Physics 5K Lecture Friday May 25, 2012

Relativity and Charged Particle Motion in Magnetic Fields

Joel Primack
Physics Department
UCSC

Einstein's Special Theory of Relativity

http://physics.ucsc.edu/~snof/er.html

Special Relativity is based on two postulates:

1. The Principle of Relativity: If a system of coordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form [i.e., K is an inertial reference system], the same laws hold good in relation to any other system of coordinates K' moving in uniform translation relatively to K.

2. Invariance of the speed of light: Light in vacuum propagates with the speed c in terms of any system of inertial coordinates, regardless of the state of motion of the light source.

About Einstein's Rocket | Thought Experiments | 1-D Space Rally | Games | Credits

About Einstein's Rocket

http://physics.ucsc.edu/~snof/er.html

General Info

This set of Java applets is designed to teach the basic principles of special relativity. The first applet goes through various thought experiments to give you an understanding of why relativity works the way it does. The second applet (1-D Space Rally) helps demonstrate why the "twin paradox" is, in fact, not a paradox. And the third applet contains several games which allow you to get a better intuition about the effects of relativity on motion at high velocities. To access the applets, simply click the links at the top of this page.

This page is best viewed in a modern browser with support for CSS. Mac: use Safari

If the applets will not load, check to make sure that your browser supports Java and has Java enabled.

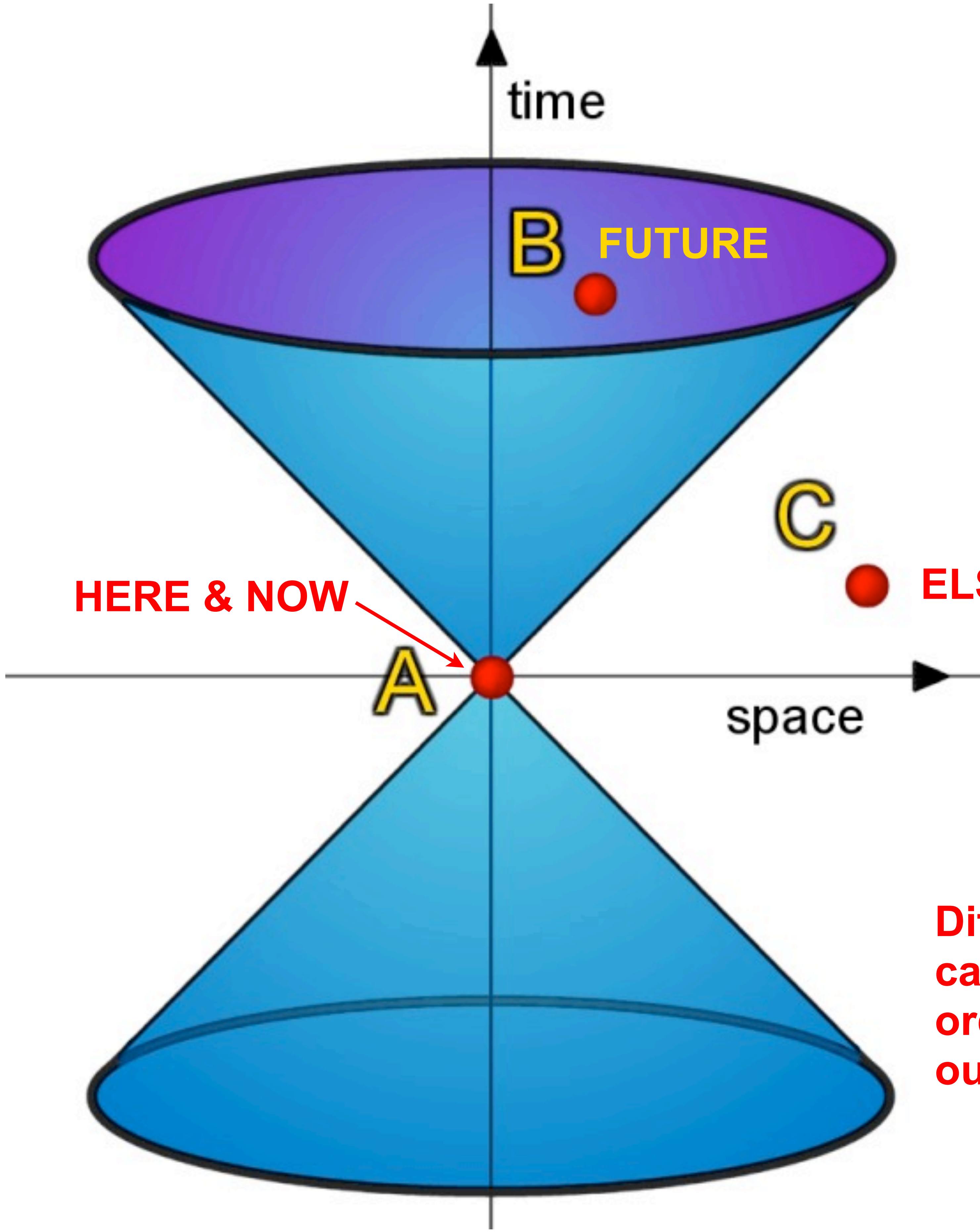
Background

These three programs were originally designed and written for an Apple II by Joel Primack and Eric Eckert in the early 1980s. The Java version was coded by Devin Kelly-Sneed under the supervision of Joel Primack.

