I. SPACETIME AND SPACETIME DIAGRAMS

In our geometrical approach to SRT, we shall emphasize the concept of *spacetime*, customarily represented by a *spacetime diagram*. The idea is a simple one, meant to draw attention to the intimate connection between the space and time coordinates that we use to describe the motion of particles in the universe.

Spacetime is the non-Euclidean four-dimensional world delineated by the three spatial axes x, y, and z, and the time axis t. A spacetime diagram is a graphical representation of spacetime. The most convenient such representation is a two-dimensional cross-section of spacetime consisting (by convention) of a plot of the time coordinate t vs. a single space coordinate such as x. It is also possible to represent a three-dimensional cross-section, in which t is plotted vs. two spatial coordinates. The actual four-dimensional world of spacetime is impossible to represent on a piece of paper, or even in three-dimensional coordinate space.

A point in spacetime is characterized by a particular position at a particular time, and is called an *event*. We often label a particular *event* by its position coordinate x and its time coordinate t like this: (x, t).

In spacetime, the existence of a particle is represented by a sequence of events, called a *world line*, which we often abbreviate by *WL*.

For most of our examples we shall consider particles moving in only one spatial dimension, which we take to be the x-direction. For such motion, spacetime is two-dimensional, and can be easily drawn on a flat piece of paper. Examples of such diagrams are shown in Fig. 1.

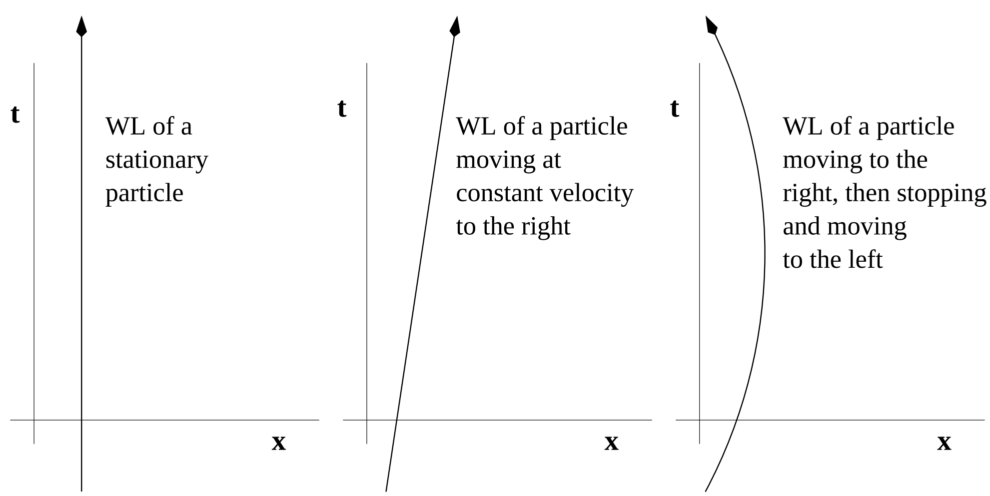

Figure 1—Spacetime diagrams
Note that the time axis is traditionally taken to be vertical.

Later in our discussion we shall occasionally consider particles moving in two dimensions, so that the spacetime diagram becomes three dimensional:

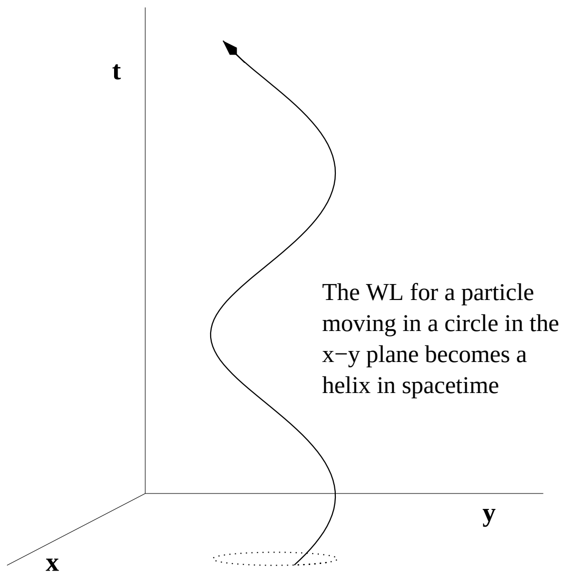

Figure 2—A three-dimensional spacetime diagram

Of course, we may also consider particles moving in three spatial dimensions, but as mentioned above, the spacetime diagram becomes four dimensional and impossible to represent. Fortunately, nearly all of the interesting behavior of SRT can be found in a two-dimensional spacetime diagram.

