Physics 5C – Practice Final Exam – Spring 2008
Solutions

Section A (40 points): Multiple choice: choose the best answer. You will get 5 points for each correct answer.

1. An electric dipole lies in a uniform electric field, and the dipole moment points perpendicular to the field. Which one of the following is FALSE?

   (a) ____ The dipole will continue to point in that direction if no other forces act on it.
   (b) ____ The net force on the dipole is zero.
   (c) ____ The electrostatic potential energy of the dipole is zero.
   (d) ____ The net charge of the dipole is zero.

   a: there is a torque $\tau = \vec{c} \times \vec{E}$ that is nonzero.

2. Einstein's theory of relativity states that $E = mc^2$, i.e. that adding energy to a system makes it more massive. Imagine a conducting sphere on a scale (but insulated from it). Now imagine adding a charge q to the sphere. Neglecting the mass of the charge carriers you use to add the charge, the mass reading on the scale will:

   (a) ____ Decrease if $q < 0$ and decrease if $q > 0$.
   (b) ____ Decrease if $q < 0$ and increase if $q > 0$.
   (c) ____ Increase if $q < 0$ and increase if $q > 0$.
   (d) ____ Increase if $q < 0$ and decrease if $q > 0$.

   c: since the potential energy is a sum over $q_i q_j / r_{ij}$ terms, and the charge will distribute uniformly on the sphere, the electric potential energy will ber positive whether or not q is, and so the mass will always increase.

3. A lightning bolt hits the ground, and creates a point of very high voltage on the ground, which is at zero potential far away from the strike. A physics 5C student and a cow are both 10 m away from the strike, *and facing toward it.* Assume that both have the same resistance to current passing through them.

   (a) ____ The cow is in greater danger than the 5C student.
   (b) ____ 5C student is in greater danger than the cow.
   (c) ____ The student and the cow are in equal danger.

   a: the cow is in greater danger because its feet are farther apart, thus they have a greater potential difference between them, and more current will flow through the cow than the 5C student.

4. If $i, R, C, B,$ and L are current, resistance, capacitance, B-field strength, and inductance, then the quantity $iRCB/L$ has units:

(a) ____ Cs/V
(b) ____ mC^2/s
(c) ____ $C^2/(m^2 s)$
(d) ____ Vm/Cs

c: iRC has units C, and B/L has units $A/m^2 = C/(sm^2)$.

5. Consider a magnetic dipole $\vec{u}$ in a magnetic field $\vec{B}$. Which of the following is FALSE?

   (a) ____ There is no net force on the dipole if B is uniform.
   (b) ____ There is a torque on the dipole unless $\vec{u} \parallel \vec{B}$.
   (c) ____ If $\vec{u} \parallel \vec{B}$, then the field due to the dipole will tend to cancel the field B.
   (d) ____ If the dipole points out of the page, it could be due to an electron circling in a clockwise fashion.

   c: It will reinforce the field.

6. To get a device with a high capacitance, it should have

   (a) ____ Large area, small plate separation, and small dielectric constant.
   (b) ____ Large area, large plate separation, and small dielectric constant.
   (c) ____ Large area, large plate separation, and large dielectric constant.
   (d) ____ Large area, small plate separation, and large dielectric constant.

   d: by the capacitance formula.

7. The diplacement current is defined as $\epsilon_0 d\Phi_E/dt$. If I was charging a capacitor with dielectric κ, then to have the displacement current between the plates equal to the current flowing into (or out of) the capactitor, I should define it as:

   (a) ____ $\epsilon_0 \kappa d\Phi_E/dt$
   (b) ____ $\epsilon_0 \kappa^{-1} d\Phi_E/dt$
   (c) ____ $\epsilon_0 d\Phi_E/dt$

   a: This can be seen by going through the argument we did in class for the capacitor.

8. If I have a transformer with a primary coil of 1000 turns and a secondary coil of 500 turns, then

   (a) ____ The primary voltage will be twice the secondary voltage and the primary current will be half the secondary current.
   (b) ____ The primary voltage will be half the secondary voltage and the primary current will be half the secondary current.
   (c) ____ The primary voltage will be twice the secondary voltage and the primary current will be twice the secondary current.