Contact Info

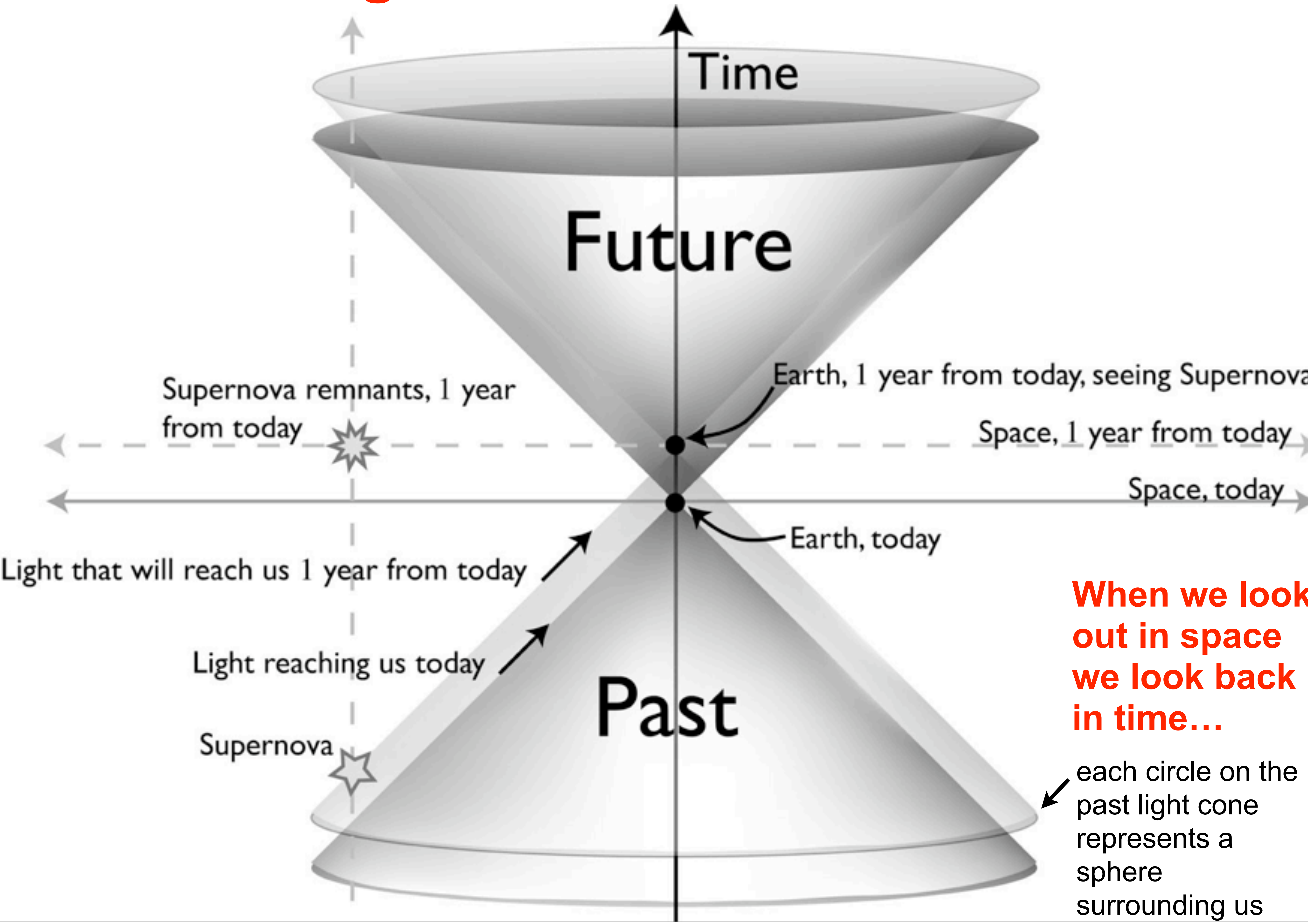
If you encounter a bug in any of the applets or in the website, please send an email describing the problem to EinsteinsRocket@gmail.com. If possible, also include what operating system and browser you're using, as well as any other info that you think may helpful to us in solving the problem. Non-bug related feedback is also welcome at the same email address.

Lightcone of Past and Future



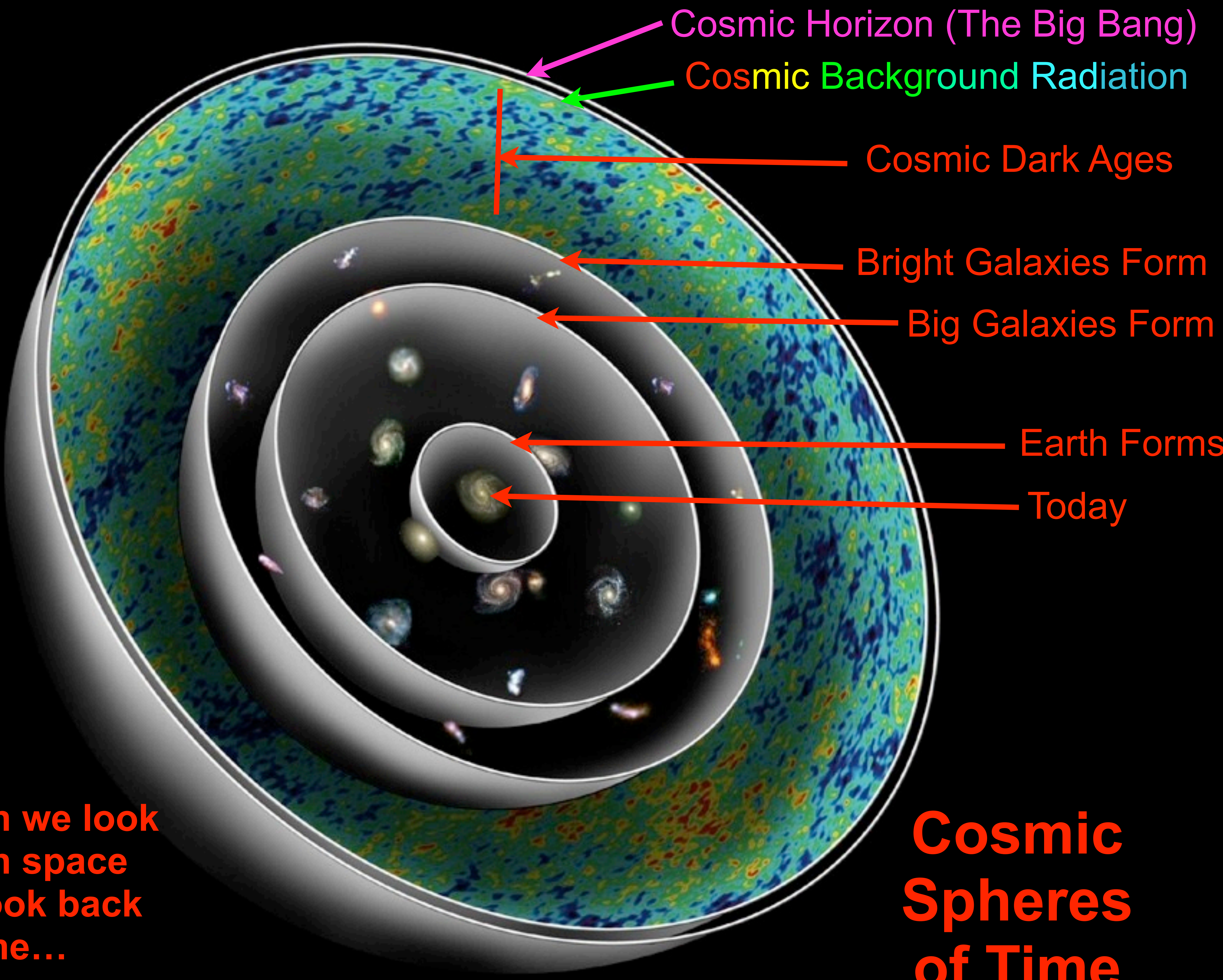
Different observers can see different time orders of events outside the lightcone!

