

$$1. \quad u = X(x) Y(y) Z(z) T(t)$$

$$\frac{X''(x)}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = \frac{1}{v^2} \frac{\ddot{T}(t)}{T}$$

$\underbrace{-k_x^2}_{\text{---}} \quad \underbrace{-k_y^2}_{\text{---}} \quad \underbrace{-k_z^2}_{\text{---}}$

$$X(x) = \left\{ \sin k_x x, \cos k_x x \right\}$$

$$\text{B.C. } X(0) = X(L) = 0 \Rightarrow X(x) = \sin \frac{n\pi x}{L} \text{ i.e. } k_x = \frac{n\pi}{L}$$

$$\text{Similarly } Y(y) = \sin \frac{m\pi y}{L}, \quad k_y = \frac{m\pi}{L}$$

$$Z(z) = \sin \frac{p\pi z}{L}, \quad k_z = \frac{p\pi}{L}$$

$$T(t) = \frac{\sin}{\cos} (\sqrt{k_x^2 + k_y^2 + k_z^2} t)$$

$$\frac{\partial u}{\partial t} \Big|_{t=0} = 0 \Rightarrow T(t) = \cos(\sqrt{k_x^2 + k_y^2 + k_z^2} \sqrt{t})$$

$$\therefore u = \sum_{n,m,p=1}^{\infty} a_{nmp} \sin \frac{n\pi x}{L} \sin \frac{m\pi y}{L} \sin \frac{p\pi z}{L} \cos(\sqrt{n^2 + m^2 + p^2} \frac{\pi v}{L} t)$$

at $t = 0$

$$\delta(x - \frac{L}{2}) \delta(y - \frac{L}{2}) \delta(z - \frac{L}{2}) = \sum_{nmp} a_{nmp} \sin \frac{n\pi x}{L} \sin \frac{m\pi y}{L} \sin \frac{p\pi z}{L}$$

1. (cont'd)

multiplied by $\int_0^L \sin \frac{v\pi x}{L} \sin \frac{\mu\pi y}{L} \sin \frac{\lambda\pi z}{L} \dots dx dy dz$

$$\sin \frac{v\pi}{2} \sin \frac{\mu\pi}{2} \sin \frac{\lambda\pi}{2} = a_{v\mu\lambda} \left(\frac{L}{2}\right)^3$$

$$a_{v\mu\lambda} = \left(\frac{2}{L}\right)^3 (-1)^{(v+1)/2} (-1)^{(\mu-1)/2} (-1)^{(\lambda-1)/2}$$

$v, \mu, \lambda \text{ odd}$

0 otherwise

$$\therefore d = \sum_{v, \mu, \lambda = \text{odd}}^{\infty} \left(\frac{2}{L}\right)^3 (-1)^{(v+\mu+\lambda-3)/2} \sin \frac{v\pi x}{L} \sin \frac{\mu\pi y}{L} \sin \frac{\lambda\pi z}{L}$$
$$\cos \left(\sqrt{v^2 + \mu^2 + \lambda^2} \frac{\pi}{L} t \right)$$

2. (a) In 1 dimension

$$D \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad \text{Now integrate}$$

$$\int_0^L D \frac{\partial^2 T}{\partial x^2} dx = \int_0^L \frac{\partial T}{\partial t} dx$$

$$\therefore \frac{\partial}{\partial t} \int_0^L T dx = D \left[\frac{\partial T}{\partial x} \right]_0^L = 0$$

$$\Rightarrow \int_0^L T dx = \text{const. in time}$$

(b) In steady state $\frac{\partial T}{\partial t} = 0$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} = 0 \quad \Rightarrow T = ax + b$$

$$T(x=0) = 0 \Rightarrow T(x=0) = b = 0, \quad T(x=L) = 1 \Rightarrow a = \frac{1}{L}$$

$$\therefore T(x) = \frac{x}{L}$$

3. 1st solve problem for T' : $0 \boxed{ }^1_0$, see CH13 Sec 2 Prob 10

$$T' = \frac{4}{\pi} \sum_{n \text{ odd}} \frac{\sin(n\pi x) \sinh(bn\pi)}{n \sinh(bn\pi)}$$

$$\therefore T(x,y) = 1 + 0 \boxed{ }^1_0 + 0 \boxed{ }^0_0 + 1 + -\boxed{ }^0_0 + 0 \boxed{ }^0_{-1} = 0 \triangle^1_0$$

$$= 1 + T'(x, y) + T'(y, -x) - T'(y, x) - T'(x, -y)$$