sunrise), Sext (6), Nones (9), Vespers (11), Compline (after sunset).

Frere Jacques, Frere Jacques,
Dormez-vous, dormez-vous?
Sonnez les matines, sonnez les matines,
Ding, ding, dong; ding, ding, dong.

Invention of mechanical clock (~1285)

Regulation of city life

The universe “is like a rare clock, such as may be that at Strasbourg, where all things are so skilfully contrived, that the engine being once set a-moving, all things proceed according to the artificer’s first design, and the motions...do not require the particular interposing of the artificer, or any intelligent agent employed by him...”

– Robert Boyle (1772)

Clockwork universe: The world is just a machine

Time is money

Relativity: synchronicity is impossible

References: David S. Landes, *Revolution in Time* (Harvard University Press, 1983).

G. J. Whitrow, *Time in History* (Oxford University Press, 1988).

Changing Cosmologies

Size R Age T Center	Composition	Unifying Ideas	God's role?
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Medieval Cosmos

Finite R Finite T Geocentric	Sublunary: Earth, Water, Air, Fire Heavens: Ether	Circular motion Great Chain of Being	Prime mover, Hierarch, Saviour
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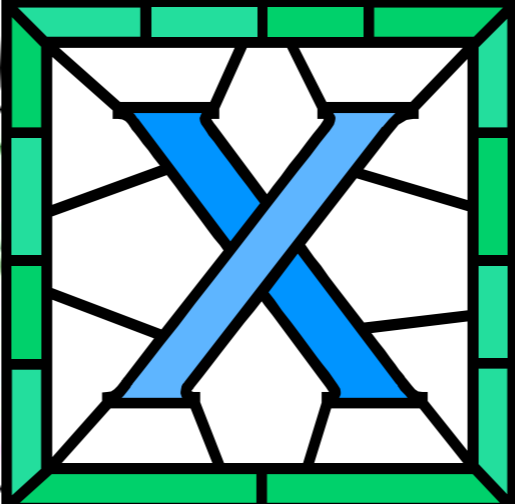
Newtonian Cosmos

Infinite R? Infinite T? No center	Atoms, Void Ether?	Deterministic mechanics Universal gravitation	Clockmaker
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Modern Cosmos

$R = 10^{28}$ cm $T = 10^{10}$ yr Homogeneous & Isotropic	Atoms quarks, electrons Radiation Dark Matter Vacuum	Gravity = space curvature Nondeterministic quantum theory Evolution	Before the Big Bang? Immanent?
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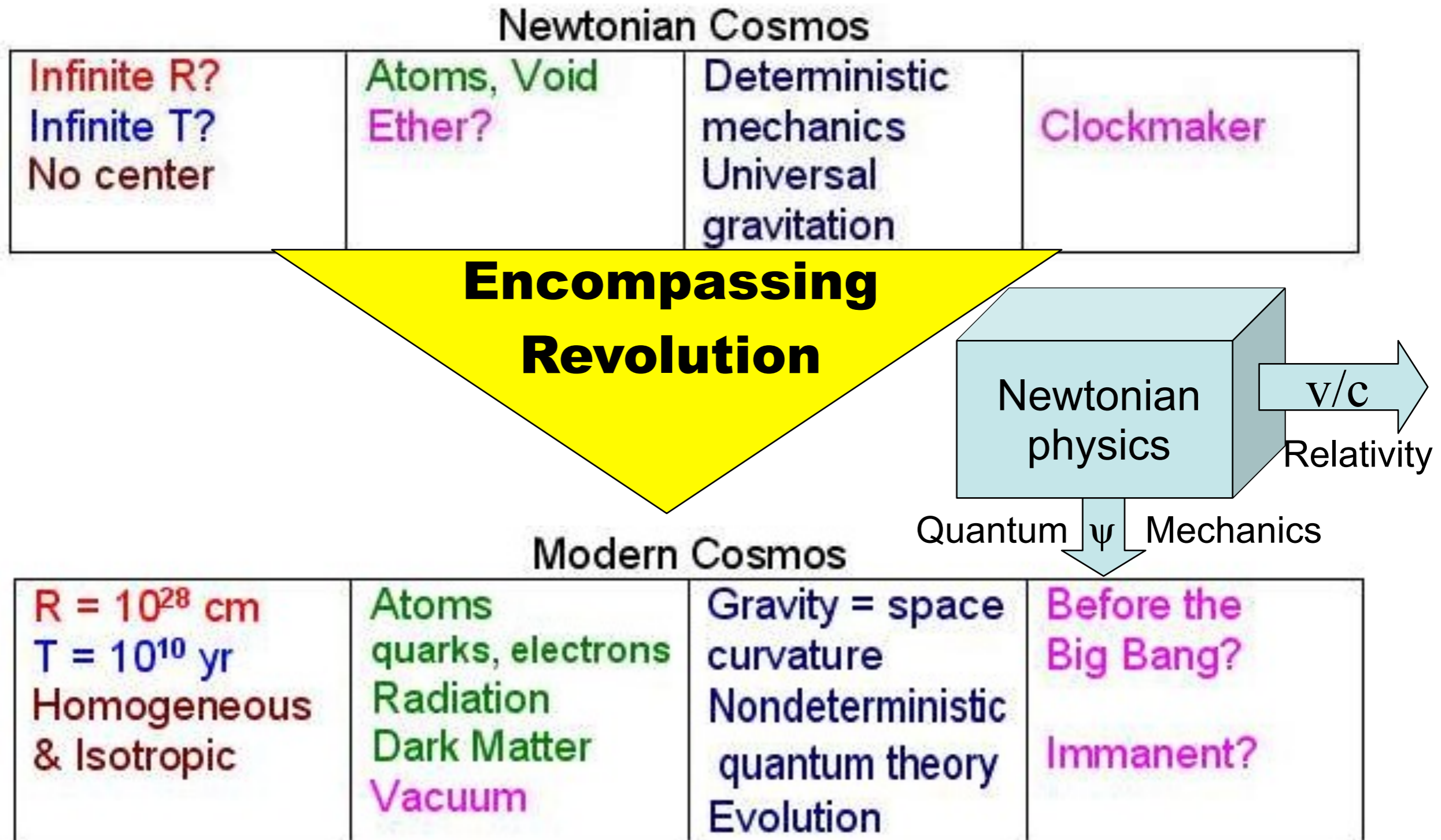
After the Copernican-Newtonian Revolution,
 the Medieval Cosmos was taught only as history, never
 again as scientific truth