Lightcone of Past and Future



When we look out in space we look back in time…

each circle on the past light cone represents a sphere surrounding us



When we look out in space we look back in time…

Cosmic Spheres of Time

Scales of Energy

Physicists measure the energies of fast-moving particles like those in cosmic rays and particle accelerators in units of electron volts, abbreviated eV. An electron volt is the amount of energy that one electron gains when it is accelerated by an electrical potential of one volt. (A flashlight battery has about 1.5 volts.) Electrons in a cathode ray tube are accelerated to an energy of about 50,000 electron volts (50 keV). When they strike the screen, they make it glow, and stopping the electrons generates X-rays.

Fermilab's Tevatron can accelerate protons to one trillion electron volts (1 TeV). The Large Hadron Collider in Geneva is now accelerating protons to 3.5 TeV. The highest-energy cosmic ray particle ever observed had an energy 100 million times higher than the protons at the LHC. Scientific notation, shown below, saves writing out the many zeros required for such large numbers.

	10^3 eV	1 keV (kilo)	thousand	
$2m_ec^2$	10^6 eV	1 MeV (mega)	million	
m_pc^2	10^9 eV	1 GeV (giga)	billion	
Tevatron E_p	10^{12} eV	1 TeV (tera)	trillion	
	10^{15} eV	1 PeV (peta)	highest energy cosmic ray:	3×10^{20} eV
	10^{18} eV	1 EeV (exa)		= 300 EeV
	10^{21} eV	1 ZeV (zetta)		= 0.3 ZeV

Relativity implies that the momentum p of a particle of rest mass m and velocity v is $p = m\gamma v$, where $\gamma = 1/(1 - v^2/c^2)^{1/2}$. The energy of the particle, including its rest energy mc^2, is $E = m\gamma c^2$.

The laws of electrodynamics formulated by Maxwell are consistent with Special Relativity, and in fact Einstein was led to this theory through electrodynamics. His great 1905 paper on relativity is titled "On the Electrodynamics of Moving Bodies."

In particular, the Lorentz force law

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

is true in any inertial reference frame. Applying this to an electron moving in circle of radius R in a magnetic field B, we write

$$F = e\, v_\perp\, B = m\gamma v^2/R = (E/R)\,(v^2/c^2)$$

It follows that the radius is given by

$$R = E\,(v^2/c^2)\,/\,(e\, v_\perp\, B)$$

For very energetic particles, with energies $E >> mc^2$, we can write the radius of the circle in which they move (the *Larmor radius*) as

$$R = E / (e\ c\ B), \quad \text{or} \quad B = E / (e\ c\ R)$$

Let's see what B field is required to keep the TeV protons at Fermilab in their R =1 km paths:

