Physics 5C Another Practice Midterm 1 April 2008

PRINT YOUR NAME ____

Take an hour and ten minutes to do this exam. You may use one page of formulas.

Section A (20 points): True or False. You will get 2 points for each correct answer and 2 additional points if you also give a correct brief explanation.

1. ___ Because electrons cannot travel from one plate of a capacitor to the other, no current can flow through a capacitor in a circuit.

   False: Electrons can pile up on one plate and depart from the other. The capacitor charges up, but from the circuit standpoint, a current is running through it.

2. ___ A dipole $\vec{p}$ points in the $\hat{y}$ direction, and is immersed in a E-field pointing in the $\hat{z}$ direction. Then positive work is done on the dipole as it turns so that it points in the $\hat{x}$ direction.

   False: to turn the dipole from the $\hat{y}$ to the $\hat{x}$ direction requires a torque in the $\hat{z}$ direction. But there is no opposing torque from the E-field, since $\vec{\tau} = \vec{p} \times \vec{E}$ is in the $\hat{x}$ direction.

3. ___ If I take two identical capacitors and connect them in series, I will get half the capacitance of each individual one, but if I connect them in parallel, I will get double.

   True: using the laws of adding capacitances.

4. ___ The usual Gauss's law fails in a dielectric; a factor of κ must be inserted in the integral to make it valid.

   False: Gauss's law is still true. But we can write a different *version* of it, involving κ, that relates the integral to the freely-specified (not induced) charges.

5. ___ The relation $V = iR$ holds if and only if Ohm's law holds.

   False: This is merely the definition of R. Ohm's law states that R, so defined, is independent of i.

Section B:

1. (14 points) Three charges, of charge Q, $2Q$, and $3Q$, form an equilateral triangle with size length a, as shown in Figure 1.

   (a) In terms of Q and a, what is the potential at point P, midway between the bottom two charges?

   (b) What are the components of $\vec{E}$ at the same point?

   (c) What is the energy of the charge configuration in Joules, if $a = 1\,m$ and $Q = 1\,C$?

   Solution:

   (a) We have

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{a/2} + \frac{2Q}{a/2} + \frac{3Q}{\sqrt{3}a/2}\right) = \frac{1}{4\pi\epsilon_0}\frac{6Q}{a}\left(1 + \frac{\sqrt{3}}{3}\right).$$

(b) E_y will clearly arise just from the top charge, so

$$E_y = \frac{3}{4\pi\epsilon_0}\frac{Q}{a^2}.$$

E_x will arise from the two bottom charges, and we get:

$$E_x = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{(a/2)^2} - \frac{2Q}{(a/2)^2}\right) = \frac{-Q}{\pi\epsilon_0 a^2}.$$

(c) There are three terms in the sum:

$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{2Q^2}{a} + \frac{3Q^2}{a} + \frac{6Q^2}{a}\right) = \frac{11Q^2}{4\pi\epsilon_0 a} \simeq 10^{10}\, J(!)$$

2. (14 points) A metal sphere levitates inside a metal shell which in turn levitates inside another metal shell, as shown cross-section in Figure 2 (the grey parts are metal, the black parts are empty space). Initially, all three spheres are neutral. Then a charge $-Q$ is added to the inner sphere and a charge Q is added to the outer sphere, where $Q > 0$.

   (a) After the charges have reached equilibrium, will the electric field in the space between the inner and middle spheres point inward, outward, or neither?

   (b) What will be the charge on the inner surface of the middle sphere?

   (c) How much charge is on the outer surface of the middle sphere?

   (d) How much charge is on the inner surface of the outer sphere?

   (e) Make a plot of E vs. radius r, where E is the radial electric field.

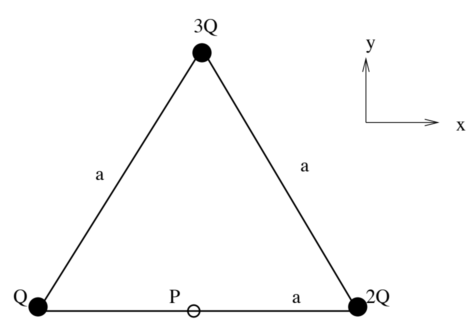

FIG. 1:

Solution:

(a) We can ignore everything outside of R_2, so E must point inward.

(b) Since E is zero inside in the metal of the middle sphere, drawing a Gaussin surface of a sphere with radius $R_2 < r < R_3$, means the charge at R_2 must be Q.

(c) From part b, this must be $-Q$ by charge conservation.

(d) This must again be Q to cancel the $-Q$ of part (c) so that the Gaussian integral at $R_4 < r < R_5$ can be zero.

(e) The field falls off as $-Q/4\pi\epsilon_0 r^2$ for $R_1 < r < R_2$. It is zero at $R_2 < r < R_3$. It then falls as $-Q/4\pi\epsilon_0 r^2$ for $R_3 < r < R_4$, is zero for $R_4 < r < R_5$, and zero outside of R_5.

