Midterm 2
Physics 116A
3/1/07

Make sure to attach this exam to rest of your work.
For each multiple choice question, mark only one answer per question. No partial credit will be given for a wrong answer to one of these, but you must briefly explain your answers.

Below z^* *denotes the complex conjugate of* z*.* i *denotes* $\sqrt{-1}$*.*

1 . (15 points) Is $\mathbf{F}(\mathbf{r}) = \mathbf{A} \times \mathbf{r}$ (where $\mathbf{A}$ is a given vector) a linear vector function? Prove your conclusion using $\mathbf{F}(\mathbf{r}_1 + \mathbf{r}_2) = \mathbf{F}(\mathbf{r}_1) + \mathbf{F}(\mathbf{r}_2)$ and $\mathbf{F}(a\mathbf{r}) = a\mathbf{F}(\mathbf{r})$.

2 . (15 points) Consider the operator that acts on matrices to find their transpose. Is it linear? Use the criteria analogous to the previous problem.

3 .

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

represents an active transformation of vectors in the (x, y) plane (axes fixed, vectors rotated or reflected).
(a) (10 points) Show that M is orthogonal.
(b) (5 points) Find $det(M)$.
(c) (15 points) Find the angle of rotation or line of reflection.

4 . Consider

$$H = \begin{pmatrix} 1 & 2i \\ -2i & -2 \end{pmatrix}$$

(a) (10 points) Show that it is Hermitian.
(b) (20 points) Find its eigenvalues and eigenvectors and write a unitary matrix U which diagonalizes H by a similarity transformation.

5 . (15 points) For an $n \times n$ Hermitian matrix H,

(a) $HH^\dagger = I$.

(b) $H_{ij} = H_{ji}$

(c) $H_{ij} = -{H_{ji}}^*$

(d) $H_{ij} = {H_{ji}}^*$

6 . (15 points) For an $n \times n$ Hermitian matrix, eigenvalues

(a) are always real.

(b) can never be 0.

(c) can be complex if the matrix elements are complex.

(d) cannot be degenerate.

problem 5: d
problem 6: a

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