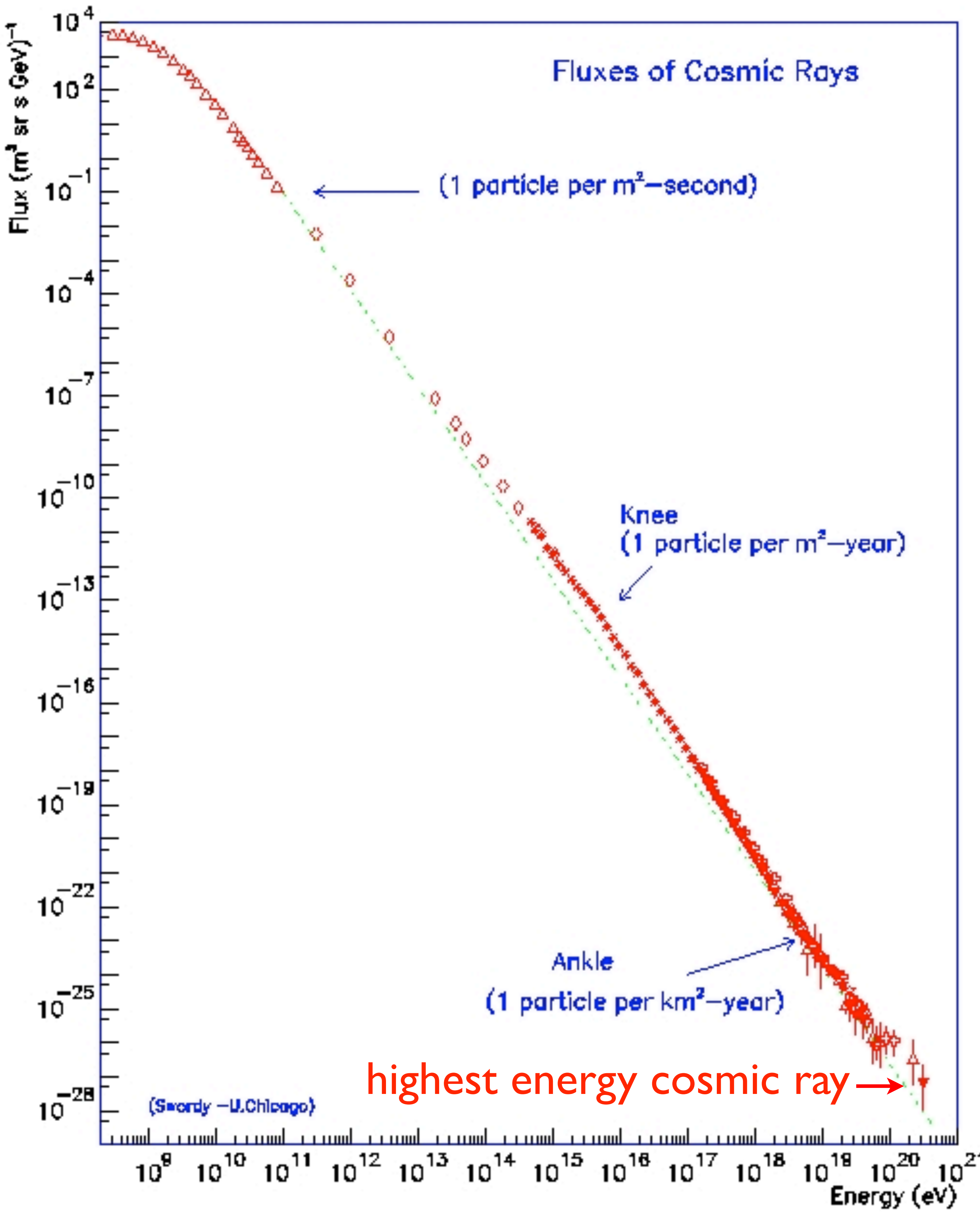
$$B = \frac{(10^{12}\ \text{eV})(1.6\text{x}10^{-19}\ \text{J/eV})}{(1.6\text{x}10^{-19}\ \text{Coul})(3\text{x}10^{8}\ \text{m/s})(10^{3}\ \text{m})} = 3.3\ \text{T}$$

Let's see now what R is for a 10 TeV particle in the Galactic magnetic field of ~3 μG = $3\text{x}10^{-10}$ T:

$$R = \frac{(10^{13}\ \text{eV})(1.6\text{x}10^{-19}\ \text{J/eV})}{(1.6\text{x}10^{-19}\ \text{Coul})(3\text{x}10^{8}\ \text{m/s})(3\text{x}10^{-10}\ \text{T})} \approx 10^{14}\ \text{m}$$

Astronomers measure distances in parsecs (pc). 1 pc = $3.1\text{x}10^{16}$ m. Thus in this case $R \approx 3\text{x}10^{-3}$ pc, much less than the distance to the nearest star (~1.3 pc).

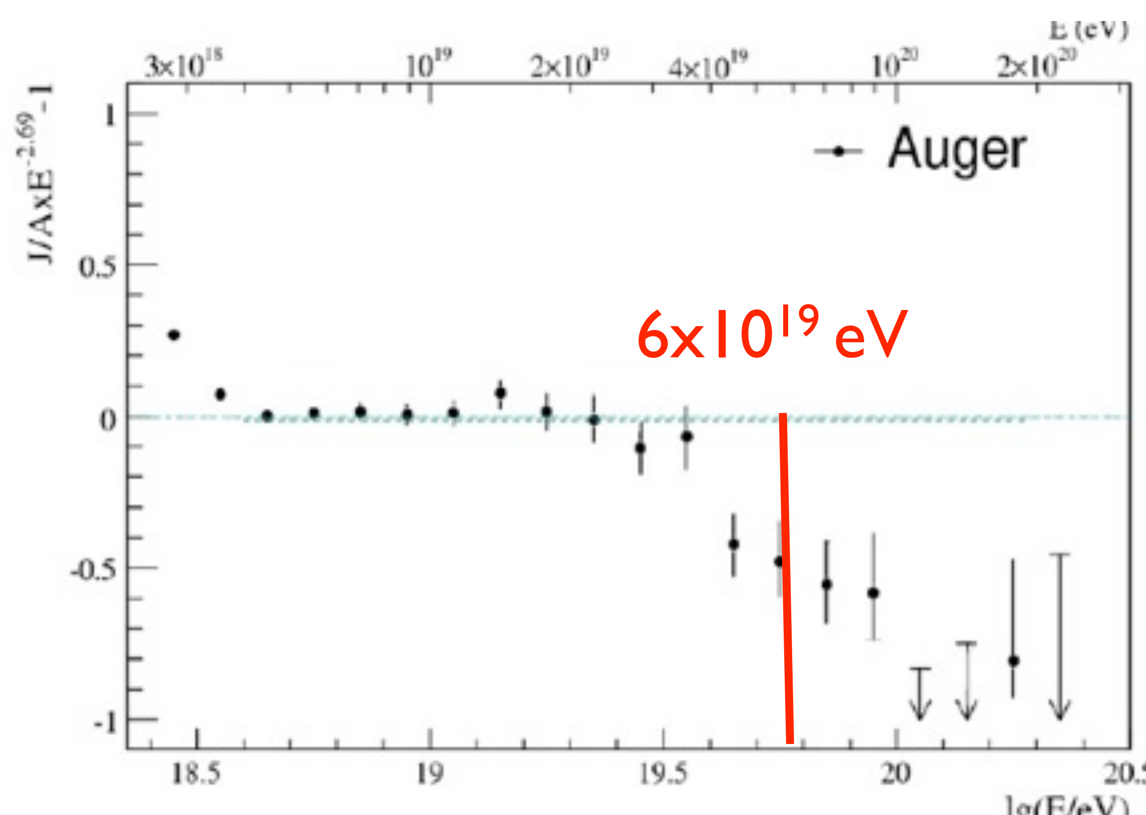
Energetic Cosmic Rays



The Larmor radius R of an energetic cosmic ray with E = 6×10^{19} eV in the inter-galactic magnetic field of 10^{-9} G = 10^{-13} T is

