VI. THE VELOCITY OF A MOVING OBJECT

In the previous section we discussed our method for using clocks and light signals to determine the space and time coordinates for any event. A slight extension of this technique allows us to determine the velocity of a moving object in any reference frame. Suppose, for example, that at $t = 0$ an object passes an observer. Then at some later time the observer can bounce a light signal off the object, determine how far away it is, and hence determine its velocity. Here is a possible construction, first for a "stationary" observer (Fig. 15) and then for a "moving" observer (Fig. 16):

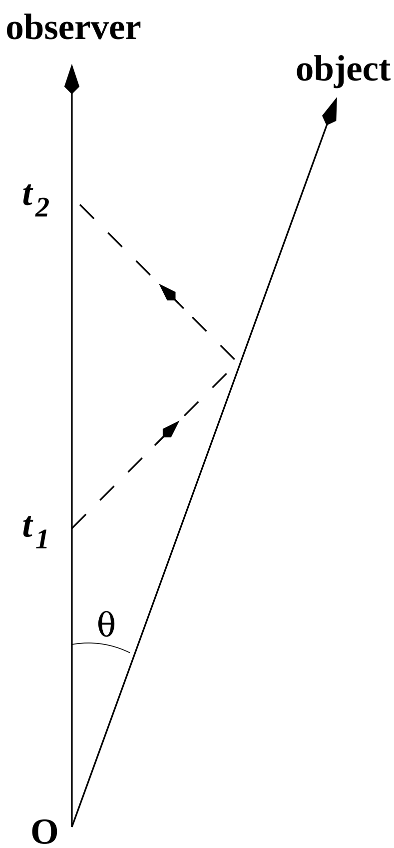


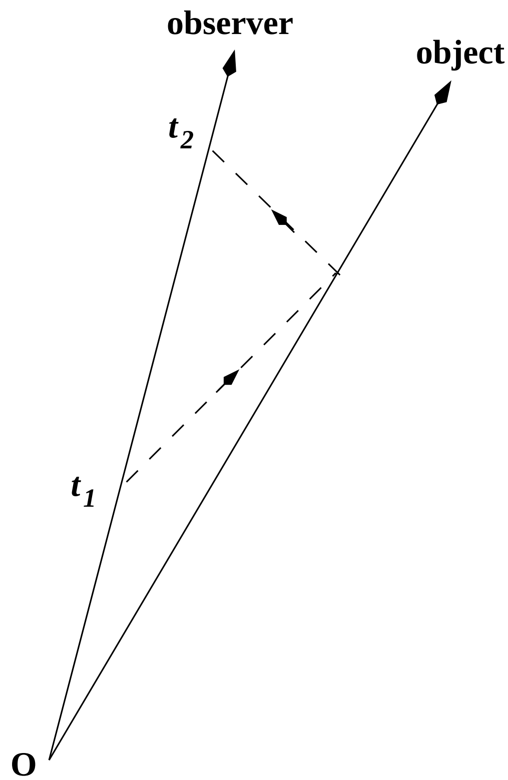
Figure 15—The velocity of an object in a "stationary" frame

Exercise

Show that

$$\frac{v}{c} = \frac{t_2 - t_1}{t_2 + t_1}$$

(In this case it is also true that $\tan\theta = v/c$.)



Here also

$$\frac{v}{c} = \frac{t_2 - t_1}{t_2 + t_1}$$

However here the Euclidean angle between the two world lines bears no simple relation to the velocity.

Figure 16—The velocity of an object in a "moving" frame

In each case, v is the velocity of the object with respect to the observer. How would you draw the observer's space axis in each figure?

Although certain quantities may be represented with greater clarity in a spacetime diagram whose space and time axes are at right angles to each other (for example in Fig. 15, $v/c = \tan\theta$), there is nothing physically special about such a spacetime diagram. Either Fig. 15 or Fig. 16 can be used to provide a complete description of an object moving at velocity v relative to an observer.