Medieval Cosmos			
Finite R Finite T Geocentric	Sublunar Earth, Water Air, Fire Heavens		Celestial motion Chain of Prime mover, Hierarch, Saviour

**Overthrowing
 Revolution**

Newtonian Cosmos			
Infinite R? Infinite T? No center	Atoms, Void Ether?	Deterministic mechanics Universal gravitation	Clockmaker

But since Newton, all scientific revolutions in physics have been encompassing revolutions



Encompassing Revolutions

Einstein's Theory of Relativity makes the same predictions for motion as Newton's theory whenever the speeds are slow compared to the speed of light, and gravity is not too strong. All the planets in the solar system move very slowly compared to the speed of light, and gravity is relatively weak. Therefore relativity makes the same predictions for the motions of the planets as Newton's theory, except for some very small differences in the motions of the fastest planets that orbit closest to the sun. Relativity encompasses Newtonian physics.

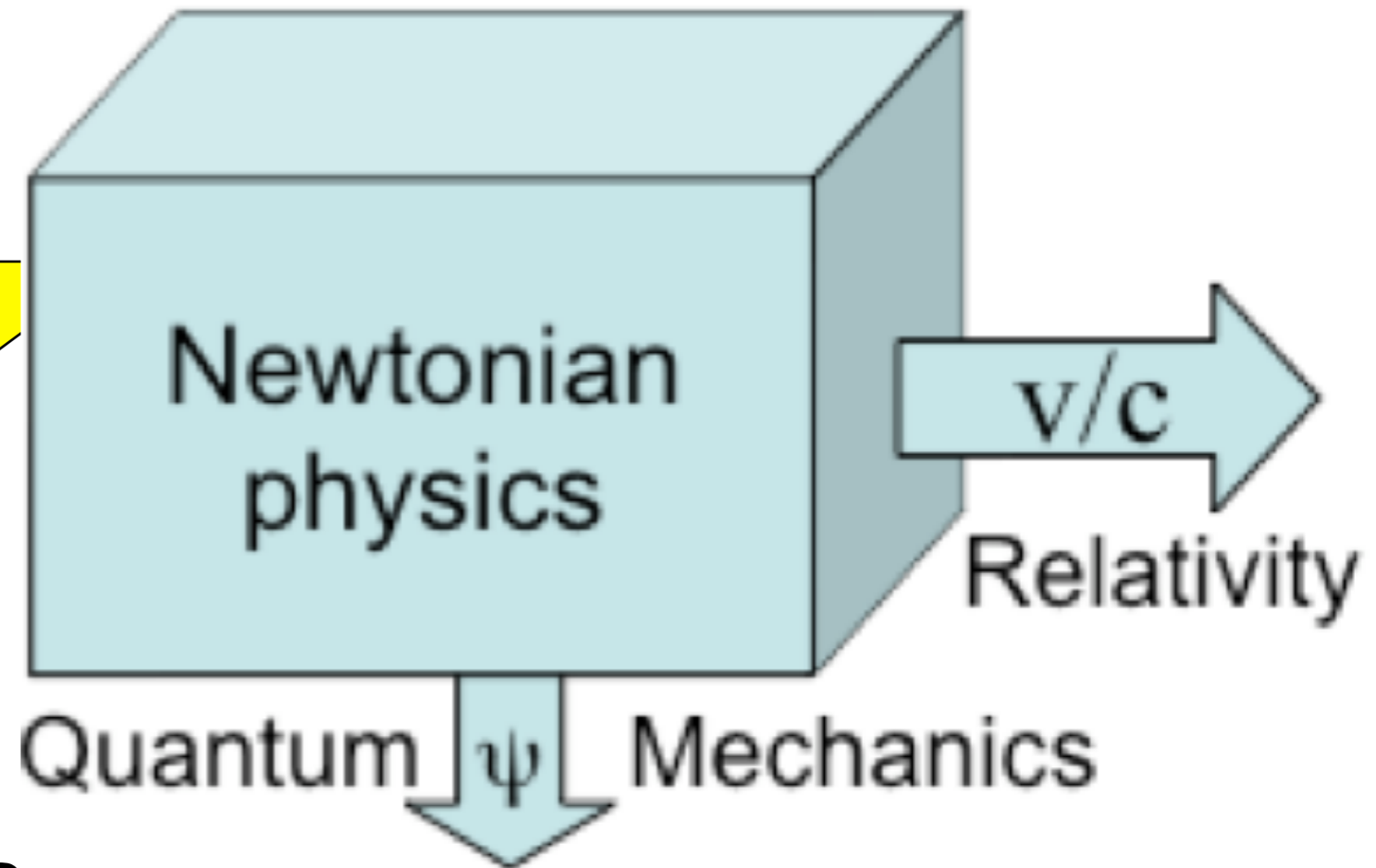
A new scientific theory ENCOMPASSES an old one when the new theory is valid for a wider class of phenomena, and the two theories make the same predictions (to some specified accuracy) for a significant range of phenomena. (We can then say that the new theory **reduces** to the old one for a subset of these phenomena.)

