Physics 5C – 2nd Practice Final Exam – Spring 2008
Solutions

Section A (40 points): Multiple choice: choose the best answer. You will get 5 points for each correct answer.

1. For a gaussian surface through which the net flux is zero, the following four statements *could* be true. Which statement *must* be true?

   (a) ____ No charges are inside the surface.

   (b) ____ The net charge inside the surface is zero.

   (c) ____ The electric field is the same everywhere on the surface.

   (d) ____ No electric field lines leave the surface.

   b: only this directly follows from Gauss's law.

2. The ends of a resistanceless inductor are labeled A and B. The potential at A is higher than at B. Which of the following is consistent with this:

   (a) ____ The current is constant and directed from A to B.

   (b) ____ The current is constant and directed from B to A.

   (c) ____ The current is directed from A to B and increasing.

   (d) ____ The current is directed from A to B and decreasing.

   c: there is only an EMF if I is changing. If I is increasing, then the EMF will be so as to decrease the current, which calls for a voltage drop as required.

3. A parallel plate capacitor with circular plates of radius r_0 is being charged. The induced magnetic field between the plates is greatest at $r =$

   (a) ____ 0 (the center)

   (b) ____ $r_0/2$

   (c) ____ r_0

   (d) ____ more than r_0

   c: The Ampere-Maxwell law relates $\oint \vec{B} \cdot d\vec{l} = 2\pi r B$ to the time-derivative of the electric flux Φ_E. $\Phi_E = \pi r^2 E$ for $r < r_0$, and $\Phi_E = \pi r_0^2 E$ for $r > r_0$. Thus for $r < r_0$, B increases with r, but for $r > r_0$ it decreases with r, so B is maximal at r_0.

4. A particle of charge q is circulating in a uniform magnetic field of strength B. Which of the following will most increase the time it takes for the charge to make a full revolution?

   (a) ____ Doubling the magnetic field.

   (b) ____ Doubling the radius of revolution.

   (c) ____ Doubling the speed of the particle.

(d) ____ Doubling the mass of the particle.

d: The time is $\propto m/qB$.

5. A hanging spring is attached to a powerful battery and a switch. When the switch is closed, a current suddenly flows through the spring. Does the spring

   (a) ____ Compress.

   (b) ____ Expand.

   (c) ____ Stay the same.

   a: consider 2 nearby parts of the coil. They are like parallel wires, with current in the same direction. Thus they attract, and the coil compresses.

6. A long thin finite solenoid having n turns per unit length is tightly wrapped in N turns of insulated wire over a short portion of its length, with the resulting mutual inductance being M. The N turns of wire are removed and replaced by N turns of identical wire having twice the radius of the original wrapping. The mutual inductance of the new configuration compared to the original is:

   (a) ____ the same

   (b) ____ greater due to the greater area of the secondary coil

   (c) ____ twice the original

   (d) ____ less than the original although possibly only slightly

   d: M is proportional to the amount of flux from the solenoid that goes through the turns of wire. In the original configuration all of this flux is in the interior of the solenoid. When the radius increases, flux from the solenoid's exterior is included, but the field there is in the *opposite* direction, so this reduces the total flux. However, the reduction is small because the field is very weak outside a good solenoid.

7. Consider the bent wire shown in Fig. 2 (see a few pages below), with current I flowing in the direction shown. Three circular loops are shown drawn to scale; loops (i) and (ii) enclose the top and bottom wires, respectively, and loop (iii) encloses both. Consider B_{max}, the maximum B-field strength around a given loop, and also the quantity I_{enc}, as it appears in Ampere's law. then:

   (a) ____ B_{max} is greater for loops (i) and (ii) than for loop (iii), but I_{enc} is largest in loop (iii).

   (b) ____ B_{max} is smaller for loops (i) and (ii) than for loop (iii), and I_{enc} is largest in loop (iii).

   (c) ____ B_{max} is smaller for loops (i) and (ii) than for loop (iii), but I_{enc} is smallest in loop (iii).

