2012 Physics 5K - Homework 1 - Solutions

1. (10 points) Giancoli 23-63 – Nuclear fission.

The two fragments can be treated as point charges for purposes of calculating their potential energy. Use Eq. 23-10 to calculate the potential energy. Using energy conservation, the potential energy is all converted to kinetic energy as the two fragments separate to a large distance.

$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad U_{\text{initial}} = K_{\text{final}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} V$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(38)(54)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(5.5\times10^{-15}\,\text{m}\right)+\left(6.2\times10^{-15}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 250\times10^6\,\text{eV}$$

$$= \boxed{250\,\text{MeV}}$$

This is about 25% greater than the observed kinetic energy of 200 MeV.

2. (5 points) Giancoli 23-75 – Photoelectric effect.

The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$\text{KE}_{\text{initial}} = \text{PE}_{\text{final}} \quad \rightarrow \quad \tfrac{1}{2}mv^2 = qV \quad \rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2\left(-1.60\times10^{-19}\,\text{C}\right)\left(-3.02\,\text{V}\right)}{9.11\times10^{-31}\,\text{kg}}} = \boxed{1.03\times10^6\ \text{m/s}}$$

3. (10 points) Giancoli 23-83 – Geiger counter.

From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$V_a - V_b = -\int_{R_b}^{R_a} \vec{\mathbf{E}}\cdot\left(d\vec{\boldsymbol{\ell}}\right) = -\int_{R_b}^{R_a} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}dR = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(R_a - R_b\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_b}{R_a}}$$

Nuclear fusion. A fusion reaction that plays a role in energy production in the sun involves capture of a proton by a carbon nucleus, which has six times the charge of the proton and a radius $r_0 \approx 2 \times 10^{-15}$ m.

4. (10 points) Estimate the Coulomb potential V experienced by the proton if it is at the surface of the carbon nucleus.

Answer: $V = (Ze^2)/(4\pi\varepsilon_0 r_0) = (9\text{x}10^9)\,(6)\,(1.6\text{x}10^{-19})^2 / (2\text{x}10^{-15}\text{ m}) = 6.9\text{x}10^{-13}\text{ J} / (1.6\text{x}10^{-13}\text{ J/MeV})$
$= \underline{4.3\text{ MeV}}$

5. (10 points) The proton is incident upon the carbon nucleus because of its thermal motion. Its kinetic energy cannot be much higher than about 10 kT, where k is Boltzmann's constant ($k = 1.38 \times 10^{-23}$ J/K) and $T \approx 10^7$ K is the temperature near the center of the sun. Estimate this kinetic energy and compare it with the height of the Coulomb barrier.

Answer: $E = 10\text{ kT} = (10)\,(1.38\text{x}10^{-23})\,(10^7)\text{ J} = 1.38\text{x}10^{-15}\text{ J} = 8.63\text{x}10^{-3}\text{ MeV} = \underline{0.002\text{ V}}$.

6. (10 points) Calculate the probability that the proton of kinetic energy 10 kT (as in problem 5) can penetrate the Coulomb barrier $V_{Coul}(r)$. Assume for simplicity that the barrier is of constant height $V = V_{Coul}(r0)$ and extends from r0 to r1, where r1 is the radius where the Coulomb potential drops to V/2 – that is, VCoul (r1) = V/2. [Note: $\hbar = 6.58 \times 10^{-22}$ MeV-sec, and the mass of the proton m times $c^2 = mc^2 = 938$ MeV.]

Answer: Since the potential falls off as r^{-1}, $r_1 = 2\, r_0$. Thus the width of the simplified rectangular barrier is $r_1 - r_0 = r_0$, and the height is V. The wavefunction for barrier penetration is then

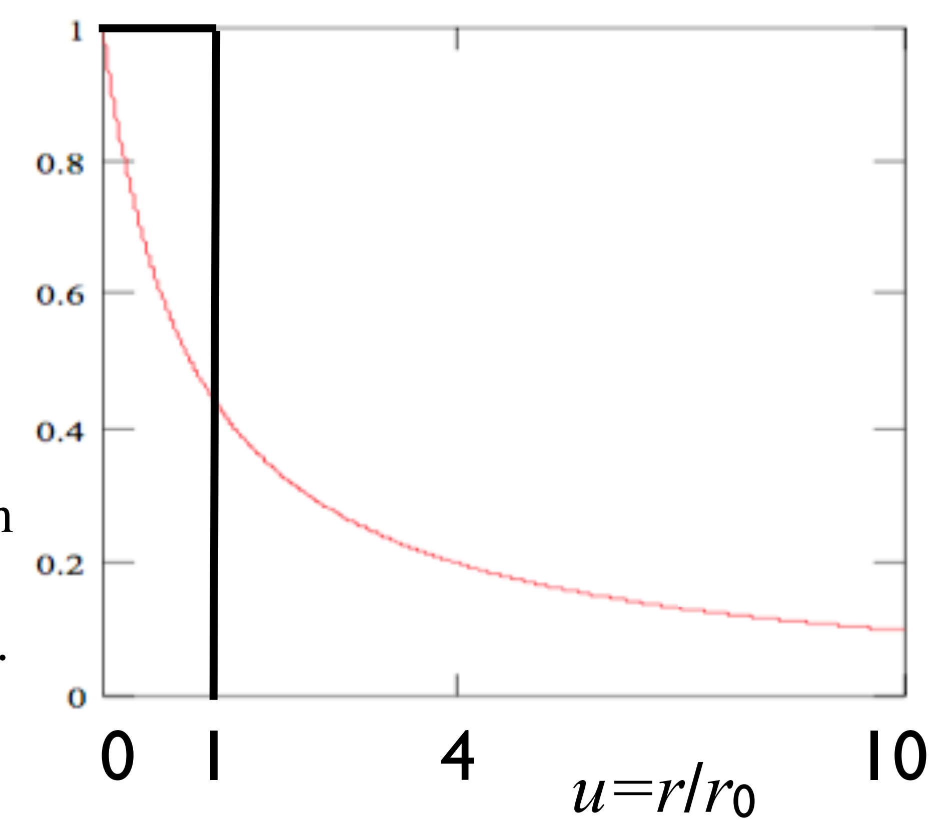
$$\psi = \exp[-(1/\hbar)\int\sqrt{2m(V-E)}\, dx] = \exp[-(1/\hbar)\sqrt{2m(V-E)}\, r_0\,] = \exp(-\,0.91)\ ,$$

so the probability of barrier penetration $|\psi|^2 = \exp(-1.82) = \underline{0.16}$.

7. (10 points) Is the penetration through the actual Coulomb barrier potential greater or less than through the rectangular barrier that we considered for simplicity in problem 6? Explain your reasoning with a diagram or calculation.

Answer: The way to answer the problem accurately is by doing the integral in problem 6 numerically. The above expression for ψ can easily be shown to be $\psi = \exp(-\,0.91\ I)$ where (setting $u = r/r_0$ and evaluating the integral numerically)

$$I = \int_1^{500} \sqrt{\frac{1}{u} - 0.002}\, du = 33.1$$



Simpler answer: The red curve on the diagram at right shows the integrand from u = 1 to 10, and it is already clear that the area under the integrand from u= 2 to 10 is larger than the area from 1 to 2 calculated in the approximation suggested in problem 6. Thus it's hardly surprising that integrating u all the way to 500 gives the much larger answer above. The approximation in problem 6 seriously underestimates the correct answer, thus overestimates the chance of barrier penetration.