

Hooke's Memorandum on the development of orbital dynamics

Abstract

I discuss a memorandum entitled " A True state of the Case and Controversy between *Sir* Isaak Newton & *Dr* Robert Hooke as the Priority of that Noble Hypothesis of Motion of ye Planets about ye Sun as their Centers", where Robert Hooke summarized his seminal contributions to the physics of orbital motion and to the theory of universal gravitation.

In a brief handwritten but undated memorandum entitled " A True state of the Case and Controversy between *Sr* Isaak Newton & *Dr* Robert Hooke as the Priority of that Noble Hypothesis of Motion of ye Planets about ye Sun as their Centers" [1], Hooke recounted his *hypothesis* for the physics of orbital motion and his theory of universal gravitation. Hooke's memorandum, which remained unpublished during his lifetime, is historically quite accurate, contradicting numerous criticisms of his contemporaries and historians of science that Hooke always claimed for himself more credit than he actually deserved. In fact, to support his "priority" Hooke quoted verbatim from several extant documents: the transcript of his lecture on *Planetary Movements as a Mechanical Problem* given at the Royal Society on May 23, 1666 [2], his short (28 pages) monograph, *An Attempt to prove the motion of the Earth by Observations* published in 1674 [3], and his lengthy correspondence in the Fall of 1679 with Isaac Newton [4]. However, Hooke did not mention his remarkable geometrical implementation of orbital motion for central force motion, see Fig. 1, based on the application of his physical principles, which was found only recently in a manuscript dated Sept. 1685 [6] [7]. Unfortunately, Hooke did not publish this manuscript and related work in spite of Edmond Halley's urging him "... that unless he produce another

differing demonstration [from Newton's], and let the world judge of it, neither I nor any one else can believe it" [5]. It can be seen that Hooke's geometrical construction is virtually the same as the one described by Newton, see Fig. 2, in connection with his proof of Kepler's area law in *De Motu*, a short draft that Newton sent to the Royal Society in 1684, which subsequently he expanded into his monumental work, the *Principia*[8].

In his memorandum, Hooke recounts that already in 1666 he had suggested that the motion of planets around the sun can be understood by the "inflection of a direct motion [inertial motion] into a curve by a supervening attractive principle" [1], the gravitational attraction of the sun. He supported this novel physical insight by a mechanical *analog*, namely, the motion of a conical pendulum, which he demonstrated experimentally by hanging a weight from the ceiling of the room where he lectured to members of the Royal Society. He also analyzed the pendulum motion mathematically, showing that the component of the net force directed towards axis of the pendulum increased linearly with the distance, recognizing that it "seems to be otherwise in the attraction of the sun..." [2]. In his 1674 monograph, which contains his first Cutlerian lecture at Gresham College given in 1670, Hooke restated his physical principles for the origin of curved motion by the action of an attractive force, and then enunciated his "supposition" for the law of universal gravitation: that "all celesstiall bodys whatsoever have an attraction or a gravitating power towards their own Centers, whereby they attract not only their own parts, & keep them from flying from them, as we may observed the Earth to do, but that they do also attract all the other Celestiall Bodies which are within the sphere of their activity" [1] [3] As far as we know, this statement is the first *published* suggestion that the gravitational force which attracts objects to the surface of the earth also acts between celestial bodies. Hooke elaborated this theory by supposing that "... not only the Sun and Moon have an influence upon the body and motion of the Earth and the Earth upon them, but that Mercury, also Venus, Mars, Saturn and Jupiter by their attractive powers, have considerable influence upon its motion as in the same manner the corresponding attractive power of the Earth hath a considerable influence upon every one of their motions also". The great novelty of this extension of terrestrial gravity to celestial bodies is underscored by the disbelief with which it was received by some of the greatest scientific minds in Europe when it was proposed again, in similar form, about