   (d) ____ B_{max} is greater for loops (i) and (ii) than for loop (iii), and I_{enc} is largest in loop (iii).

c: For small distances d from the wire, the B-field goes like $B = \mu_0 I/2\pi d$, so the field is quite large on parts of loop (*iii*). However, $I_{\rm enc} = 0$ for this loop, while it is nonzero for the others.

8. Some stuff happens, involving capacitors, currents, resistors and inductors of values C, I, R, and L, respectively. A quantity results, which is:

$$X = 4\pi \frac{Q^2}{(\epsilon_0 \mu_0)^{1/2} I^2} \frac{R}{L^{3/2} C}.$$

This quantity is:

(a) ____ A current

(b) ____ A velocity

(c) ____ A force

(d) ____ A capacitance

(e) ____ A power

b: My apologies for this one. There was a typo, as the C should have been $C^{1/2}$. were this the case, the solution would have been:

Leaving out the 4π, we can decompose this as:

$$v = \left(\frac{Q^2}{\epsilon_0}\right) \left(\frac{1}{\mu_0 I^2}\right) (\epsilon_0 \mu_0)^{1/2} \left(\frac{R}{L}\right) \sqrt{\frac{1}{LC}}$$

Respectively, these have units: Nm^2 (Coulomb's law), $1/N$ (force between two wires), s/m ($= c^{-1}$), s^{-1} (from L-R circuit time constant L/R), and s^{-1} (L-C circuit frequency). The product then gives m/s.

Section B: Problems

1. (12 points) On the island of Pala, AC power is delivered at 23 Hz and 42 Volts rms. You have brought a hotpot that draws 600 W when plugged in at home (with 60 Hz, 110 V rms power).

   (a) If you consider your hotpot to be a pure resistance R, how much power is delivered to your it in Pala?

   (b) Suppose the same hotpot also had a capacitor of capacitance C in it, connected in series to the resistor. What would be the current amplitude I_0 and power burned in Pala in this case? (Don't bother numerically evaluating – just write the symbolic expression.)

Solution:

(a) The power is just $V_{\rm rms}^2/R$, since the resistor is in phase with the voltage source. So in Pala the power would be $(42/110)^2 \simeq 15\%$ as much. We note also that $R = V_{\rm rms}^2/P = (12100/600)\,\Omega = 20\Omega$.

(b) We now have a total impedence $Z = \sqrt{R^2 + (1/\omega C)^2}$. Then $I_{\rm rms} = V_{\rm rms}/Z$, and we still have $P = I_{\rm rms}^2 R$, or

$$P = \frac{V_{\rm rms}^2 R}{R^2 + (1/\omega C)^2}$$

2. (16 points) Figure 1 shows a circuit with EMF $\mathcal{E} = 7\,V$. If $R = 1\,\Omega$, which is the current I' in the indicated wire segment?

   Solution:

   Let us denote by I the current in the segment containing the EMF (heading in the EMF's direction), and by I_1 (heading down) the current in the segment containing the top-left resistor.

   Then, the bottom-left resistor sees current $I_1 - I'$, so the loop around the EMF and the left two resistors gives:

$$\mathcal{E} - 2I_1 R - (I_1 - I')R = 0.$$

   Then, the current through the top-right resistor is $I - I_1$, and that through the bottom-right one is $I - I_1 + I'$, so a loop around the EMF and the right two resistors gives:

$$\mathcal{E} - (I - I_1)R - (I - I_1 + I')R = 0.$$

   Finally, we can make a loop over the top two resistors and the I' segment, getting:

$$-2I_1 R + (I - I_1)R = 0$$

   we have three unknowns (I, I_1 and I') and three equations, so we can solve. The last equation gives $I = 3I_1$. Putting this into the first two yields $I' = -I/6$. Then finally we plug back into the first to get $\mathcal{E} = \frac{7}{6}IR$, so:

$$I' = I/6 = \frac{\mathcal{E}}{7R} = 1\,A.$$

3. (16 points) Consider the 'hairpin' of wire shown in Fig. 2, with current $I = 1\,A$ flowing in the direction shown, and the radius $R = 0.4\,m$. Note that the wires continue indefinitely to the right.