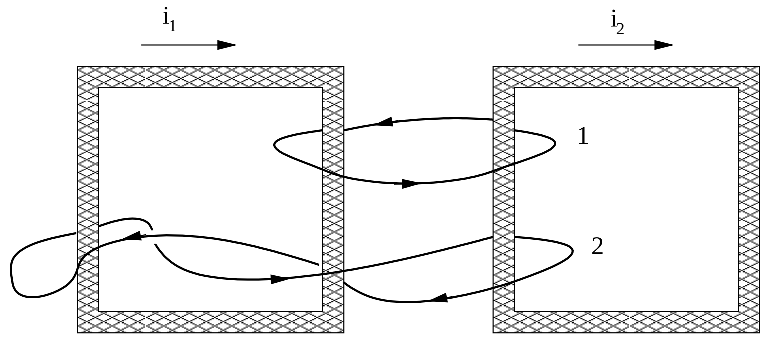

FIG. 1:

(d) ____ The primary voltage will be half the secondary voltage and the primary current will be twice the secondary current.

a: Since $V_s/V_p = N_s/N_p$, and $I_p/I_s = N_s/N_p$

Section B: Problems

1. (14 points) Figure 1 shows two closed paths wrapped around two conducting loops carrying currents $i_1 = 5.0\,\text{A}$ and $i_2 = 3.0\,\text{A}$. What is the value of the integral $\oint \vec{B} \cdot d\vec{l}$ for

   (a) Path 1?

   (b) Path 2?

   Solution:

   (a) $\oint \vec{B} \cdot d\vec{l} = \mu_0(i_2 - i_1)$

   (b) $\oint \vec{B} \cdot d\vec{l} = \mu_0(-i_2 - 2i_1)$

2. (14 points) A long, straight wire of length l carries current i. A right-angle bend formed at the middle of the wire is in the shape of a circular arc of radius $r \ll l$, as shown in Fig. 2 (the wire is the bent line).

   Determine the approximate magnetic field (magnitude and direction) at the center of the arc, using $r \ll l$.

   Solution:

   The Biot-Savart law says:

$$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}.$$

   We see that $d\vec{l} \times \vec{r}$ is always in the $-\hat{z}$ direction (into the page) for any segment of the wire, so there will be no cancellations, etc. Then the top segment contributes as 1/2 of an infinite line, as does the bottom long segment, for a total of $B_{\text{seg}} = \frac{\mu_0 i}{2\pi r}$. The arc's contribution can be obtained from B-S:

$$B_{\text{arc}} = \frac{\mu_0 i}{4\pi} \frac{2\pi r}{4} \frac{r}{r^3} = \frac{\mu_0 i}{8r},$$

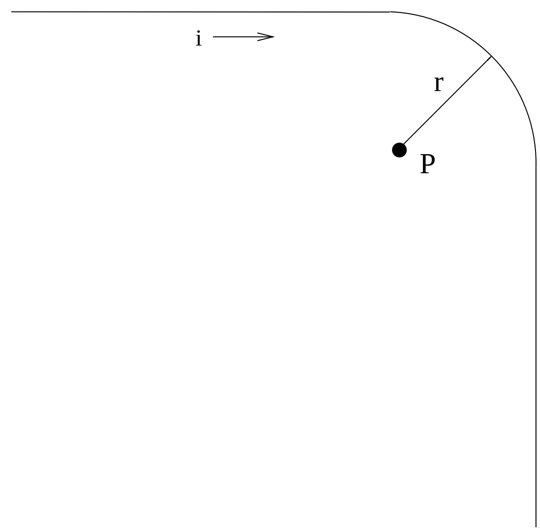

FIG. 2:

so

$$\vec{B} = \vec{B}_{\text{seg}} + \vec{B}_{\text{arc}} = -\frac{\mu_0 i}{r}\left(\frac{1}{2\pi} + \frac{1}{8}\right)\hat{z}.$$

3. (16 points) Scientists working on the SETI program have just detected a signal from another star. It is a radio wave of intensity $10^{-22} W/m^2$ (near Arecibo's sensitivity limit), and appears to come from a star $\sim$ 20 light-years (2×10^{17} m) away.

   (a) What are the maximum E and B fields at the SETI antenna due to this signal?

   (b) What is the power, in Watts, of the Alien emitter, if it beams signals equally in all directions? How does this compare to the $10^{13}\, W$ used (on average) by all of humanity?