3. (16 points) You are given a mysterious black box with two metal terminals sticking out. You X-ray it, and deduce that inside there is an EMF E connected in series to 2 identical resistors, each with resistance r, which are in parallel. You hook the thing up to a 21 V battery and find (using an ammeter) that there is a current of 1 A between the terminals. If you reverse the connections to the battery you find a current of 2 A in the opposite direction. Find E and r.

   Solution:

   Draw a circuit diagram with the internal EMF E, the resistance $R = r/2$, and the external EMF E_0 in series. Choose i to run 'against' the internal EMF. Then we have two cases.

   Case 1: the external EMF is parallel to the current, and

$$E_0 - E - i_1 R = 0.$$

   Case 2: the external EMF is anti-parallel to the current, and

$$-E_0 - E - i_2 R = 0.$$

   here, $i_1 = 1\,A$, and $i_2 = -2\,A$. Adding these two, solving for E and plugging into the first yields:

$$R = \frac{2E_0}{i_1 - i_2} = \frac{42\,V}{3A} = 14\,\Omega.$$

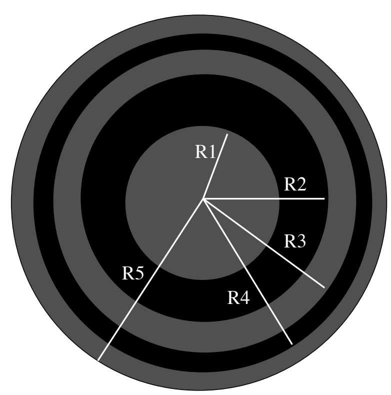

FIG. 2:

Thus $r = 28\,\Omega$.

Putting this back in to the first gives $E = 7\,\mathrm{V}$. We can also check that this makes sense: with parallel EMFs we have $28\,V/14\,\Omega = 2\,A$, and the other way we have $14\,V$ and $14\,\Omega$ for $1\,\mathrm{A}$.

4. (18 points) Two conducting spheres of radius R and R are separated by a large distance. Initially, sphere 1 is neutral and sphere 2 has a charge Q. Then then the two spheres are connected by a long, thin wire.

   (a) What fraction of the initial potential energy of the configuration disappears?

   (b) Where does it go?

Solution:

(a) Initially, the energy is $U = CV^2/2$, where $C = 4\pi\epsilon_0 R$, and $V = \frac{Q}{4\pi\epsilon_0 R}$, so

$$U_i = \frac{Q^2}{8\pi\epsilon_0 R}.$$

After they are connected, they must each (by symmetry) have charge $Q/2$, thus total energy:

$$U_f = 2\frac{(Q/2)^2}{8\pi\epsilon_0 R} = U_i/2,$$

so half of the energy goes away.

(b) It goes into heat, powered by the resistance of the thin wire (or perhaps the spheres if they are bad conductors compared to the wire).

5. (18 points) A capacitor that allows some current to flow from one plate to the other is sometimes called a 'leaky capacitor'. We shall call it a 'resipacitor'. A parallel-plate resipacitor is made by taking a banana cream pie in an aluminum pie plate, and sticking a second pie plate on top of it. The pie plates have area A and the pie has a thickness d. Let κ and ρ be the dielectric constant and resistivity of banana cream (neglect the crust on the bottom and the meringue on the top of the pie, and assume $d^2 \ll A$).

   (a) Find RC where R is the resistance of the whole mess, and C is its capacitance.

   (b) (5 points extra credit) Consider a banana resipacitor composed of a (cylindrical) banana segment wrapped in foil, with a metal skewar through it. Show that the same expression for RC holds for it in terms of the resistivity and dielectric constant of (cream-free) banana.

Solution:

(a) $R = V/i$ and $C = Q/V$ give $RC = Q/i$. Then, $i = Aj = AE/\rho$, where j is the current density and E is the electric field. For parallel plates, $E = \sigma/\kappa\epsilon_0$, where $\sigma = Q/A$ is the surface charge density. Thus

$$RC = \frac{Q\rho}{AE} = \frac{Q\rho}{A(Q/A)/\epsilon_0\kappa} = \epsilon_0\kappa\rho.$$

(b) We have

$$C = 2\pi\kappa\epsilon_0 \frac{L}{\log b/a}$$

for a cylinder, where b and a are the inner and outer radius. Then, $Q = CV$ and $R = V/i$ give $RC = Q/i$ as before. $Q = L\lambda$, where λ is the charge/length of the skewar. At a radius $a < r < b$,

$$E = \frac{\lambda}{2\pi\epsilon_0\kappa r},$$

so

$$i = (2\pi r)LE/\rho = L\lambda/\epsilon_0\rho\kappa,$$

and

$$RC = \epsilon_0\kappa\rho$$