Physics 5C   Practice Midterm 1 SOLUTIONS   April 2008

PRINT YOUR NAME ____

Take an hour and ten minutes to do this exam. You may use one page of formulas.

Section A (20 points): True or False. You will get 2 points for each correct answer and 2 additional points if you also give a correct brief explanation.

1. ___ Object A has a charge of +2 C, and object B has a charge of +6 C. If $\vec{F}_{\text{AB}}$ is the force on object B due to object A, then $\vec{F}_{\text{AB}} = -\vec{F}_{\text{BA}}$.

   True: Nothing we've learned in this class violated Newton's third law, which is all this is.

2. ___ Tomorrow morning, you awake in the twilight zone, where the linear dimensions of every object in your bedroom have doubled but nothing outside your room has changed. If the resistivities of material are the same in the twilight zone, then your bedside lamp shines more dimly.

   False: Thinking of the light bulb as a 'wire' resistor with resistivity ρ, we have $R = \rho l/A$, where the area A quadruples and the length l doubles in the Twilight Zone. So R is cut in half. Since the wall will still give the same voltage V as before, the power V^2/R would increase.

3. ___ The electric field does positive work on a negative charge as the charge moves from low to high potential.

   True: The electric field points towards low potential, and a negative charge feels a force opposite the electric field, thus toward high potential, so it gets positive work done on it as it goes there.

4. ___ If a fully-charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates, then the energy stored in the capacitor decreases.

   False: The energy is $CV^2/2$, and V is fixed. But inserting the dielectric sends $C \to \kappa C > C$, so the energy goes *up*.

5. ___ In an RC-circuit (an EMF, a capacitor, and a resistor hooked up in series), the total work done by the EMF from when the circuit is closed until a very long time afterward is equal to the energy stored in the capacitor at that late time.

   False: some is lost to the resistor.

Section B:

1. (15 points) Two resistors, R_1 and R_2, may be connected either in series or in parallel across a (resistanceless) battery of EMF $\mathcal{E}$. We would like the power dissipated by the parallel combination to be 4 times that of the series combination. If $R_1 = 100\,\Omega$, what is R_2?

   Solution:

   In series, we have $R = R_1 + R_2$, while in parallel we have $R = (R_1^{-1} + R_2^{-1})^{-1}$. In both cases, we have $P = V^2/R = \mathcal{E}^2/R$. Thus comparing the powers, we desire:

$$4\mathcal{E}^2/(R_1 + R_2) = \mathcal{E}^2(R_1^{-1} + R_2^{-1})$$

which gives

$$4 = (R_1 + R_2)(R_1^{-1} + R_2^{-1}).$$

This yields a quadratic in R_2 which is most easily solved by setting $a \equiv R_2/R_1$, to get $a^2 - 2a + 1 = 0$, so that $a = 1$, and $R_2 = R_1$.

This makes sense intuitively (and in fact we could just about solve it this way): two equal resistors in series vs. parallel will have effective resistances that differ by a factor of four, as will the dissipated powers.

2. (15 points) A point charge q is located at the center of a cube of length d.

   (a) What is the value of the electric flux $\int \vec{E} \cdot d\vec{a}$ over any *one face* of the cube?

   (b) The charge q is moved to one corner of the cube. What is now the value of the electric flux through each of the faces of the cube? (*hint:* use another symmetry argument.)

   Solution:

   (a) By Gauss's law, $\Phi = q/\epsilon_0$ for the whole cube. By symmetry, one face must have just 1/6th of this value, so

$$\Phi_E = q/6\epsilon_0.$$

   Note: It is *not* true that the flux though one side is EA; $\int \vec{E} \cdot \vec{da} = EA$ only when $\vec{E}$ is both constant and always parallel to $\vec{da}$; neither is true in this case.

   (b) Since the field is radial it is perpendicular to $\vec{a}$ on the 3 sides adjacent to the charge, the total flux through these three sides is zero. The other three sides, by symmetry, have the same flux. We can compute it by imagining 7 other identical cubes, with the charge at one corner of each, so that the 8 cubes together form one big cube of side length $2d$. Then the flux through each side of this big cube is, by part 1, $\Phi_E = q/6\epsilon_0$. But clearly the flux through one of the non-adjacent sides of the original cube is just 1/4th of this, so

$$\Phi_E = q/24\epsilon_0.$$

3. (14 points) Figure 1 shows a system equivalent to two parallel-plate capacitors in series, the rigid (conductive) center section of length b being movable vertically. If the plate area is A, find the equivalent capacitance of the series combination, and note that it is independent of the position of the center section.

   Solution:

   Let d be the separation between the plates of the top capacitor (keeping $d < a - b$). Then the separation of the bottom one is $a - b - d$. The capacitances are then, respectively $C = \epsilon_0 A/d$ and $C = \epsilon_0 A/(a - b - d)$. Adding these in series we get

$$C_{\text{tot}}^{-1} = d/A\epsilon_0 + (a - b - d)/A\epsilon_0 = (a - b)/A\epsilon_0,$$

   so

$$C_{\text{tot}} = A\epsilon_0/(a - b),$$

   independent of d.

