V. SPACETIME COORDINATES, SIMULTANEITY, AND INERTIAL REFERENCE FRAMES

In our earlier discussion of clock rates, we imagined a situation in which a moving clock travels between two stationary clocks, and noted that the moving clock lags behind the stationary clocks by an amount depending upon the velocity of the moving clock—the *time dilation* effect (see Fig. 9). There we stated, somewhat glibly, that we can use the pair of synchronized stationary clocks to measure the spacetime coordinates of the event at which the moving clock passes the second stationary clock. How do we synchronize the stationary clocks? If we put them side by side, set them to the same time and then move one of them to a new spatial position, they will no longer be synchronized because the one we move will lag behind. Perhaps we could move both clocks symmetrically, one to the right and one to the left, but this will lead to difficulties when we try to extend the process. What we need is a clean unambiguous procedure for synchronizing a number of clocks at different spatial positions, a procedure that doesn't involve moving clocks around. We describe such a procedure now, or more generally, a procedure for measuring the spacetime coordinates of a distant event. In the process we shall be led to a unique definition of *simultaneity*—what we mean when we say that two spatially separated events are simultaneous.

Our procedure involves the use of only a single clock and some *light signals*, along with a means of sending and receiving such light signals. By a light signal we mean a little pulse of light, such as might be generated by blinking a flashlight once. So far we have discussed the role of light only peripherally, pointing out only that it plays an essential role in linking space and time. Now we shall begin to examine this role more carefully.

Suppose we send out a light signal to a distant mirror, where it is reflected and returned to us, and using our clock, we record the clock reading when the signal is sent (t_1) and again when it is returned (t_2). The reflection of the light signal from the mirror constitutes an event, whose spacetime coordinates we wish to determine. Figure 12, on the next page, shows an appropriate spacetime diagram.

What does our clock read when the light signal strikes the mirror? There is only one simple answer: Half way between t_1 and t_2. That is, the time coordinate of the event E is given by

$$t = \frac{1}{2}\left(t_1 + t_2\right)$$

Another way of stating this is to say that the reflection of the light signal from the mirror and the reading of time t on our clock are *simultaneous events*. We have drawn a line on our spacetime diagram through these two events; clearly all the points on this line are simultaneous with the clock reading of t. We therefore call such a line a *line of simultaneous events*.

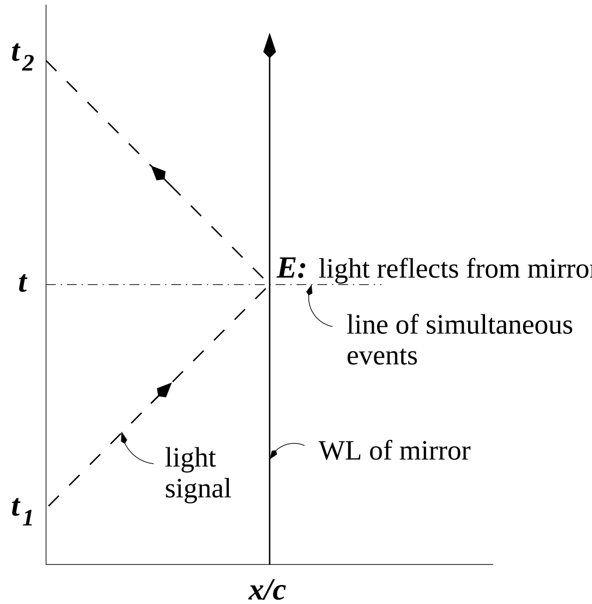

Figure 12—Determining spacetime coordinates of an event

How far away is the mirror when the light signal is reflected from it? In other words, what is the space coordinate of the event E? Clearly, since the light signal goes out and back at velocity c, the distance to the mirror is given by

$$\frac{x}{c} = \frac{1}{2}\,(t_2 - t_1)$$

Thus we have outlined a procedure for determining the spacetime coordinates of the event E. Note that the line of simultaneous events is parallel to the space axis.

One of the advantages of using clocks and light signals to determine spacetime coordinates is that they can be just as easily understood by a moving observer as by one who is stationary. What if the spacetime coordinates of the event we have just considered (the reflection of a light signal from a distant mirror) are determined by an observer moving along with velocity v? Figure 13, on the next page, shows a spacetime diagram, redrawn to include such a moving observer.

