Physics 5K Lecture Friday May 11, 2012

Radioactivity, Radioactive Dating, Band Theory of Solids, and Quantum Electronics Devices such as LEDs

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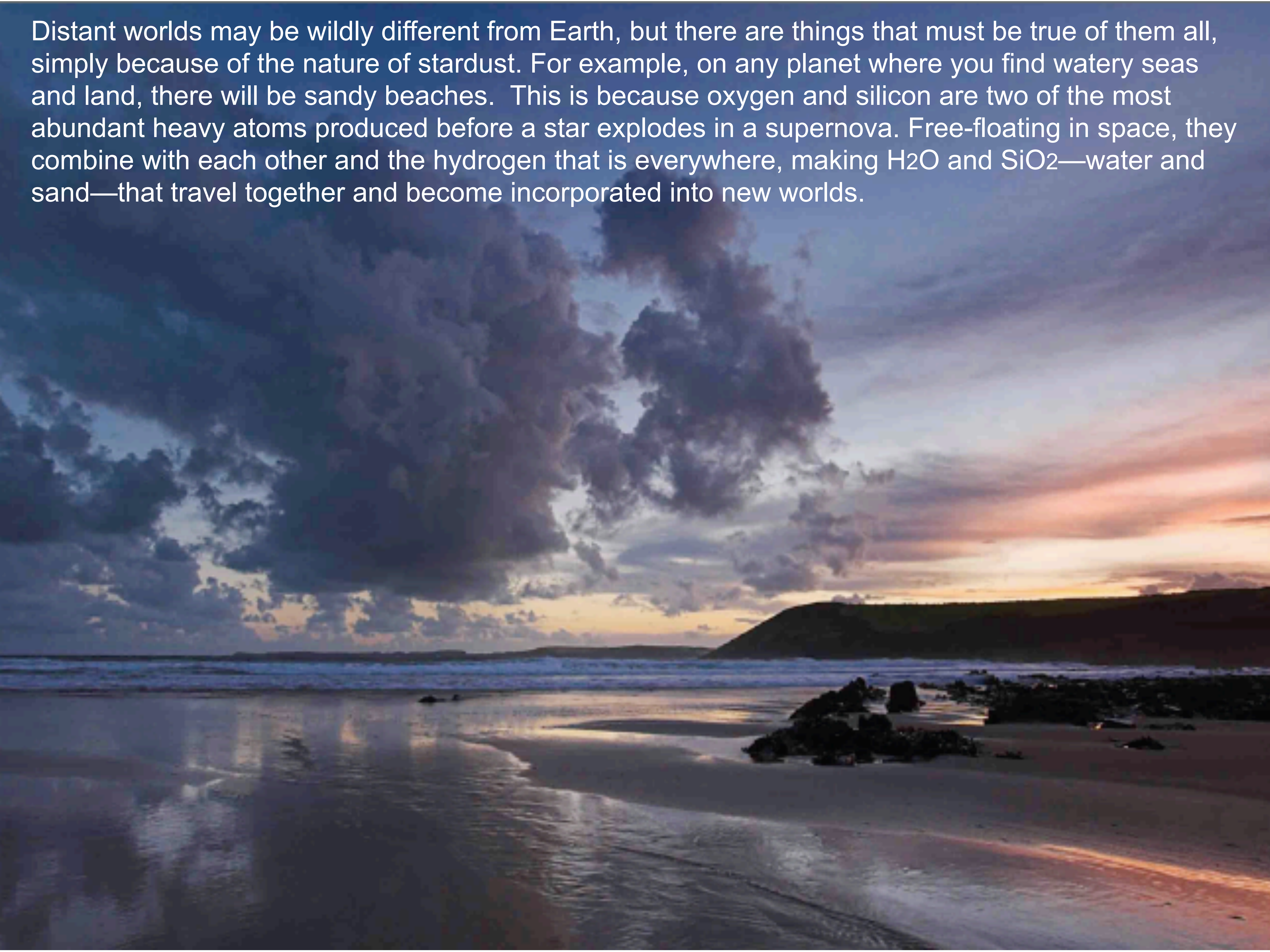
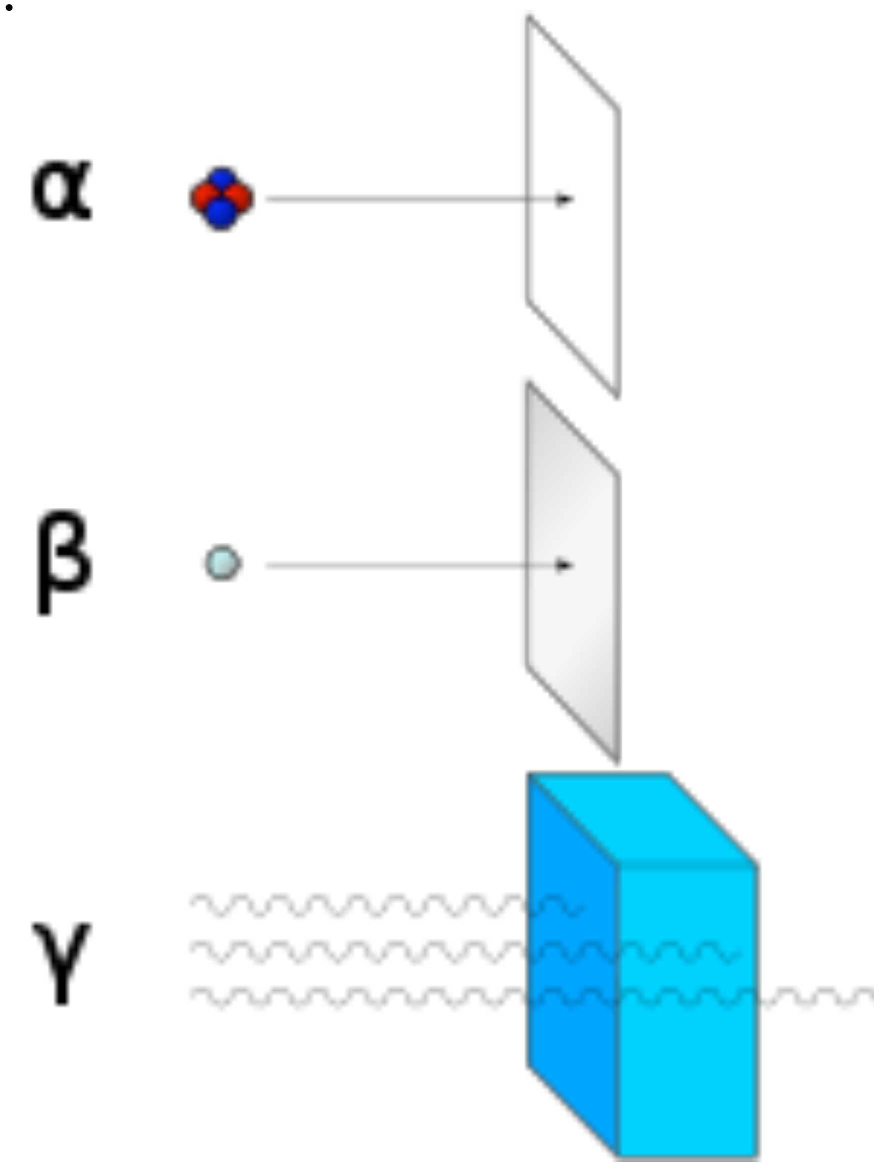
Distant worlds may be wildly different from Earth, but there are things that must be true of them all, simply because of the nature of stardust. For example, on any planet where you find watery seas and land, there will be sandy beaches. This is because oxygen and silicon are two of the most abundant heavy atoms produced before a star explodes in a supernova. Free-floating in space, they combine with each other and the hydrogen that is everywhere, making H_2O and SiO_2—water and sand—that travel together and become incorporated into new worlds.



Radioactivity

Radioactivity was first discovered in 1896 by the French scientist Henri Becquerel. At first it seemed that the new radiation was similar to the then recently discovered X-rays. Further research by Becquerel, Marie Curie, Pierre Curie, Ernest Rutherford and others discovered that radioactivity was significantly more complicated. Different types of decay can occur, but Rutherford was the first to realize that they all are described by the same exponential formula.

Rutherford also discovered the three main types of radioactivity:



Alpha particles (helium nuclei) may be completely stopped by a sheet of paper.

Beta particles (electrons or positrons) are stopped by a thin sheet of aluminum

Gamma rays can only be reduced by much more substantial barriers, such as a very thick layer of lead.

The genetic effects of radiation and the cancer risk were recognized much later. In 1927 Hermann Joseph Muller published research showing genetic effects, and in 1946 was awarded the Nobel prize for his findings.