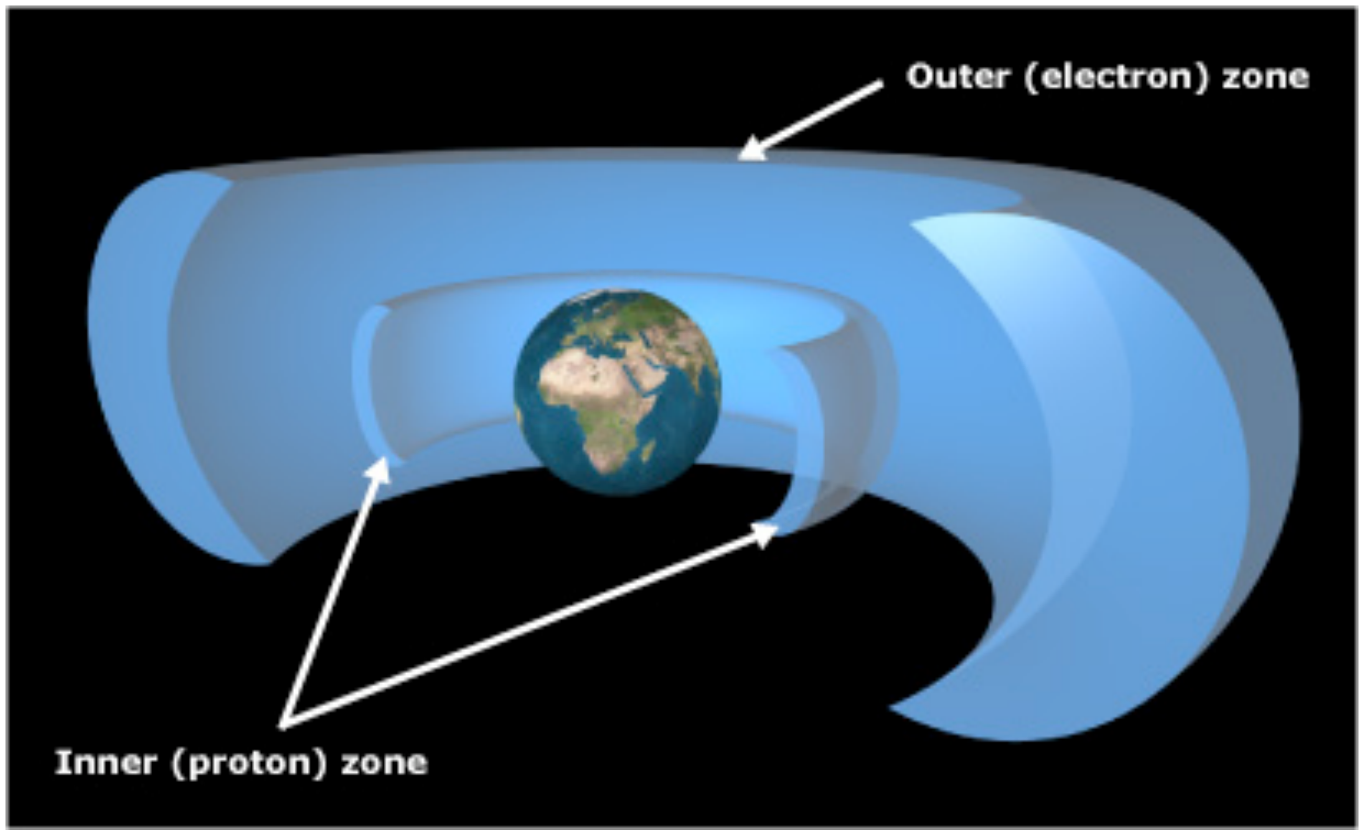
$$R \approx 60 \text{ Mpc}$$

Thus cosmic rays from the Virgo Cluster, 20 Mpc from us, will not point back toward their source.



The Van Allen Radiation Belts

The earth's magnetic field traps energetic electrons and protons from the sun in radiation belts around the earth.



These charged particles spiral around the earth's magnetic field lines as discussed in Giancoli.

