Period relation for the 2:1 resonance in the GJ876 planetary system

Michael Nauenberg
Department of Physics
University of California, Santa Cruz, CA 95064

Abstract

The recent radial velocity Keck data for GJ876 in Laughlin et al. (2005) is shown to be in good agreement, apart from a 6 m/s scatter, with a theoretical calculation (Nauenberg, 2002) based on orbital parameters from a fit to the earlier Keck data. The time variation of the periods of the inner and outer planets, which are locked in a near 2:1 resonance, are evaluated, and their mean values, P_i and P_o, are shown to satisfy closely the resonance relation $P_o/P_i = 2 + P_o/P$, where P is the common mean period for the retrograde precesion of the periastron of each of these planets.

Recently, Laughlin et al. (2005) have given a progress report on the extrasolar planetary system GJ976, which consists of two Jupiter size planets in a near 2:1 resonance, and provided new high precision radial velocities obtained with the Keck telescope during the period 2001 to 2004. This new report discusses a fit to a coplanar configuration of the two planets which includes a 6 meters/second stellar jitter. The possible occurrence of such a jitter was suggested in (Nauenberg 2002) because of the excesively large values of the reduced chi-square in the fit to the data [1], and it appeared to have further confirmation by studies of chromospheric activity in M3-M4 stars similar to GJ876 (Laughlin et al. 2005). There also may be, however, other sources, e.g. additional smaller planets of order 6/200 of a Jupiter mass

[1] In earlier fits to the data (Marcy et al. 2001; Laughlin & Chamber 2001; Rivera & Lissauer 2001), the reduced chi square was reported incorrectly

which can also contribute to the estimated 4 to 6 meter/second variations in the maximum radial velocity of about 200 meters/sec [2]. In Figs. 1-3 we show that the radial velocities in the new Keck data are in good agreement, apart from a mean 6 meter/second scatter, with the values predicted from theoretical calculations (Nauenberg 2002) based on the orbital parameters obtained from a fit to the early Keck data of Marcy et al. (2001) including an additional 9 data points released previously. The scatter shown in Fig. 4 is larger than expected from the errors quoted in the data, but the relatively good agreement is surprising in view of the fact that in the new report (Laughlin et al. 2005) the velocities from the earlier data have been modified. This modification includes a substantial shift in the mean velocity of the central star of about 70 meters/second which is included in comparing the new data with our predictions.

It has been customary to characterize the orbital motion of the two planets of GJ876 by the parameters of the osculating ellipse at a given epoch. In particular, the two parameteres P_i and P_o associated with the periods of the inner and outher planets are approximately 30 and 60 days, respectively, which was the original basis for the assertion that these two planets move in a near 2:1 resonance. But the actual periods, defined as the time for a sidereal revolution of the planets, are not constants, but exhibit oscillations[3]. The inner planet moves alternatively between two nearby orbits, Fig. 5, with two different oscillating periods which are shown in Fig. 6 . The oscillations of the period of the outer planet is somewhat more regular and is shown in Fig. 7. These oscillations have a period of about 660 days. Of special interest is the mean value of the orbital periods, because the condition of a near 2:1 resonance implies the relation

$$\frac{P_o}{P_i} = 2 + \frac{P_o}{P} \tag{1}$$

where P is the common mean period for the rotation of the periastron of these planets, which is moving retrograde relative to the rotation of the planets . For completness, a derivation of this relation (Murray 1999) , which reveals the reason why $P_o/P_i > 2$, is given in the following Appendix.

[2]After the completion of this paper, a new planet of 7.5 earth- mass orbiting GJ876 has been reported (Rivera et al., to be published)

[3]The oscillations of the period, defined as the time elapsed between consecutive periastrons of the orbit, is shown in (Nauenberg 2002)

We have calculated the mean values of these periods by counting the number of rotations for a given time, and increasing this time t until there are no further changes in the period to a chosen accuracy. Increasing t from 1000 to 8000 days, we obtain $P_o = 61.035$ days, $P_i = 30.220$ days and $P = 3125$ days. Hence, $P_o/P_i - 2 = .0197$ and $P_o/P = .0195$, which is in very good agreement with the period relation, Eq. 1. Previously we had estimated that $P = 3200$ days (Nauenberg 2002),while Lee and Peale (2002) obtained $P = 3100$ days, and more recently Laughlin et al. (2005) reported $P = 3205$ days (quoted as a rotation rate of 41^o per year). Actually, we have evaluated P from the orbital parameters given in (Laughlin 2005), and obtained instead $P = 3136$ in better agreement with our results.

This analysis indicates that GJ876 offers a remarkable opportunity for a detail quantitative study of a near 2:1 resonance in a planetary system, which will greatly improve as more data becomes available in the future.