Radioactive Decay (Review)

Lifetime τ and half-life $T_{1/2}$

$$N(t) = N_0\, e^{-t/\tau}$$

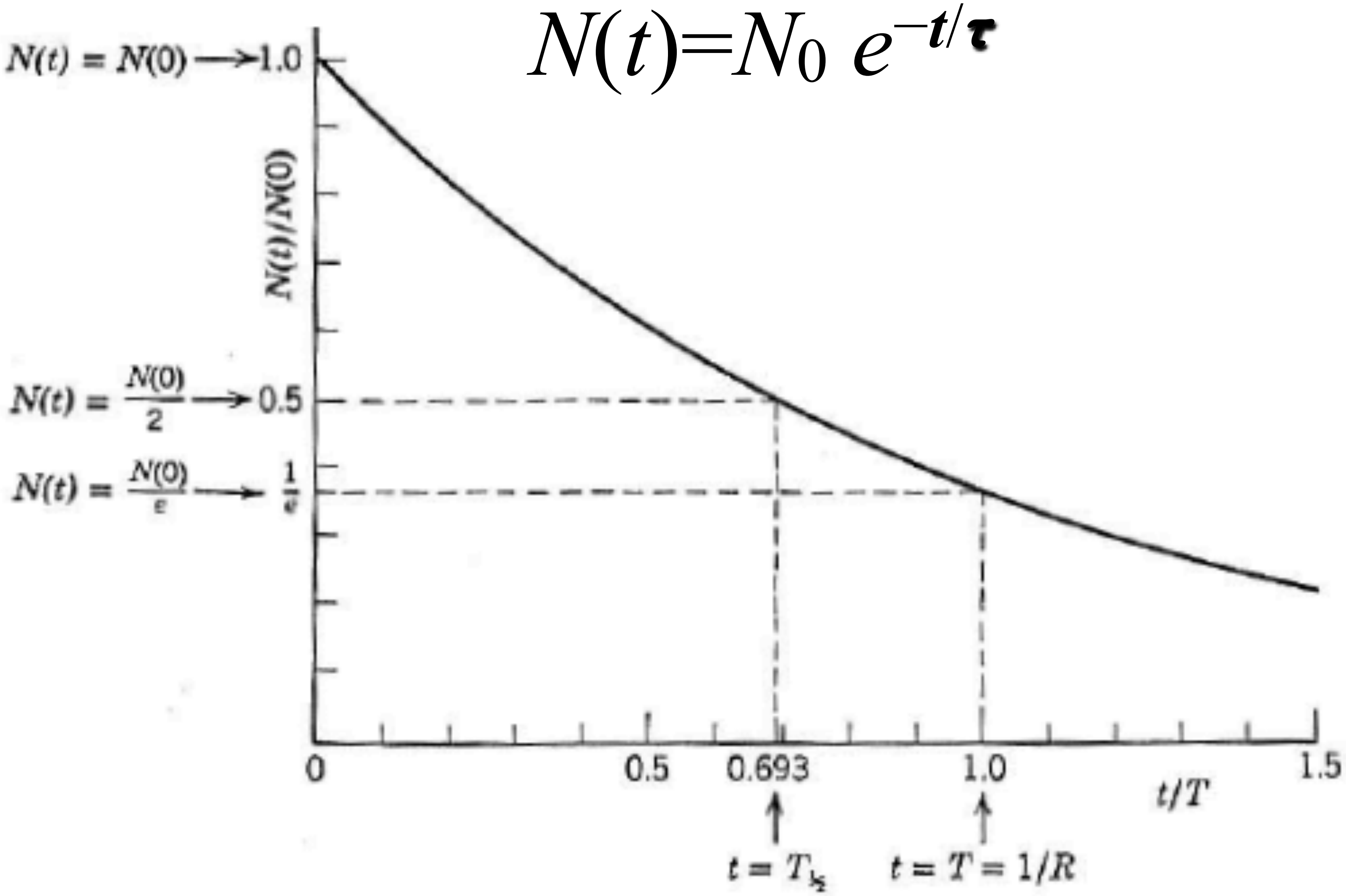


Figure 16-3 The exponential decay law for $N(t)$, the number of nuclei surviving at time t. Also shown are the lifetime T and half-life $T_{1/2}$. Note that $N(t)$ is expressed in units of the original number of nuclei $N(0)$, while time is expressed in units of the lifetime T.

Note:

$$e^{-\lambda t} = e^{-t/\tau}$$

implies that the "lifetime" $\tau = 1/\lambda$.

The relation between lifetime τ and half-life $T_{1/2}$ follows from

$$e^{-t/\tau} = 2^{-t/T_{1/2}}$$

Taking natural logs,

$$t/\tau = (t/T_{1/2}) \ln 2$$

so

$$\tau = T_{1/2} / \ln 2$$

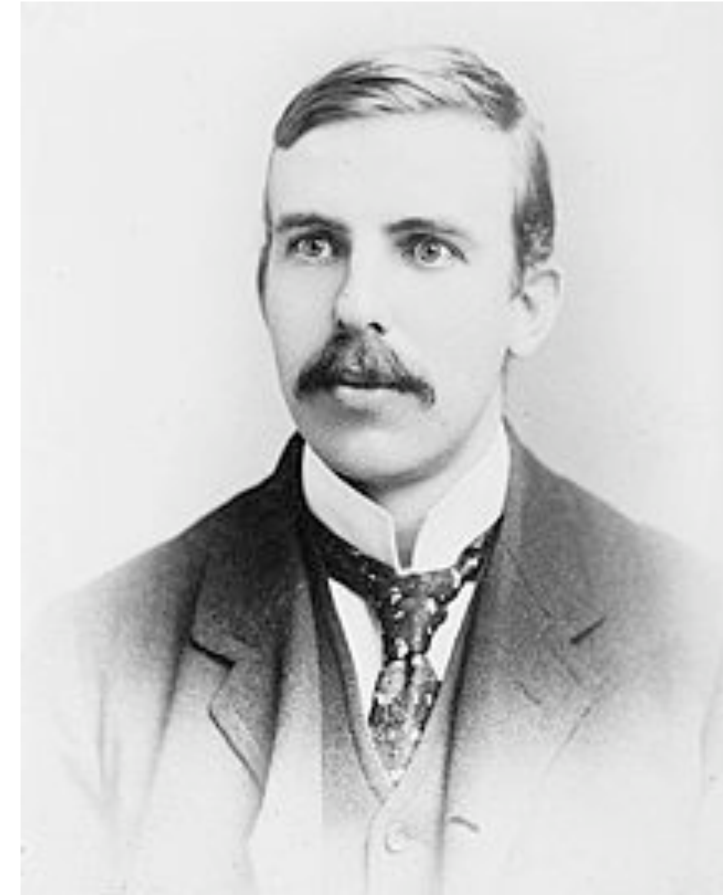
or

$$T_{1/2} = \tau \ln 2 = 0.693\, \tau$$

The figure is from Robert Eisberg and Robert Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* (Wiley, 1985).

Radioactive Dating

Geologists quickly realized that the discovery of radioactivity upset the assumptions on which most calculations of the age of Earth were based. These calculations assumed that Earth and Sun had formed at some time in the past and had been steadily cooling since that time. Radioactivity provided a process that generated heat.



Ernest Rutherford in 1908.

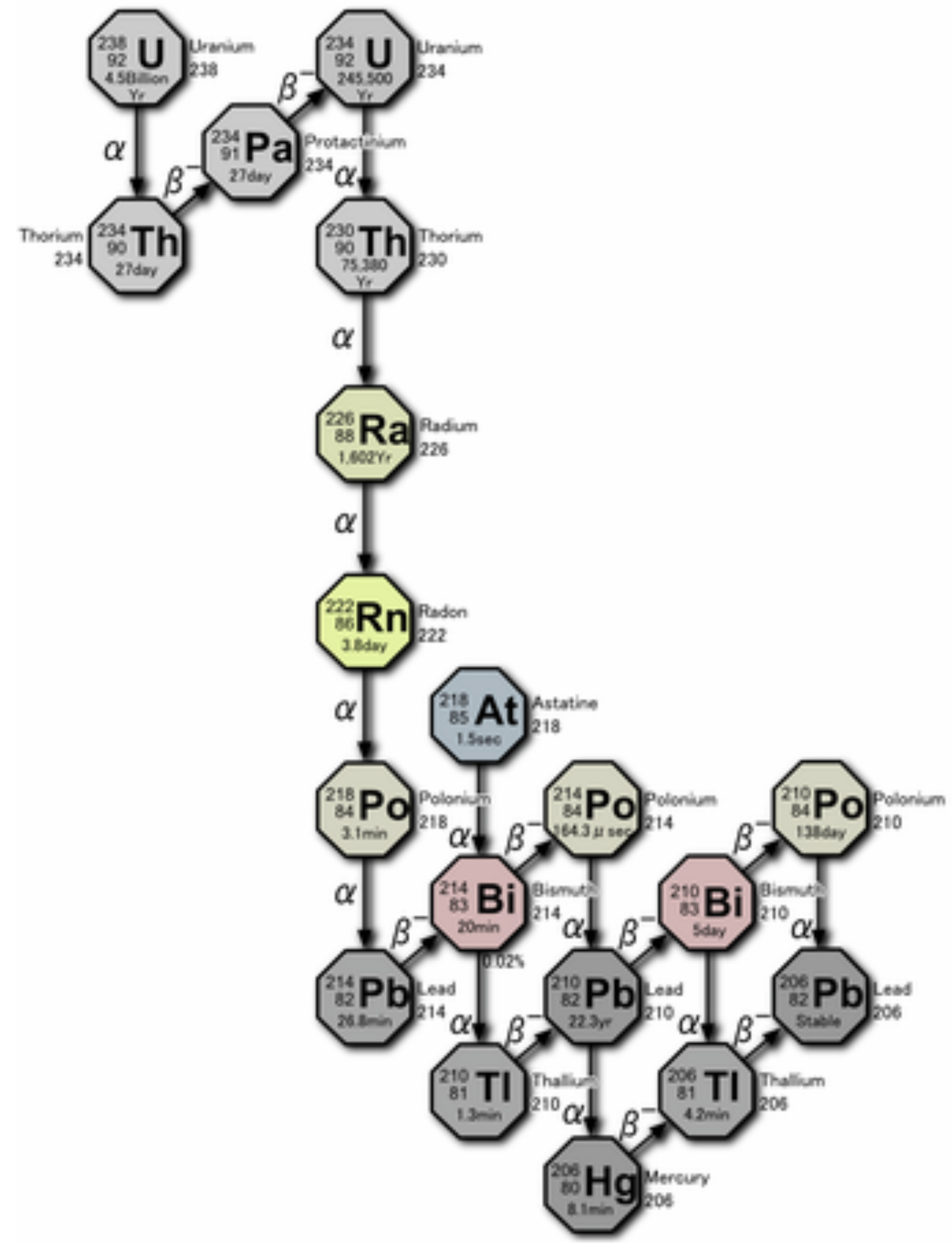
Radioactivity, which had overthrown the old calculations, yielded a bonus by providing a basis for new calculations, in the form of radiometric dating. Late in 1904, Rutherford took the first step toward radiometric dating by suggesting that the alpha particles released by radioactive decay could be trapped in a rocky material as helium atoms. He dated a rock in his possession to an age of 40 million years by this technique.

