PHYSICS 112
Midterm, 2012

In class, Thursday February 9, 10:00 - 11:10 pm.

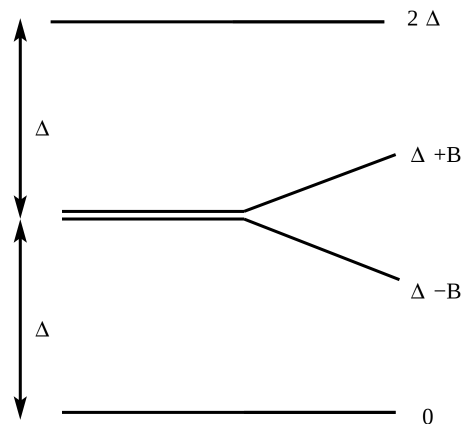
Closed book. You may bring in one sheet of notes if you wish.
No calculators are allowed. **You must show your working**

There are questions of both sides of the sheet.

1. [**40 points**]
Consider an atom with four energy levels as shown in the picture. The ground state has energy $E = 0$ and spin $S = 0$. There is an excited state with energy 2Δ and also spin $S = 0$. There is also a "doublet" half way in between, i.e. with energy Δ above the ground state. The doublet consists of two states with spin 1 and -1. A magnetic field B splits the "degeneracy" of the doublet, and the energy levels become

$$\begin{aligned} E &= 0, & \text{spin} &= 0 \\ E &= \Delta - B, & \text{spin} &= 1 \\ E &= \Delta + B, & \text{spin} &= -1 \\ E &= 2\Delta, & \text{spin} &= 0\,, \end{aligned}$$

see the figure.



(a) For the case of *zero magnetic field* determine:
   i. the partition function
   ii. the probability that the system has energy 0, the probability that it has energy Δ, and the probability that it has energy 2Δ,.
   iii. the free energy
   iv. the entropy
   v. the average energy
   vi. the expectation value of the spin, $\langle S \rangle$. Explain this result intuitively.

(b) For the case of a *magnetic field* B, determine
   i. the partition function
   ii. $\langle S \rangle$.

Note: Use the Boltzmann distribution.

2. [**35 points**]
Consider photons contained in a box of volume V. We showed in class that the density of photon states is

$$\rho(\epsilon) = \frac{V}{\pi^2} \frac{1}{(\hbar c)^3} \epsilon^2 \,, \tag{1}$$

where $\epsilon = \hbar\omega = \hbar c k$ is the photon energy.
Note: You are NOT required to derive this here.

Using Eq. (1) determine

(a) the mean number of photons, and

(b) the average energy,

as a function of temperature.
Note: You may leave any *dimensionless* integrals unevaluated.

3. [**25 points**]
Consider a system which has four levels as follows:

- One level with no particles. This has energy 0.
- Two levels with one particle. These each have energy ϵ.
- One level with two particles. This has energy $2\epsilon + U$.

(a) Determine the mean number of particles.
Note: This is a problem involving Boltzmann statistics, there is no chemical potential.

(b) Give simplified forms of your results for

i. $U = 0$

ii. $U \to \infty$.