

INTRODUCTION

Special Relativity Theory (SRT) provides a description for the kinematics and dynamics of particles in the four-dimensional world of spacetime. For mechanical systems, its predictions become evident when the particle velocities are high—comparable with the velocity of light. In this sense SRT is the physics of high velocity. As such it leads us into an unfamiliar world, quite beyond our everyday experience. What is it like? It is a fantasy world, where we encounter new meanings for such familiar concepts as time, space, energy, mass and momentum. A fantasy world, and yet also the real world: We may test it, play with it, think about it and experience it. These notes are about how to do those things.

SRT was developed by Albert Einstein in 1905. Einstein felt impelled to reconcile an apparent inconsistency. According to the theory of electricity and magnetism, as formulated in the latter part of the nineteenth century by Maxwell and Lorentz, electromagnetic radiation (light) should travel with a velocity c , whose measured value should not depend on the velocity of either the source or the observer of the radiation. That is, c should be *independent* of the velocity of the reference frame with respect to which it is measured. This prediction seemed clearly inconsistent with the well-known rule for the addition of velocities, familiar from mechanics. The measured speed of a water wave, for example, will depend on the velocity of the reference frame with respect to which it is measured. Einstein, however, held an intuitive belief in the truth of the Maxwell-Lorentz theory, and worked out a resolution based on this belief. He was driven to SRT mostly by aesthetic arguments, that is, by arguments of simplicity.

In his famous 1905 paper,¹ Einstein lays out the essential features. His paper contains an introduction, five sections on kinematics followed by five sections on electrodynamics, no references, and one acknowledgement. His development is based entirely on two postulates. Here they are as he wrote them, translated from the German:

- (1) The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion.
- (2) Any ray of light moves in the “stationary” coordinate system with the determined velocity c , whether the ray be emitted by a stationary or by a moving body.

¹ A. Einstein, “Zur Elektrodynamik bewegter Körper” (On the electrodynamics of moving bodies), *Annalen der Physik* **317**, 891 (1905). This volume of *Annalen der Physik* contains, in addition to the paper on special relativity, Einstein’s original papers on the photoelectric effect (for which he was awarded the Nobel Prize), and on Brownian motion. If you look on the shelves of McHenry Library you will find, where Vol. 17 should be, a block of wood. The volume is in Special Collections. It is indeed special. Look at it if you have a chance.

In modern language,² these two postulates may be stated as follows:

- (1) The laws of physics take the same form in all inertial reference frames. [The Principle of Relativity].
- (2) In empty space, light travels with a velocity c that is independent of the velocity of either the source or the observer. [The Principle of the Constancy of the Speed of Light].

The development of SRT in most textbooks follows that of Einstein. From the two postulates, the Lorentz transformation equations may be derived, and from them a variety of strange new predictions, like how moving meter sticks are shorter, how moving clocks tick more slowly, and how nothing can go faster than the speed of light.

A pedagogical difficulty with this approach stems from the fact that only the second of the two postulates lends itself to direct experimental verification (the Michelson-Morley experiment, for example). The first postulate is difficult to justify except on the basis of one's belief in the simplicity of nature. A result is that suspicion develops among students (and occasionally instructors) about the validity of the various conclusions. Lengthy discussions commonly ensue about whether moving clocks "really" tick more slowly, and whether moving sticks "really" are shorter. In general, clear resolution of the common paradoxes of special relativity is difficult, the "twin paradox" being the chief example.

These notes represent an alternative approach to learning and understanding SRT. To start with, we do not rely on the "Principle of Relativity". Instead, we focus on the concept of a *clock*, and a description of the observed behavior of real clocks, and how such clocks are observed to tick more slowly when they are moving. Second, our approach is highly graphic, taking great care to understand the four-dimensional spacetime geometry within which SRT takes place, and making liberal use of spacetime diagrams to describe, illustrate and clarify each concept and observation. With this geometrical approach based on solid experimental evidence, coupled with definitions of certain primitive concepts such as *event*, *light signal* and *inertial reference frame*, it is easy to discuss the meaning of a spacetime interval, the meaning of simultaneity, the meanings of length contraction and time dilation, and finally to derive the Lorentz transformation equations. Descriptions of the various "strange" phenomena follow hard on the experimental evidence, and the arguments are clear and easily defended. The "twin paradox" in particular follows with clarity; indeed, it becomes difficult to see that there is even a paradox involved.