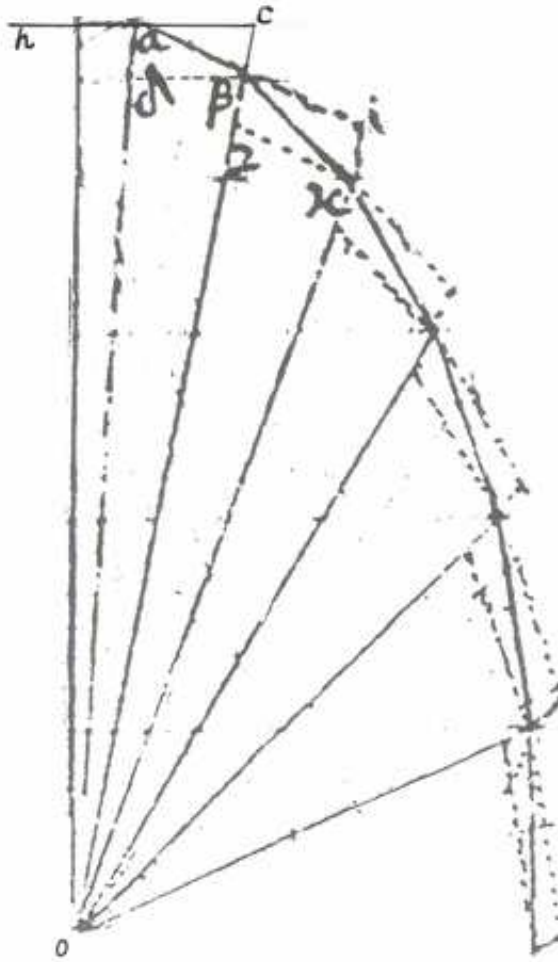


Figure 1: The upper right hand part of Hooke's Sept. 1685 diagram, with some auxiliary lines deleted, showing his geometrical construction for a discrete approximation to an elliptic orbit rotating clockwise under the action of a sequence of radial impulses which vary linearly with the distance from the center at O . For details, see ref. [7]

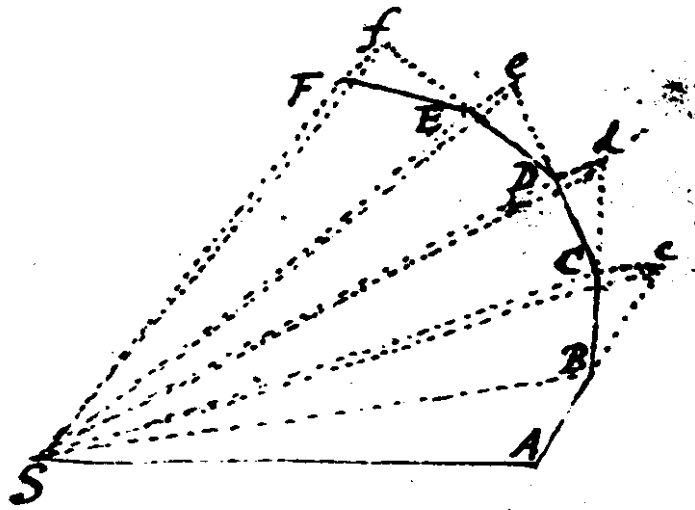


Figure 2: Diagram in *De Motu* associated with Newton's proof of Kepler's area law, showing the construction of a discrete orbit rotating counterclockwise under the action of a sequence of radial impulses of unspecified magnitude with center at S .

13 years later by Newton in Book 3 of his *Principia*¹. After first reading the *Principia*, Christiaan Huygens admitted that it never occurred to him "to extend the action of gravity to such great distances as those between the sun and the planets or between the moon and the earth" [9], while Gotfried Leibniz declared that "the introduction of gravitation of matter towards matter is in effect to return to occult qualities and, even worse, to inexplicable ones [10]. Actually, an account of Hooke's 1674 monograph introducing the idea of universal gravitation had appeared in *The Philosophical Transactions*, Vol IX, 101, 12, (1674), and four issues later there appeared extracts of several letters containing comments including one by Huygens. Evidently, after the publication of the *Principia* in 1687, Hooke's priority in proposing universal gravitation had been forgotten.