Barringer Crater, AZ, where the Canyon Diablo meteorite was found.

Radiometric dating continues to be the predominant way scientists date geologic timescales. Techniques for radioactive dating have been tested and fine-tuned for the past 50+ years. Forty or so different dating techniques are utilized to date a wide variety of materials, and dates for the same sample using these techniques are in very close agreement on the age of the material. Today's accepted age of Earth of 4.54 billion years was determined by C.C. Patterson using uranium-lead isotope dating on several meteorites including the Canyon Diablo meteorite and published in 1956. It's been confirmed by a great deal of evidence since then.

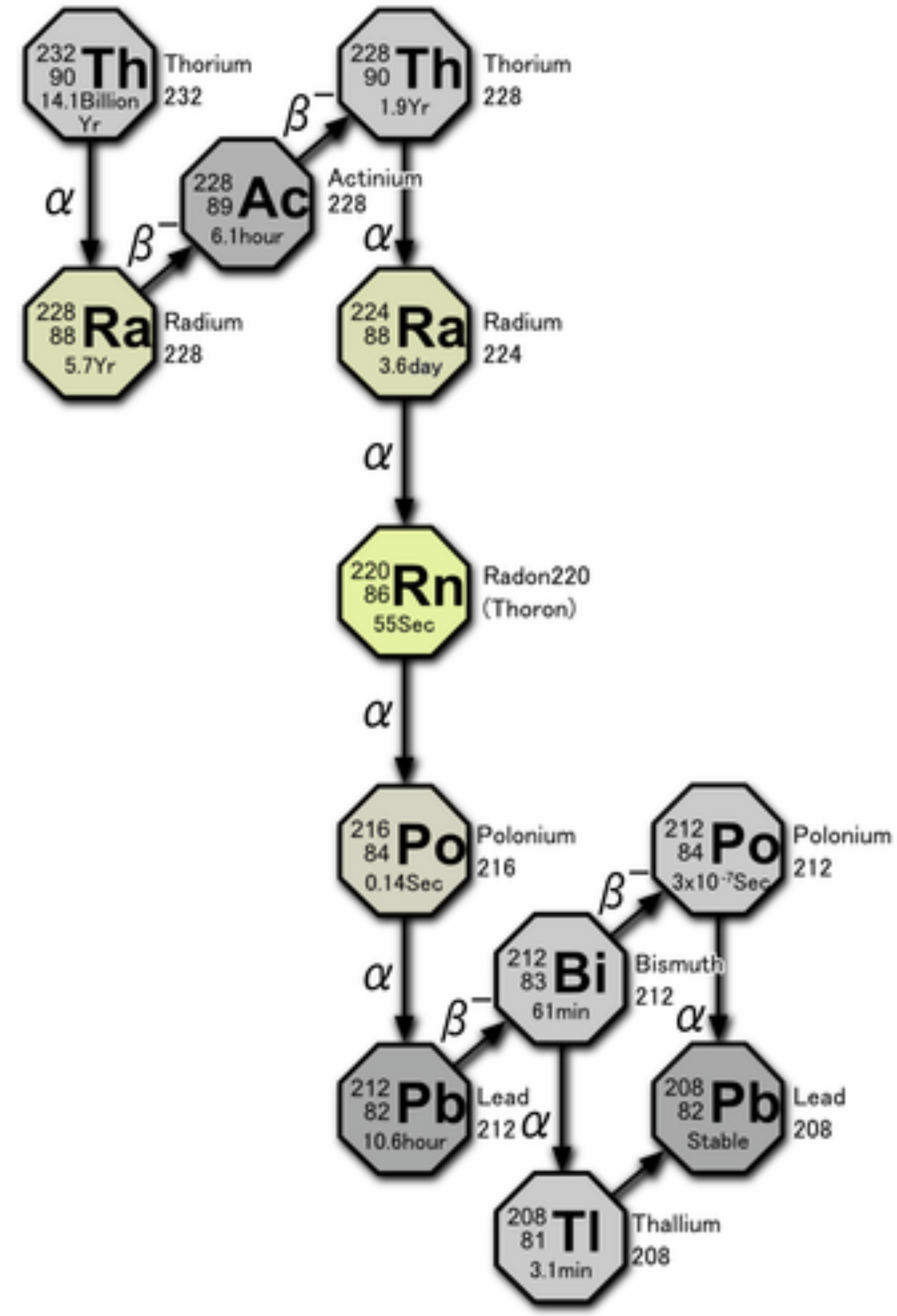
^{238}U Decay Chain

nuclide	historic name (short)	historic name (long)	decay mode	half life	MeV	product of decay
^{238}U	U	Uranium	α	$4.468 \cdot 10^{9}$ a	4.270	^{234}Th
^{234}Th	UX_1	Uranium X1	β^-	24.10 d	0.273	^{234m}Pa
^{234m}Pa	UX_2	Uranium X2	β^- 99.84 % IT 0.16 %	1.16 min	2.271 0.074	^{234}U ^{234}Pa
^{234}Pa	UZ	Uranium Z	β^-	6.70 h	2.197	^{234}U
^{234}U	U_{II}	Uranium two	α	245500 a	4.859	^{230}Th
^{230}Th	Io	Ionium	α	75380 a	4.770	^{226}Ra
^{226}Ra	Ra	Radium	α	1602 a	4.871	^{222}Rn
^{222}Rn	Rn	Radon	α	3.8235 d	5.590	^{218}Po
^{218}Po	RaA	Radium A	α 99.98 % β^- 0.02 %	3.10 min	6.115 0.265	^{214}Pb ^{218}At
^{218}At			α 99.90 % β^- 0.10 %	1.5 s	6.874 2.883	^{214}Bi ^{218}Rn
^{218}Rn			α	35 ms	7.263	^{214}Po
^{214}Pb	RaB	Radium B	β^-	26.8 min	1.024	^{214}Bi
^{214}Bi	RaC	Radium C	β^- 99.98 % α 0.02 %	19.9 min	3.272 5.617	^{214}Po ^{210}Tl
^{214}Po	RaC'	Radium C'	α	0.1643 ms	7.883	^{210}Pb
^{210}Tl	RaC"	Radium C"	β^-	1.30 min	5.484	^{210}Pb
^{210}Pb	RaD	Radium D	β^-	22.3 a	0.064	^{210}Bi
^{210}Bi	RaE	Radium E	β^- 99.99987% α 0.00013%	5.013 d	1.426 5.982	^{210}Po ^{206}Tl
^{210}Po	RaF	Radium F	α	138.376 d	5.407	^{206}Pb
^{206}Tl	RaE"	Radium E"	β^-	4.199 min	1.533	^{206}Pb
^{206}Pb			-	stable	-	-



^{232}Th Decay Chain

nuclide	historic name (short)	historic name (long)	decay mode	half life	energy released, MeV	product of decay
^{232}Th	Th	Thorium	α	$1.405 \cdot 10^{10}$ a	4.081	^{228}Ra
^{228}Ra	$MsTh_1$	Mesothorium 1	β^-	5.75 a	0.046	^{228}Ac
^{228}Ac	$MsTh_2$	Mesothorium 2	β^-	6.25 h	2.124	^{228}Th
^{228}Th	RdTh	Radiothorium	α	1.9116 a	5.520	^{224}Ra
^{224}Ra	ThX	Thorium X	α	3.6319 d	5.789	^{220}Rn
^{220}Rn	Tn	Thoron	α	55.6 s	6.404	^{216}Po
^{216}Po	ThA	Thorium A	α	0.145 s	6.906	^{212}Pb
^{212}Pb	ThB	Thorium B	β^-	10.64 h	0.570	^{212}Bi
^{212}Bi	ThC	Thorium C	β^- 64.06% α 35.94%	60.55 min	2.252 6.208	^{212}Po ^{208}Tl
^{212}Po	ThC'	Thorium C'	α	299 ns	8.955	^{208}Pb
^{208}Tl	ThC"	Thorium C"	β^-	3.053 min	4.999	^{208}Pb
^{208}Pb		.	stable	.	.	



The Age of the Universe

In the mid-1990s there was a crisis in cosmology, because the age of the old Globular Cluster stars in the Milky Way, then estimated to be 16±3 Gyr, was higher than the expansion age of the universe, which for a critical density ($\Omega_m = 1$) universe is 9±2 Gyr (with the Hubble parameter h=0.72±0.07). But when the data from the Hipparcos astrometric satellite became available in 1997, it showed that the distance to the Globular Clusters had been underestimated, which implied that their ages are 12±3 Gyr.