Appendix, Period relation for a near $r : s$ resonance

Since Laplace's pioneering work on the 2:1 resonances among three of the satellites of Jupiter (Laplace 1798), the period relation given in Eq. 1 has been extensively discussed (Murray 1999). In this appendix we derive this relation in a form which is directly related to the numerical approach which we applied to evaluate these periods. Consider the general condition that two planets are in a near $r : s$ resonance, where r and s are two integers. Let $\omega_i(t)$ and $\omega_o(t)$ be the time dependent angular frequencies for the inner and outer planets, and $\Omega_i(t)$ and $\Omega_o(t)$ the corresponding angular frequencies for the rotation of the periastron of these two planets. Then the number of rotations n_i and n_o of the inner and outer planet from one periastron [4] to the the next one, during a given time t, is given by

$$n_i = (\omega_i - \Omega_i)t/2\pi, \tag{2}$$

and

$$n_o = (\omega_o - \Omega_o)t/2\pi \tag{3}$$

[4] The periastron is the position on the orbit nearest to the central star. It is assumed that this point is connected to the farthest point on the orbit by a line, referred to as the line of apsides (Lee 2000; Laughlin 2005). However, when the mean periastron is rotating, as in GJ876, these two points are found to be 2^o to 3^o from such an imaginary line

where ω_i, ω_o, Ω_i and Ω_o are the mean values of these angular frequencies during t,

$$\omega_i = \frac{1}{t}\int_o^t dt'\omega_i(t'), \tag{4}$$

$$\omega_o = \frac{1}{t}\int_o^t dt'\omega_o(t'), \tag{5}$$

$$\Omega_i = \frac{1}{t}\int_o^t dt'\Omega_i(t'), \tag{6}$$

and

$$\Omega_o = \frac{1}{t}\int_o^t dt'\Omega_o(t'). \tag{7}$$

The condition for a near r:s resonance leads to an asymptotic condition for large t, which requires that

$$\frac{n_i}{n_o} = \frac{r}{s}, \tag{8}$$

and

$$\Omega_i = \Omega_o = \Omega. \tag{9}$$

Hence, according to Eqs. 2 and 3, $\Omega = r\omega_o - s\omega_i)/(r-s)$ which for a 2:1 resonance gives $\Omega = 2\omega_o - \omega_i$ (Murray 1999). For negative values of Ω, which corresponds to retrograde motion for the mean position of the periastron of both planets, the mean periods $P_i = 2\pi/\omega_i$, $P_o = 2\pi/\omega_o$ and $P = 2\pi/|\Omega|$ satisfy the relation

$$\frac{P_o}{P_i} = \frac{r}{s} + (\frac{r}{s} - 1)\frac{P_o}{P}. \tag{10}$$

For the case of GJ876, $r = 2$ and $s = 1$, we obtain Eq. 1

The angular oscillations of the periastron of the inner and outer planets relative to its mean precession, which have been discussed in (Lee 2002;Nauenberg 2002;Lee 2003;Laughlin 2005), is given by

$$\theta_i = \int_o^t dt'(\Omega_i(t') - \Omega) \tag{11}$$

$$\theta_o = \int_o^t dt'(\Omega_o(t') - \Omega) \tag{12}$$

Our numerical results are shown in Figs. 8 and 9, giving maximum displacements $|\theta_i| = 6^0$ and $|\theta_o| = 50^0$ which are somewhat different from those reported in (Laughlin 2005).

References

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[3] Laughlin, G., & Chambers, J.E. 2001, ApJ, 551,L109

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[5] Lee, M.H. 2003, ApJ, 611, 517

[6] Marcy, G.W., Butler, R.P., Fischer, D.A., Vogt, S.S., Lissauer, J.J.,& Rivera, E.J. 2001, ApJ, 556, 396

[7] Murray, C.D. & Dermott, S.F. *Solar Dynamics System* (Cambridge Univ. Press, 1999) pp. 321-408