In the first edition of the *Principia* Hooke's early proposal for universal gravitation was not mentioned, while in the second edition (1713), Newton left it to his editor, Roger Cotes, to admit in an editor's preface, "that the force of gravity is in all bodies universally others have suspected or imagined, but Newton was the first and only one who was able to demonstrate it from phenomena and to make it a solid foundation for his brilliant theories". Even this small concession to "others", was left out in Newton's third and final edition (1726) of the *Principia*. Apparently, after hearing of Hooke's priority complains, Newton eliminated many references to Hooke in earlier drafts of his *Principia*. In a letter to Halley, Newton complained that "... he [Hooke] knew not how to go about it. Now is not this very fine? Mathematicians that find out, settle & and do all the business must content themselves with being nothing but dry calculators & drudges & and another that does nothing but pretend & grasp at all things must carry away all the invention as well as those who were to follow him as of those that went before him" [11]. Probably prompted by Newton, Cotes added the remark that "I can hear some people disagreeing with this conclusion[about universal gravitation] and muttering something or other about occult qualities. They are always prattling on and on to the effect that gravity is something occult, and that occult causes are to be banished completely from philosophy." [12]. But this remark applies only to Huygens and Leibniz, and not to Hooke who nowhere in his writings

¹This book is entitled *The System of the World* which are the words that Hooke used to introduce his theory of universal gravitation in his 1674 tract *An Attempt to prove the Motion of the Earth by Observations*

considered gravity to be an *occult* quantity.

In his memorandum, Hooke did not claim to know how the gravitational force varies with distance, supposing only "that these attractive powers are so much the more powerful in operating, by how much the nearer the body wrought upon is to their own Center". Instead, Hooke proposed that this dependence be determined experimentally, and predicted that it "will mightily assist the Astronomer to reduce all Celestiall Motions to a certain rule, which I doubt will never be done true without it" [3]. Finally, Hooke recalled that he communicated his principles for orbital motion in a correspondence with Newton. In a letter dated Nov. 24, 1679, he explicitly asked Newton "as a great favour if you shall please to communicate your objections against my Hypothesis or opinion of mine particularly if you will let me know your thoughts of that compounding of celestiall motions of the planets of a direct motion by the tangent & an attractive motion towards the centrall body" [3] [4]. Hooke notes that "in answer to this Newton pretends he knew not Hooke's Hypoth." ², referring to Newton's response on Nov. 28, 1679 that "...perhaps you will incline the more to believe me when I tell you that I did not before the receipt of your last letter, so much as heare (that I remember) of your Hypothesis of compounding the celestial motions of the Planets, of a direct motion by a tangent to the curve..." [3] [4] But in the same letter Newton also remarked that "I am glad to heare that so considerable a discovery you have made of the earth's annual parallax is seconded by Mr. Flamstead's Observations". Since Hooke had not mentioned his own role in this supposed discovery, this remark indicates that Newton already was familiar with Hooke's 1674 monograph, where Hooke had published his observations which he incorrectly had interpreted as due to the earth's annual parallax. But in this monograph Hooke also enunciated his principles of orbital dynamics, which Newton "pretended" not to have heard.

The question still remains what, if anything, did Newton learn from his 1679 correspondence with Hooke? Newton's early notebook, the *Waste book*, indicates that by 1664 he was already analysing uniform circular motion by the action of a sequence of impulses on a moving body that are directed towards the center of the circular orbit [13]. Therefore, it is incorrect to assert, as several historians of science have done, that Newton had learned this approach to orbital motion from Hooke [14]. But it is surprising that in

²this comment also appears as an insertion Hooke made in his copy of Newton's letter.

his Nov. 28 letter to Hooke, Newton claimed that he was unaware that Hooke had had similar views on orbital motion, because Newton had read Hooke's 1674 monograph. In his memorandum, Hooke recalled that in his response letter he had reminded Newton that "I could add many other considerations which are consonant with my Theory of Circular motions compounded by a Direct motion and an attractive one to a center"[1], [15]. Later on, in his 1686 correspondence with Halley regarding what he had heard about Hooke's priority claims, Newton focused mainly on the discovery of the inverse square dependence of the gravitational force, neglecting to mention Hooke's earlier formulation of the principles of orbital dynamics and the theory of universal gravitation.