Several lines of evidence now show that the universe does not have $\Omega_m = 1$ but rather $\Omega_{tot} = \Omega_m + \Omega_\Lambda = 1.0$ with $\Omega_m \approx 0.3$, which gives an expansion age of about 14 Gyr. (Ω_m means average density of matter in units of critical density.)

Moreover, a new type of age measurement based on radioactive decay of Thorium-232 (half-life 14.1 Gyr) measured in a number of stars gives a completely independent age of 14±3 Gyr. A similar measurement, based on the first detection in a star of Uranium-238 (half-life 4.47 Gyr), gives 12.5±3 Gyr.

All the recent measurements of the age of the universe are thus in excellent agreement. It is reassuring that three completely different clocks – stellar evolution, expansion of the universe, and radioactive decay – agree so well.

Band Theory of Solids (Review)

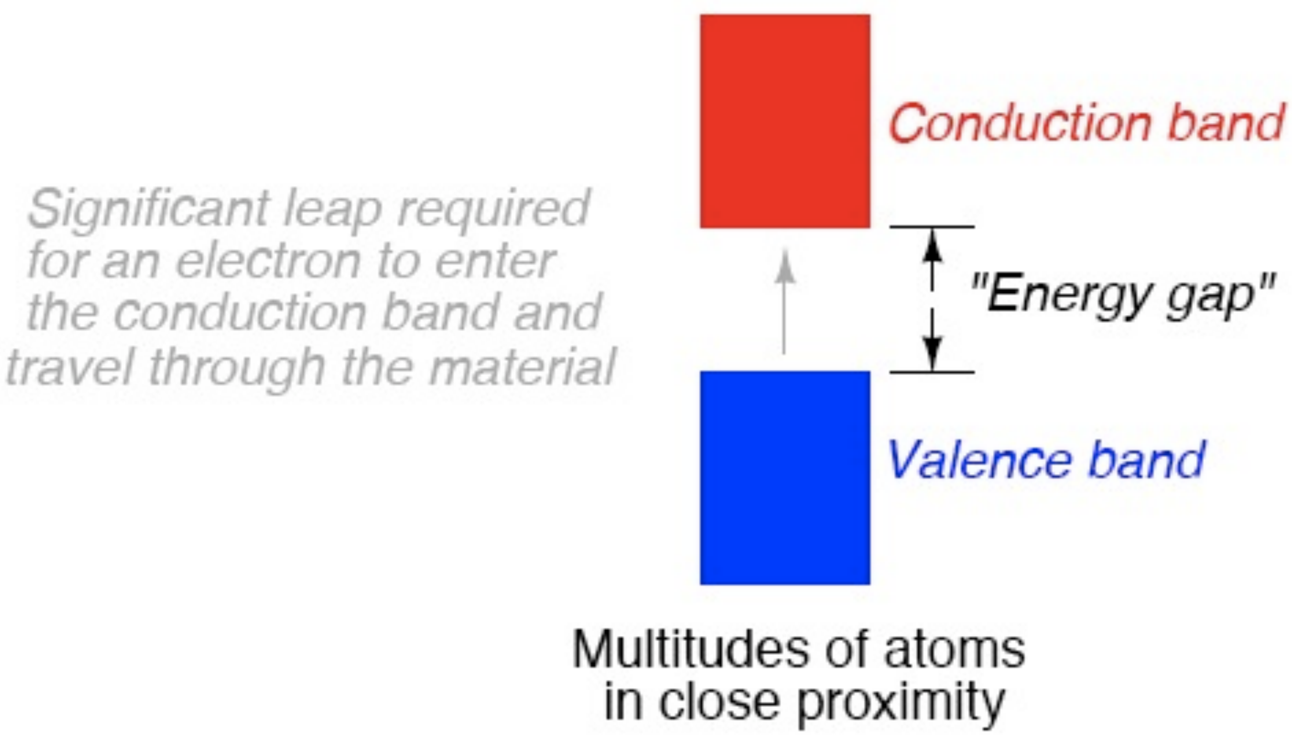
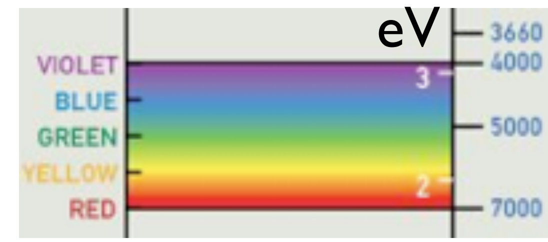


Figure 2.21: *Electron band separation in insulating substances.*



If the Energy gap is larger than the energy of visible light (~3.2 eV), then the insulator will be transparent. Glass is an example. A very small percentage of impurity atoms in the glass can give it color by providing specific available energy levels which absorb certain colors of visible light.

Another example: Ruby is aluminum oxide with a small amount (about 0.05%) of chromium, which gives it its characteristic red color by absorbing green and blue light.

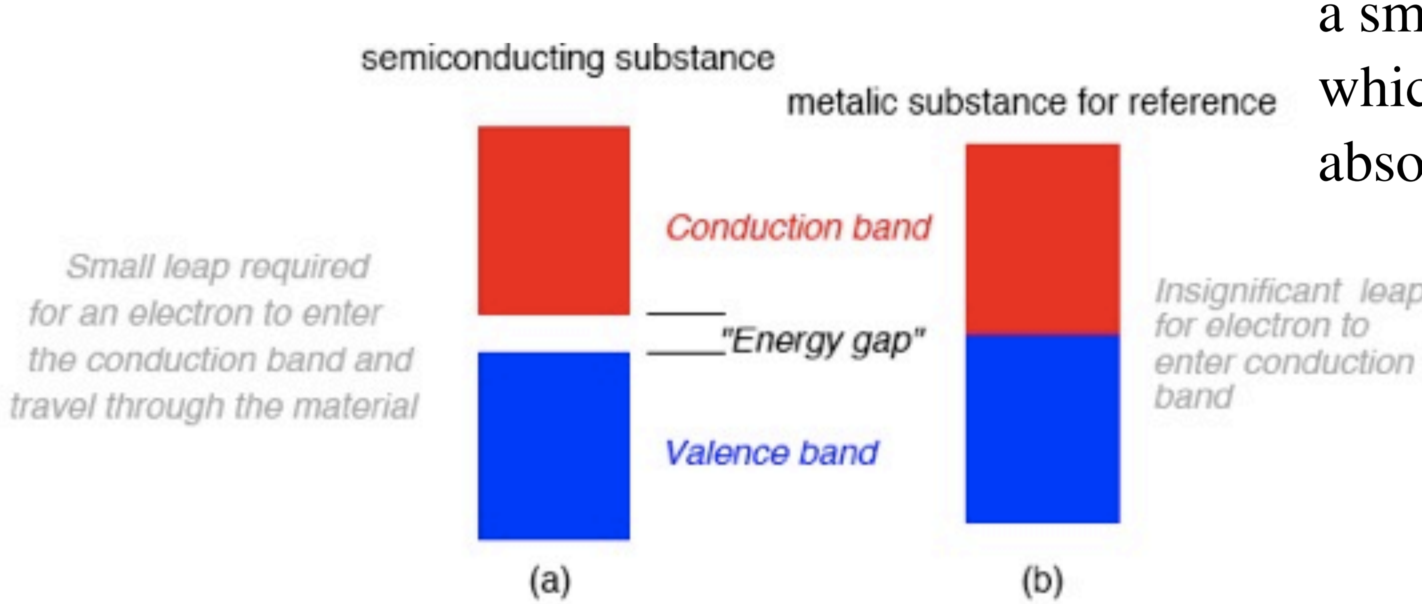
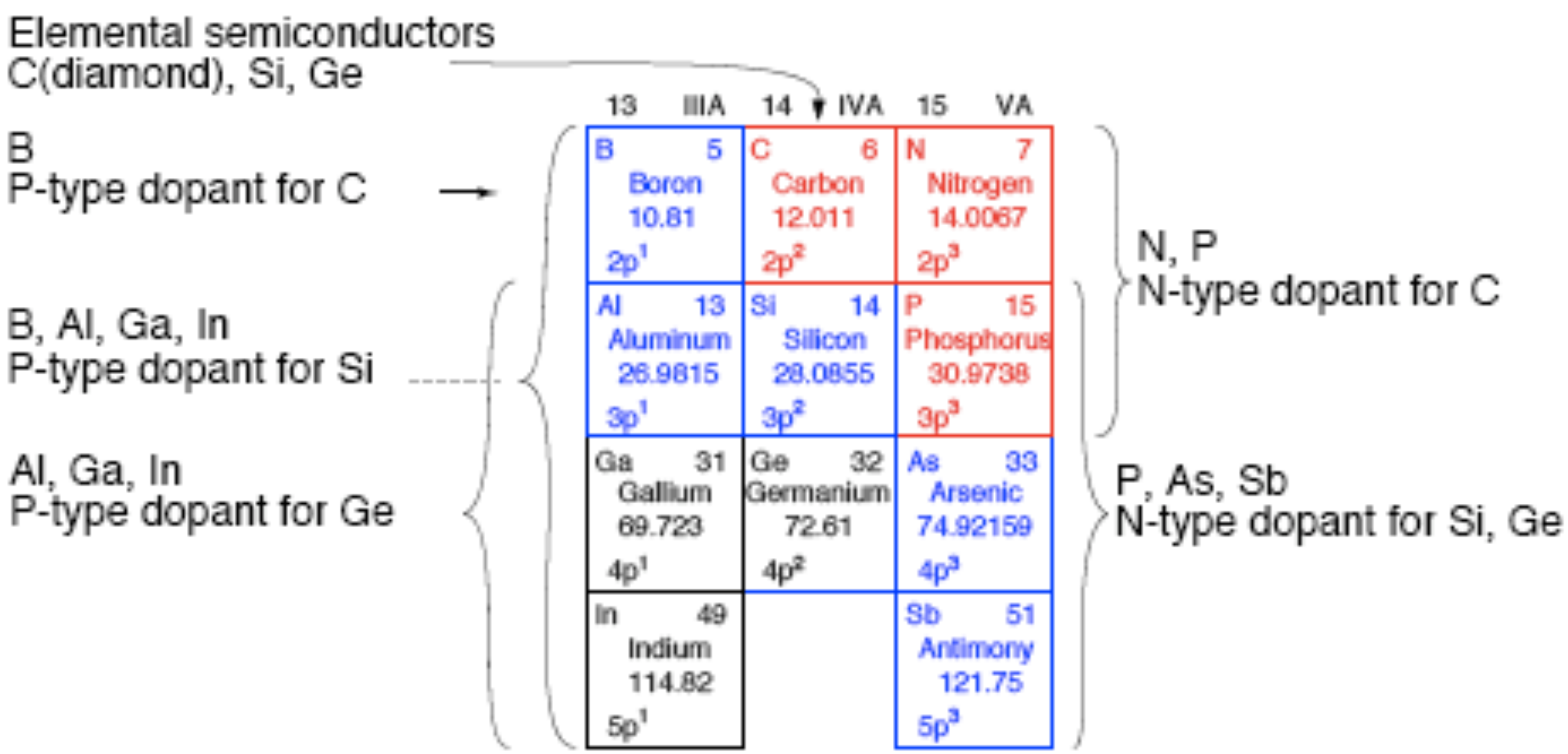
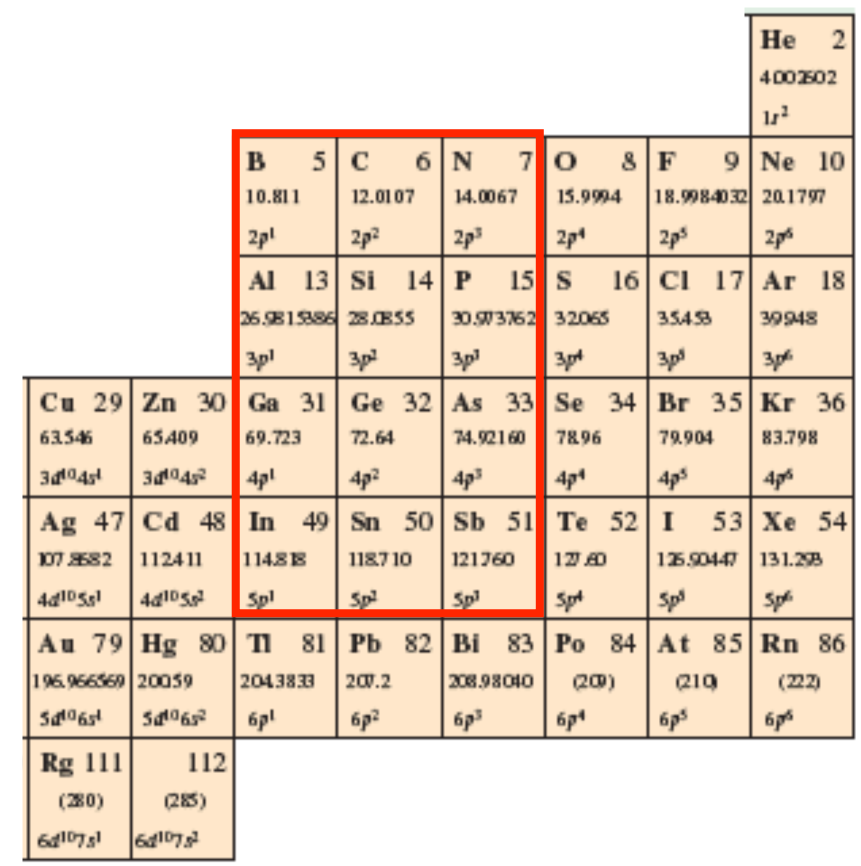