Solution:

(a) $S = EB/\mu_0$ and $E = cB$ for an EM wave, so

$$S = cB^2/\mu_0 = E^2/c\mu_0,$$

where E and B are r.m.s. quantities. which gives:

$$B_0 = \sqrt{2}\sqrt{\mu_0 S/c} = 9.3 \times 10^{-19}\, T, \quad E = \sqrt{2}\sqrt{c\mu_0 S} = 2.8 \times 10^{-9}\, V/m.$$

(b) The luminosity (or power emitted) is related to the intensity f at radius r by:

$$f = L/4\pi r^2,$$

so

$$L = 4\pi r^2 f = 5.0 \times 10^{13}\, W,$$

pretty close to humanity's total power output.

4. (18 points) Figure 3 shows a portion of a circuit through which there is a curernt $I = 6\,\text{A}$. The resistances are $R_1 = R_2 = 2.00R_3 = 2.00R_4 = 4.00\,\Omega$. What is the current i_1 through resistor 1?

Solution:

Let $R_5 = R_3 + R_4$. Then one loop gives $i_1R_1 = i_2R_2$, and a loop around the outer perimeter gives $i_1R_1 = i_5R_5$, where i_5 runs through R_5. The junction conditions give $I = i_1 + i_2 + i_3$. Plugging the loops into this yields:

$$I = i_1 + i_1R_1/R_2 + i_1R_1/R_5 = i_1(1 + R_1/R_2 + R_1/R_5) = i_1(1 + 1 + 1),$$

so $i_1 = I/3 = 2\,A$.

5. (18 points) The voltage in a power outlet is about 110V rms, at 60Hz in the US and Canada.

   (a) What is the maximum voltage that comes out of the wall socket at any instant of time?

   (b) What is the resistance of a 4-slice, 1600W toaster?

   (c) What is the capacitance that (with no resistor present) will allow the same amplitude current to flow when plugged into the wall socket?

Solution:

(a) $V_{\text{rms}} = V_0/\sqrt{2}$, where V_0 is the amplitude and also the maximum, so $V_0 = 156\,V$.

(b) $\langle P \rangle = V_{\text{rms}}^2/R$, so $R = 7.6\,\Omega$.

(c) $i_0 = V_0/Z$, where the impedence $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. Thus if $L = 0$, a capacitance of $C = 1/\omega R = C/2\pi\nu$ is equivalent to a resistance of R; in this case $C = 3.5 \times 10^{-4}\,F$.

6. (18 points) Figure 4 shows, in cross-section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centers of the spheres, at radial distance $R/2.00$ from the center of sphere 1. If the net

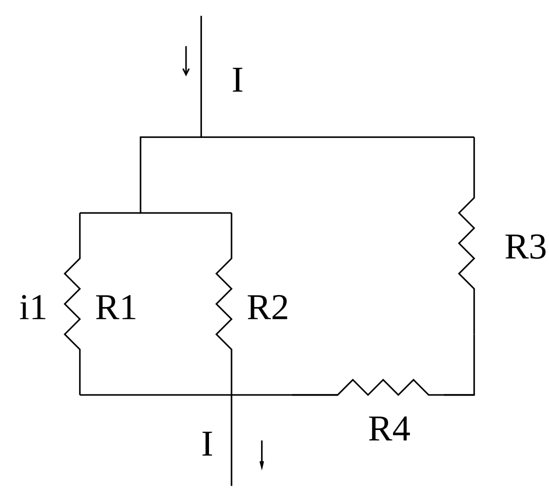

FIG. 3:

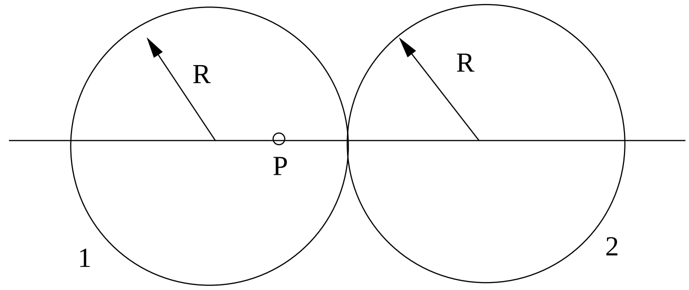

FIG. 4:

electric field at point P is zero, what is the ratio q_2/q_1 of the total charge q_2 in sphere 2 to the total charge q_1 in sphere 1?