[8] Nauenberg, M. 2002, ApJ, 568, 369

[9] Rivera, E.J., & Lissauer, J.J. 2001, ApJ, 559, 392

Figures

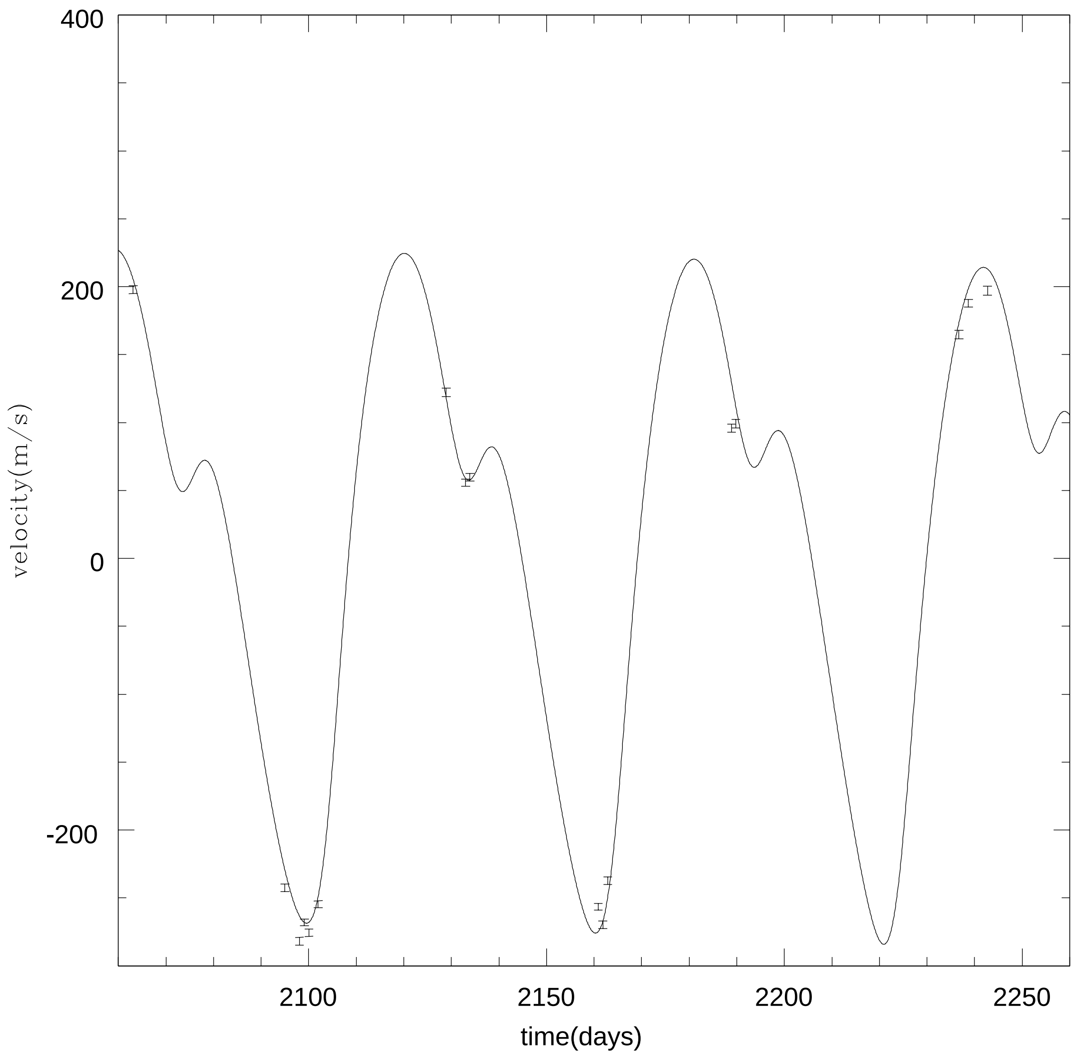

Figure 1: New Keck radial velocities (Laughlin et. al 2005) and calculation based on a fit (Nauenberg 2002) to the early Keck data (Marcy et al. 2001)