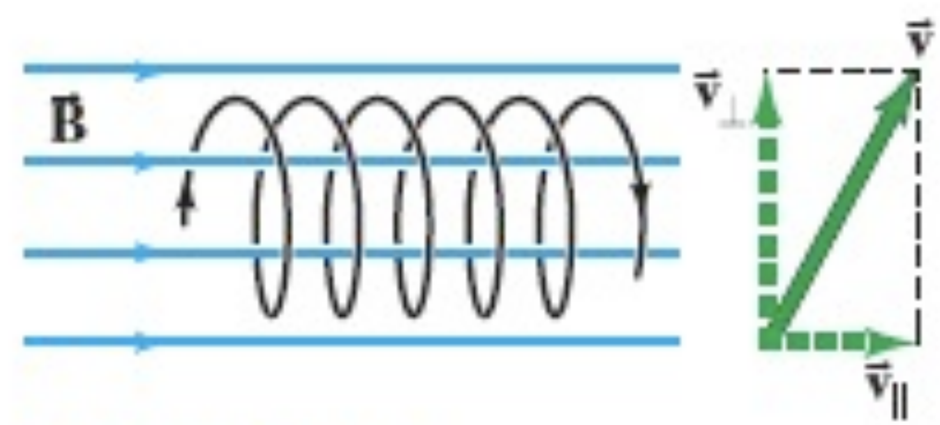
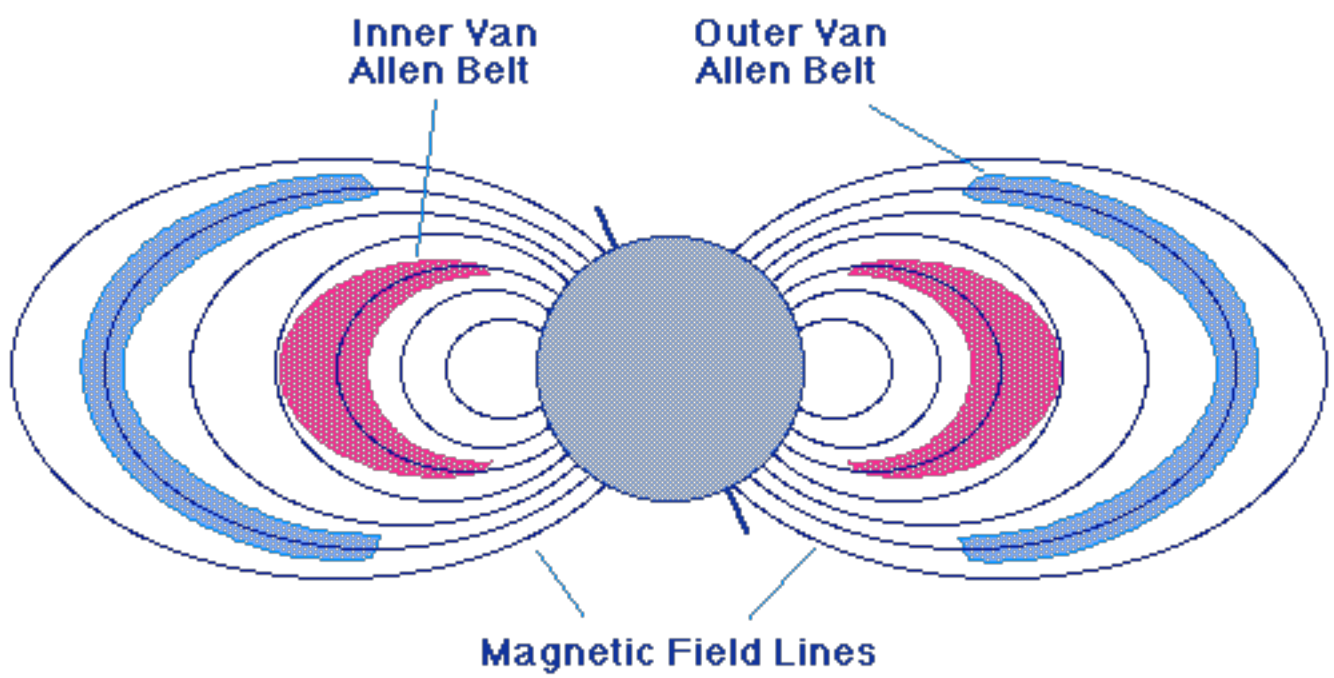