The light signal passes the moving observer at time t'_1 (measured by his clock), and passes him again, after being reflected from the mirror, at time t'_2. What is the time coordinate of E as recorded by the moving observer? Here again, there is only one simple answer: Halfway between t'_1 and t'_2. That is, the moving observer's time coordinate of the event E is given by

$$t' = \frac{1}{2}\,(t'_1 + t'_2)$$

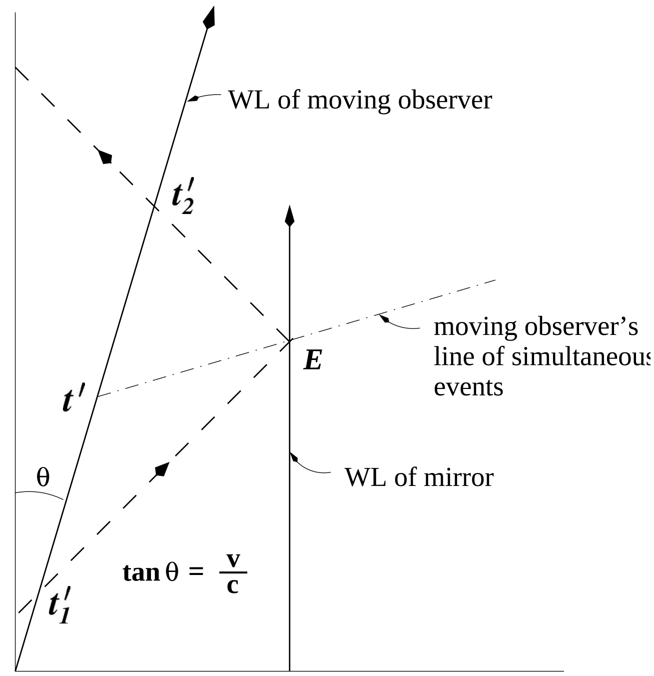

Figure 13—A moving observer determines spacetime coordinates

This is a powerful statement. It says that the reflection of the light signal from the mirror and the reading of time t' on the observer's clock are simultaneous events for the moving observer. To emphasize this, we have drawn another *line of simultaneous events* on our spacetime diagram, this time for the moving observer. It is clear from our diagram that events that are simultaneous for the moving observer are *not* simultaneous for the stationary observer. For the *stationary* observer, the event E occurs later than the reading of time t' on the moving observer's clock, while for the *moving* observer, the event E occurs earlier than the reading of t on the stationary observer's clock.

Exercise

Verify the above statement by drawing appropriate light signal world lines on Fig. 13.

Thus we have abandoned the Newtonian idea of absolute simultaneity. Clearly simultaneity is a relative concept. Whether or not two spatially separated events are simultaneous depends upon the velocity of the observer, *i.e.*, upon the direction of the observer's world line.

To gain an appreciation for this definition of simultaneity, it is helpful to dwell for a moment on the rather extraordinary but simple behavior of light signals. When we think of a little pulse of light traveling through space, we are apt to draw analogies from our past experience with moving things, such as a golf ball, a water wave, or even an elementary particle traveling along from one point to another. We often think, for example, of a light signal as a particle-like photon, which we suspect ought to behave like a moving electron. Alternatively, since we normally think of light as a wave, we might think that a light pulse behaves like a pulse of sound or a pulse of water waves, such as might be formed by dropping a pebble into a still pond.

In fact, light pulses are not completely analogous either to moving particles or to other moving wave pulses familiar to us. In the first place, light pulses are unlike particles in that their velocity is independent of the motion of the source. For example, some unstable particles emit a pulse of light as they decay; such a pulse will travel no faster if the decaying particle is moving, even if the particle moves at speeds comparable to the light velocity. In this respect, light pulses are like sound or water wave pulses: A pulse of noise given off in the forward direction by a moving (subsonic) airplane travels along through the air ahead of the plane no faster than if the plane were stationary. Similarly, a stone thrown into a quiet pond from a moving boat makes a wave pulse that travels with a velocity independent of the boat's speed.

There is, furthermore, something else. The speed of a sound pulse is measured with respect to the air through which it travels, and the speed of a water wave pulse is measured with respect to the water. Each requires the presence of a medium for its propagation. Sound waves require a gas, a liquid or a solid, and we can't have water waves without the water. Light wave pulses are different. They travel through empty space with no trouble at all. In fact, the emptier the space, the better the propagation of the light pulses. No medium is required for their propagation.

While this observation was hard to accept—a large number of attempts have been made to detect the so-called "luminiferous ether" as a supposed medium required for the propagation of light—the lack of such a medium provides a great simplification when we discuss the propagation of light signals. It implies, in particular, that since there is no medium with respect to which the speed of light is to be measured, there are only two possible directions for light signal world lines on our spacetime diagrams: Left-going and right-going, lines which for simplicity we have taken to be at $\pm 45°$ relative to the spacetime axes. A light signal world line in any other direction would imply either the existence of a medium or that the light signal velocity depended upon the velocity of the source. In short, the abandonment of the notion of a "luminiferous ether", along with the observation that light travels with a speed independent of its source, leads to the description of nature that is simple, beautiful and in consonance with experiment.

For us, this constancy of the light velocity, together with the curious behavior of clocks, provide a firm basis from which all of SRT may be developed.

Now, returning to the determination of the spacetime coordinates of the event E by our moving observer, we conclude that the space coordinate of the event E must be given by

$$\frac{x'}{c} = \frac{1}{2}\,(t'_2 - t'_1)$$

just as would be determined for the stationary observer. Both observers use the same procedure for determining the spacetime coordinates of a distant event.