Figure 2.22: *Electron band separation in semiconducting substances, (a) multitudes of semiconducting close atoms still results in a significant band gap, (b) multitudes of close metal atoms for reference.*

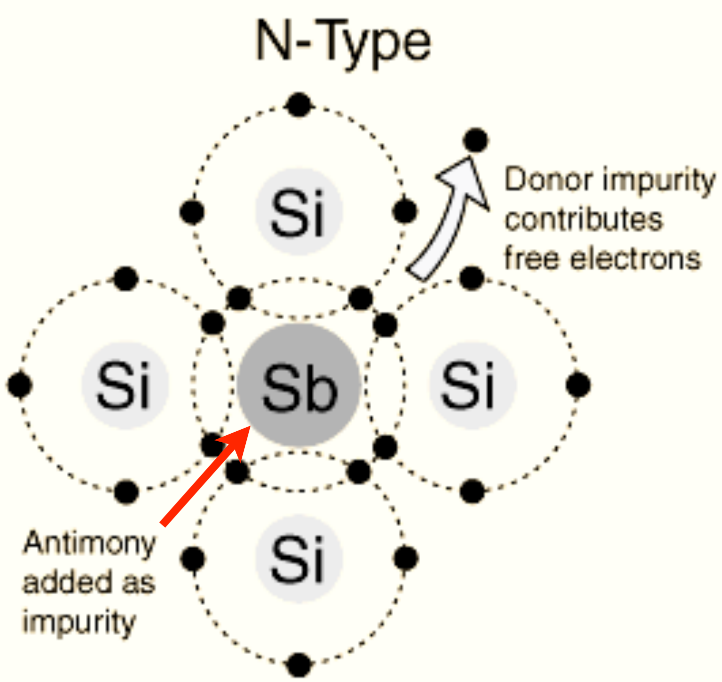
• Intrinsic semiconductor materials, pure to 1 part in 10 billion, are poor conductors.
• N-type semiconductor is doped with a pentavalent impurity to create free electrons. Such a material is conductive. The electron is the majority carrier.
• P-type semiconductor, doped with a trivalent impurity, has an abundance of free holes. These are positive charge carriers. The P-type material is conductive. The hole is the majority carrier.



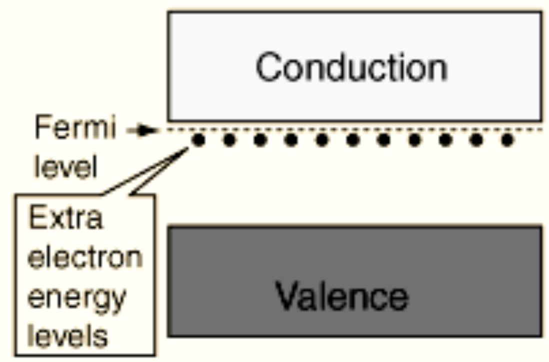


Semiconductor material may have specific impurities added at approximately 1 part per 10 million to increase the number of carriers. The addition of a desired impurity to a semiconductor is known as *doping*. Doping increases the conductivity of a semiconductor so that it is more comparable to a metal than an insulator. It is possible to increase the number of negative charge carriers within the semiconductor crystal lattice by doping with an electron *donor* like Phosphorus. Electron donors, also known as *N-type* dopants include elements from group VA of the periodic table: nitrogen, phosphorus, arsenic, and antimony. Nitrogen and phosphorus are N-type dopants for diamond. Phosphorus, arsenic, and antimony are used with silicon.

N-Type Semiconductor

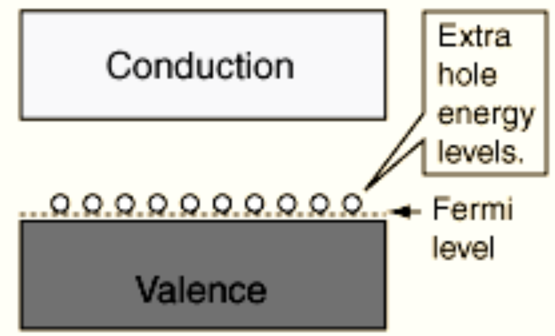


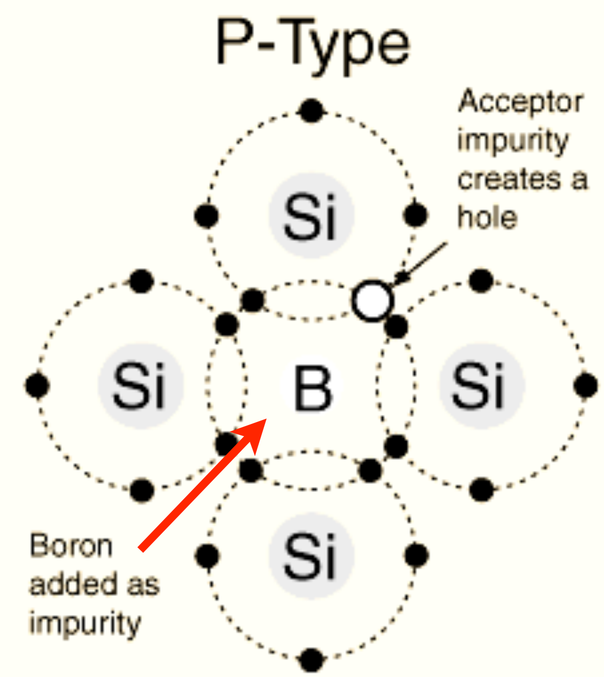
The addition of pentavalent impurities such as antimony, arsenic or phosphorous contributes free electrons, greatly increasing the conductivity of the intrinsic semiconductor. Phosphorous may be added by diffusion of phosphine gas (PH3).