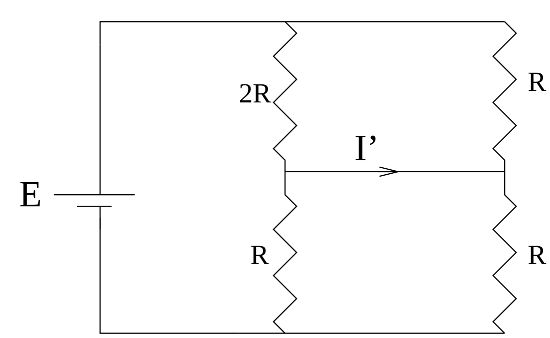

FIG. 1: For problem 2

(a) What is the field amplitude and direction at point a, very far away from the turn in the hairpin?

(b) What is the field amplitude and direction at point b *due to the two straight wire segments only*?

(c) Would the field direction due to just one of the two wire segments alone be in the same direction as the combined field of both? Why or why not?

(d) What is the total field amplitude and direction at point b?

Solution:

(a) Far from the turn, we can think of this as an infinite wire. Then ampere's law gives $B = \mu_0 I/2\pi R$, out of the page ($\hat{z}$ direction). The bottom wire gives just the same contribution, so we have $B = \mu_0 I/\pi R$. Then, since $I = 1\,A$ and $\mu_0 = 4\pi \times 10^{-7}\,T \cdot m/A$, we have

$$\vec{B} = 10^{-6}\,T\,\hat{z}.$$

(b) We could do this using Biot-Savart, but it is far easier to use symmetry. We can consider the contribution from *one-half* of an infinite line, so $B = \mu_0 I/4\pi R$ for each segment, or

$$\vec{B}_{\text{wires}} = \mu_0 I/2\pi R\,\hat{z} = 5 \times 10^{-7}\,T\,\hat{z}.$$

However, we have to think in terms of Biot-Savart to justify this, as the next part shows.

(c) We can ask why there is no $\hat{x}$ or $\hat{y}$ component in part (b). The reason is not symmetry of the infinite line, but rather that (in Biot-Savart) the vectors $d\vec{l}$ and $\vec{d}$ (connecting the bit of wire to the point b) are both in the x-y plane, so the contribution to $d\vec{B}$ must be in the $\hat{z}$ plane. So the answer is yes.

(d) Here again, we can use symmetry: we know the field due to a current loop (given in prob. 6 with $x = 0$, so $B = \mu_0 I/2R$. This can be considered two semi-circles, each giving:

$$\vec{B}_{\text{semi}} = \mu_0 I/4R\hat{z} = \pi \times 10^{-6}\,T\,\hat{z},$$

which can be added to $\vec{B}_{\text{wires}}$ for the total.

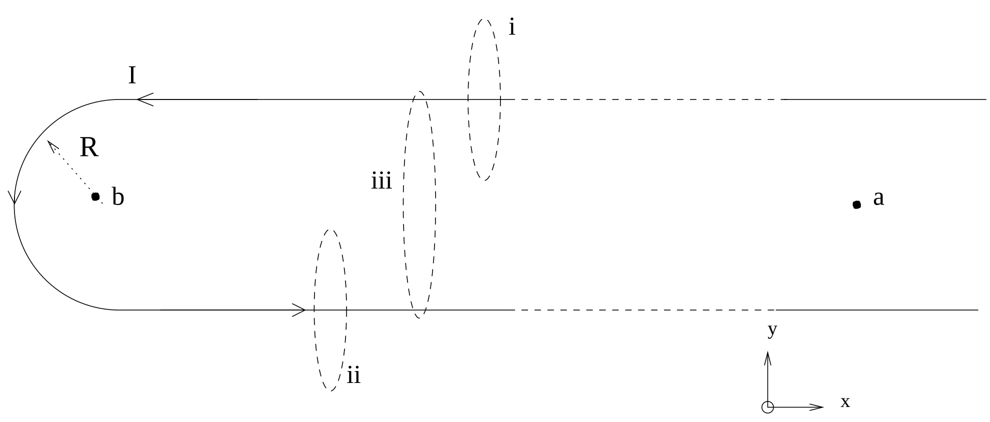

FIG. 2: For multiple-choice #7 and problem 3.