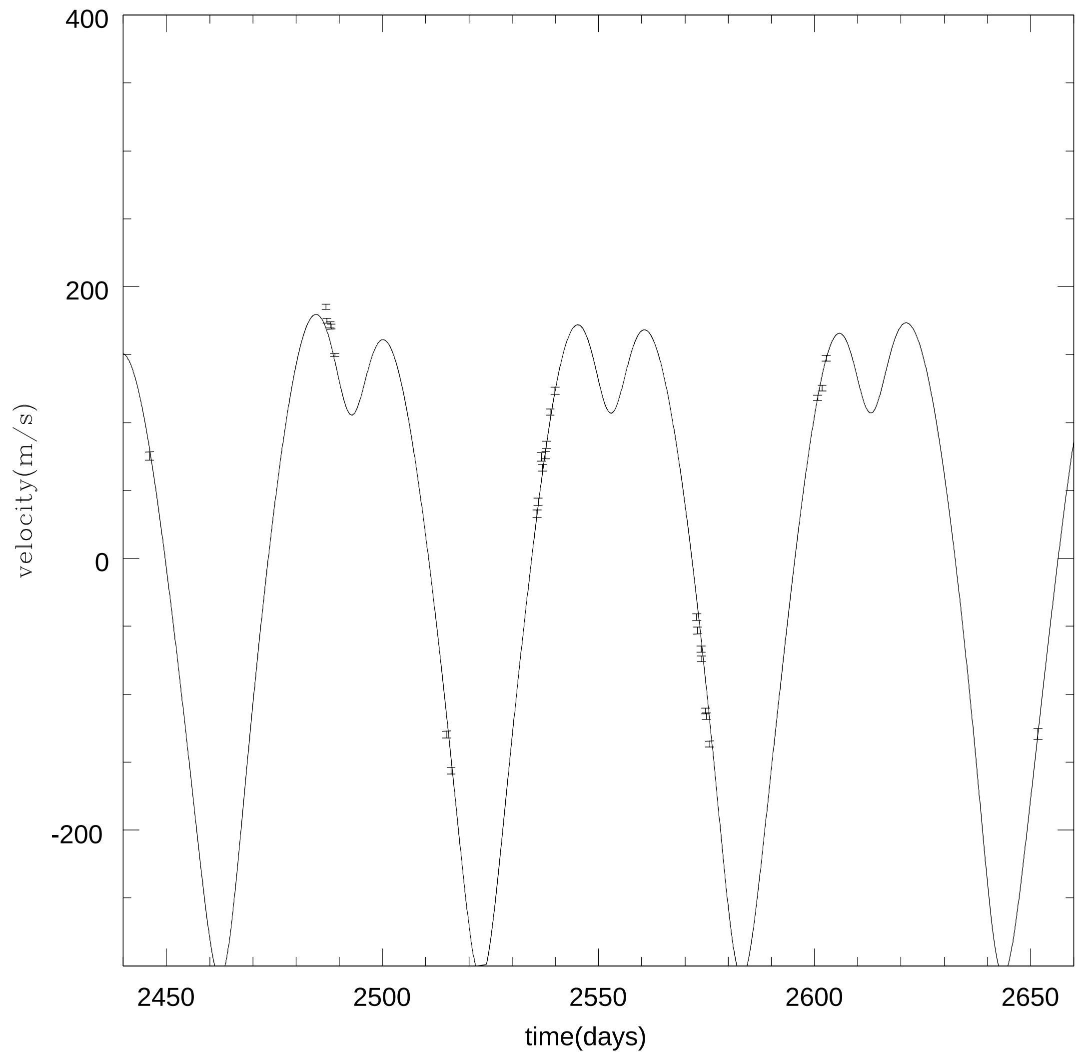

Figure 2: New Keck radial velocities (Laughlin et. al 2005) and calculation based on a fit (Nauenberg 2002) to the early Keck data of Marcy et al. (2001)

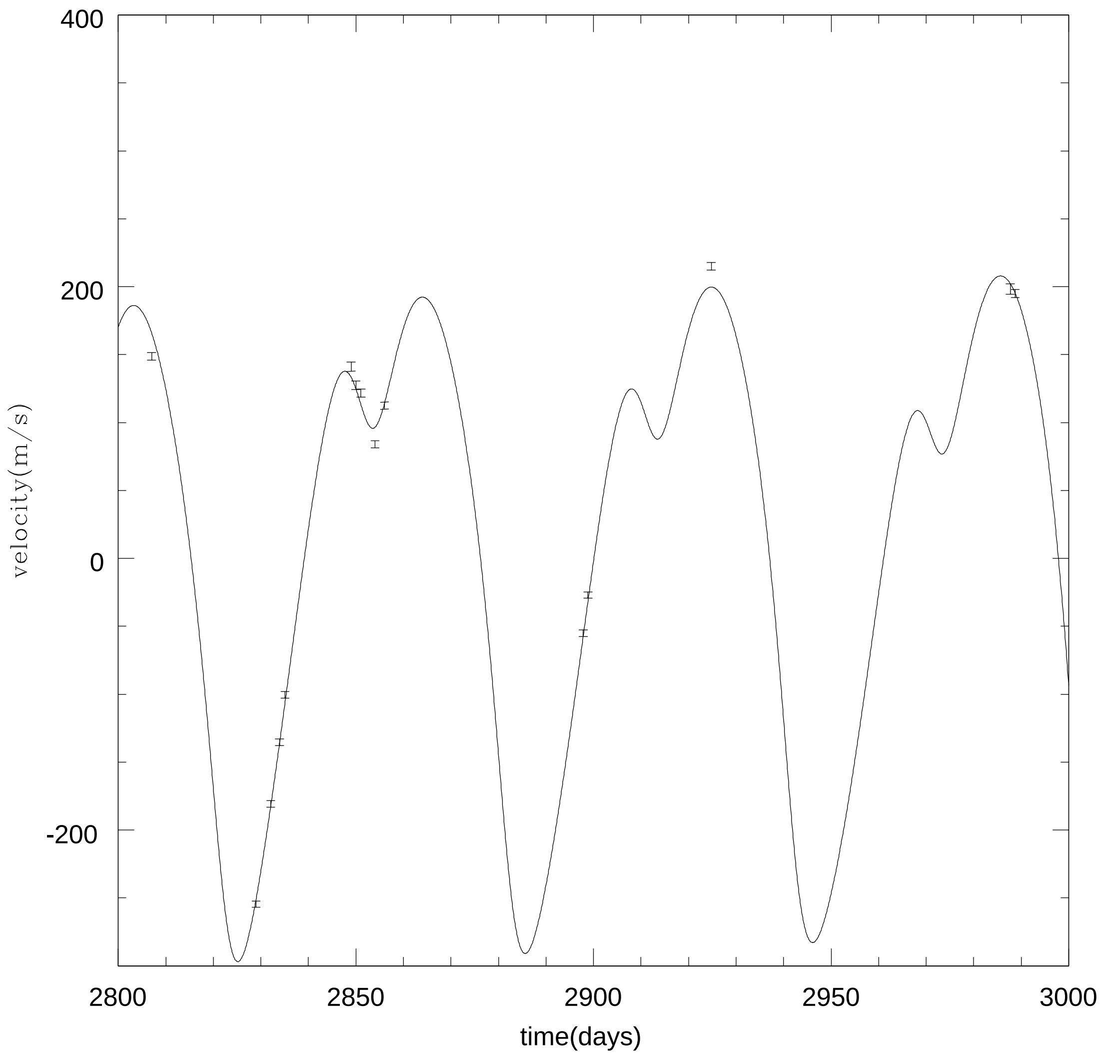

Figure 3: New Keck radial velocities (Laughlin et. al 2005) and calculations based on a fit (Nauenberg 2002) to the early Keck data Marcy et al. (2001)