P-Type Semiconductor

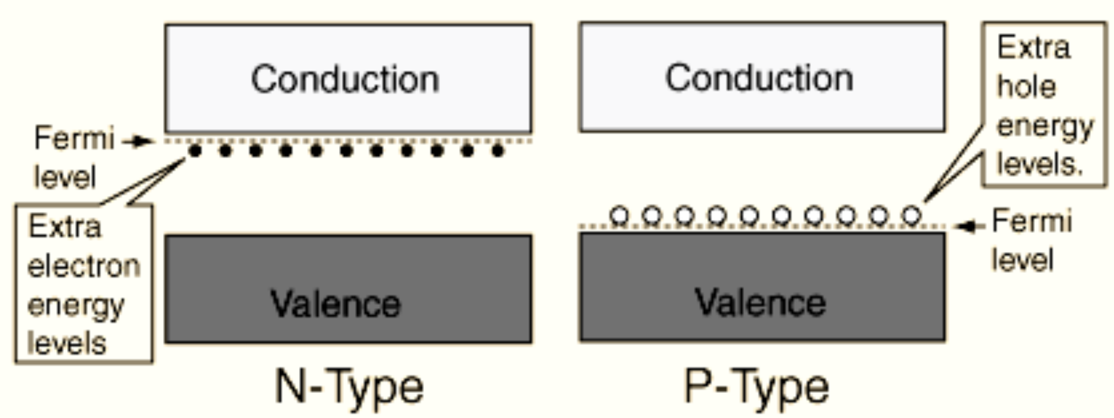
The addition of trivalent impurities such as boron, aluminum or gallium to an intrinsic semiconductor creates deficiencies of valence electrons,called "holes". It is typical to use B_2H_6 diborane gas to diffuse boron into the silicon material.





Bands for Doped Semiconductors

The application of band theory to n-type and p-type semiconductors shows that extra levels have been added by the impurities. In n-type material there are electron energy levels near the top of the band gap so that they can be easily excited into the conduction band. In p-type material, extra holes in the band gap allow excitation of valence band electrons, leaving mobile holes in the valence band.



N-type and P-type Semiconductor Diode

In Figure 2.29(a) the battery is arranged so that the negative terminal supplies electrons to the N-type material. These electrons diffuse toward the junction. The positive terminal removes electrons from the P-type semiconductor, creating holes that diffuse toward the junction. If the battery voltage is great enough to overcome the junction potential (0.6V in Si), the N-type electrons and P-holes combine annihilating each other. This frees up space within the lattice for more carriers to flow toward the junction. Thus, currents of N-type and P-type majority carriers flow toward the junction. The recombination at the junction allows a battery current to flow through the PN junction diode. Such a junction is said to be *forward biased*.

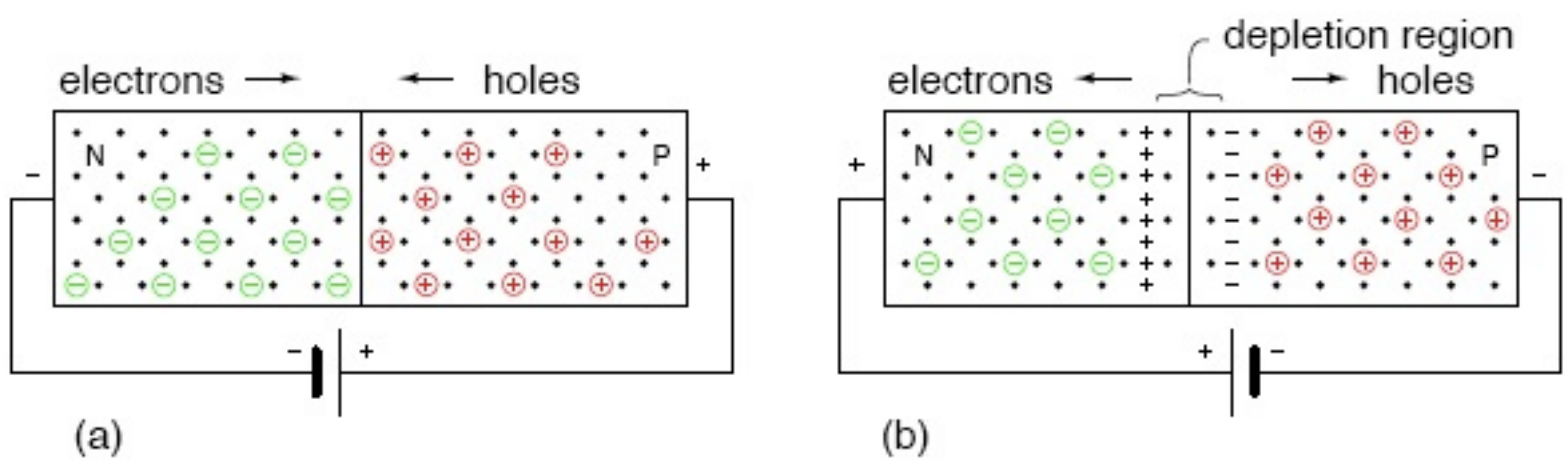
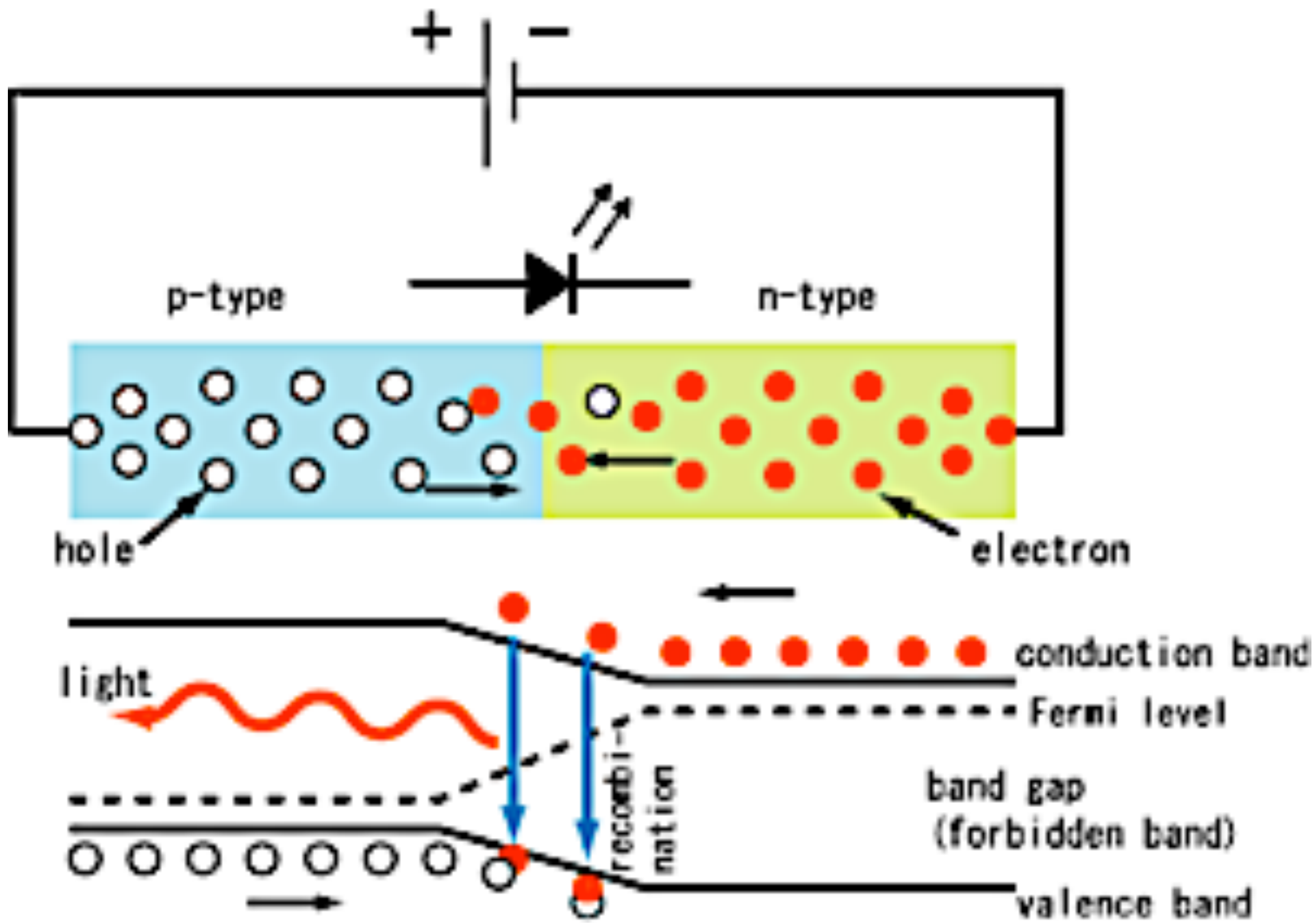
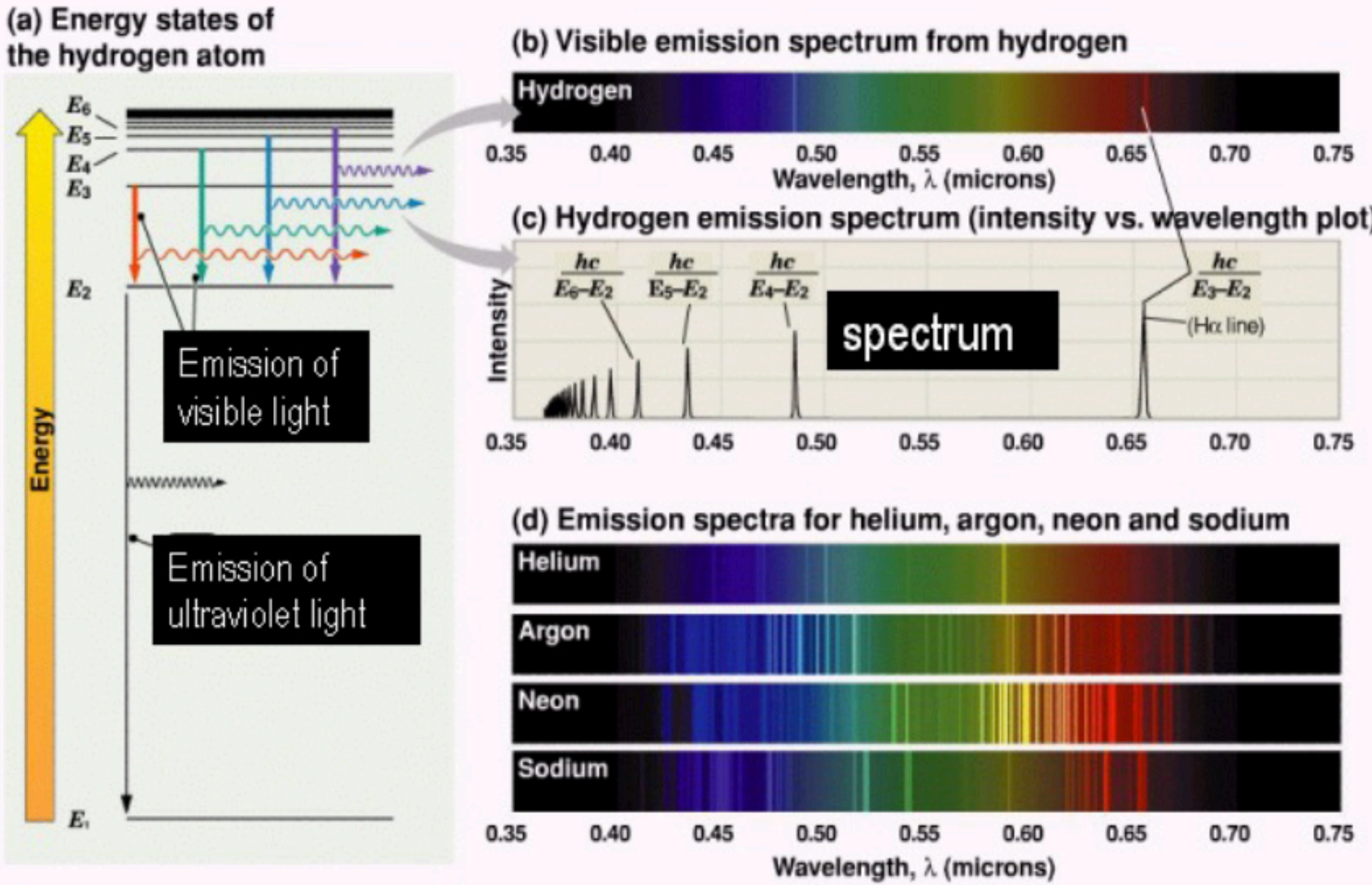