Newtonian Cosmos

Deterministic mechanics

Universal gravitation

**Encompassing
Revolution**



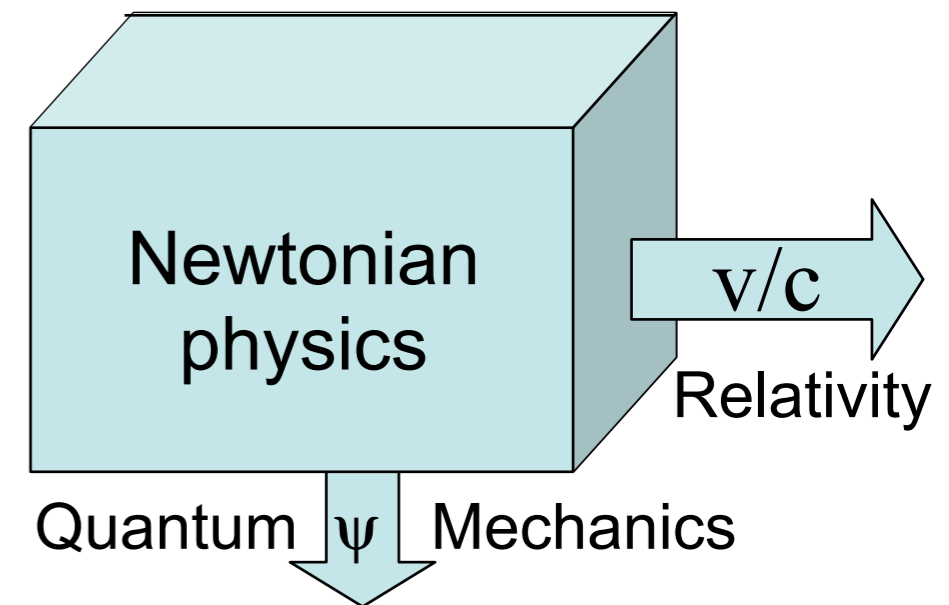
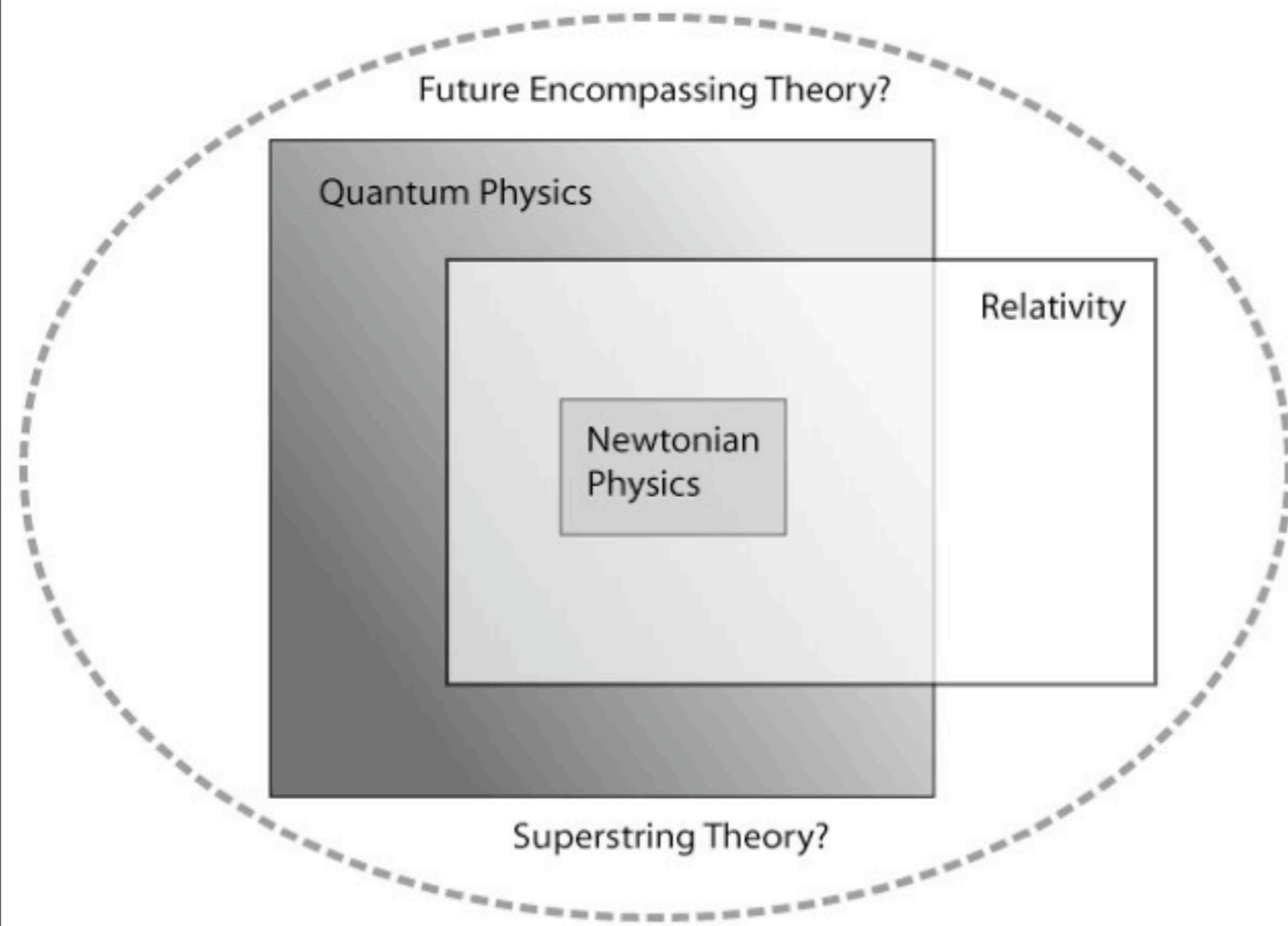
21st Century Cosmos