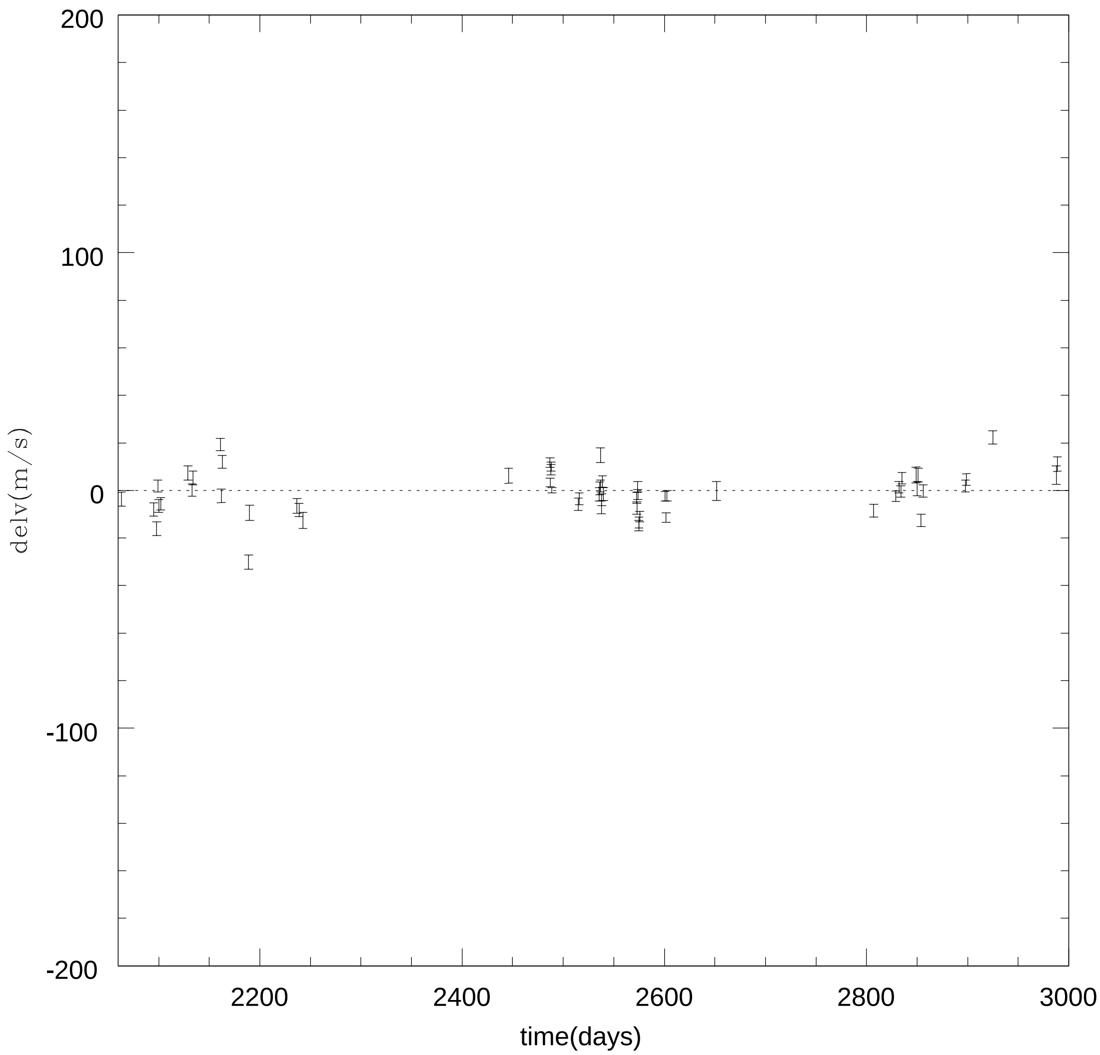

Figure 4: Scatter of new Keck radial velocities from a fit (Nauenberg 2002) to the early Keck data of Marcy et al.(2001)

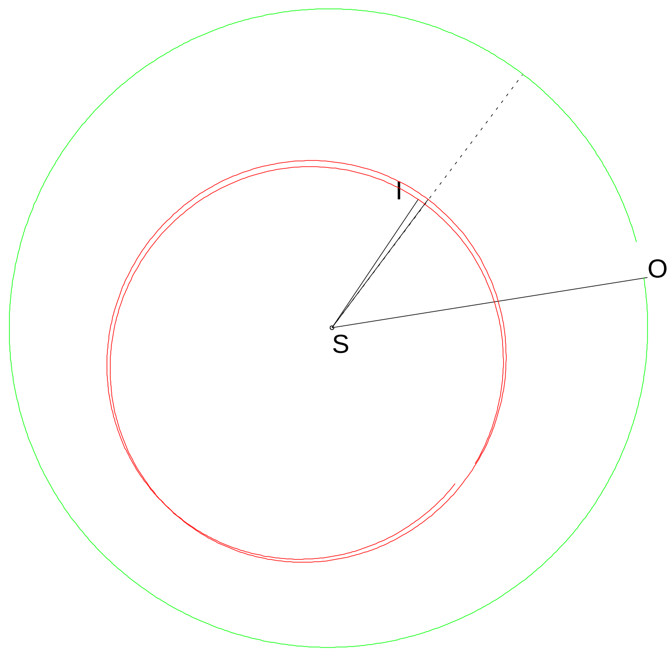

Figure 5: Two consecutive orbits of the inner planet (red curve) during a single orbit of the outer planet (green curve). The black lines indicate the positions of periastron for the inner planet at I, and the outer planet at O.

