Physics 219
Homework 8

Read Gould and Tobochnik Thermal and and Statistical Physics Chapter 5. Look at the simulation applet problems, they're very instructive.

Do problems:

Sethna 9.5 (see Chapter 9 section 9.5 of Gould and Tobochnik for a treatment of many aspects of this problem),

Reif 10.1,

Gould and Tobochnik 5.22. This is most easily accomplished by employing the canonical ensemble and converting a term similar to $\exp(M^2/2)$ to a term proportional to

$$\int_{-\infty}^{\infty} e^{\lambda M} e^{-\lambda^2/2} d\lambda$$

1. Consider the one dimensional Ising model for N spins. The Hamiltonian

$$H = -J \sum_{i=1}^{N-1} s_i s_{i+1} , \qquad s_i = \pm 1$$

(a) Find the formula for the correlation function $\langle s_i s_j \rangle$, as a function of $i - j$. *Hint: Use the change of variables $\sigma_i = s_i s_{i+1}$. You should end up with a Hamiltonian in terms of these variables and possibly s_1, and find that they are all non-interacting.*

(b) How does the correlation length in the above formula depend on temperature? How does the correlation function depend on the sign of J?

2. Consider the one dimensional Ising model Hamiltonian

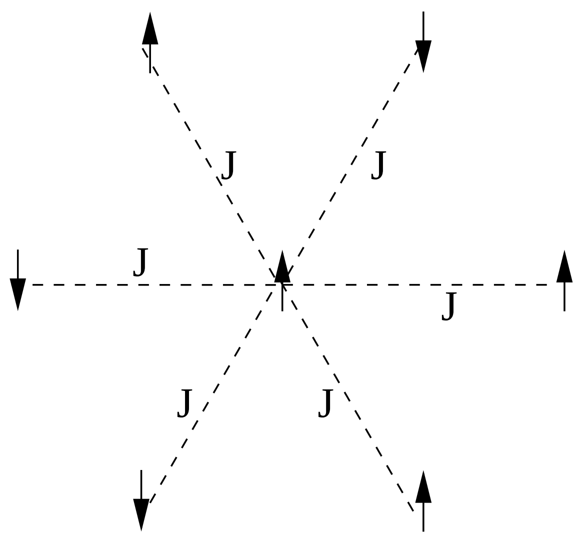
$$H = -J \sum_{i=1}^{N} s_i s_{i+1} , \qquad s_i = \pm 1$$

(a) What is $\langle s_i \rangle$?

(b) The Hamiltonian is now changed, by adding an extra term acting only on the first spin $H_1 = -hs_1$. Now what is $\langle s_i \rangle$? *See hint to problem 1(a)*

(c) How is $\langle s_i \rangle$ related to the correlation function $\langle s_1 s_i \rangle$?

3. N Ising spins $S_i = \pm 1$, $i = 1 \dots N$ are all connected to a central spin S_0, but not each other, through a ferromagnetic coupling J as shown below.



They are all in a uniform magnetic field. The Hamiltonian for the system is

$$H = -J \sum_{i=1}^{N} S_0 S_i - h \sum_{i=0}^{N} S_i$$

(a) Calculate the partition function for arbitrary N.

(b) For very large N calculate $\langle S_0 \rangle$ as a function of β and h.

(c) In the large N limit, state if there is a discontinuity in $\langle S_0 \rangle$ as a function of h, and for what temperature(s) it appears. Calculate the size of the discontinuity.

(d) In the large N limit, calculate $\langle S_i \rangle$ for all $i \leq N$, as a function of β and h.

4. A system of spins is described by the vector model

$$H = -\frac{J}{2} \sum_{i,j} \mathbf{s_i} \cdot \mathbf{s_j}$$

where the $\mathbf{s_i}$'s are unit vectors living in two dimensions and i, j are nearest neighbors on a two dimensional square lattice of dimensions $L \times L$. Take $J > 0$ and ignore quantum effects.

(a) What is the ground state and ground state energy?

(b) Find an approximate expression for H at low temperatures by writing the spin variables in terms of the angular deviation from the ground state θ_i. Your answer should involve terms no higher than second order in the θ_i's.

(c) Using this write down the partition function at low temperatures. What is the specific heat?

5. Consider the Hamiltonian

$$H = -J\sum_{i=1}^{N} S_i S_{i+1} - h\sum_{i=0}^{N} S_i$$

but now where S_i can take on the values -1, 0, and 1. Calculate the free energy for large N in this case. *Hint: read Gould and Tobochnik 5.4.4.*