In his book ‘Reading Newton’s Principia’, Niccolo Guicciardini recounts a well known legend about a student in Cambridge who was overheard to say, while Newton was passing by, that “ there goes the man who has writt a book that neither he nor anyone else understands” Indeed, the ‘Principia’ has always been a very difficult book to understand, and several guide books have been written to help the reader along the way. In the same spirit, in the first few chapters of his book, Guicciardini presents a helpful introduction to Newton’s novel combination of geometry and calculus which provides the mathematical foundations of the ‘Principia’. Shortly after its publication in 1687, mathematicians began to recast Newton’s propositions and proofs in terms of the analytic formulation of the calculus which had also been developed by Newton and by Leibniz, but not discussed in Newton’s book. This process picked up some speed after the publication in 1696 of the first textbook on the calculus written by Guillaume de L’Hospital from lectures by Johann Bernoulli. The heated debates which subsequently ensued over these diverse mathematical methods constitute the main subject of Guicciardini’s book. By the time of the appearance of the second edition of the Principia in 1713 Newton’s physical principles had been adopted, but his geometrical approach to the calculus had been mostly discarded. As William Whewell eloquently described it, “the ponderous instrument of synthesis, so effectively in Newton’s hands, has never since been grasped by anyone who could use it for such purpose: and we gaze at it with admiring curiosity, as some gigantic implement of war, which stands idle among the memorials of ancient days, and makes us wonder what manner of man he was who could wield as a weapon what we can hardly lift as a burden."

Guicciardini presents a very lucid and detailed account of this development in the history of mathematics. Continental mathematicians like Johann Bernoulli, Jacob Hermann and Pierre Varignon originally learned the calculus from Leibniz who in turn had been stimulated by Christiaan Huygens. Leibniz stressed the advantages of the analytic or algebraic formulation of the calculus as a powerful algorithm to solve difficult mathematical problems with great ease. While Newton reluctantly revealed his fluxional formulation of the calculus only in private correspondence, Leibniz actively promoted his equivalent differential formulation and published it in the Act Eruditorum in two papers in 1684 and 1686. Therefore, it was natural that after the appearance of the 'Principia' in 1687, Leibniz and his disciples began to reformulate Newton's propositions and proofs in their own mathematical language. Ultimately, this program was completed by a student of Johann Bernoulli, the great mathematician Leonhard Euler, and today what we call Newton's equations of motion correspond actually to Euler's formulation of Newton's dynamics. Significantly, one of the most recent guide books, ' Newton's Principia for the Common Reader', published in 1995 by the distinguished astrophysicist Chandrasekhar, is the most complete reformulation of Newton's propositions and proofs into Euler's equations.

To give an illustration of the nature of the debates over mathematical methods, one of the examples recounted by Guicciardini is Hermann's re-casting of Newton's proof that Kepler's area law is a consequence of central forces. This fundamental proof appears as Proposition 1 in the 'Principia', and it is based on a geometrical construction where the force consists of discrete impulses. In Proposition 6, however, Newton applied this law to obtain the radial dependence of the force by using a different geometrical construction where the force acts continuously. Hermann was dissatisfied with Proposition 1, and developed a proof of the area law based on the same geometrical construction for a continuous force which appears in Proposition 6. It should be pointed out, however, that in Proposition 1 the area law is manifest, while this is not the case with Hermann's proof. Indeed, prior to his 1679 correspondence with Robert Hooke, Newton had missed discovering the area law precisely because earlier he had developed a theory of orbital motion based on his curvature ideas along the same lines later rediscovered and applied by Hermann. This example also illustrates the difference which sometimes occur between the method by which a mathematical discovery is made and the proof subsequently given. Another interesting illustration given by Guicciardini is associated with Newton's Proposition 41 which gives a general solution for the inverse problem: given a central force and initial conditions, to obtain the resulting orbit "granting the quadrature of curvilinear figures". In modern language, Newton's expression for the orbit can be rewritten in the form of an indefinite integral (quadrature) in polar coordinates. He did not, however, give even a hint how to carry out such integrations, and the only concrete result presented was for the case of an inverse cubic force, rather than the physically interesting case of an inverse square force. This was typical of the problems that challenged contemporary mathematicians who had to develop their own methods of integration. One of the first to rise to this challenge was Johann Bernoulli who published a method to integrate Newton's expression in Proposition 41 for the case of an inverse square force. But Newton was not impressed, and his disciple John Keill unfairly dismissed Bernoulli's result as "le morceau de philosophie qui a jamais paru"

In the conclusion of his book, Guicciardini discusses the interesting question of ultimate choices in the history of science from the perspective of the debates on Newton's mathematical methods. While certain methods ultimately prevail, one should not assume that those methods which are discarded have only a historical value. In fact, Newton's geometrical approach to dynamics continues to provide valuable insights, and one gains a deeper understanding of dynamics by reading the 'Principia' which is greatly elucidated by Guicciardini's book.

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