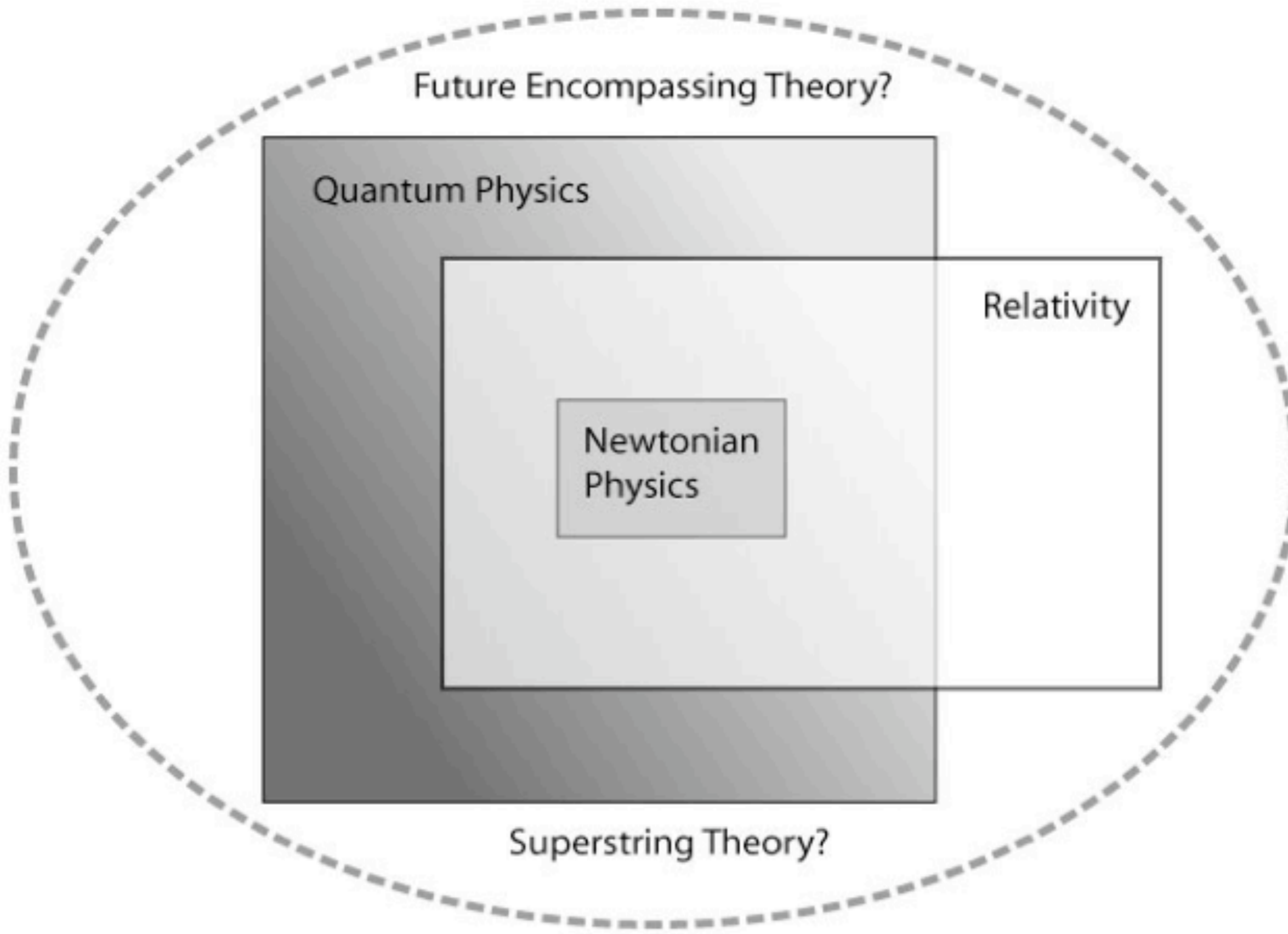
Nondeterministic quantum mechanics

Gravity = space-time curvature

Evolution

Encompassing Scientific Revolutions





Future Encompassing Theory?

Quantum Physics

Relativity

Newtonian
Physics

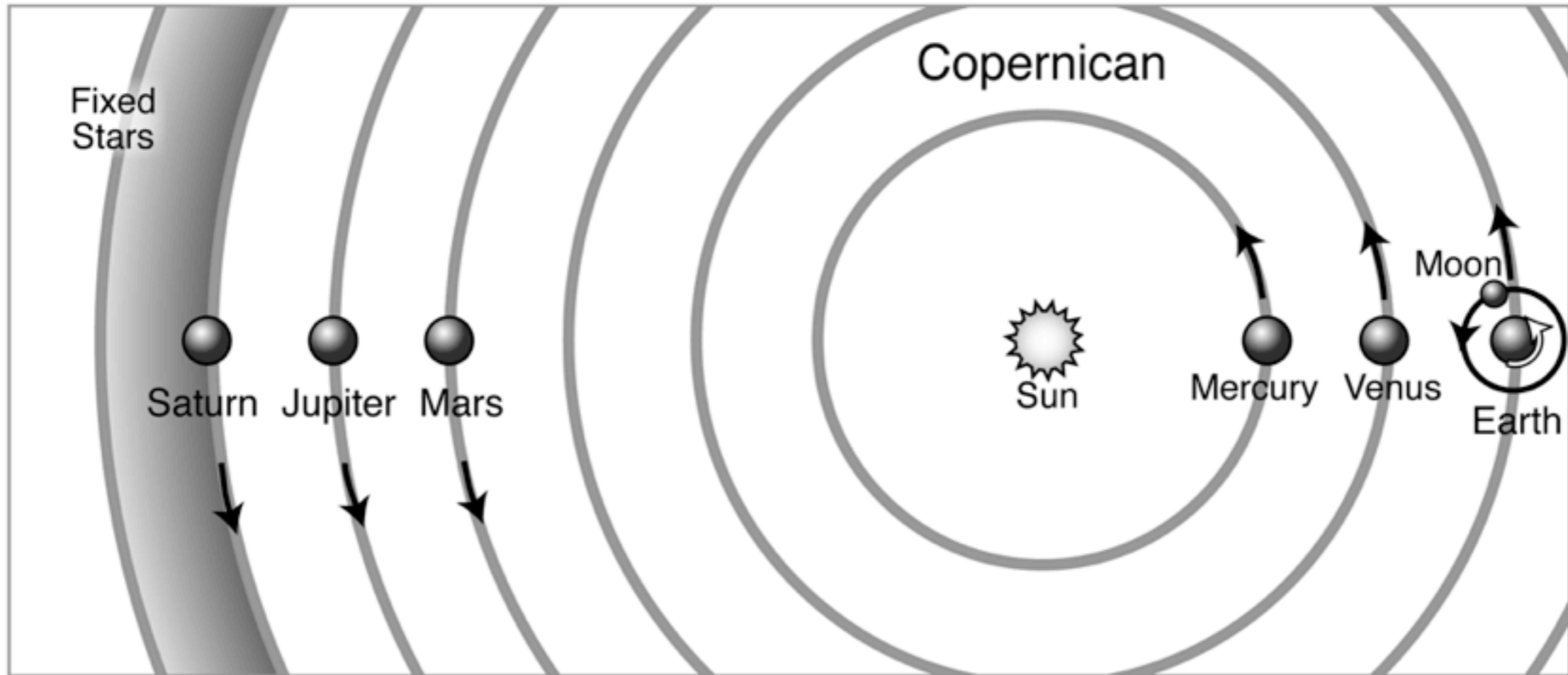
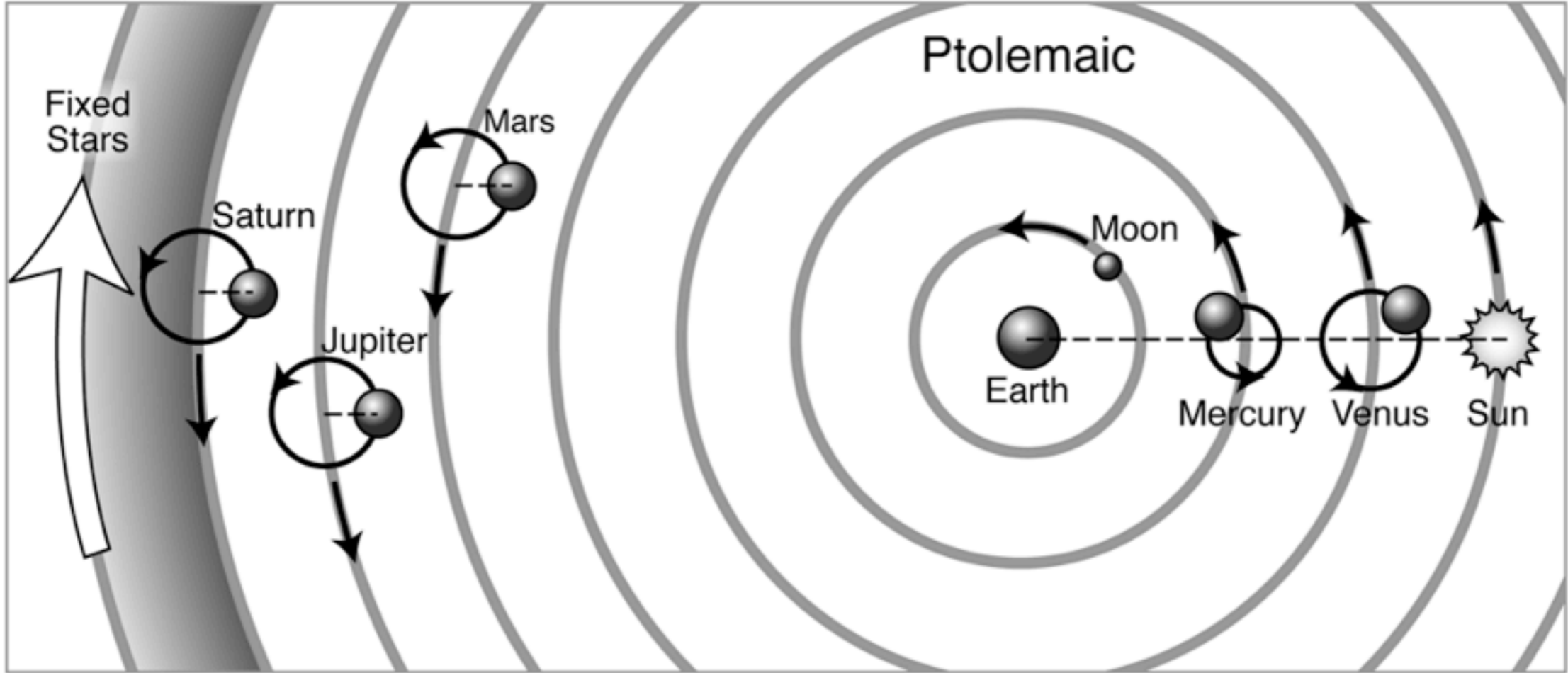
Superstring Theory?