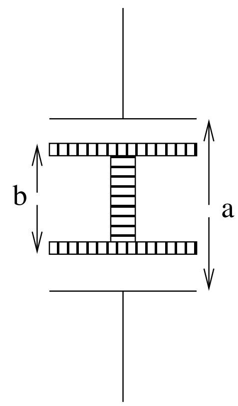

FIG. 1: For problem 3.

4. (16 points) Consider the system shown in Fig. 2, consisting of a large flat non-conducting plate with charge $Q > 0$ and several charges in above it, which are attached together by an (uncharged, non-conducting) rigid bar. This bar lies along the x-axis. Assume that the distances a separating the charges, and the distance of the charges from the plate, are all very small compared to plate scale L, and let $q > 0$. (*Hint:* the best way to think about system of 3 charges is *not* as 3 individual charges.)

   (a) Calculate the (vector) net force on the system of charges.

   (b) Calculate the (vector) net torque on the system as it is shown.

   (c) *Describe qualitatively* the motion of the system if released from rest in the configuration shown. Assume there is no gravity. (A concise one- or two-sentence description will suffice.)

   Solution:

   Let $\Sigma = Q/L^2$ be the surface charge density on the plate. Then $\vec{E} = (\Sigma/2\epsilon_0)\hat{z} = (Q/2\epsilon_0 L^2)\hat{z}$ is the field near the plate.

   (a) Since the field is uniform, the forces on the $+q$ and $-q$ charges cancel out, so

$$\vec{F}_{\rm net} = q\vec{E} = (qQ/2\epsilon_0 L^2)\hat{z}.$$

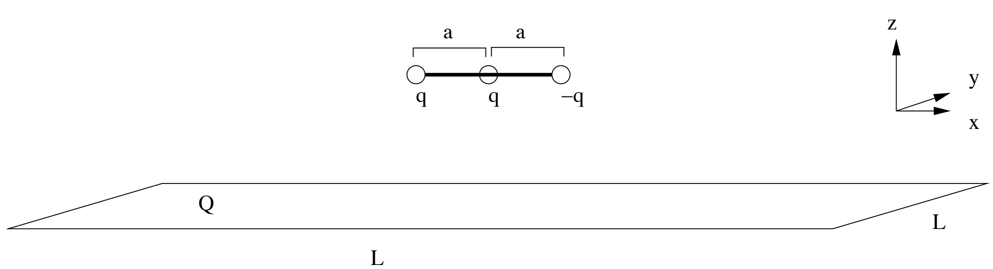

FIG. 2: For problem 4

(b) Consider the thing as a monopole $+q$, *plus* a dipole with moment $\vec{p} = -2aq\hat{x}$. Then the torque, $\vec{\tau} = \vec{p} \times \vec{E}$ is

$$\vec{\tau} = 2pE\hat{y} = (aqQ/\epsilon_0 L^2)\hat{y}.$$

(c) The thing will oscillate, with an amplitude of 180 degrees, about the $\hat{y}$ axis, while the center-of mass accelerates in the $\hat{z}$ direction.

5. (20 points) A lightning bolt transfers $\sim 10\,\mathrm{C}$ of (negative) charge from the clouds to the Earth. The charge of the Earth is determined by an equilibrium between this charge transfer due to lightning, and the steady diffusion of electrons up through the atmosphere, which has an average conductivity of $\sigma \approx 10^{-14}(\Omega \cdot m)^{-1}$. If the Earth is taken to be a conductor, with potential $V \approx -10^9\,\mathrm{V}$ (where $V_\infty = 0$), and the radius of the Earth is $6.4 \times 10^6\,\mathrm{m}$, approximately how many lightning strikes are there per second on the Earth, on average?

Solution:

Let R be the Earth radius, then $Q = CV$, where $C = 4\pi\epsilon_0 R$ is the Earth's capacitance. Then $E = Q/4\pi\epsilon_0 r^2 = V/R$, and it is radially inward.

Now, we have a current density $\vec{j} = \sigma\vec{E}$ for the electrons going up (i.e. a current down). The total current is then $I = 4\pi R^2 j$, going into the Earth. This is balanced by N bolts per second, inward, with $Q = -10\,\mathrm{C}$ each, so:

$$\begin{aligned} 4\pi R^2 j &= NQ && (1) \\ 4\pi R\sigma V &= NQ \\ N &= 4\pi R\sigma V/Q = 80\,\mathrm{bolts}/s! \\ & && (2) \end{aligned}$$

Note: You might try to do this problem using $V = IR$. This can be done, but you would need to calculate the resistance correctly, which in this case involves an integral. In particular, the formula $R = \rho l/A$ does *not* apply. Coincidentally, you do get the right answer if you use this formula and plug in the Earth's area and radius for A and l; but this makes no sense, as the current is not flowing through the Earth. It is also only partly coincidence: just by dimensional analysis, the only way to combine R, σ and V to get a current is $I \sim R\sigma V$, which is right up to a factor of 4π.