Figure 2.29: (a) Forward battery bias repells carriers toward junction, where recombination results in battery current. (b) Reverse battery bias attracts carriers toward battery terminals, away from junction. Depletion region thickness increases. No sustained battery current flows.

If the battery polarity is reversed as in Figure 2.29 (b) majority carriers are attracted away from the junction toward the battery terminals. The positive battery terminal attracts N-type electrons majority carriers away from the junction. The negative terminal attracts P-type majority carriers, holes away from the junction. This increases the thickness of the nonconducting depletion region. There is no recombination of majority carriers; thus, no conduction. This arrangement of battery polarity is called *reverse bias*. Such a device is called a *diode*.

Light-emitting diode





Each element has a characteristic spectrum

Homework 2 Solutions

1. (15 points) (a) Find the radius a_0 of the $n = 1$ orbit, and in terms of a_0 find the radii r_n of the orbits for the higher values of n.

Answer Applying Newton's law $F = ma$ sets the Coulomb force equal to the centripetal force:

$$\frac{e^2}{4\pi\epsilon_0 r^2} = m_e \frac{v^2}{r} \ .$$

Bohr's angular momentum quantization assumption is $m_e r v = n\hbar$. Combining these equations,

$$r = \frac{n\hbar}{m_e v} = \frac{e^2}{4\pi\epsilon_0 v^2 m_e} \quad ,$$

which implies that in the n^{th} orbit

$$v_n = \frac{e^2}{4\pi\epsilon_0 n\hbar} = \frac{\alpha c}{n} \ , \quad r_n = n^2 \frac{4\pi\epsilon_0 \hbar^2}{e^2} = n^2 \alpha^{-1} \frac{\hbar}{m_e c} \quad .$$

The Bohr radius $a_0 = r_1 = \alpha^{-1}\hbar/(m_e c) = 0.529 \times 10^{-10}$ m, and the radii of the higher orbits are $r_n = n^2 a_0$.

[Note that we have neglected the motion of the proton since its mass m_p is nearly 2000 times that of the electron m_e. Including the proton motion just replaces m_e by the reduced mass $m_r = m_e/(1 + m_e/m_p)$.]

(b) Show that the ground state (i.e., $n = 1$) energy is $E_1 = \frac{1}{2}\alpha^2 m_e c^2$ and express E_1 in units of electron volts (eV).

Answer The energy is the sum of the kinetic and potential energies:

$$E_1 = \mathrm{KE}_1 + \mathrm{PE}_1 = \frac{1}{2}m_e v_1^2 - \frac{e^2}{4\pi\epsilon_0 r_1} = \frac{1}{2}\alpha^2 m_e c^2 - \alpha^2 m_e c^2 = -\frac{1}{2}\alpha^2 m_e c^2 \ .$$

Numerically, $m_e c^2 = 0.511$ MeV and $E_1 = -13.6$ eV. (That the energy is negative just means that the binding energy of the ground state of the hydrogen atom is 13.6 eV.)

(c) Find an expression for the energy levels E_n of the states of the hydrogen atom labeled by Bohr's quantum number n.

Answer Using r_n and v_n in $E_n = \mathrm{KE_n} + \mathrm{PE_n}$ above gives $E_n = E_1/n^2$.

Note that $\mathrm{KE_n} = -(1/2)\mathrm{PE_n}$. That $< \mathrm{KE} >= -(1/2) < \mathrm{PE} >$ (where $< ... >$ means time average) for an inverse-square-law (i.e., $1/r^2$) force is a consequence of a general theorem of mechanics known as the "Virial theorem". The **Virial theorem** states that $< \mathrm{KE} >= (n/2) < \mathrm{PE} >$ where n is the power of r in the potential: $V(r) = ar^n$. An r^{-2} force corresponds to an r^{-1} potential, so here $n = -1$.

2. (15 points) The light emitted by the hydrogen atoms transition from the $n = 3$ to the $n = 2$ energy level is called Hα.

(a) Find the energy E of the photons of this light in eV.

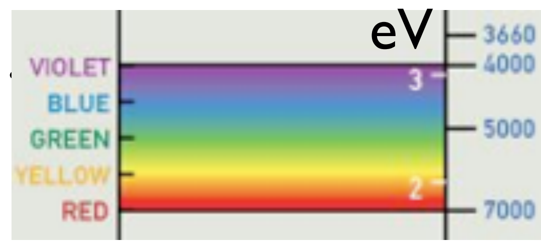
Answer $E_{\mathrm{H}_\alpha} = E_3 - E_2 = (1/9 - 1/4)E_1 = (-5/36)E_1 = 1.89$ eV .

(b) Find the frequency ν of this light by using Planck's formula $E = h\nu$.

Answer $\nu_{\mathrm{H}_\alpha} = E_{\mathrm{H}_\alpha}/h = (1.89\ \mathrm{eV})(1.602 \times 10^{-19}\mathrm{J})/(6.63 \times 10^{-34}\mathrm{J} \cdot \mathrm{s}) = 4.56 \times 10^{14}$ Hz.

(c) Find the wavelength λ. What is the color of this light?

Answer $\lambda_{\mathrm{H}_\alpha} = c/\nu_{\mathrm{H}_\alpha} = 6.58 \times 10^{-7}$ m $= 658$ nm, which is *red* light. Visible light ranges from 400 nm (blue) to 700 nm (red)



3. (10 points) (a) What is the energy required to reach the n = 3 energy level from the ground state (the n = 1 energy level)? What sort of light has this much energy?

Answer $E_3 - E_1 = (1/9 - 1)E_1 = -(8/9)E_1 = 12.1$ eV $= 1.94 \times 10^{-18}$ J. This is ultraviolet light.

(b) What is the minimum energy required to ionize a hydrogen atom [in its ground state] (i.e., free the electron)?

Answer The binding energy in the ground state, 13.6 eV $= 2.18 \times 10^{-18}$ J, is the minimum energy required to free the electron in its ground state. If the electron in the ground state absorbs more energy than 13.6 eV from a photon, it also has kinetic energy.

