## Homework 4

*Due Friday June 1 in Class*

The following problems concern relativity and particles moving in magnetic fields. We discussed relativity in class May 25, including using the Einstein's Rocket website (http://physics.ucsc.edu/~snof/er.html). We showed that when an object is moving at velocity $v$, its length is contracted in the direction of motion by a factor $\gamma = (1 - v^2/c^2)^{-1/2}$, its clocks are slowed by the same factor $\gamma$, and its momentum and energy are increased by the same factor $\gamma$. Thus the momentum and energy of a particle of rest mass $m$ are given by $p = mv\gamma$ and $E = mc^2\gamma$ .

1. (15 points) An electron is in the ground ($n = 1$) state of Bohr's model of a hydrogen atom.

(a) Find the velocity $v$ of the electron in the $n = 1$ state of the Bohr atom in terms of its mass $m_e$, the speed of light $c$, and the fine structure constant $\alpha \equiv e^2/(4\pi\varepsilon_0\hbar c) = 1/137.036$ .

(b) Find the numerical value of the electron's velocity $v$ in m/s.

(c) Determine the fractional error that you make if you use the classical expression for the kinetic energy $K = \frac{1}{2} m_e v^2$ rather than the relativistic expression $K = m_e c^2 (\gamma - 1)$.

2. (10 points) (a) Show that the speed $v$ of a particle of mass $m$ and energy $E$ is given by $v/c = [1 - (mc^2/E)^2]^{1/2}$ and that, if $E$ is much greater than $mc^2$, it is a good approximation to take $v/c = 1 - \frac{1}{2}(mc^2/E)^2$.

(b) Find the speed of an electron of kinetic energy $K = 0.511$ MeV and that of an electron of kinetic energy $K = 10$ MeV. (Note: the rest energy of an electron $m_e c^2 = 0.511$ MeV.)

3. (15 points) A muon is almost identical to an electron except for its mass, which is 105.7 MeV/$c^2$, and the fact that it is unstable, with a lifetime of about 2.2 microseconds. Suppose that muons of energy 10 GeV are produced when cosmic rays strike nitrogen nuclei at an altitude of 35 km, and that the muons are traveling straight downward.

(a) If we neglect relativistic time dilation (the relativistic slowing of moving clocks), how far would the muons go in 2.2 microseconds?

(b) Now taking into account time dilation, calculate how far the muon will go in 2.2 microseconds.

(c) Calculate the fraction of the muons produced at 35 km altitude that reach sea level.

4. (15 points) If it is found that charged particles of energy 100 TeV are coming from a particular direction in space, how far away can their source be so that they are not deflected by more than about 10 degrees? Assume that the galactic magnetic field in which they move is 3 microgauss, and express your estimate of the distance in both meters and parsecs (pc).

5. (15 points) An alternative explanation for the charged particles of 100 TeV energy that come from the same direction is that they actually come from the decay of neutrons with 200 TeV energy, since neutrons are neutral and therefore not deflected by magnetic fields. The lifetime of a free neutron is $\tau = 885$ seconds. How far would such neutrons travel before only 1/8 of the original neutrons are left?

6. (10 points) Draw a diagram of charged particles spiraling around lines of magnetic field and drifting toward a region where the strength of the magnetic field is increasing (so that the field lines are getting closer together). Explain using the diagram why the charged particle drift direction reverses if the field becomes sufficiently strong. (This theory applies to the charged particles trapped in the earth's Van Allen radiation belts, since the magnetic field gets stronger as the particles approach the north and south magnetic poles.)