The discussion of relativistic dynamics is likewise clarified and simplified though the use of the geometrical approach, with a focus on the use of momentum-energy diagrams. With this technique, it becomes possible to present clear discussions of particle interactions, including pair annihilation, pair production and the Compton effect. Inherent in this treatment is the concept of the *4-vector*, and in particular, the *4-momentum* and

² The expression "inertial reference frame" was not used by Einstein in his 1905 paper. Not until several years later did it come into vogue.

the observation that 4-momentum is conserved. Clarity also ensues with the abandonment of the useless concept of “relativistic mass” often referred to in textbooks. In our treatment the “rest mass” is the only mass that appears. Finally, our approach lends clarity to the relations between energy and mass, a matter which is often somewhat obscure in the traditional approach.

We originally wrote these notes for use during a three-week segment of the second quarter of our three-quarter sequence entitled “Introduction to Physics”. The material is preceded only by standard Newtonian mechanics, which is covered during the first quarter.

SRT deals with the most basic physical concepts. In the grandest sense, it provides us with a unifying framework encompassing not only mechanics, but also electricity and magnetism and quantum mechanics. Hence, it makes sense to put SRT near the beginning of our physics education. It’s nice to lay down the framework before building the rest of the structure. Perhaps the strongest reason, however, lies in the notion that a study of SRT can teach all of us something at a more basic level—a way of *thinking*. To start with the most elementary concepts we know, such as time, distance, velocity, momentum and energy, and to have our understandings of them drastically altered by the observational evidence, can only expand the mind, loosen up the brain, and cause us to distrust and re-evaluate all sorts of basic premises. In short, it teaches us to distrust our “common sense”—that set of predictions based on our past everyday experience. Since this kind of creative skepticism lies at the heart of the study of physics, it is appropriate that SRT be studied early on in the physics curriculum.

A great many people have written about special relativity. On the shelves of our library there are over 40 books with the words “special relativity” in the title, and numerous textbooks contain sections devoted to SRT. A thorough bibliography, though by no means encompassing all of the literature, appears in Gerald Holton’s “Resource Letter SRT-1 on Special Relativity Theory”, published in January 1962 in the American Journal of Physics. In addition, a collection of 16 reprints of journal articles listed in this bibliography has been compiled and published (along with the Resource Letter) by the American Institute of Physics under the title *Special Relativity—Selected Reprints*. Excellent translations of a number of Einstein’s papers (including his famous paper of June, 1905), along with translations of other papers by Lorentz, Minkowski and Weyl, are contained in *The Principle of Relativity*, A. Sommerfeld, Ed. (Dover, 1923). A comprehensively annotated version of a portion of an earlier translation of the June 1905 paper is included in that marvelous collection entitled *Great Experiments in Physics*, edited by Morris H. Shamos (Holt, Rinehart and Winston, 1959). Those wishing to trace the historical roots of SRT should take a look at Pais’ marvelous biography of Einstein, *Subtle is the Lord...* (Oxford University Press, 1982).

For those who wish to supplement these notes with further explanations, exercises, problems, stories and paradoxes, by far the best source is *Spacetime Physics*, by Edwin F. Taylor and John Archibald Wheeler (W.H. Freeman and Co., 1966). (Particular attention should be paid to a little paragraph on page 60). Another more traditional

treatment, from which some of the ideas for these notes were taken, is *Einstein's Theory of Relativity*, by Max Born (Dover Publications, 1962). Holton calls it (prior to the existence of Taylor and Wheeler) "without doubt the best elementary account. Thoroughly works out everything, from how to plot graphs through Maxwell, Minkowski, to GRT, with very few rabbits being pulled out of the hat." It is indeed a very nice little book. A third book, *Essential Relativity*, by Wolfgang Rindler (Springer-Verlag, 1979), lays considerable stress on the geometrical aspects of relativity, and includes a number of problems and exercises, some of which we have incorporated into these notes. Both Born and Rindler also provide excellent introductions to General Relativity for those who may be inclined to make further explorations in that direction.

Included with these notes are two pages of very heavy paper with curves drawn on them. These curves are calibration hyperbolae, and are meant to be carefully cut out with a sharp pair of scissors, and used as guides or templates to draw accurate calibration hyperbolae on the many spacetime diagrams which appear from time to time in the exercises and problems in these notes. They are as useful in SRT as a calibrated scale is in Euclidean geometry. Also included is a 45-degree triangle which is useful for drawing light lines on spacetime diagrams.

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