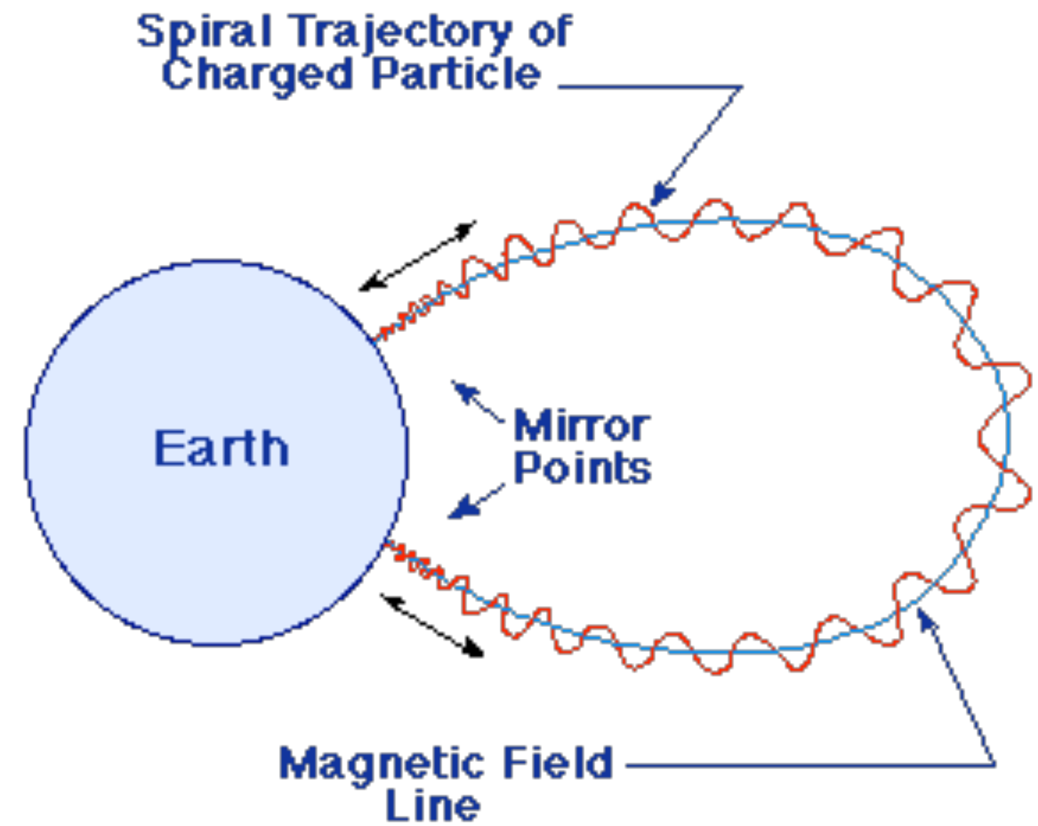
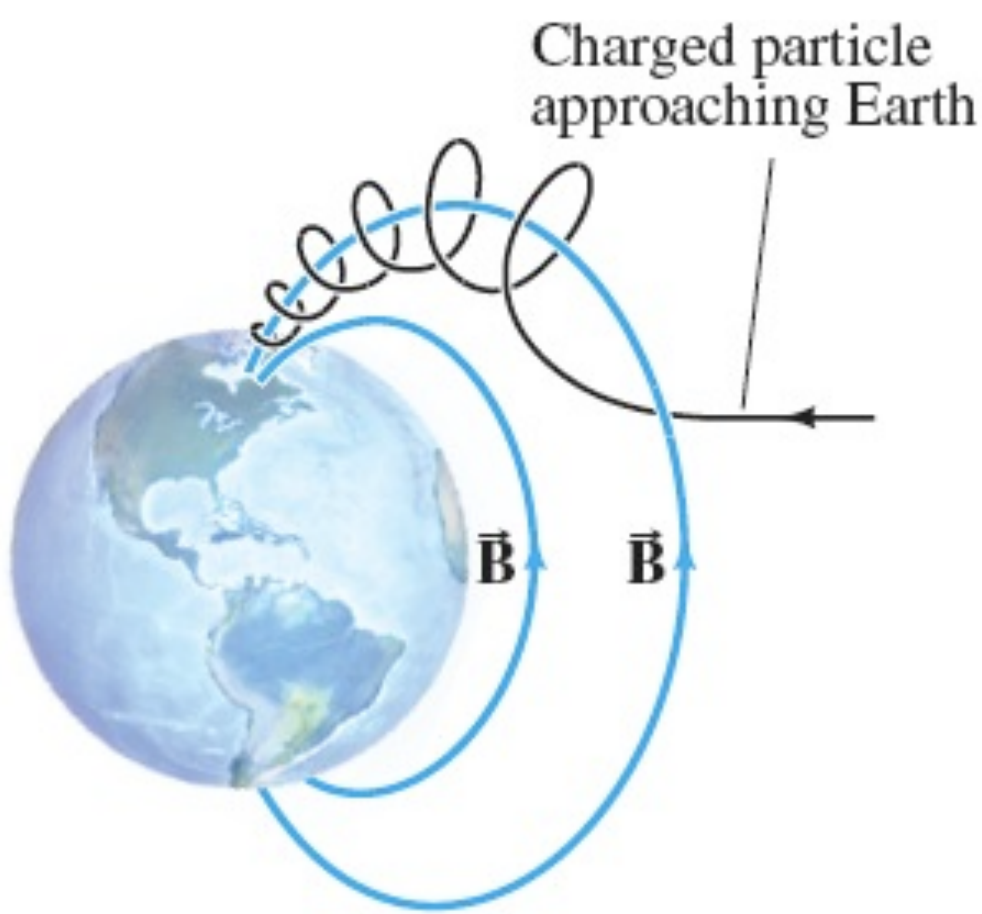