Solution:

We can treat the charge on sphere 2, and the charge within $R/2$ in sphere 1, to be at the centers of the spheres. If q_1 and q_2 are both positive (or both negative), they can cancel at P. We can then equate the magnitudes of the forces:

$$\frac{q_2}{(3R/2)^2} = \frac{q_1(1/2)^3}{(r/2)^2},$$

where some constants are cancelled, and charge within $R/2$ in sphere 1 is $q_1/8$. Then simplifying,

$$q_2/q_1 = 9/8.$$

7. (25 points) A perfectly conducting rod of length l moves on two horizontal, frictionless, resistanceless rails as shown in Fig. 5. Connecting the rails are a capacitor C and resistor R. At time $t = 0$, the capacitor is uncharged and the bar is moving at a constant speed v through a magnetic field $\vec{B}$ that is directed into the page. It is kept at this speed by an external force F.

   (a) What is the current i through the resistor an instant after $t = 0$?

   (b) What is the charge on the capacitor a very long time after $t = 0$?

   (c) What is the current i through the resistor when the capacitor has attained half its final charge?

   (d) What is the total work done after $t = 0$ by the agent exerting the external force F?

   Solution:

   (a) Initially the capacitor is uncharged so there is no voltage across it. The loop theorem then gives $i = \mathcal{E}/R$, where $\mathcal{E} = lvB$ is the motional EMF induced in the rod. Thus $i(t = 0) = lvB/R$.

   (b) After a long time, the capacitor will be fully charged, so no current will be flowing and there will be no voltage across the resistor. Thus $q(t \to \infty) = CV = C\mathcal{E} = ClvB$.

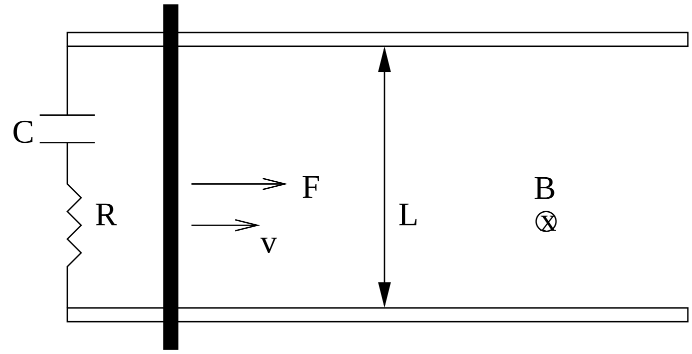

FIG. 5:

(c) This is really just an RC circuit, so

$$i = \frac{\mathcal{E}}{R} e^{-t/RC}, \quad q = C\mathcal{E}\left(1 - e^{-t/RC}\right).$$

when $q = C\mathcal{E}/2$, we have $1 - e^{-t/RC} = 1/2$, so $e^{-t/RC} = 1/2$, and $i = lvB/2R$. Another way to do this is with a loop:

$$\mathcal{E} - ir - q/C = 0,$$

which relates q to i at any time.

(d) The total work done will equal the energy in the capacitor at the end plus the energy burned by the resistor.

The power dissipated in the resistor at any time is

$$P = i^2 R = \frac{\mathcal{E}^2}{R} e^{-2t/RC}.$$

To get the energy dissipated in the resistor, we must integrate this from $t = 0$ to $t = \infty$:

$$W_R = \int_0^\infty P(t)dt = \frac{\mathcal{E}^2}{R} \int_0^\infty e^{-2t/RC} dt = -\frac{\mathcal{E}^2 C}{2} e^{-2t/RC}|_0^\infty = \frac{\mathcal{E}^2 C}{2} = \frac{(lvb)^2 C}{2}$$

The capacitor gives an exactly equal contrubution $W_C = C\mathcal{E}^2/2 = \frac{(lvB)^2 C}{2}$, so

$$W = W_R + W_C = (lvB)^2 C.$$

An easier way to do all this is to note that the induced EMF is waht supplies the power to both the capacitor and resistor. Since $\mathcal{E} = dW/dq$, we can do:

$$W = \int_0^{q_{fin}} \mathcal{E} dq = \mathcal{E} q_{\text{fin}} = (vBl)(CvBl) = (lvb)^2 C.$$

8. (25 points) A Sith Lord is able to fry a Jedi Knight at a distance of $L \approx 5\,m$ with electric lightning shooting from his fingertips. See Figure 6. Assess the following.

(Note that you can answer later parts in terms of the variables given in earlier parts, so don't stop if you get stuck.)