4. (16 points) A solid nonconducting sphere carries a uniform charge per unit volume ρ. Let $\vec{r}$ be the vector from the sphere's center to a general point P within the sphere.

   (a) Show that the electric field at P is given by $\vec{E} = \rho\vec{r}/3\epsilon_0$.

   (b) A spherical cavity is created in the sphere; the vector $\vec{a}$ points from the sphere's center to the cavity's center. Find the electric field at all points within the cavity; your result should depend *only* on $\vec{a}$, ρ, and ϵ_0. (Hint: use superposition, and keep careful track of you vectors.)

   Solution:

   (a) By symmetry, $\vec{E} = E\hat{r}$. Then at radius r, by Gauss's law,

$$E_r = \frac{Q_{\text{enc}}}{4\pi\epsilon_0 r^2} = \frac{4\pi\rho r^3/3}{4\pi\epsilon_0 r^2} = \frac{r\rho}{3\epsilon_0}$$

   thus:

$$\vec{E} = \frac{\vec{r}}{3\epsilon_0}$$

   (b) We can use superposition:

$$\vec{E} = \vec{E}_{\text{sphere}} - \vec{E}_{\text{cavity}} = \frac{\vec{r}}{3\epsilon_0} - \frac{(\vec{r}-\vec{a})}{3\epsilon_0} = \frac{\vec{a}}{3\epsilon_0},$$

   which is independent of P. (In the third term, $\vec{r}-\vec{a}$ is just the vector pointing from the center of the cavity to the point P.)

5. (19 points) Two small metal spheres, of charge $\pm Q$, are floating in space. They have a potential difference ΔV and a separation d.

   (a) Using our usual formulas for electric potential, what is ΔV?

   (b) What is the capacitance of the configuration?

   (c) How much energy is stored in this capacitor?

   (d) What is the potential energy of the configuration, considering the two spheres to be point charges?

   (e) The last two quantities both represent amounts of energy stored by the system. Do they match? If not, discuss why not.

   Solution:

   (a) This problem, upon reflection, was a bit flawed because if these spheres are close together and conductive (and the latter must be true so that there is a single potential associated with each one), then they will induce a non-uniform charge distribution on each other which makes the problem very hard. Also, I should have specified the radius R of the spheres.

   So we accepted a variety of answers. One reasonable answer would be to neglect the charge induction, either because you have to or because you take R to be small compared to d. Then you find

$$\Delta V = \frac{\pm Q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{d-R}\right).$$

if $d \gg R$, then this is

$$|\Delta V| \simeq \frac{Q}{2\pi\epsilon_0 r}$$

another, less right, thing to do would be to think of the spheres as close together, and consider them as a parallel-plate capacitor, but what to assume for the surface-charge distribution is rather unclear.

(b) Taking the $d \gg R$ result above, we have $C = Q/\Delta V = 2\pi\epsilon_0 r$. Note that this is *not* double the result of a single charged-sphere capacitor: the idea there is that the other 'charge' is off at infinite. Here it is provided by the other sphere.

(c) In terms of (a) and (b), $U = C(\Delta V)^2/2 = Q^2/4\pi\epsilon_0 r$.

(d) $U = -\frac{1}{4\pi\epsilon_0}\frac{Q^2}{d}$.

(e) The basic difference results because we don't associate a potential energy with a single point charge in the usual formula used in part (d), whereas if we consider all the energy in the field, it does have an energy associated with it. In fact, for a point charge that energy would be infinite! (As you can see by taking $R \to 0$.) This is an old puzzle in E&M that is really only resolved by quantum mechanics.

6. (70 points, 8 parts) Somewhere in the south Pacific ocean, in a bunker under a Hatch, there is an enormous magnet (hereafter "The Device") of unknown nature. It is possible that an Event involving this electromagnet led to the crash of Oceanic Flight 815 from Sydney to Los Angeles. You are to investigate this.

   (Note: you can answer later parts in terms of the variables given in earlier parts, so don't stop if you get stuck.)

   (Note: while finding the numerical answers may be fun, remember that you will get almost all of the credit for the correct symbolic expressions. If you do evaluate, don't get hung up on precise numbers, since the spirit of this problem is *estimating* things.)

