Physics 5C **Midterm 1 - Solutions** April 23, 2008

PRINT YOUR NAME ____

You have an hour and ten minutes to do this exam. You may use one page of formulas.

Section A (25 points): True or False Questions. You will get 3 points for each correct answer and 2 additional points if you also give a correct brief explanation.

i. _T_ The electric field is always perpendicular to the equipotential surfaces, and it points in the direction of decreasing electric potential.

$\boldsymbol{E} = -\nabla V$ implies that $\boldsymbol{E}$ is perpendicular to surfaces of constant V, and the minus sign means that E points toward decreasing V.

ii. _T_ If a nonconducting ball of radius R has a uniform charge density throughout its volume, the ratio of the electric potential at its center to that at its edge is 1.5.

Inside the ball, $E_r = kQ\,(r/R^3)$, so $\Delta V = V(0) - V(R) = \int_0^R E_r\,dr = ½\, kQ/R$. Since $V(R) = kQ/R$, it follows that $V(0) = kQ/R + \Delta V = 1.5\, V(R)$. [It's also OK to say that we showed this in a lecture.]

iii. _T_ If a dielectric is inserted between the plates of a disconnected fully-charged parallel plate capacitor, the energy stored in the capacitor is decreased.

The energy stored in a capacitor $U = ½\, Q^2/C$, and inserting the dielectric changes C to KC, where the dielectric constant $K > 1$. Thus the energy stored decreases by a factor of $1/K$.

iv. _F_ If you double the resistance R but keep the voltage V constant, the power dissipated in this resistance decreases by a factor of 4.

The power P dissipated when a current I flows through a resistor R with voltage V is $P = IV = V^2/R$, so doubling R decreases the power by a factor of 2, not 4.

v. _F_ Ten 100 W bulbs that are on for an hour use a kilowatt-hour of power.

The 10 bulbs use a kWh of *energy*, not power.

Section B (75 points): Calculation problems. Show *all* your work and make your method clear in order to get full credit. Use the backs of these pages, and if you need additional paper write your name and the problem number on each sheet.

1. (15 points) Suppose that a charge $-Q$ is located at $x = -a$, a charge $+2Q$ is located at $x = 0$, and a charge $-Q$ is located at $x = +a$.
(a) Calculate the electric potential $V(x)$ on the x-axis for x >> a.
(b) Calculate the electric field E_x on the x-axis for $x >> a$ using Coulomb's law.
(c) Calculate the electric field E_x on the x-axis for $x >> a$ by differentiating the potential.

(a) Recall the binomial formula $(1+\varepsilon)^n = 1 + n\varepsilon + ½\, n(n-1)\varepsilon^2 + O(\varepsilon^3)$. Since $x >> a$, $\varepsilon = x/a << 1$ and it will suffice to work to order ε^2. $V(x) = kQ\,[-(a+x)^{-1} + 2/x - (a-x)^{-1}] = (kQ/x)\,[-(1+\varepsilon)^{-1} + 2 - (1-\varepsilon)^{-1}] = (kQ/x)[-(1-\varepsilon+\varepsilon^2) + 2 - (1+\varepsilon+\varepsilon^2) = (kQ/x)(-2\varepsilon^2) = \underline{-2kQ/x^3}$.
(b) $E_x(x) = kQ\,[-(a+x)^{-2} + 2/x^2 - (a-x)^{-2}] = (kQ/x^2)\,[-(1+\varepsilon)^{-2} + 2 - (1-\varepsilon)^{-2}] = (kQ/x)[-(1-2\varepsilon+3\varepsilon^2) + 2 - (1+2\varepsilon+3\varepsilon^2) = (kQ/x^2)(-6\varepsilon^2) = \underline{-6kQ/x^4}$. [Note: alternatively one could add dipoles to solve each part.]
(c) $E_x(x) = -dV/dx = -(-2kQ)(-3x^{-4}) = \underline{-6kQ/x^4}$, in agreement with (b).

2. (10 points) A spherical rubber nonconducting balloon carries a total charge Q uniformly distributed on its surface. At time t = 0 the balloon has radius r_0. It is then slowly blown up so that r increases linearly to $2r_0$ in a time T. Determine the electric field as a function of time (a) just outside the balloon, and (b) at r = 3.2 r_0 .

If the radius is to increase from r_0 to $2r_0$ linearly during an elapsed time of *T*, then the rate of increase must be r_0/T. The radius as a function of time is then $r = r_0 + \frac{r_0}{T}t = r_0\left(1+\frac{t}{T}\right)$. Since the balloon is spherical, the field outside the balloon will have the same form as the field due to a point charge.

(*a*) Here is the field just outside the balloon's surface:

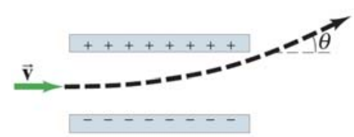
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2\left(1+\frac{t}{T}\right)^2}}$$

(*b*) Since the balloon radius is always smaller than $3.2r_0$, the total charge enclosed in a gaussian surface at $r = 3.2r_0$ does not change in time.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(3.2r_0\right)^2}}$$

3. (15 points) An electron is accelerated in the *x* (horizontal) direction from rest in a television picture tube by a potential difference of 5500 V. (a) What is the speed v_x of the electron?

