Midterm 1
2/1/07
Physics 116A

Note: Throughout this test the symbol i denotes $\sqrt{-1}$.

1. Test the following series for convergence or divergence. Decide for yourself which test is easiest to use.
   (a) (10 points)
$$\sum_{n=1}^{\infty} \frac{n^5}{5^n}$$
   (b) (10 points)
$$\sum_{n=2}^{\infty} \frac{1}{n^2 - n}$$

2. Use series to calculate
   (a) (10 points)
$$\frac{d^4}{dx^4} \ln(1 + x^3) \quad \text{at} \quad x = 0$$
   (b) (10 points)
$$\frac{d^{10}}{dx^{10}} (x^8 \tan^2 x) \quad \text{at} \quad x = 0$$

3. (10 points) Find the following limit using Maclaurin series.
$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$$

Hint: First combine fractions. Then find the first term of the denominator series and the first term of the numerator series.

4. (20 points) Using $\exp i\theta = \cos\theta + i\sin\theta$, prove that
$$\cos\theta + \cos 3\theta + \cos 5\theta + \ldots + \cos(2n-1)\theta = \frac{\sin 2n\theta}{2\sin\theta}$$

Hint: For a complex number z,
$$\sum_{k=0}^{n-1} z^k = \frac{1 - z^n}{1 - z}$$

5. (15 points) Find all values of $\sqrt[3]{-1}$.

6. (15 points) Find the disk of convergence of the series $\sum_{n=1}^{\infty} (z - 2i)^n / n$.

7. (10 points) Calculate the determinant
$$\begin{vmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{vmatrix}$$