4. (5 points) Suppose that, on average, a hydrogen atom will exist in the $n = 2$ state for about 10^{-8} second. How many revolutions does the electron make in this time.

Answer The time per revolution in the n$^{\text{th}}$ Bohr energy level is $T_n = 2\pi r_n / v_m$. Recall from Problem 1 above that

$$v_n = \frac{e^2}{4\pi\epsilon_0 n\hbar} = \frac{e^2}{4\pi\epsilon_0 \hbar c}\frac{c}{n} = \frac{\alpha c}{n}\ , \quad r_n = n^2\alpha^{-1}\frac{h}{m_e c}\ .$$

Thus

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi n^2 \hbar}{\alpha m_e c}\frac{n}{\alpha c} = n^3\alpha^{-2}\frac{\hbar}{m_e c^2} = n^3(4.55\times 10^{-8})\ \text{s}\ .$$

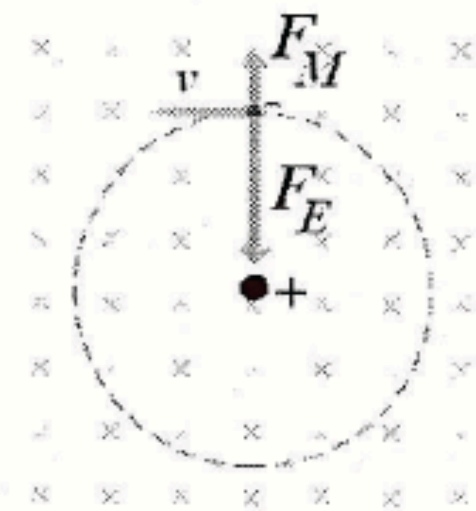
Then $T_2 = 3.64\times 10^{-7}$ s, and the number of revolutions of an electron in the $n = 2$ Bohr orbit in 10^{-8} s is $(10^{-8}\ \text{s})/T_2 = 0.027$ revolutions.

[Note that the electron can't slow down, since as long as it is in the $n = 2$ state it's velocity is v_2. What actually happens as the electron transitions to the ground state remained a mystery in the Bohr theory. Then quantum mechanics showed that the electron wavefunction becomes a sum of the $n = 1$ and $n = 2$ wavefunctions during the transition; the interference of their respective frequencies is the frequency of the light emitted during the transition.]

5. (10 points) Giancoli, problem 27-72 on the **Zeeman effect**. In the Bohr model of the hydrogen atom, the electron is held in its circular orbit of radius r about its proton nucleus by electrostatic attraction. If the atoms are placed in a weak magnetic field $\mathbf{B}$, the rotation frequency of electrons rotating in a plane perpendicular to $\mathbf{B}$ is changed by an amount $\Delta f = \pm eB/4\pi m$ where e and m are the charge and mass of an electron.

(a) Derive this result, assuming the force due to $\mathbf{B}$ is much less than that due to electrostatic attraction of the nucleus.

As the electron orbits the nucleus in the absence of the magnetic field, its centripetal acceleration is caused solely by the electrical attraction between the electron and the nucleus. Writing the velocity of the electron as the circumference of its orbit times its frequency, enables us to obtain an equation for the frequency of the electron's orbit.



$$\frac{ke^2}{r^2} = m\frac{v^2}{r} = m\frac{(2\pi r f_0)^2}{r} \rightarrow f_0^2 = \frac{ke^2}{4\pi^2 m r^3}$$

When the magnetic field is added, the magnetic force adds or subtracts from the centripetal acceleration (depending on the direction of the field) resulting in the change in frequency.

$$\frac{ke^2}{r^2} \pm q(2\pi r f)B = m\frac{(2\pi r f)^2}{r} \rightarrow f^2 \mp \frac{qB}{2\pi m} f - f_0^2 = 0$$

We can solve for the frequency shift by setting $f = f_0 + \Delta f$, and only keeping the lowest order terms, since $\Delta f \ll f_0$.

$$(f_0 + \Delta f)^2 \mp \frac{qB}{2\pi m}(f_0 + \Delta f) - f_0^2 = 0$$

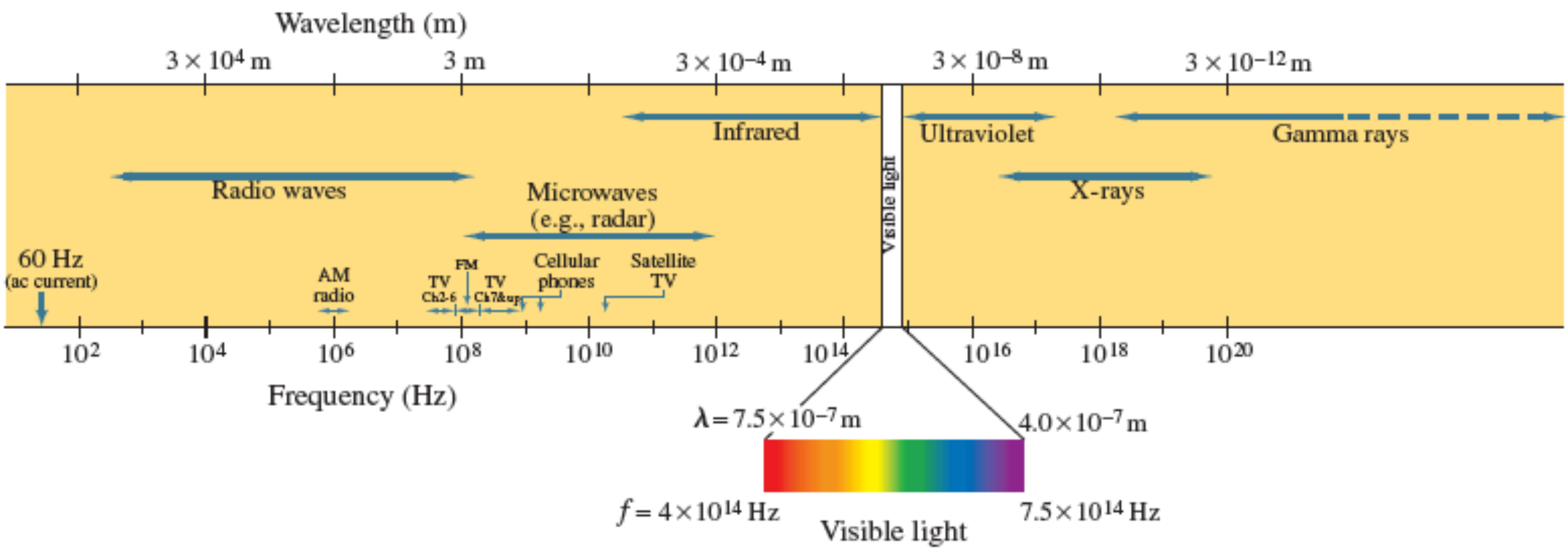
$$\cancel{f_0^2} + 2 f_0 \Delta f + \cancel{\Delta f^2} \mp \frac{qB}{2\pi m} f_0 \mp \cancel{\frac{qB}{2\pi m}\Delta f} - \cancel{f_0^2} = 0 \rightarrow \boxed{\Delta f = \pm\frac{qB}{4\pi m}}$$

(b) What does the $\pm$ sign indicate?

The "$\pm$" indicates whether the magnetic force adds to or subtracts from the centripetal acceleration. If the magnetic force adds to the centripetal acceleration, the frequency increases. If the magnetic force is opposite in direction to the acceleration, the frequency decreases.

Giancoli **FIGURE 31–12**

Electromagnetic spectrum.



6. (15 points) It is possible for a muon to be captured by a proton to form a muonic atom. A muon is identical to an electron except for its mass, which is 105.7 MeV/c^2, and the fact that it is unstable, with a lifetime of about 2.2 microseconds.

(a) Calculate the radius of the first Bohr orbit of a muonic atom.

Answer To a first approximation, the radius is $R_1 = \alpha^{-1}\hbar/(m_\mu c) = a_0(m_e/m_\mu) = 0.529 \times 10^{-10}\text{m})(m_e/m_\mu) = 2.55 \times 10^{-13}$ m.

3 points Extra Credit: A more accurate treatment uses the reduced mass of the muon $m_{\mu,r} = m_\mu/(1+m_\mu/m_p) = m_\mu/(1+105.7/938) = 95.0\text{MeV}/c^2$. This is a bigger correction here than using the reduced electron mass was in Problem 1, since the muon mass is so much larger. The first Bohr radius of a muonic atom is thus more accurately 2.85×10^{-3} m

(b) Calculate the magnitude of the lowest energy state in eV.

Answer Replacing m_e by m_μ in the expression for E_1 in Problem 1 gives $E_1 = -(1/2)\alpha^2 m_\mu c^2$ so $E_1 = (m_\mu/m_e)(-13.6\text{eV}) = 2.81$ keV.

3 points Extra Credit: Using instead the reduced muon mass gives the more accurate answer 2.53 keV.

(c) What is the wavelength of the $n = 2$ to $n = 1$ transition in a muonic atom?

Answer The energy of the transition is $(1/4 - 1)E_1 = (3/4)E_1 = 2.11$ keV. The frequency $\nu = E/h$ and the wavelength is $\lambda = c/\nu = hc/E = (1.24 \times 10^{-6}\text{eVm})/E = 5.88 \times 10^{-10}$ m $= 0.59$ nm. This is an x-ray wavelength. (See Fig. 31-12 on p. 823 of Giancoli for a picture of the entire electromagnetic spectrum.)

3 points Extra Credit: Using instead the reduced muon mass gives the more accurate answer 0.655 nm.