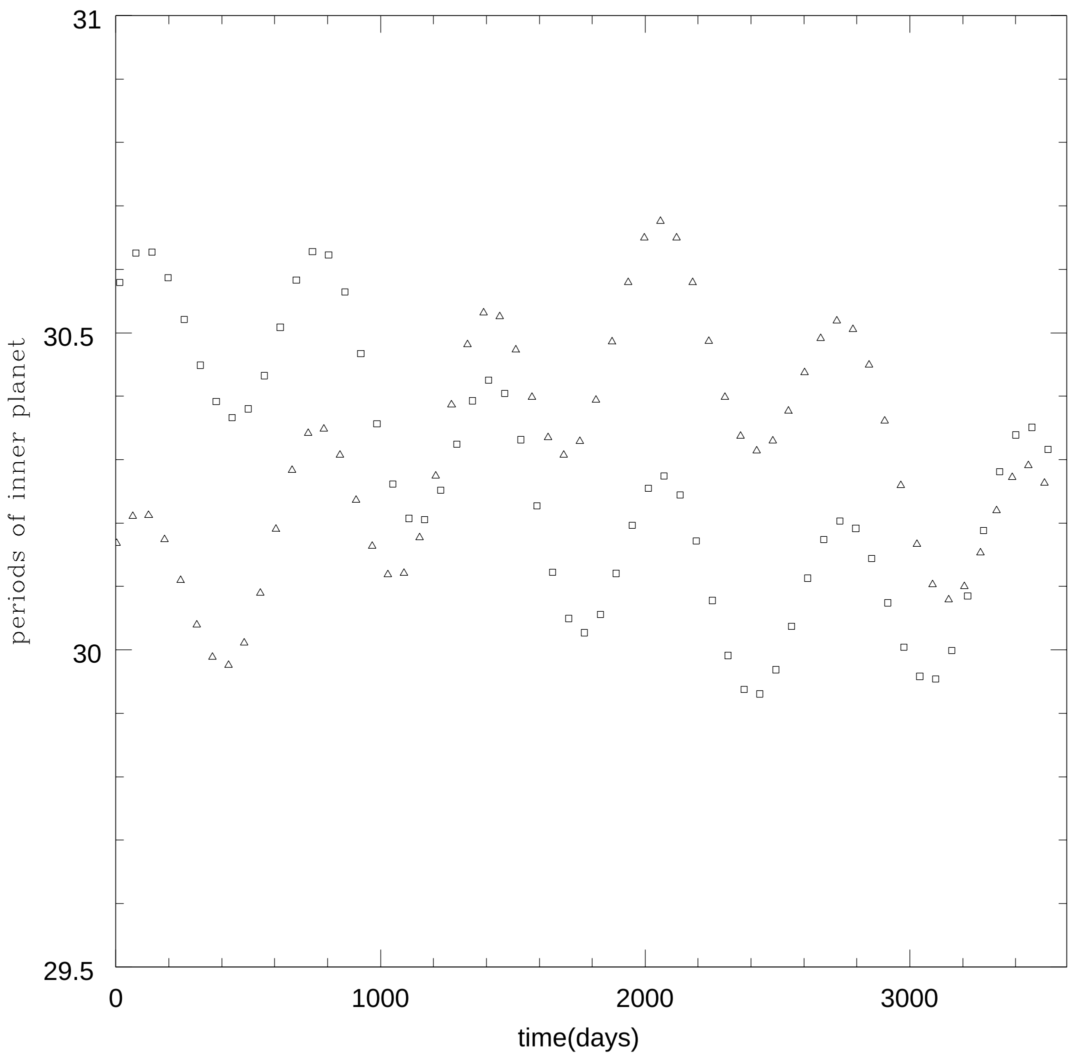

Figure 6: Oscillations of the two periods of the inner planet in its consecutive orbits, indicated by triangles and squares.

