Homework Set 1

DUE: Wednesday January 28

1. Cosmological constant. (a) Calculate the value of the cosmological constant Λ, and also calculate $\Omega_\Lambda = \Lambda/3H_0^2$, the corresponding vacuum energy density in units of critical density ρ_c. Assume that $\Omega_m = 0.3$ and that the curvature vanishes ($k = 0$). Express Λ and $\rho_\Lambda \equiv \Omega_\Lambda \rho_c$ in several different units, including natural units ($\hbar = c = 1$; take $\rho_\Lambda \equiv \Omega_\Lambda \rho_c$ to have units of eV^4).

(b) The acceleration due to the cosmological constant equals $\Lambda r c^2/3$. Numerically compare that to the sun's gravitational acceleration $GM_\odot/r^2$ in order to find the distance r at which they are equal.

2. For a flat universe with $\Omega_{m,0} < 1$ and positive cosmological constant $\Omega_{\Lambda,0} = 1 - \Omega_{m,0}$, the density contributions of the matter and cosmological constant are equal when the scale factor has the value $a_{m\Lambda} = (\Omega_{m,0}/\Omega_{\Lambda,0})^{1/3}$. This equals 0.75 for the Benchmark Model: $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$. Show that for this case the Friedmann equation can be integrated to give the expression

$$H_0 t = \frac{2}{3\sqrt{1 - \Omega_{m,0}}} \ln[y^{3/2} + \sqrt{1 + y^3}] \,,$$

where $y \equiv a/a_{m\Lambda}$. Show that for $a \ll a_{m\Lambda}$, this reduces to

$$a(t) \approx (\frac{3}{2}\sqrt{\Omega_{m,0}} H_0 t)^{2/3} \,,$$

and for $a \gg a_{m\Lambda}$, it reduces to

$$a(t) \approx a_{m\Lambda} \exp(\sqrt{1 - \Omega_{m,o}} H_0 t) \,.$$

Show finally that the age of the universe today in this case is

$$t_0 = \frac{2}{3H_0\sqrt{1 - \Omega_{m,o}}} \ln\left[\frac{\sqrt{1 - \Omega_{m,o}} + 1}{\sqrt{\Omega_{m,0}}}\right] \,,$$

and that for the Benchmark Model this is $t_0 = 0.964 H_0^{-1}$.

3. Popularizations of cosmology often talk about the "size of the universe" at some earlier time, but they usually mean by this the size that the present-epoch horizon was at that time. (For example, in the *Cosmic Voyage* IMAX film, Rocky Kolb says that everything

that we can presently see was once only as big as a marble that he holds in his hand.) To clarify this, make a log-log plot in which the horizontal axis is time since the Big Bang, from 10^{-30} s to 10^{20} s, and the vertical axis is length, and plot curves representing both (a) the size in the past of the present-epoch horizon, and (b) the size of the current horizon (i.e., the distance that light has travelled since the Big Bang), as a function of the time since the Big Bang. (These curves cross at t_0.) Use any cosmology that you like, for example Einstein-de Sitter, but specify which one you are using. Briefly discuss some implications of your plot.

4. Geometry. (a) Show that if the curvature constant $K = 0$ and the scale factor a grows as $t^{2/3}$, the apparent angular sizes of distant objects of the same linear size have a minimum at $z = 1.25$.

(b) Under the same assumptions, suppose that a galaxy is observed at $z = 1.25$. For what fraction of the Hubble time has its light been travelling toward us? (*Be careful to answer this question, don't just calculate something similar.*)

(c) Consider a galaxy of physical (visible) size 5 kpc. What angle would this galaxy subtend if situated at redshift 0.1? 1? 5? Do the calculation in a flat universe, first with zero cosmological constant, and then in the Benchmark Model with $\Omega_{m,0} = 0.3$. (*You are welcome to use Ned Wright's Javascript cosmology calculator – see below.*)

5. The astronomical convention is that the relationship between apparent magnitude m and absolute magnitude M is

$$m - M = 5\log(\frac{d_L}{10\text{pc}}) + K$$

where d_L is the luminosity distance and K is a correction for the redshifting of the spectrum of the source (Weinberg, *Cosmology*, p. 54). Plot $m - M$ as a function of redshift for a flat matter-dominated universe (this can be done analytically) and for the Benchmark Model (you need to evaluate numerically a 1D integral). Neglect the K-correction. Compare with a plot showing high-redshift supernova data, for example Weinberg, *Cosmology*, Fig. 1.2, or the more recent data shown in class.

6. Suppose astronomers measure the age of a galaxy with redshift $z = 2.5$. How old would this galaxy have to be (at the tme the light from it was emitted) in order to rule out the hypothesis that $\Omega_M = 1$ with negligible vacuum and radiation energy density. Use $H_0 = 70$ km/sec/Mpc. (This is Weinberg, *Cosmology*, page 569, problem 4.)

Note: Ned Wright's Cosmology web page, with many useful links, is

http://www.astro.ucla.edu/~wright/cosmolog.htm .

His Javascript distance calculator is at

http://www.astro.ucla.edu/~wright/CosmoCalc.html .