Homework 6 Physics 231

Ashcroft and Mermin: 12.2, 12.4 12.6 Ashcroft and Mermin: 17.1, 17.2

1. Consider the tight binding Hamiltonian in one dimension:

$$H = \sum_{i=-\infty}^{\infty} [\epsilon |i\rangle \langle i| + V(|i+1\rangle \langle i| + |i\rangle \langle i+1|)]$$

(a) Calculate the eigenstates and eigenvalues of H.

(b) If the electron is initially localized on the site i=0, i.e. $|\psi(t=0)\rangle = |0\rangle$, calculate the variance of the wavefunction as a function of time.

2. Consider the tight binding Hamiltonian in one dimension in an applied field:

$$H = \sum_{j=-\infty}^{\infty} \frac{V}{2} (|j+1\rangle\langle j| + |j\rangle\langle j+1|) - ej|j\rangle\langle j|$$

(a) According to the semiclassical model, how does the crystal momentum of a particle, k, depend on time? Assume that at t = 0, $k = k_0$.

(b) Now forget about the semiclassical model and consider that initially the wave function is a plane wave with crystal momentum k_0 . Calculate the subsequent wave function for the particle. How does the expectation value of k compare with the k found in the semiclassical approximation? Note the similarity of this problem and Ashcroft and Mermin 12.6 above.