   (Note: if you are not a *Lost* fan you can do just as well on this, so don't worry.)

   (a) Let's model the Device as a coil of many current loops each of radius $R = 10\,\mathrm{m}$. As you recall from class, the magnetic field strength a distance x from the center of one such loop, in a direction perpendicular to the plane of the loop, is:

   $$B = \mu_0 I R^2/2(R^2 + x^2)^{3/2}.$$

   Since the plane was at an altitude $h \simeq 10,000\,m \gg R$, write down this field strength for $x \gg R$, if there are N turns of wire around the loop.

   (b) One theory is that the plane was simply pulled down out of the sky by the magnet. I won't make you show it (you could), but this turns our to require a field of $B_{\text{yank}} \sim 1\,T$ in the vicinity of the plane. Another possibility is that the field somehow disrupted the electronics in the airplane. Suppose, for example, that a some key components of the airplane's circuitry are connected by a piece of wire of length $L = 10\,\mathrm{m}$ that runs perpendicular to the plane's flight direction, and which normally has a potential difference across it of $V = 5\,V$. If the airplane is flying at $v = 200\,m/s$, what magnitude B_{disrupt} of a magnetic field (assume it is uniform and pointed up and thus perpendicular to the wire) would be required to induce a similar EMF across the wire? Is B_{disrupt} or B_{yank} larger?

(c) We might also theorize that a magnetic field significantly larger than that of the Earth simply threw the plane's navigational system into total confusion. Let's call this field $B_{\text{confuse}} \sim 5 \times B_{\text{earth}} \sim 2.5 \times 10^{-4}\,T$. This is certainly much smaller than B_{yank}. Let B_{crash} be the smaller of B_{disrupt} and B_{confuse}. Let's think about what could cause such a B-field. If $N = 10000$, what current is required in the loop to produce this field? (Express in terms of B_{crash}, then numerically).

(d) We'd like to know the inductance of the Device, but we don't know the inductance of a current loop. For a decent estimate, assume the loop is instead a solenoid of radius $r = 10\,m$ and length r, and compute its inductance. Then, assume for the moment that the Device is made up of steel wire of resistivity $\rho = 10^{-7}\,\Omega\,m$ and cross-sectional area $A = 10^{-4}\,m^2$; what would its resistance R be? Does this make sense when combined with the current calculated above? Perhaps the Device is superconducting...

(e) Consider, then, the Device to be an EMF E attached in series to an almost perfectly *superconducting* loop that has resistance $R = 10^{-5}\,\Omega$ and inductance L. Suppose the EMF is connected at time $t = 0$ (as a result, say, of someone failing to enter certain numbers into a computer), and that the current necessary to supply B_{crash} to the airplane is reached 30 seconds later. What is the amplitude of the EMF? (You may want to use the approximation $\exp x \simeq 1 + x$ for small x.)

(f) In a second, later Event, things go on a bit longer, leading to a catastrophic explosive 'discharge' initiated by a failsafe system. Among other things, this discharge released a lot of energy. Using the same assumptions as in the last question, calculate the total energy stored in the magnetic field at time $t = 200\,s$.

(g) Suppose the discharge lasts 10 s, during which the energy U_B you just calculated is released at a uniform rate into a spherically outgoing electromagnetic wave. 20 km away, at a small suburban town oddly situated on this mysterious island, what is the intensity (in W/m^2) of radiation during the discharge?

(h) So far away, we can model the wave as a plane wave. What is the maximum amplitude E_0 of the electric field in this wave? Does it seem possible that this can create enough voltage (say 50 V) across say, a 1 cm circuit element to "fry" it in this electromagnetic pulse?

Solution:

(a) Counting all N loops, and using $x \gg R$, we get: $B \simeq \mu_0 N I R^2 / 2x^3$.

(b) From the motional EMF we have $emf = E = V = BLv$, so we require $B_{\text{disrupt}} = V/Lv \simeq 2.5 \times 10^{-3}\,T$. This is much smaller than B_{yank}.