The space coordinate of E can be scaled off on an axis parallel to the moving observer's "line of simultaneous events". This is hence the x'-axis. Thus we are able to construct spacetime axes, x' and t', that are appropriate to the moving observer. Furthermore, by examining our spacetime diagram, we can show that the world line of the observer (the t'-axis) and his line of simultaneous events (the x'-axis) make equal Euclidean angles with the light signal world line. That is, we can show that the x'-axis is tilted inwards from the x-axis by the same angle that the t'-axis is tilted inwards from the t-axis and that this angle is such that $\tan\theta = v/c$.

To show this, we redraw, in Fig. 14, our spacetime diagram, taking t'_1 for convenience at the origin of our diagram.

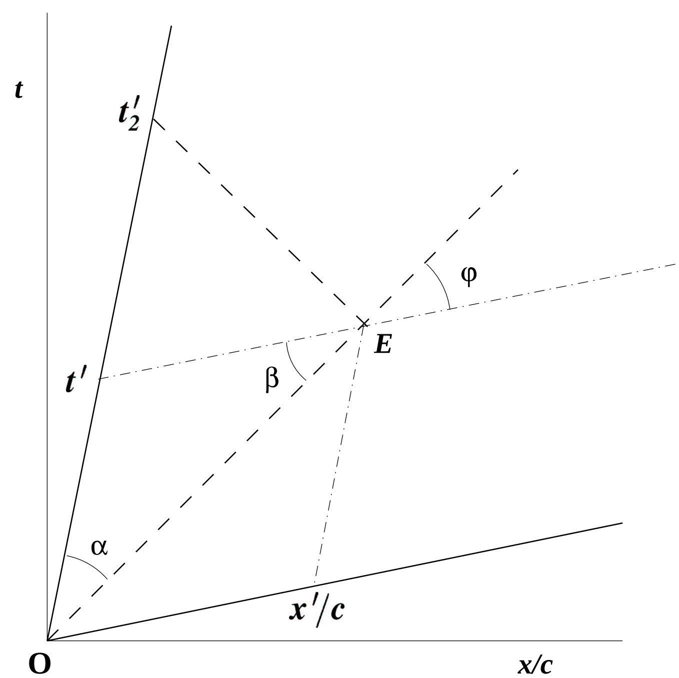

Figure 14—Determining the x'-axis

Since t' bisects the interval $O\ t'_2$, $O\ t' = t'\ t'_2$. Also $O\ t' = t'\ E$, since $t'\ E$ is just x'/c, the time taken for the light signal to reach the mirror at E. Hence, $O\ t'\ E$ is an isosceles triangle in the Euclidean sense, and $\alpha = \beta$. Hence, since β is clearly equal to φ, $\alpha = \varphi$, and our assertion is proved.

The result of this bit of geometrical analysis is that we now have a quick method for drawing spacetime diagrams for moving observers as well as for stationary observers. We simply draw the spacetime axes for the moving observer at an angle $\theta = \tan^{-1}(v/c)$ from the (perpendicular) axes for the stationary observer.

Note that although we have termed one of our observers "stationary" and the other "moving", we have introduced no basis for determining which is which. We could as well have thought of the second observer as "stationary" and the first one as "moving". It is only the *relative* motion of one observer with respect to the other that matters. We shall soon dwell further on this point, when we discuss a procedure by which such relative motion can be measured. Before doing so, however, we shall introduce the notion of an *inertial reference frame*.

We are accustomed to thinking of a *reference frame* (RF) as a set of spatial coordinate axes (usually Cartesian) with respect to which we can measure the position of a particle. We frequently envision a laboratory, a moving train, an elevator, a spaceship, the earth, or the system of "fixed" stars as examples of such reference frames. Further, we term a RF *inertial* if it is not accelerated, that is, one in which Newton's First Law (the "Law of Inertia") holds: Particles upon which no forces act are observed to travel in straight lines at constant velocities.

In our discussion of SRT we expand upon this concept to include the time dimension: By a reference frame, we mean a set of spacetime axes, with respect to which the coordinates of events can be measured. Thus, when we draw a set of axes on a spacetime diagram we imply the existence of a reference frame. Further, an inertial reference frame is one for which the world lines of free particles (particles upon which no forces are acting) are straight lines. In our discussions, we shall not normally consider non-inertial frames, and so will usually omit the word "inertial". We may think of a RF as a classical Newtonian frame with a clock added. Often, of course, there is an observer, which we may think of as traveling along in his own RF, equipped with a clock and a means of sending and receiving light signals—the tools for measuring the spacetime coordinates of events.

Problem

If two events occur at the same spatial position in some reference frame S, prove that their temporal sequence is the same in all reference frames, and that the least time separation is assigned to them in S. This illustrates that local *causality* is not violated by SRT. Solve this problem geometrically, using a spacetime diagram.