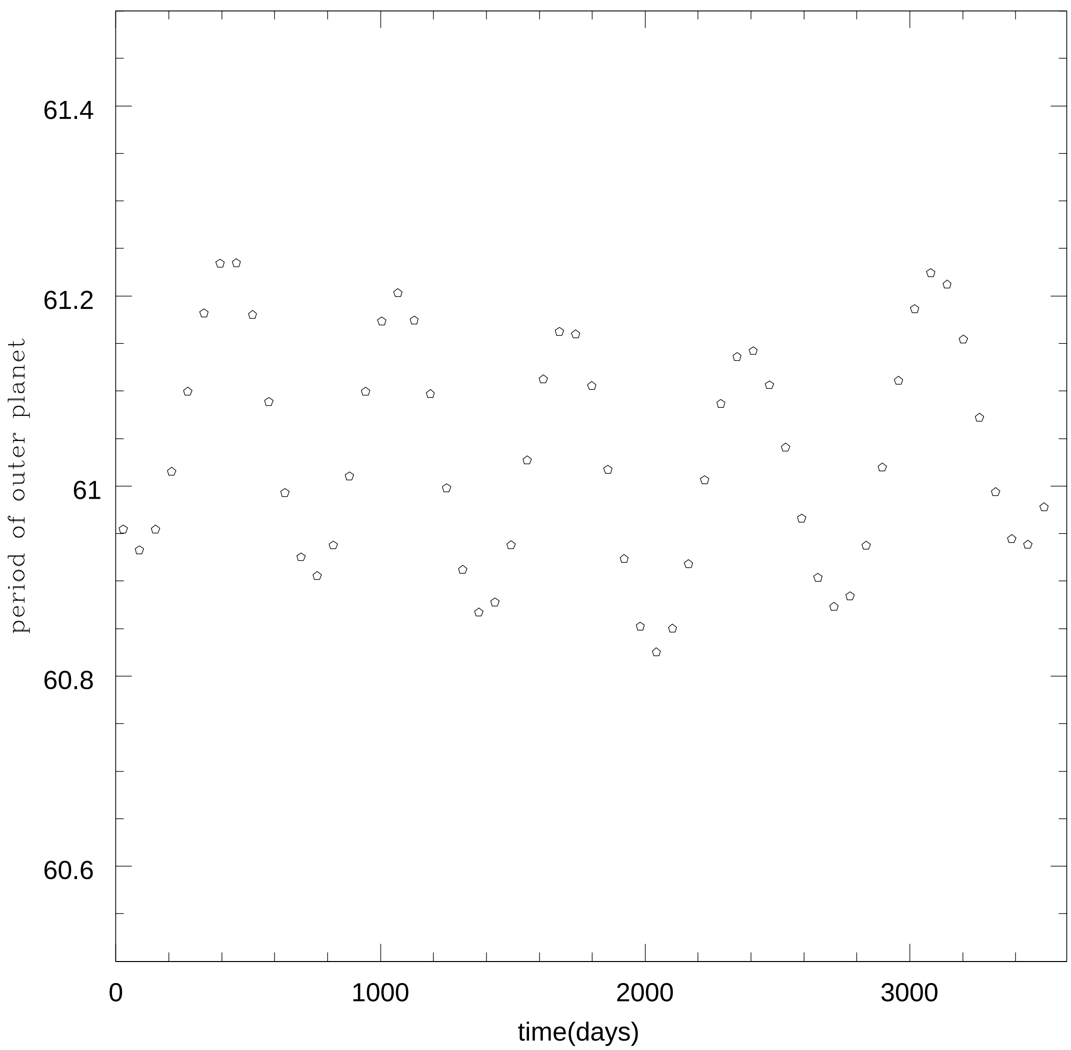

Figure 7: Oscillations of the period of the outer planet

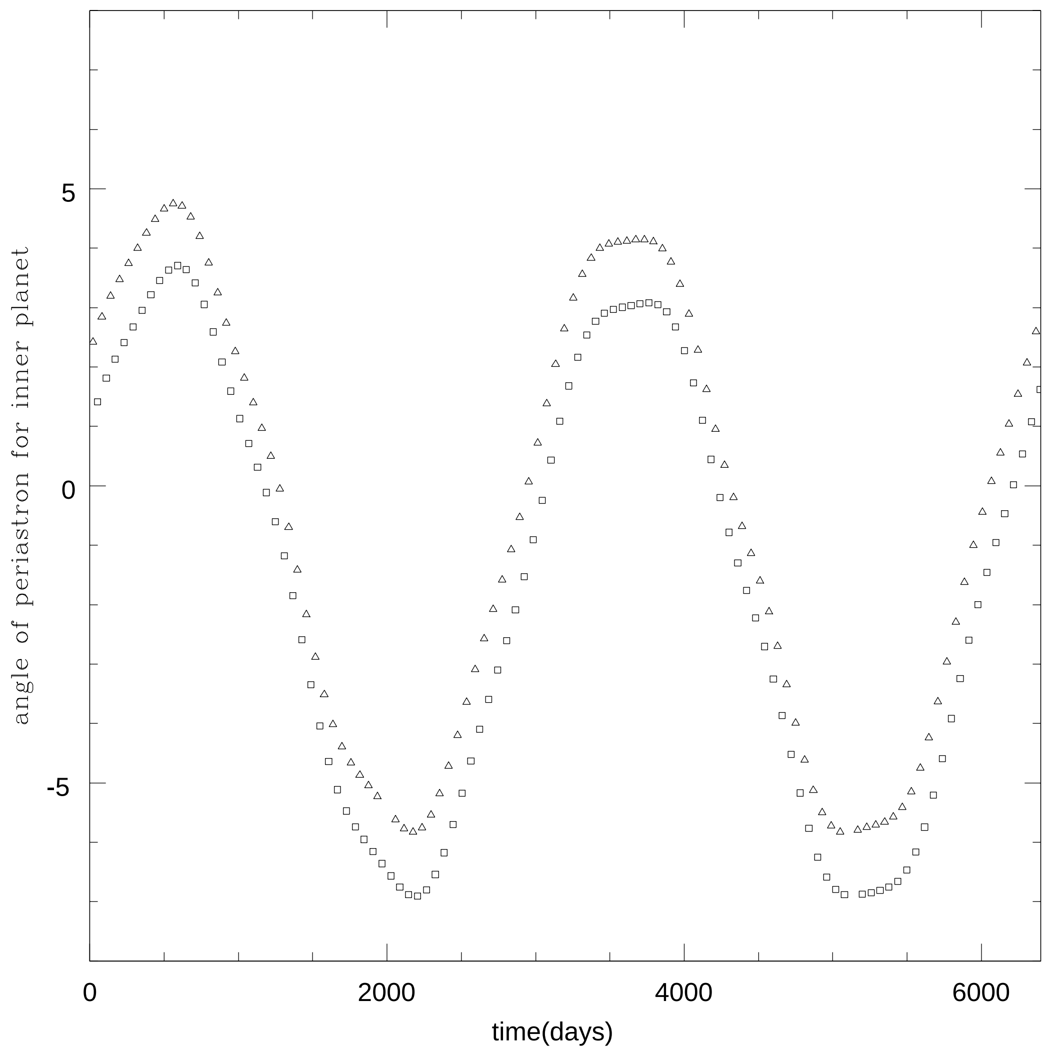

Figure 8: The periastron angle