Physics 231
HOMEWORK 2
for quiz 2 10/14/2009

Ashcroft and Mermin: 2.1,

2.2(a-e). In 2(a) do not use the method of derivation they suggest, but derive the answer more simply from the "p log p" formula for the entropy.

2.3(a,b) 4.1, 4.2

In this problem you'll work out the density of states for the simplest version of the "tight binding model" in one dimension. This is a lattice discretization of the Schrödinger equation for free particles. Instead of describing a wavefunction ψ as a function of a continuous variable, it only takes values at discrete points $i = 0, \ldots, N-1$, where N is the total number of points:

$$E\psi_i = \frac{-1}{2m}(\psi_{i+1} - 2\psi_i + \psi_{i-1}). \tag{1}$$

The term in the parenthesis is a discretized second derivative, so there is a direct analogy with Schrödinger's equation. Choose periodic boundary conditions, that is $\psi_0 = \psi_N$. As usual, this is an eigenvalue value problem, with E representing an eigenvalue of the Hamiltonian.

(a)How many solutions to this eigenvalue problem do you expect?

(b) Find all possible solutions for ψ and E given the above boundary conditions. These are just the eigenvalues and eigenvectors of the Hamiltonian.(*Hint: What is ψ for the usual free particle Schrödinger equation? Try using this kind of functional form for this problem.*)

(c) For large N, find the density of states of this system.