According to David Gregory, who visited Newton at Cambridge in 1694, "I saw a manuscript [written] before 1669 ... where all the foundations of his philosophy are laid down: namely the gravity of the Moon to the Earth, and of the planets to the Sun. And in fact all these even then are subject to calculation..." [16]. This manuscript, which is found among Newton's still existing papers, indicates that by 1669 Newton had gone further than Hooke, having rediscovered the mathematical relation for the radial acceleration or central force in the case of uniform circular motion that had been found earlier by Huygens, but not published by him until 1673, when his *Horologium Oscillatorium* first appeared. Newton applied this relation to planetary motion, and by assuming that it satisfied Kepler's harmonic law, he found that "the endeavours of receding from the Sun will be reciprocally as the squares of the distance from the Sun" [16]. Newton assumed that such a dependence on distance applied also to the force attracting the moon to the earth, which he attempted to identify with the gravitational force acting on bodies at the surface of the earth. But due to an error in the value for the radius of the earth which he used in his calculations, he was misled into thinking that an inverse square dependence was not accurate for terrestrial gravity. Actually, it was not until after he discovered his error around 1685, by applying Piccard's correct value for the earth's radius to his earlier calculation, that he proved, by a remarkable mathematical feat that this dependence is valid up to the surface of any spherical body.³ Without this proof, however, the moon test for the universality of gravity is not possible, although usually

³In his book, *Newton's Principia for the Common reader*, Chandrasehkar calls it one of Newton's "superb theorems".

this is forgotten. In his 1686 correspondence with Halley, in which he rejected accusations that he had learned about the inverse square dependence from Hooke, Newton remarked that " Mr. Hook without knowing what I have found since his letters to me, can know no more than the proportion was duplicate *quam proximé* [approximately] at great distances from the center, & only guessed it to be so accurately, & guess amiss in extending that proportion down to the very center..." . Newton repeatedly used the word *guess* to indicate that Hooke had not provided any mathematical proof for his supposition "that the Attraction always is in a duplicate proportion to the Distance fromt he Center Reciprocally" as Hooke had written to him [17]. In a letter to Halley, Newton pointed out that in this "Theory I am plainly before Mr Hook. For he, about a year after [1673], in his *Attempt to prove the Motion of the Earth*, declared expressly that the degrees by which gravity decreased he had not then experimentally verified, that is he knew not how to gather it from phenomena, & therefore he there recomends it to the prosecution of others" [18]. Newton also asserted that Hooke had extended the inverse square proportion to the interior of the earth. But, instead, Hooke had correctly pointed that inside the earth the gravitational force varies linearly with the distance from the center, stating that " I rather Conceive that the more the body approaches the Center, the lesse will it be Urged by the attraction- possibly somewhat like the Gravitation on a pendulum or a body moved in a Concave Sphere where the power Continually Decrease the neerer the body inclines to a horizontal motion..." [17].

On Dec. 13, 1679 Newton wrote a remarkable letter to Hooke [19], which demonstrates that by that time Newton had gained a deep understanding of the physics of central force motion, and provides evidence that he had developed a very good approximate mathematical method to calculate the orbits for various central forces [20]. The letter includes a diagram, Fig. 3, which shows the trajectory of a body moving under the action of a central force which has a constant magnitude. Hooke promptly responded that "Your Calculation of the Curve by a body attracted by an equall power at all Distances from the center Such as that of a ball Rouling in an inverted Concave Cone is right and the two auges [farthest points from the center of force] will not unite by about a third of a revolution" [21]. Hooke must have been astounded that Newton could calculate a trajectory which previously he had observed in one of his mechanical experiments to understand orbital motion. In Fig. 4 I show a stroboscopic photograph of a steel ball rolling inside an inverted cone