for the two consecutive orbits of the inner planet shown by triangles and squares

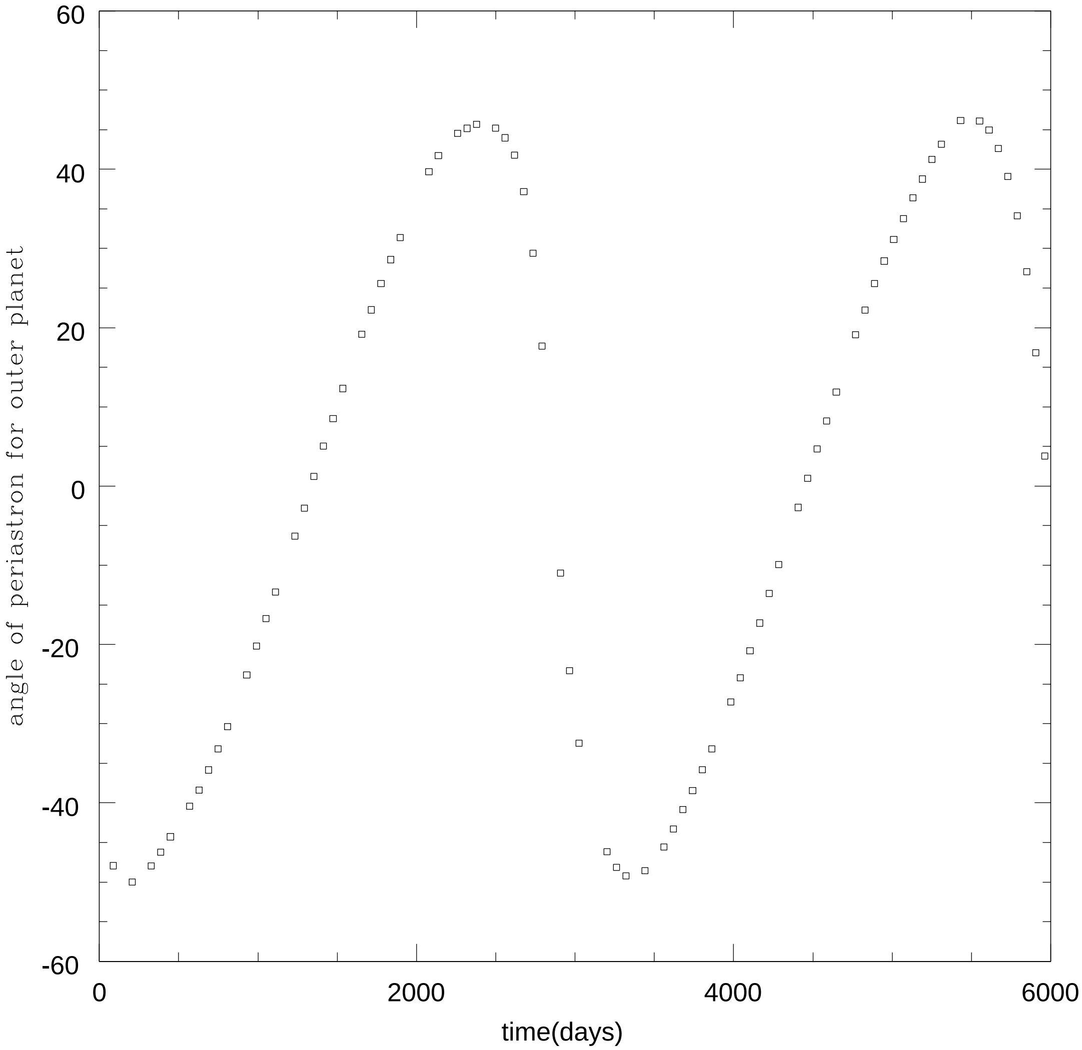

Figure 9: The periastron angle for the outer planet