The highest grade of truth

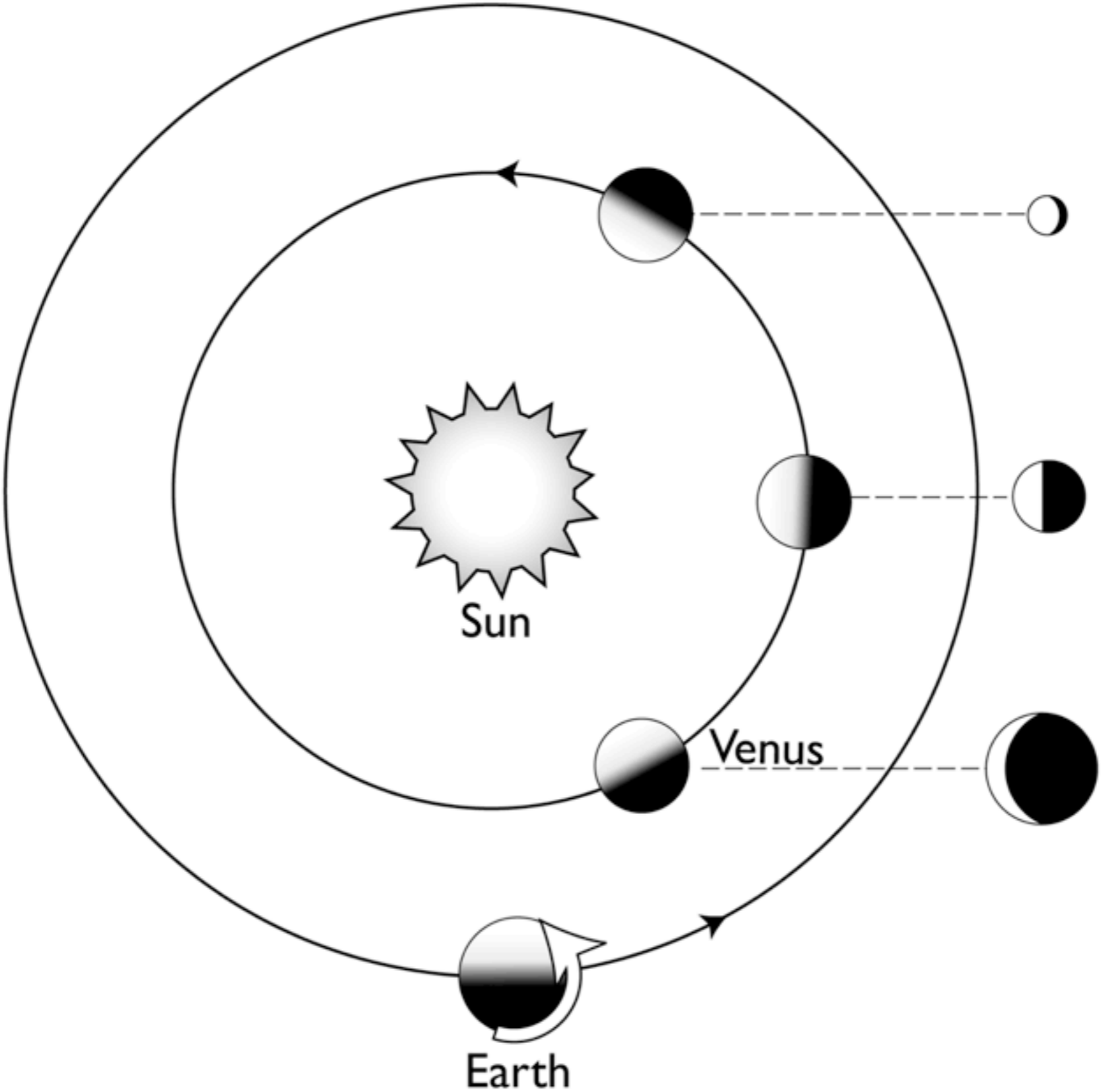
Charles Misner has pointed out a deep insight about scientific truth: the only sort of theory we can know to be “true” is one which has been shown to be false – in the sense that its limitations are known. As philosophers of science from Hume to Popper have emphasized, we can never prove that a scientific theory is true, since there is always the possibility that new data will be discovered that disprove it. But when a scientific theory has been encompassed by a more comprehensive theory that itself has been well tested, we can have considerable confidence that the encompassed theory is “true” within its known limits. This is the highest grade of scientific truth that is available.