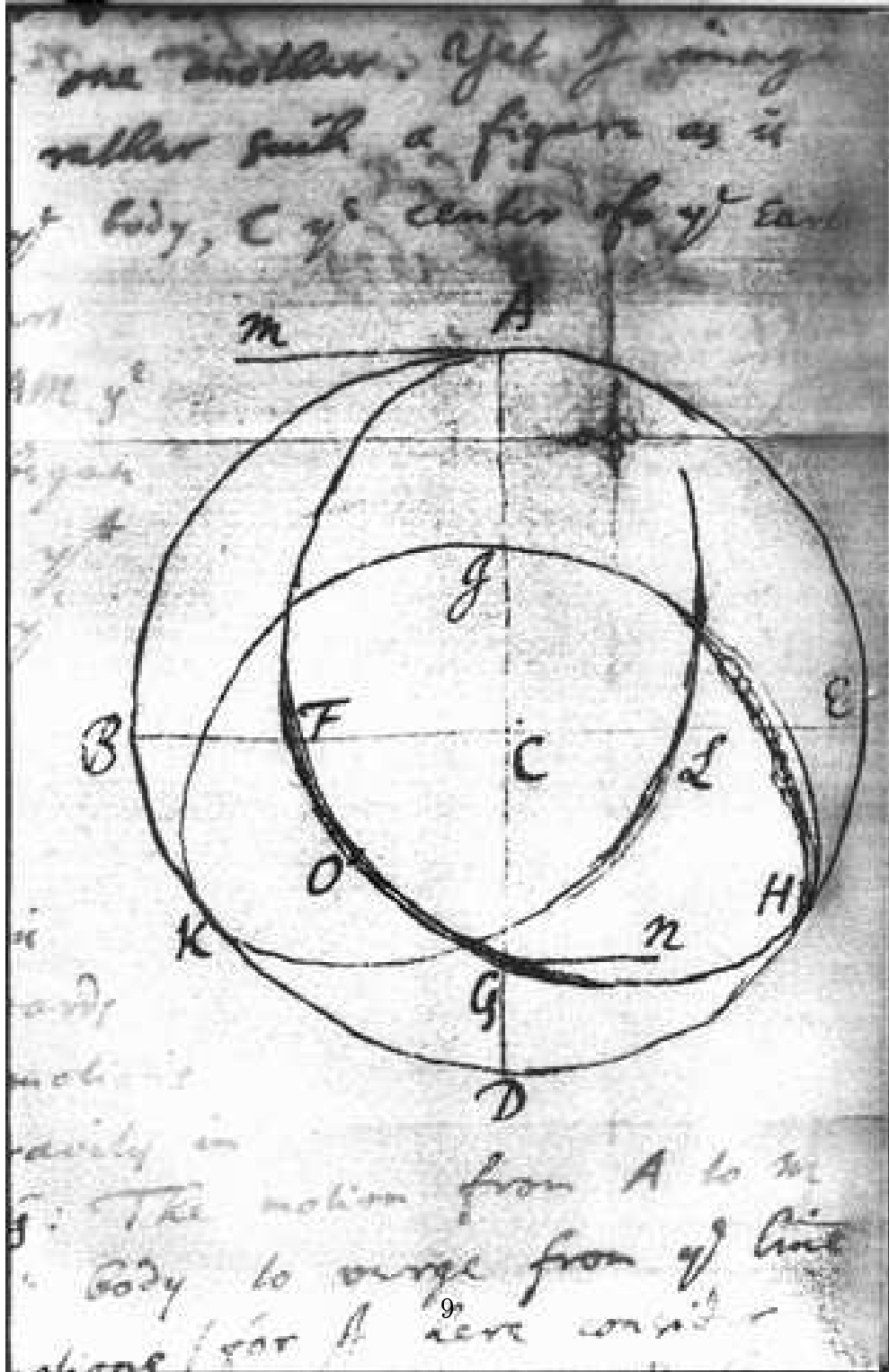


Figure 3: Diagram in Newton's Dec. 13, 1679 letter to Hooke, showing a curve *AFOGHIKL* for the approximate orbit of a body moving under the action of a constant central force.