(b)The electron then passes between two horizontal plates 6.5 cm long and 1.3 cm apart that have a potential difference in the *y*-direction of ΔV = 250 V, as shown in the figure.



What is the electric field between the plates, neglecting fringing?

(c) At what angle will the electron be traveling after it passes the plates?

(a) The horizontal component of the electron's velocity can be found using conservation of energy. That component is not changed as the electron passes through the plates.

$$\mathrm{PE}_{\text{inital}} = \mathrm{KE}_{\text{final}} \rightarrow qV = \tfrac{1}{2}mv_x^2 \qquad t = \frac{\Delta x}{v_x}$$

Thus $v_x^2 = 2e(5500\text{ V})/m_e = 11\text{ keV}/m_e = 11\text{ keV}\,c^2/(m_ec^2) = (11\text{ keV}/511\text{ keV})\,c^2 = 0.0215\,c^2$, so $v_x = 0.147c =$ <u>4.4×10^7 m/s</u> .

(b) The electric field $E_y = \Delta V/\Delta y$ = 250 V / 0.013 m = <u>19 kV/m</u> .

(c) The vertical component of the electron's velocity can be found using the vertical acceleration due to the electric field E_y between the plates, and the time *t* that the electron spends between the plates.

$$F_E = qE_y = ma = m\frac{\left(v_y - v_{y0}\right)}{t} \rightarrow v_y = \frac{qE_yt}{m} = \frac{qE_y\Delta x}{mv_x}$$

We calculate the angle using $\tan\theta = v_y / v_x$, as follows:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\dfrac{qE_y\Delta x}{mv_x}}{v_x} = \frac{qE_y\Delta x}{mv_x^2} = \frac{qE_y\Delta x}{2qV} = \frac{E_y\Delta x}{2V} = \frac{\left(\dfrac{250\,\text{V}}{0.013\,\text{m}}\right)(0.065\,\text{m})}{2(5500\,\text{V})} = 0.1136$$

$$\theta = \tan^{-1} 0.1136 = \boxed{6.5°}$$

4. (10 points) In nuclear fission, a large nucleus splits into two unequal smaller nuclei plus a few neutrons. Suppose one of the smaller nuclei has charge $q_1 = +38e$ and radius $r_1 = 5.5$ fm and the other has $q_2 = +54e$ and $r_2 = 6.2$ fm. As the fission fragments fly apart their electric potential energy is all converted to kinetic energy. What is the resulting kinetic energy of these fragments in MeV? (Note: fm = 10^{-15} m, 1 MeV = 10^6 eV.)

The two fragments can be treated as point charges for purposes of calculating their potential energy. By energy conservation, the potential energy is all converted to kinetic energy as the two fragments separate to a large distance.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad U_{\text{initial}} = K_{\text{final}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}V$$

$$= \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(38)(54)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(5.5\times10^{-15}\,\text{m}\right)+\left(6.2\times10^{-15}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 250\times10^6\,\text{eV}$$

$$= \boxed{250\,\text{MeV}}$$

5. (10 points) There is an electric field near the earth's surface of about 150 V/m. How much energy is stored per cubic meter in this electric field?

$$u = \tfrac{1}{2}\,\varepsilon_0 E^2 = \tfrac{1}{2}\,(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\,(150\ \text{V/m})^2 = \underline{1.0\times10^{-7}\ \text{J/m}^3}$$

6. (15 points) The average person in the U.S. consumes 1.40 kW of electricity (averaged over time), but Californians use less energy per capita than residents of any other state, only about 0.80 kW.
(a) At the PG&E current average cost of residential electricity of $0.17 per kWh, how much does an average Californian's electricity cost per person per year?
(b) On average, each kWh of electricity generation in the U.S. produces 1.35 lb of carbon dioxide, but California uses less fossil fuel to generate electricity and as a result each kWh of electricity generated by PG&E produces about 0.52 lb of carbon dioxide.
How much CO_2 per year is produced by the average American's electricity use?
(c) How much CO_2 per year is produced by the average PG&E customer's electricity use?

(a) There are (24 h/day)(365.25 days/yr) = 8766 h/yr, so per capita annual electric use in California is (0.80 kW)(8766 h) = <u>7000 kWh</u>.
(b) The per capita annual CO_2 production due to electricity use in America is (1.40 kW)(8766 h)(1.35 lb/kWh) = <u>16,600 lb</u> = <u>8.3 tons</u>.
(c) The per capita annual CO_2 production due to electricity use by PG&E customers is (7000 kWh)(0.52 lb/kWh) = <u>3600 lb</u> = <u>1.8 tons</u>, less than ¼ the national average!