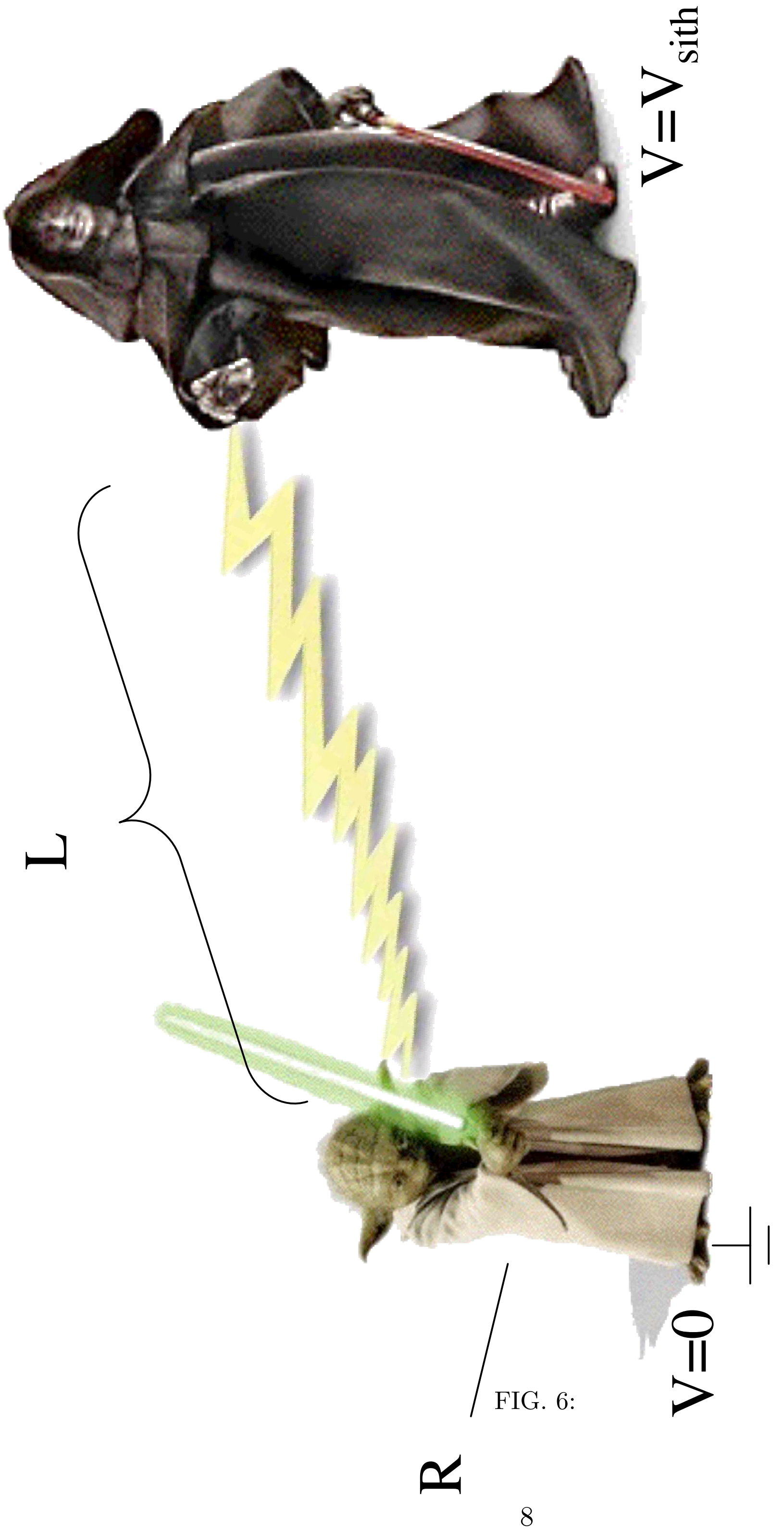

FIG. 6:

(d) $F = (Q_{\text{sith}}/2)^2/4\pi\epsilon_0 L^2 = 0.6\,N$. Not too much, though in the realistic case where the current keeps flowing, Yoda's sphere would accumulate a lot of charge.

(e) $R = \rho l/A$, and he needs $R = (1.5 \times 10^6\,V)/(0.1\,A) = 15\,M\Omega$. Thus $\rho = AR/l = 3.75 \times 10^6\,\Omega m$. Those mitochloria (or whatever) must be working hard!