The Scientific Revolution

Medieval astronomy rested on the idea that the motions observed in the heavens actually take place, and that the earth is stable. **Modern science was born in the denial of common sense.** Copernicus became dissatisfied with the Ptolemaic system because it was not geometrical enough. Galileo's observations with the telescope of the phases of Venus disproved the Ptolemaic system. They fit more smoothly into the heliocentric system, but they did not prove that it was true. Kepler's three laws of planetary motion further showed how economically the motion of the planets could be described in the heliocentric system, but if the Earth really were rotating on its axis and hurtling through space, why do we not perceive the motion? Galileo and Descartes tried to answer this question by reformulating the theory of motion, so that not motion but *change* in motion was what must be accounted for. Newton completed this reformulation.



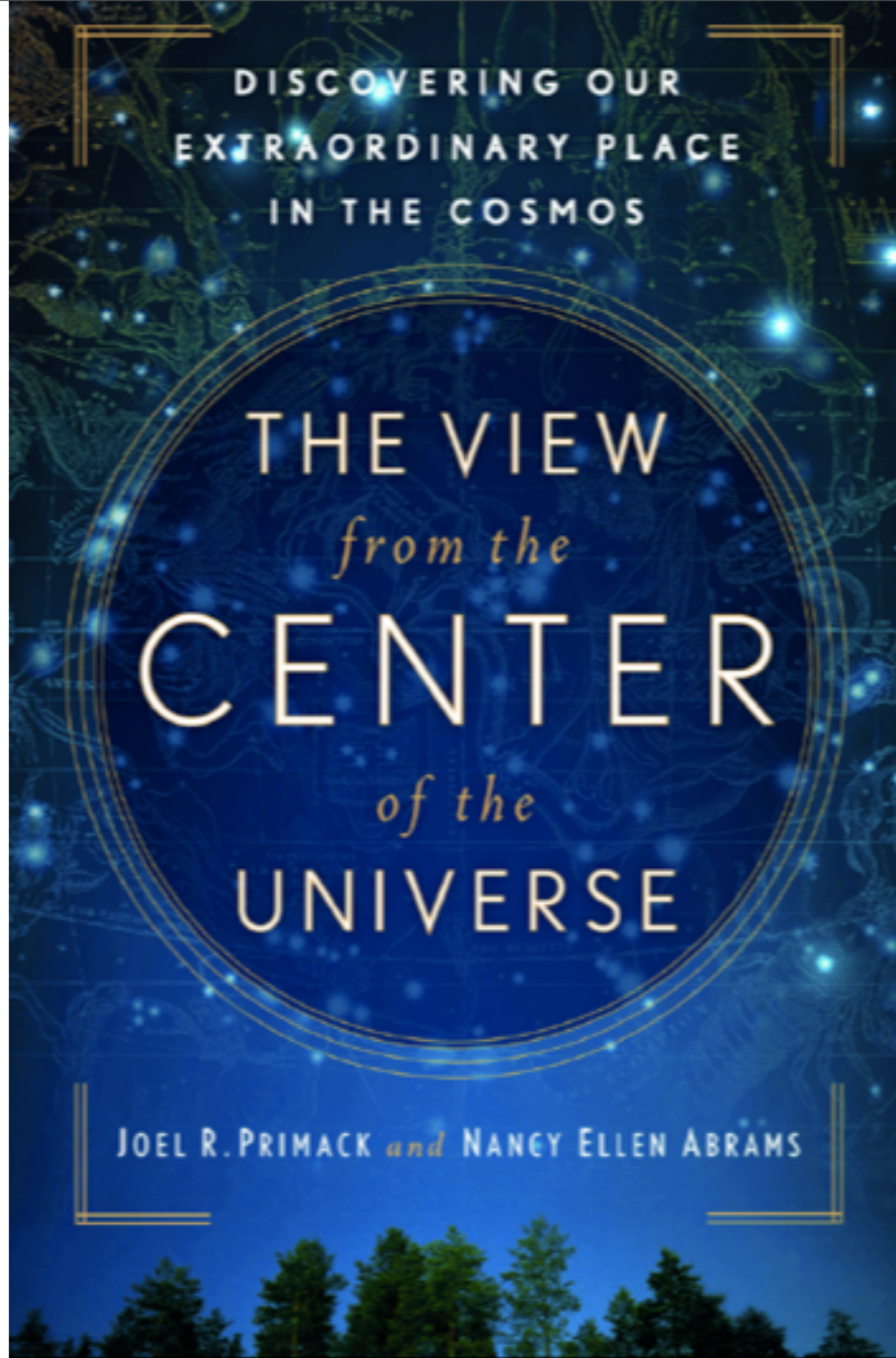
Venus as Seen from Earth



Newton and the Scientific Revolution

Isaac Newton was born in 1642, and by 1666 he had invented calculus and begun his development of mechanics. But chemistry and alchemy dominated his interests until 1684, when correspondence with Robert Hooke and Edmund Halley's question about the orbit of an object moving under the influence of gravity as an inverse-square-law force drew Newton back to mechanics. Newton's *Mathematical Principles of Natural Philosophy*, published in 1687, was immediately recognized as a monumental work – perhaps the most influential in the entire history of science. By describing the motion of all bodies, on Earth and in the heavens, as arising from the same principles, he simultaneously provided a unified conception of nature and an answer to all the objections to the heliocentric system. No one could fail to be impressed by the astounding precision with which Newton's system predicted such phenomena as the tides and the motions of the planets and comets.

