Physics 120/240
Homework 5
Due 5/28/08

1. Consider the Rouse equation:

$$\frac{\partial r}{\partial t} - \frac{\partial^2 r}{\partial s^2} = \eta(s,t) \tag{1}$$

where s is arclength, t time, $r(s,t)$ is position, and η is random noise with zero mean and autocorrelation function

$$\langle \eta(s,t)\eta(s',t')\rangle = \delta(s-s')\delta(t-t') \ . \tag{2}$$

Take r to be a scalar. The extension to the vector case is straightforward. This problem illustrates how to calculate $\langle (r(s,t)-r(0,t))^2\rangle$, and $\langle (r(s,t)-r(s,0))^2\rangle$.

(a) Define the Fourier transform with respect to both s and t of $r(s,t)$, Call it $\hat{r}(k,\omega)$. Write down the inverse, that is, how to express $r(s,t)$ in terms of integrals of $\hat{r}(k,w)$. Do the same for η. Call the result $\hat{\eta}$.

(b) Fourier transform equation 1 to obtain an algebraic equation.

(c) Fourier transform eqn. 2 with respect to *two* sets of variables $s \to k$ and $s' \to k'$ and similarly for t and t'. Calculate $\langle \hat{\eta}(k,\omega)\hat{\eta}(k',\omega')\rangle$.

(d) Solve for $\langle \hat{r}(k,\omega)\hat{r}(k',\omega')\rangle$.

(e) Consider $\langle r'(s,t)r'(s',t)$, where $r' = \partial r/\partial s$. Relate this to $\langle (r(s,t) - r(0,t))^2\rangle$, through an integral formula.

(f) Relate the Fourier transform of $r'(s,t)$ to $\hat{r}$ through the simple relation between the Fourier transform of a function and its derivative.

(g) Because the Rouse equation is translationally invariant in time, we can set $t = 0$ and calculate instead $\langle r(k,t=0)r(k',t=0)\rangle$. Calculate this by expressing it in terms of integrals over ω and ω'.

(h) Transform back to obtain $\langle r'(s,t=0)r'(s',t=0)\rangle$.

(i) From this obtain $\langle (r(s,t=0) - r(0,t=0))^2\rangle$ using part 1e.

(j) Compare this with the prediction for $\langle (r(s) - r(0)^2\rangle$ according to the equilibrium statistical mechanics of a random walk.

(k) Follow a similar procedure to obtain $\langle (r(s,t) - r(s,0))^2\rangle$. Note that there is translational invariance in arclength also so that you can set $s = 0$.