(c) The confusion B is smaller than the disruption B, so let's use the former, and set it equal to our B field from the solenoid in part (a). We then have $2.5 \times 10^{-4}\,T = B_{\text{crash}} = \mu_0 N I R^2 / 2x^3$, where $x = 10^4$ m, $N = 10000$, $R = 10$ m. Then

$$I = 2B_{\text{crash}} x^3 / R^2 \mu_0 N \simeq 4 \times 10^8\,A \text{ (!)}$$

This should start to feel rather unrealistic.

(d) The inductance is $L = \mu_0 N^2 A/l = \mu_0 N^2 \pi r \simeq 4000\,\text{H}$. The resistance is $R = \rho l/A = \rho N 2\pi r/A \simeq 600\,\Omega$. We might be worried about the power dissipated in the coil, since $P = I^2 R \simeq 10^{20}\,W$. This is a hundred gigatons of energy released in one second, completely impossible. For comparison, the total US+Soviet nuclear arsenal at its all time high was about 50 gigatons.

(e) For an L-R circuit we have $I = \frac{E}{R}(1 - \exp(-t/\tau)$, where $\tau = L/R \simeq 4 \times 10^8$ s. If we desire I_{disrupt}, then we require

$$I_{\text{disrupt}} = \frac{E}{R}[1 - \exp(-t/\tau)] \simeq \frac{E}{R}(t/\tau),$$

where in the last step the small-x approximation for e^x is used. Thus

$$E = RI_{\text{disrupt}}(\tau/t) \simeq 5 \times 10^{10}\, V\ \ (!)$$

(f) $I = I_{\text{disrupt}}(200/30)$, and $U = LI^2/2 \simeq 1 \times 10^{22}$ J. Again, enough to pretty much destroy the world.

(g) The luminosity (energy emitted per unit time) $\mathcal{L}$ is given by $\mathcal{L} = U/\tau$ and the intensity is related to it via the inverse-square law: $I = \mathcal{L}/4\pi d^2$, so $I = 3 \times 10^{11}\, W/m^2$. This is about 10^8 times brighter than the sun...

(h) We have $I = \epsilon_0 c E_0^2/2$, so

$$E_0 = \sqrt{\frac{2I}{\epsilon_0 c}} \simeq 10^7\, v/m.$$

Over 1 cm this would create a potential of $\sim 10^5$ V, and fry just about anything.

Note 1: It was heartbreaking to see so much struggling with numbers. Yes, calculators were forbidden, but I also specifically noted that almost all credit would follow from symbolic expressions. Advice for the future: *never, ever plug in numbers until you have the complete symbolic expression written down.*

Note 2: From all this we can conclude that the electromagnetic anomaly that apparently brought down the plane was *not* caused by some sort of huge electromagnet. We might also imagine it being caused by a giant ferromagnet in which all the dipoles are aligned. This is still hard. For example, if had a mole of particles, all aligned, with one Bohr magneton of magnetic dipole ($\mu_B = eh/4\pi m_e \simeq 10^{-23}\, J/T$) each, that gives about $\mu \sim 10\, J/T$ per gram. Then if we took a 100-ton block ($10^8\, g$) and put into the dipole field formula $B = \mu_0\mu/2\pi x^3$, we get (at $x = 10,000$ m) $B = 2 \times 10^{-7}\, T$, still short by a factor of about 1000.

Or, another fun idea is that the anomaly is really a generator of magnetic monopoles (monopoles of the opposite charge would have to be moved away somewhere else far away so as not to violate conservation of magnetic charge). It would take a huge effort to contain these, but they could readily create a large field, and the 'discharge' would mean simply letting the monopoles flow away, creating a huge electric field (sourced by the monopole current) that turns the sky purple. A rough estimate I did shows that if we have monopoles in a region of size $\sim a$, then we can relate the B field a distance r away to the energy U of the full field configuration, using

$$B \sim \frac{1}{r^2}\sqrt{\mu_0 U a}, \text{ or } U \sim B^2 r^4/\mu_0 a$$

Using our numbers, we find $U \sim 5 \times 10^{13}\, J$ for $a = 10$ m. This is still huge (about 10 kilotons), but much smaller than the other numbers we have seen, and not totally crazy. The big advantage comes from the inverse-square (rather than inverse-cube) that comes into the force law if we have monopoles.