Physics 231
HOMEWORK 5
for quiz 5 11/4/2009

Ashcroft and Mermin: 10.2(a,b), 10.3

1. Consider the tight binding model in one dimension in an applied field e:

$$H \;=\; -\sum_{i=-\infty}^{\infty} [ei|i\rangle\langle i| \;+(|i+1\rangle\langle i| \;+\; |i\rangle\langle i+1|)]$$

Find the eigenfunctions of H .

Hints: First write the difference equation associated with the eigenvalue equation. It should look similar (but not identical) the the tight binding model in homework 2. To solve this equation consider the discrete fourier transform

$$\hat{\psi}_q = \sum_j \psi_j e^{-iqj}$$

This gives a **first order differential equation** *for $\hat{\psi}_q$. Solve this, and transform back to real space. The final integral can be done by using the identity:*

$$J_n(x) \;=\; \frac{1}{2\pi}\int_0^{2\pi} e^{i(x\sin\theta - n\theta)}$$

2. Consider the tight binding model on a two dimensional square lattice with only nearest neighbor interactions. There is a divergence in the density of states at half filling. Calculate the form of the divergence. Don't worry about prefactors, just obtain the functional form to leading order.