which closely resembles the trajectory shown in Newton's diagram, Fig. 3, and attests to Hooke's careful observation. In the text to his letter to Hooke, Newton also discussed the changes in the orbit when the force increases with decreasing distance to the center, or in his words, " Thus I conceive it would be if gravity were the same at all distances from the center. But if it be supposed greater nearer the center the point O [nearest to the center C] may fall in the line CD or in the angle DCE or in other angles that follow, or even nowhere. For the increase of gravity in the descent may be supposed such that the body shall by an infinite number of spiral revolutions descend continually till it cross the center by motion transcendently swift" Although in his letter to Hooke, Newton did not identify the force law that would lead to such an extraordinary orbit with "an infinite number of spiral revolutions", later in 1684 he amplified his description in a Scholium to an early draft of the *Principia* [22], and he revealed that this force depends inversely as the cube power of the radial distance. This Scholium, however, was not included in the final draft of the *Principia*, and it has been generally ignored in the past. Thus, it is evident that by the time of his correspondence with Hooke, Newton already had developed a fairly sophisticated method to calculate orbital motion for central forces. I have given arguments [20] that Newton's method was based on his observation that for central forces the component of the force normal to the orbit determines its radius of curvature by the Huygens-Newton relation for circular motion, provided that the velocity is known. Newton indicated this connection in a cryptic remark in his 1664 notebook: " If the body b moves in an Ellipsis, then its force in each point (if its motion [velocity] be given) may be found by the tangent circle of equal crookedness [curvature] with that point of the Ellipsis" [13]. But in this curvature approach it is difficult to see that Kepler's area law [conservation of angular momentum] is a consequence of the action of central forces [20]. Newton discovered this fundamental connection, which became a cornerstone of his *Principia* as Proposition 1 in Book 1, only after his correspondence with Hooke. To prove this theorem Newton first had to discretize the continuous central force by a series of impulses, and then apply to general orbital motion the principles advocated for a long time by Hooke, which he had explained to Newton in his 1679 correspondence as " compounding the direct motion with an inflection towards the center of force". Although previously Newton had applied such a decomposition to uniform circular motion, evidently the impetus for considering it for general motion came from Hooke, yet Newton