This material is from *The View from the Center of the Universe*, a 2006 book that grew out of our teaching Cosmology and Culture at UCSC since 1993.

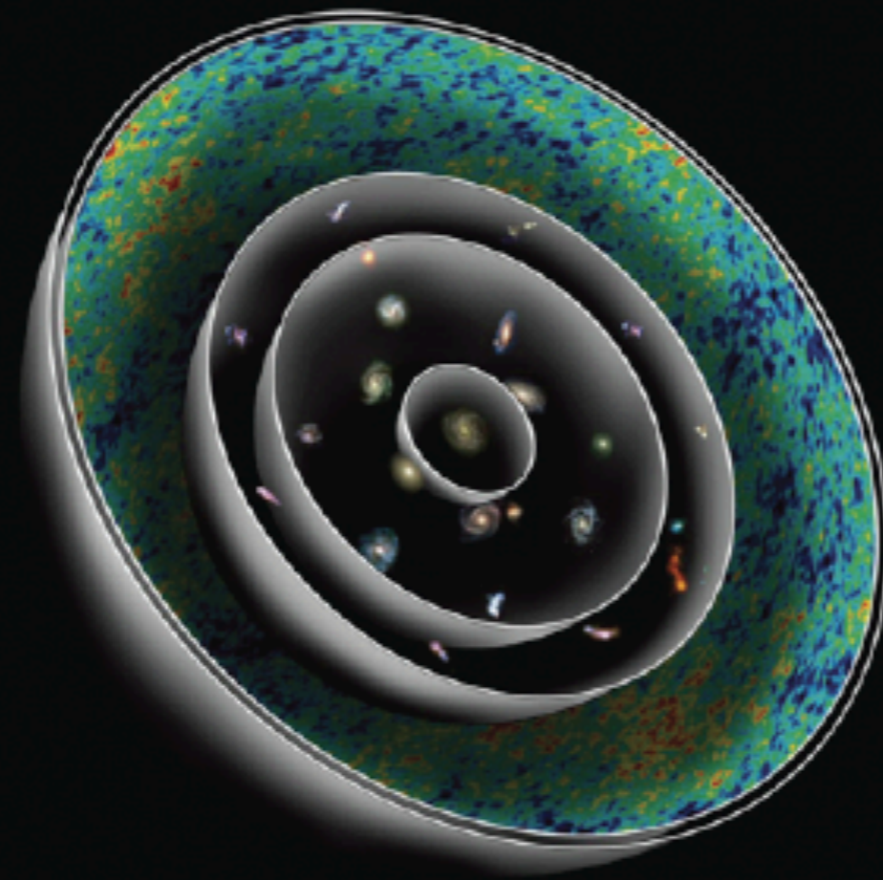


It's available at the library and bookstores -- and probably more cheaply at Amazon.com

See also

[http://
viewfromthecenter.com](http://viewfromthecenter.com)

Our new book
*The New
Universe and
the Human
Future* is an
updated
version with
beautiful
color
illustrations.



THE NEW UNIVERSE AND THE HUMAN FUTURE

How a Shared Cosmology Could Transform the World

NANCY ELLEN ABRAMS AND JOEL R. PRIMACK

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illustrations and
many great
astronomical
videos are on
our new
website
[http://new-
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(The videos are
linked to the
Kindle and
Nook electronic
editions.)

Galileo's Inclined Plane Experiment

Using equipment that we might now call simple or even rudimentary, Galileo revolutionized basic scientific principles which were posited by Aristotle and held firmly by scholars of the High Middle Ages and Renaissance. One of his most important experiments was the inclined plane experiment. Galileo used his inclined plane, a simple board with a groove down which he rolled a small metal ball, to examine Aristotelian ideas about motion. Galileo's inclined plane experiment radically changed these ideas by concentrating on acceleration, a stage of motion ignored by Aristotle and most of his followers.

We decided to replicate Galileo's inclined plane experiment because it was so fundamental to new concepts of motion in Galileo's time. We based our experiment on Galileo's own description of the inclined plane in his book [*Discourses on Two New Sciences*](#) (1638):

A piece of wooden moulding or scantling, about 12 cubits [about 7 m] long, half a cubit [about 30 cm] wide and three finger-breadths [about 5 cm] thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball.

Our own construction entailed planing at a 45 degree angle one edge each on two 16-foot two by fours, which when nailed together formed a groove. We sanded and oiled the groove to create a low-friction effect like Galileo's parchment. Instead of a small bronze ball, we used a three-quarter inch steel ball bearing. We added a metal piece to the end of the inclined plane, against which the ball struck at the end of each run, to make our timing precise.

We also replicated Galileo's apparatus for timing the inclined plane experiment. Galileo describes his water clock in *Discourses on Two New Sciences* (1638):

For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent... the water thus collected was weighed, after each descent, on a very accurate balance; the difference and ratios of these weights gave us the differences and ratios of the times...

Our water clock consisted of a plastic bucket with a small hole drilled in the bottom, into which we placed a length of plastic tubing. When filled with water the bucket emitted a thin stream of water through the plastic tubing. We controlled the flow of water by clamping the tubing with a small metal clamp.

We marked our inclined plane at one quarter, one half, and three quarters its length. Starting with the full length of the plane, we rolled the ball twenty times down each length, timing each trial with our water clock. Like Galileo, we weighed the water from each trial so as to determine the ratio of times for each length.

Our experiment proved that Galileo could have attained the accuracy which he claimed for this experiment. Our findings also point clearly to the concept of acceleration: the ball travels down one quarter of the plane in half the time it takes to traverse the entire plane. Aristotle would have posited, of course, that the ball's time would be directly proportional to the distance it traveled.