As can be seen from the examples in Fig. 1, the greater the angle between the t-axis and the world line, the greater the speed of the particle. To represent this speed quantitatively, it is necessary to scale the position and time axes appropriately. In the Newtonian picture, where space and time are not related to each other, we may freely scale the space and time axes to suit the problem at hand. For example, when considering the motion of a golf ball, we might choose position in meters and time in seconds. In SRT, however, the speed of light will play a crucial role. The light velocity provides a close connection between space and time, and it is helpful to scale the space axes so that distance is measured in units of time. Such scaling is already familiar to astronomers who often measure time in years, and distances in *light years*. Alternatively, if time is in seconds, distance will be in *light seconds*, or simply *seconds*. With the light velocity $c \approx 3 \times 10^8$ meters/sec, this means that 1 light second is 3×10^8 meters. A foot is about a nanosecond (10^{-9} sec), the length of a football field is about 1/3 of a microsecond, the moon is about 1.3 seconds away, and the sun is about 500 seconds away. In our diagrams, we'll make this explicit by plotting $t\ vs x/c$, rather than $t\ vs x$. In such diagrams, a light signal must travel along either the line $t = x/c$ or $t = -x/c$, that is, along lines at 45° to the axes. We shall commonly draw such *light lines* as dashed lines as shown in Fig. 3.

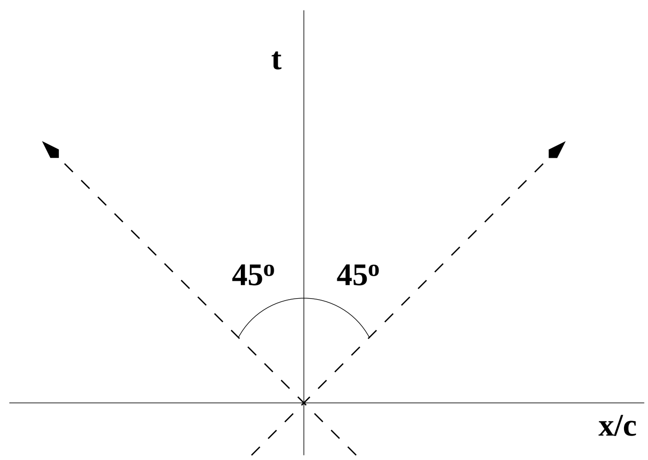

Figure 3—Light signals, moving left and right from the origin

Should we have occasion to consider motion in two spatial dimensions, where the spacetime diagram becomes three-dimensional, light signals must travel along the surface of a *cone* making an angle of 45° with the t-axis.

This cone is commonly called the *light cone*:

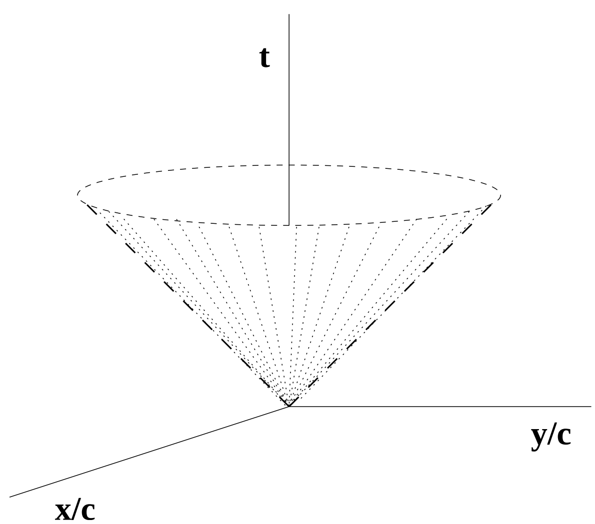

Figure 4—The light cone

Note that in a spacetime diagram, a particle traveling at a constant velocity v will be represented by a world line making an angle θ with the t-axis, where $\tan\theta = v/c$, as shown in Fig. 5.

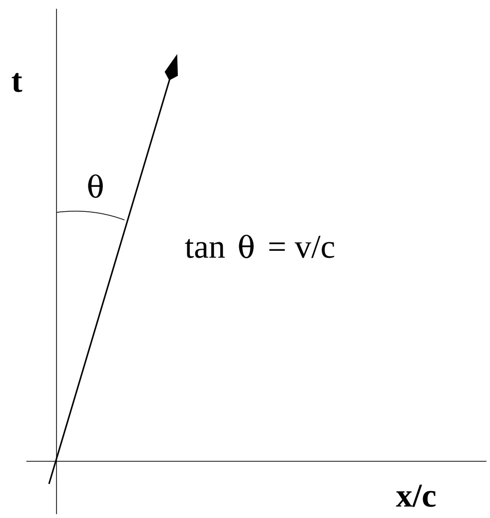

Figure 5—The world line of a moving particle

Later on in our discussion of SRT, we shall find it cumbersome to keep writing "c" in the denominator every time we wish to express a length or velocity. At that time we shall set $c = 1$, and write x instead of x/c, and v instead of v/c. After all, c is simply 1 light-second per second, and it is usually redundant to carry "c" along in all our formulas. In setting $c = 1$ we imply that x is measured in seconds and v is measured as a fraction of the light velocity. For example, for a particle traveling from the earth to the moon at half the speed of light, we would mean that it would travel a distance $x = 1.3$ seconds at a speed $v = 0.5$.

However, most of us find it confusing and inhibiting to be suddenly forced to think in terms of measuring distances in seconds. If I am asked "How tall are you?", it is not likely that I would say "6.3 nanoseconds", and I might be confused by a highway sign reading "SAN FRANCISCO 410 MICROSECONDS". Hence for the next few sections, we shall put "c" in explicitly. This will serve to emphasize the important role played by the light velocity, as well as making it self-evident that it is x/c that is measured in seconds and v/c that is a fraction of the light velocity.