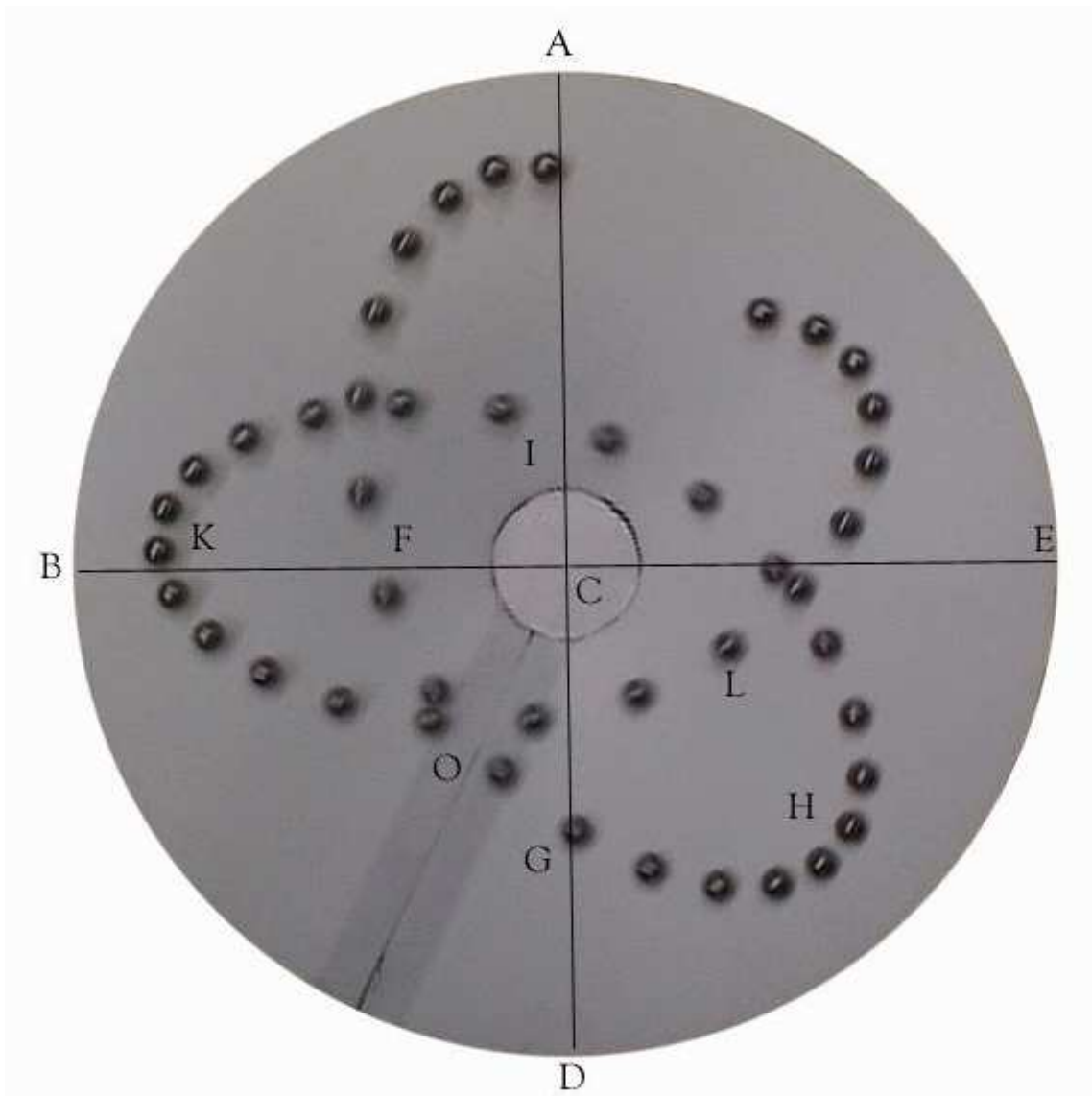


Figure 4: Stroboscopic photograph reproducing Hooke's observation of " a ball Roulling in an inverted Concave Cone".

vehemently denied that he learned anything from him, admitting only that " ...his correcting my Spiral occasioned my finding the Theorem by which I afterward examined the Ellipsis; yet I am not beholden to him for any light into that business but only for the diversion he gave me from my other studies and for his domaticalness in writing as if he had found the motion in the Ellipsis, which inclined me to try it after I saw by what method it was to be done..." [18]. But without Hooke's crucial intervention in 1679, it is most likely that Newton would have continued with his "other studies" [alchemy and theology]. Then, when in the Fall of 1684 Halley was travelling back to London after burying his father in Lincolnshire [23], and decided to visit Newton in Cambridge, he would not have been prepared to respond " an ellipsis" to Halley's famous question ⁴ "what he thought the Curve would be that would be described by the Planets supposing the attraction towards the Sun to be reciprocal to the square of their distance from it" [24].

References

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⁴according to Newton's later recollection, as told by Abraham De Moivre

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- [16] J. Herivel, *The Background to Newton's Principia* ref. [13] pp. 192-198
- [17] Letter from Hooke to Newton, Jan. 6, 1680, *The Correspondence of Isaac Newton* ref. [4] p. 309
- [18] Letter from Newton to Halley, July 27, 1686, *The Correspondence of Isaac Newton* ref. [4] p. 446
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