[Experiment Group Home Page](#)



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Vector calculus

From Wikipedia, the free encyclopedia

Vector calculus (or **vector analysis**) is a branch of [mathematics](#) concerned with [differentiation](#) and [integration](#) of [vector fields](#), primarily in 3 dimensional [Euclidean space](#) \mathbf{R}^3 . The term "vector calculus" is sometimes used as a synonym for the broader subject of [multivariable calculus](#), which includes vector calculus as well as [partial differentiation](#) and [multiple integration](#). Vector calculus plays an important role in [differential geometry](#) and in the study of [partial differential equations](#). It is used extensively in [physics](#) and [engineering](#), especially in the description of [electromagnetic fields](#), [gravitational fields](#) and [fluid flow](#).

Vector calculus was developed from [quaternion](#) analysis by [J. Willard Gibbs](#) and [Oliver Heaviside](#) near the end of the 19th century, and most of the notation and terminology was established by Gibbs and [Edwin Bidwell Wilson](#) in their 1901 book, *Vector Analysis*. In the traditional form using cross products, vector calculus does not generalize to higher dimensions, while the alternative approach of [geometric algebra](#), which uses [exterior products](#) does generalize, as [discussed below](#).

[Contents](#) [\[show\]](#)

Basic objects

[\[edit\]](#)

The basic objects in vector calculus are [scalar fields](#) (scalar-valued functions) and [vector fields](#) (vector-valued functions). These are then

Topics in Calculus

[Fundamental theorem](#)
[Limits of functions](#)
[Continuity](#)
[Mean value theorem](#)

[Differential calculus](#) [\[show\]](#)

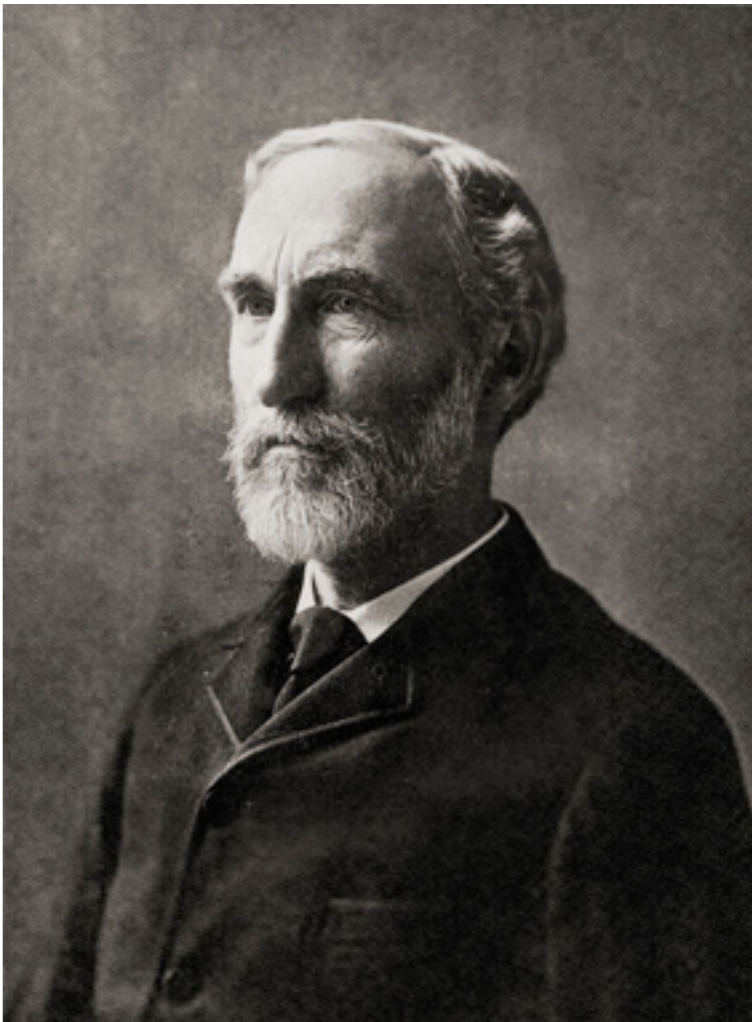
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[Vector calculus](#) [\[hide\]](#)

[Gradient](#)
[Divergence](#)
[Curl](#)
[Laplacian](#)
[Gradient theorem](#)
[Green's theorem](#)
[Stokes' theorem](#)
[Divergence theorem](#)

[Multivariable calculus](#) [\[show\]](#)

Josiah Willard Gibbs (February 11, 1839 – April 28, 1903) was an American theoretical [physicist](#), [chemist](#), and [mathematician](#). He devised much of the theoretical foundation for [chemical thermodynamics](#) as well as [physical chemistry](#). As a [mathematician](#), he invented [vector analysis](#) (independently of [Oliver Heaviside](#)). [Yale University](#) awarded Gibbs the first American [Ph.D.](#) in [engineering](#) in 1863, and he spent his entire career at Yale.



In 1901, Gibbs was awarded the highest possible honor granted by the international scientific community of his day, granted to only one scientist each year: the [Copley Medal](#) of the [Royal Society of London](#), for his greatest contribution, that being "the first to apply the [second law of thermodynamics](#) to the exhaustive discussion of the relation between chemical, electrical, and thermal energy and capacity for external work."

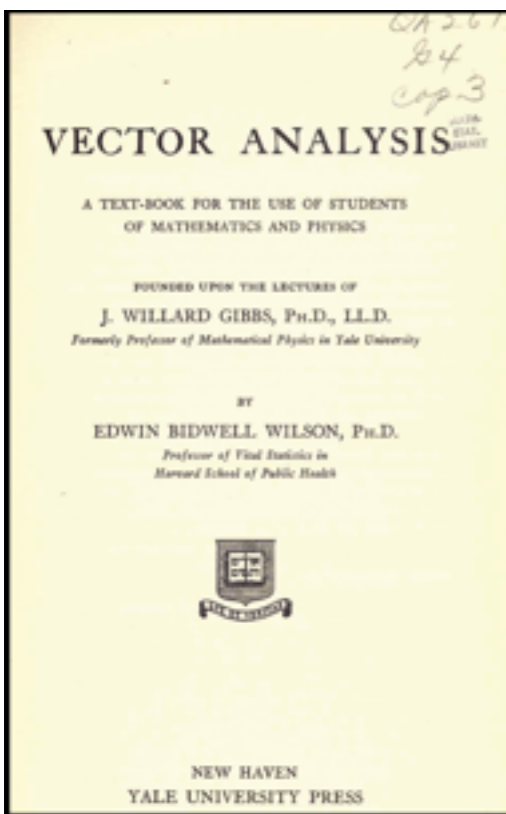


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