FIGURE 27–19 Example 27–10.



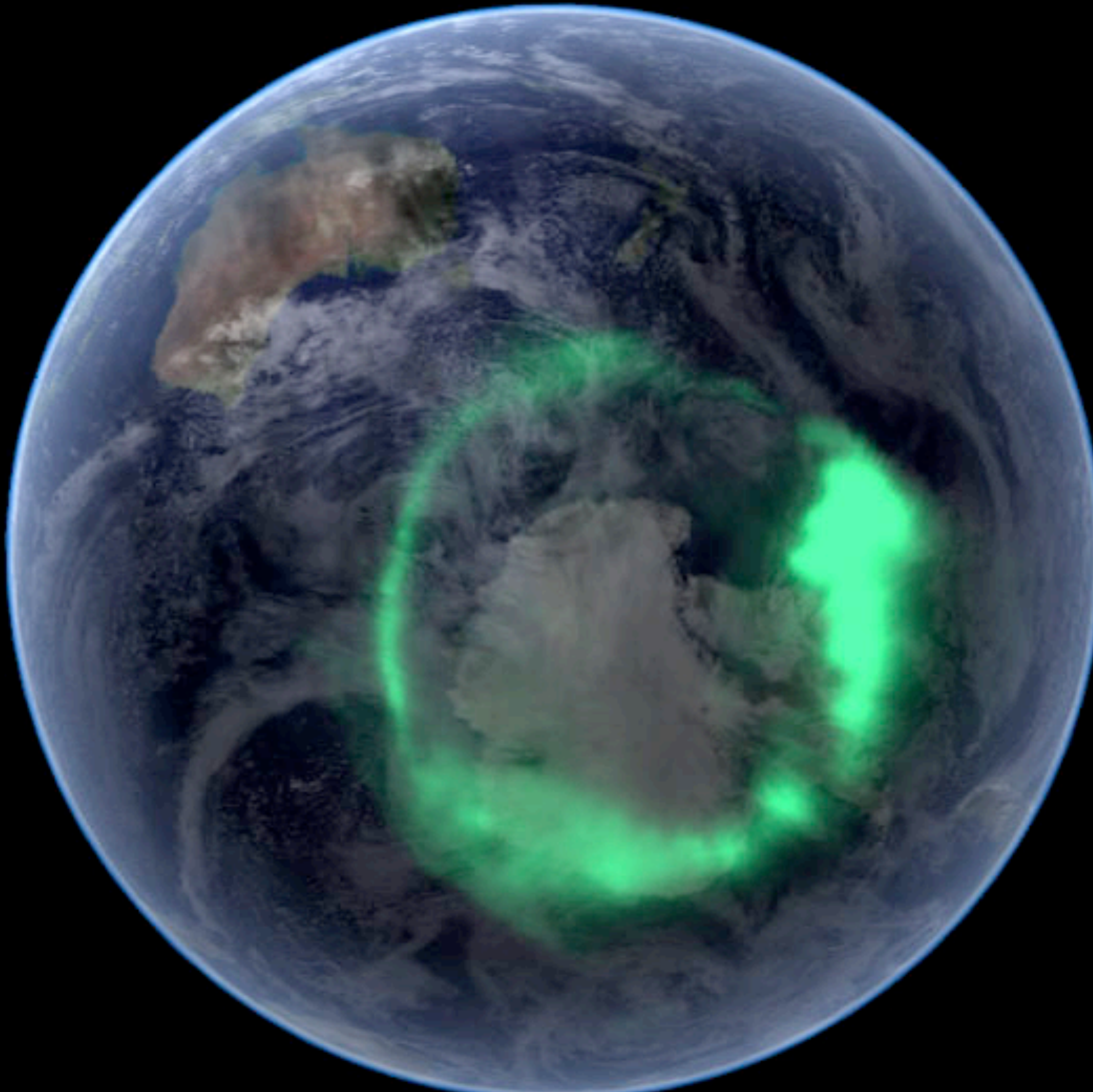


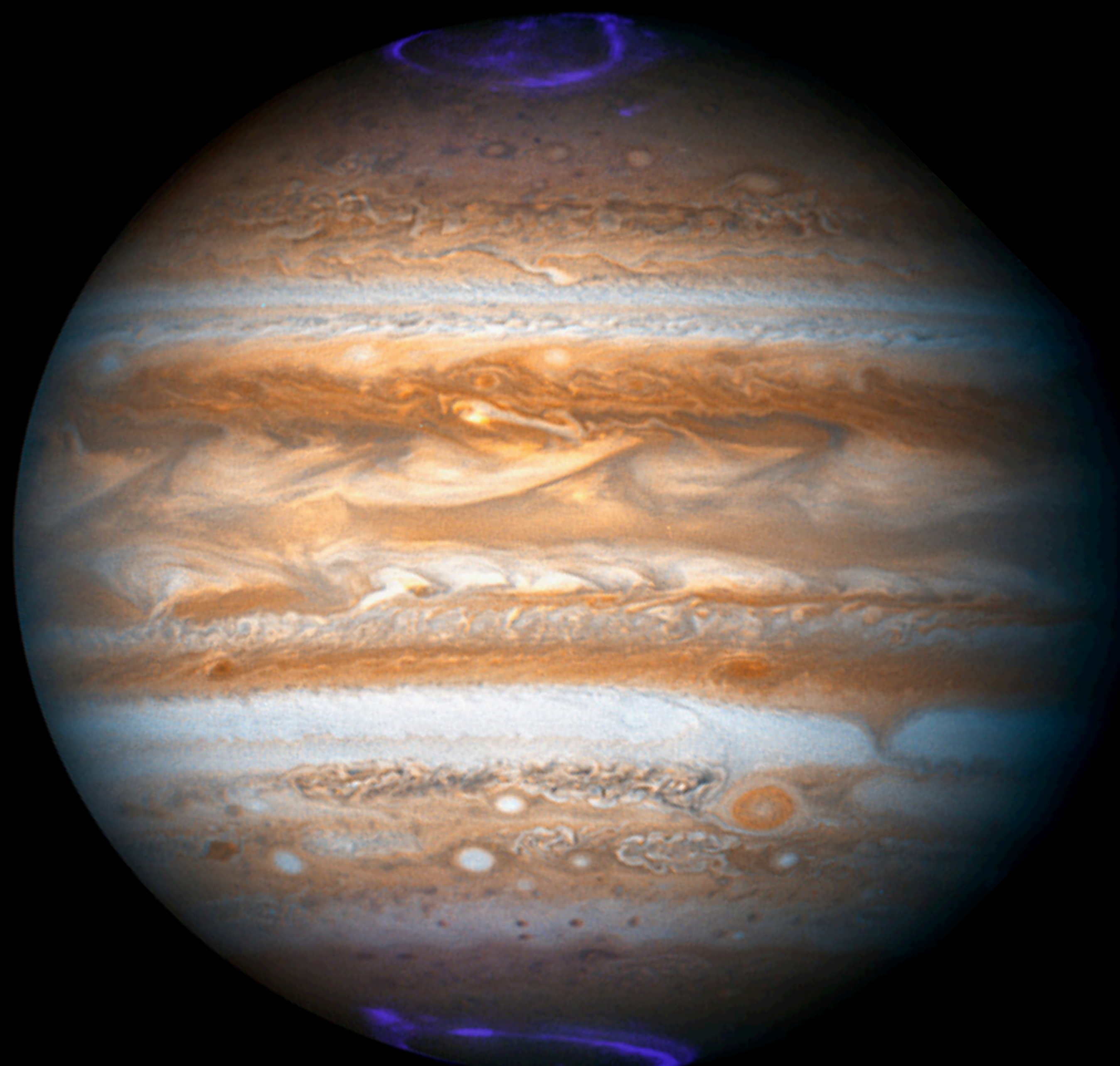


When a particle spirals around magnetic field lines in an increasing B field, the particle can be reflected - a "magnetic mirror." Particles that aren't reflected excite atoms in the upper atmosphere, producing auroras.

Aurora Australis: This view of the Aurora Australis, or Southern Lights, which was photographed by an astronaut aboard Space Shuttle Discovery (STS-39) in 1991, shows a spiked band of red and green aurora above the Earth's Limb. Calculated to be at altitudes ranging from 80 - 120 km (approx. 50-80 miles), the auroral light shown is due to the "excitation" of atomic oxygen in the upper atmosphere by charged particles (electrons) streaming down from the magnetosphere above.

From space, the aurora is a crown of light that circles each of Earth's poles. The IMAGE satellite captured this view of the aurora australis (southern lights) on September 11, 2005, four days after a record-setting solar flare sent plasma—an ionized gas of protons and electrons—flying towards the Earth. The ring of light that the solar storm generated over Antarctica glows green in the ultraviolet part of the spectrum, shown in this image. The IMAGE observations of the aurora are overlaid onto NASA's satellite-based Blue Marble image. From the Earth's surface, the ring would appear as a curtain of light shimmering across the night sky.





This image is a composite made from ultraviolet- and visible-light images of Jupiter taken with Hubble from February 17-21, 2007. The glowing aurorae near Jupiter's North and South Poles were imaged using Hubble's Advanced Camera for Surveys' surviving ultraviolet camera. Jupiter's ever-changing cloudtops are seen through blue and red filters with Hubble's Wide Field Planetary Camera. In this dramatic image, Jupiter shows a novel array of cloud features including the recently formed Little Red Spot, a smaller version of Jupiter's well-known and long-lived Great Red Spot. Atmospheric features as small as 100 miles (160 km) across can be discerned.